AUSTRALIAN ATOMIC ENERGY COMMISSION RESEARCH ESTABLISHMENT LUCAS HEIGHTS

DESIGN OF CONCENTRIC TUBULAR REACTOR FUEL ELEMENTS FOR UNIFORM COOLANT CONDITIONS

by

IAN M. BINNS R. LICCIARDO

ABSTRACT

Concentric tubular reactor fuel element geometries to give equal coolant outlet temperatures are presented. Oscillations from tube to tube in thickness and temperatures generally occur but it is possible to eliminate them by choice of the centre element. This may be a fuel rod or a nonheat-producing rod with or without a surrounding annulus of fuel. The geometries and temperatures are dependent on the voidage and on a non-dimensional parameter equivalent to a Biot number based on the channel equivalent diameter.

CONTENTS

Page

1

1 3

3 3

33

4

5

6

6

1. INTRODUCTION

2. GENERAL EQUATION AND ITS SOLUTION

2.1 Surface and Maximum Fuel Temperatures

- 3. DISCUSSION OF THE ANALYSIS
 - 3.1 The Equality of Coolant Temperatures
 - 3.2 Coolant Mass Flow Per Unit Area
 - 3.3 Equality of Heat Transfer Coefficients

4. RESULTS OF CALCULATIONS

5. DISCUSSION OF RESULTS OF CALCULATIONS

- 6. CONCLUSIONS
- 7. NOTATION
- APPENDIX 1 Derivation of the General Equation
- APPENDIX 2 Derivation of Equation 5
- APPENDIX 3 Limiting Values

Figure 1 Temperatures and radii as used in calculations

Figure 2 Variations in tube thickness within clusters of concentric tubes having centre coolant channels

Figure 3 Temperatures at inner tube surfaces within clusters of concentric tubes having centre coolant channels

Figure 4 Temperature differences across tubes within clusters of concentric tubes having centre coolant channels

Figure 5 Maximum temperatures of tubes within clusters of concentric tubes having centre coolant channels

Figure 6 Centre boundary conditions for tube clusters of uniform tube thickness and coolant temperatures

Figure 7 Fuel temperatures for various centre boundary conditions for tube clusters of uniform tube thickness and coolant temperatures

Figure 8 Curve-illustrating relation between boundary values of rmo and roo to give tubes of equal thickness and uniform coolant conditions

1. INTRODUCTION

In design of nuclear fuel elements it is difficult to make compatible the temperature distribution, heat conduction, and surface heat transfer, so a trial-and-error method is commonly used to obtain a configuration of fuel elements involving concentric tubes to give a practical temperature distribution. However a more efficient method which would lead to a better understanding of the problem would be the evolution of a set of simultaneous equations whose solution for specified conditions would uniquely determine the geometry of the fuel element.

In the following analysis the specified condition is that the coolant channels should produce equal coolant cutlet temperatures from equal inlet temperatures.

The analysis involves a study of the dimensions of individual tubes, relations between neighbouring tubes, and the effect of boundaries.

2. GENERAL EQUATION AND ITS SOLUTION

For the analysis the fuel elements, a series of concentric tubes, are identified as in Figure 1.

The following assumptions are made;

- 1. The fuel element material is homogeneous having uniform heat generation and thermal properties, both independent of temperature.
- 2. The model including its boundaries is axially symmetric, with constant heat flux at the boundaries.
- 3. Steady state conditions have been achieved.
- 4. There is negligible conduction longin.dinally in the fuel elements.

Using the notation given in Section 7 the resulting general equation for the rate of increase in coolant temperature in the n-th channel is as follows:

$$\frac{d T_{fn}}{dl} = \frac{q}{m_n C_{pn}} \left[\frac{r_{mn}^2 - r_m(n-1)}{\Lambda_{n/\pi}} - 1 \right] + \frac{2K}{m_n C_{pn}} \frac{1}{\Lambda_{n/\pi}} \left[a_{(n-1)} \left(T_{f(n-1)} - T_{fn} \right) + a_n \left(T_{f(n+1)} - T_{fn} \right) \right], \quad (1)$$

where

1

$$r_{mn}^{2} = a_{n}K \left[\frac{r_{on}}{h_{on}} + \frac{r_{in}}{h_{in}} - \frac{(r_{on}^{2} - r_{in}^{2})\left(\frac{1}{r_{on}h_{on}} + \frac{1}{r_{in}h_{in}}\right)}{2\ln\left(\frac{r_{on}}{r_{in}}\right)} \right] + \frac{r_{on}^{2} - r_{in}^{2}}{2\ln\left(\frac{r_{on}}{r_{in}}\right)}$$

$$a_{n} = \frac{1}{\ln\frac{r_{on}}{r_{in}} + K\left(\frac{1}{r_{on}h_{on}} + \frac{1}{r_{in}h_{in}}\right)}$$

and

rmn is the radius at which the temperature gradient in the n-th fuel element is zero if surrounding coolant temperatures Tf(n+1) and Tfn are equal.

NOTE: The derivation of equation 1 is given in Appendix 1.

If the coolant comperatures are equal then

$$\frac{d T_{fn}}{dl} = \frac{q}{m_n C_{pn}} \frac{t_{mn}^2 - t_{m(n-1)}^2}{A_{n/n}} - 1 =$$

and this can be made equal for all values of s by design choice.

(4)

(2)

(3)

It is shown in Appendix 2 that for equal mass flow per unit area the successive values of rmn are given by

$$r_{mn} = r_{m(n-1)} + \frac{r_{in} - r_{o(n-1)}}{C}$$

where C is the voidage defined as flow cross-section area divided by the sum of the flow and fuel crosssection areas.

For similar gas conditions in each channel the equivalent diameter De is made equal for all n.

WHITE LIDE WHITE PARTY BUT APRIL

(5)

(6)

(7)

Therefore

I and a di su

$$f_{in} = f_{o(n-1)} = \frac{De}{2}$$

and the dimensions of a tube bundle can be calculated using equations 2, 3, 5, and 6 assuming a known boundary.

A major simplification can be made by assuming the heat transfer coefficients hon and hin to be equal.

Equation 7 and 3 then give

$$r_{mn}^{a} = r_{in}^{a} X_{n} \left[\frac{1 + r_{in} \frac{h}{K} \frac{X_{n} - 1}{2}}{1 + r_{in} \frac{h}{K} \frac{X_{n} \ln X_{n}}{X_{n} + 1}} \right]$$

where Xn + ton

.

The solution can be made non-dimensional by making the following substitutions:

$$-\frac{Deh}{K}$$
, $R = \frac{rh}{K}$, $A' = A(\frac{h}{K})$

The above equations now become

$$R_{in} = R_{o}(n-1) + \frac{D}{2}$$

$$R_{mn}^{s} = R_{m}^{s}(n-1) + \frac{1}{C} \left(R_{in}^{s} - R_{o}^{s}(n-1) \right)$$

$$R_{mn}^{s} = R_{in}^{s} - X_{n} \left[\frac{1 + R_{in} \frac{X_{n} - 1}{2}}{1 + R_{in} \frac{X_{n} \ln X_{n}}{X_{n} + 1}} \right] .$$
(B)
(9)
(10)

Equation 10 is transcendental and the following method of solution is suggested:

Since in (X_n) =
$$\frac{1 + R_{in} \frac{X_n - 1}{2}}{1 + R_{in} \frac{X_n \ln X_n}{X_n + 1}}$$

Rin Rin

which is approximately equal to one,

-2-

$$(X_n)_2 = \frac{(X_n)_1}{f[(X_n)_1]}$$

$$(X_n)_2 = \frac{(X_n)_1}{f[(X_n)_2]}$$

until the required accuracy is obtained.

2.1 Surface and Maximum Fuel Temperatures

The surface temperature is given relative to the fluid bulk temperatures in equations (v) and (vi) of Appendix 1. Substituting tmn for r n the non-dimensional equations are

-3-

$$\frac{T_{in} - T_f}{\frac{D_e q}{h}} = \left[\frac{1}{2} \frac{R_{mn}^*}{R_{in}^*} - 1\right] \frac{R_{in}}{D} \qquad (11)$$

$$\frac{T_{on} - T_f}{\frac{D_e q}{h}} = \frac{1}{2} \left[1 - \frac{R_{mn}^*}{R_{in}^*} \frac{1}{X_n^*}\right] \frac{X_n R_{in}}{D} \qquad (12)$$

These equations in the above form can be used in conjunction with the geometric analysis.

The maximum fuel element temperature occurs at r = rpn = rmn and from equations (iii) and (iv) of Appendix 1 .

$$\frac{T_m - T_f}{\frac{D_e q}{h}} = \frac{1}{4} \frac{R_{in}^3}{D} \left[\left(\frac{R_{mn}}{R_{in}^3} \right) \ln \left(\frac{R_{mn}}{R_{in}^3} \right) - \frac{R_{mn}^3}{R_{in}^3} + 1 \right] + \frac{T_{ni} - T_{fn}}{\frac{D_e q}{h}} .$$

The limiting surface temperature parameters derived in Appendix 3 both equal Ron - Rin

The similar limit on the maximum fuel element parameter is found to equal

$$\frac{R_{on}-R_{in}}{2D}\left[1+\frac{R_{on}-R_{in}}{4}\right].$$

3. DISCUSSION OF THE ANALYSIS

3.1 The Equality of Coolant Temperatures

This results from the assumption of uniform inlet conditions. If there is a coolant temperature gradient in the radial direction, the effect of the temperature terms in equation 1 is to adjust the coolant temperature increase until the temperature is equal to that in the two neighbouring chaunels.

3.2 Coolant Mass Flow Per Unit Area

It is desirable to have the pressure drop equal for each channel and this could be very closely achieved by specifying equal equivalent diameters. Secondary variations result from surface temperature variations.

As a result of this, or by using entrance constrictions, equal mass flows per unit area can be assumed.

3.3 Equality of Heat Transfer Coefficients

In the simplification of equations 2 and 3 heat transfer coefficients at the inside and outside surfaces of the fuel element are assumed equal. It seems generally accepted that the heat transfer

(13)

coefficient at the outer boundary of an annular coolant channel is that given by the normal tabe equation using the equivalent diameter while for the inner boundary the equation is usually modified by a factor depending on the curvature of the surface. No consistently accurate equation seems to be available in the literature but a simple illustrative example is that given by Monrad and Pelton."

For the outer boundary on the inside surface of tubes Monrad and Pelton justify the following:

and for the inner boundary on the outside surface of tubes,

It seems likely that for gas-cooled systems douter will be within the range 1.0 to 2.0 yielding

ration of heat transfer coefficients from 1.0 to 1.255. Except for the centre the ratio should be very class to 1.0 and so it is reasonable to assume that the heat transfer coefficients are equal.

4. RESULTS OF CALCULATIONS

Calculations were made for voidages of 15, 30, and 45 per cent. and values of D from 0.01 to 100.0, to determine the geometric configurations and corresponding temperatures. The centre boundary consisted of a channel of a non-dimensional diameter equal to D. The results indicated in Figure 2 showed oscillations in tube thickness about a mean of 1-C. De as predicted by the approximate

analysis in Appendix 3. The magnitude of these oscillations increased with increasing D and decreasing voidage. Similar results are obtained for the temperatures, Figures 3 to 5.

It was possible to eliminate these thickness variations by determining the centre boundary values of r_{mo} and r_{oo} to give a tube thickness of $\frac{1-C}{C} \cdot \frac{De}{2}$. Eliminating r_{mn} from equations 10 and 11 and putting n = 1 and $r_{o1} = r_{11} = \frac{1-C}{C} \cdot \frac{De}{2}$,

$$r_{mo} = r_{i1}^{1} X_{1} \left[\frac{1 + \frac{r_{i1}}{K} \frac{X_{1} - 1}{2}}{1 + \frac{r_{i1}}{K} \frac{X_{1} \ln X_{1}}{X_{1} + 1}} \right] = \frac{r_{i1}^{1} - r_{0}}{C}$$

and tis " too "

Hence this gives a direct relation between r_{mo}^2 and r_{oo}^2 in the form shown in Figure 8. The portion AB of this curve corresponding to r_{mo}^2 r_{oo} has no physical significance since for equal coolant temperatures r_m is the radius at which the temperature gradient is zero within the fuel material. (Refer equation 4).

It follows that the portion BC of the curve corresponding to rmo " roo gives solutions for a nonheat-producing rod surrounded by an annulus of fuel, and point C where rmo =0 gives an all fuel rod.

The portion CD where the is negative gives imaginary configurations, point D is equivalent to point C, and DE is equivalent to BC.

 Lapides, M.E. and Goldstein, M.B. (1957) Heat Transler Source File Data APEX 425

-4-

The real solutions, as shown in Figure 6 where curves have been plotted for various voidages, therefore include the following cases

(a) a central non-heat-producing rod, i.e. tmo = too ,

(b) a central fuel rod, i.e. Imo = 0, too > 0 ,

(c) a central non-heat-producing tod of radius rmo surrounded by an annulus of fuel,

i.e. 1mo + 0

1mo < 100

Case (a) occurs only in the region of small sizes.

The corresponding tube temperature behaviour is indicated in Figure 7 for a voidage of 30 per cent. and D = 1.0. The temperatures including those of the centre element (n = o) are presented for a range of sizes of the centre element beginning at a fuel rod where $\frac{Imo}{De/2} = 0$ and $\frac{roo}{De/2} = 2.19$ up to a fuel annulus

De/2 15 surrounding a non-heat-producing rod. The corresponding values of rmo can be obtained from

Figure 6. All temperatures are presented with respect to the bulk coolant temperature in a non-dimensional form.

These boundary configurations gave no thickness variations for the series of concentric tubes.

5. DISCUSSION OF RESULTS OF CALCULATIONS

The tube cluster involving a centre channel represents the general case of concentric tubular fuel elements of constant coolant channel equivalent diameter and uniform voidage to give uniform coolant conditions. The results for this case indicate the significance of the non-dimensional equivalent diameter $D = \underline{Deh}$; to achieve the above specification at high values of D, large oscillations from one tube to the

next in thickness and temperature must result. These oscillations decrease significantly for lower values of D and also for higher voidages.

The variations of the surface temperatures seem to invalidate the assumption that the heat transfer coefficients are the same in each channel since it is necessary to include a temperature term in the heat transfer coefficient equation. The variation in the coefficient would not amount to more than approximately 5 per cent, for gas-cooled systems which operate at values of D from 0.1 to 1.0 but at low voidages. High values of D are usually associated with fast reactors having high voidages, and therefore giving lower temperature variations.

For the boundary conditions for clusters of tubes of uniform thickness, since c_{mo} must be positive and within the fuel material, any solutions involving negative or unreal values of r_{mo} or values of $r_{mo} > r_{oo}$ have no physical significance. It should also be noted that no attempt was made to start with defined outside boundaries because of the possibility of obtaining unreal configurations at the centre.

Only the solutions having physical significance are presented in the results (Figure 6). They are in two separate ranges and except at low voidages and high values of D one of these ranges would not be practical because of the very small sizes involved. Within this range the solution where $t_{mo} = t_{oo}$, that is a central non-heat-producing rod, would require some modification to the heat transfer assumption since, unlike the other channels, heat is only being transferred at one surface.

The more practical range begins with values of t_{00} a little less than the tube thickness of $\frac{1-C}{C}$. De and $t_{00} = 0$. As t_{00} increases t_{00} becomes finite and the difference between them, that is

the thickness of the fuel annulus surmunding the central non-heat-producing rod, asymptotically approaches <u>1-C</u>. De .

The temperatures do not have the oscillatory effect present in the general case and the temperature drops across the fuel elements are markedly decreased. It is evident from all the results that the larger the tube the closer it approaches the limiting geometry and temperatures.

-6-

6. CONCLUSIONS

Concentric tubular reactor fuel elements to give equal coolant outlet temperatures can be designed if the voidage and Biot number based on the channel equivalent diameter are known.

If the oscillations from tube to tube in thickness and temperature, which generally result, cannot be tolerated, they can be eliminated by specifying a particular centre fuel element.

7. NOTATION

a as defined in equation 3

A area of coolant channel

 A^{i} non-dimensional area = $A\left(\frac{h}{L}\right)^{2}$

c coolant flow voidage with respect to fuel element area

- c1)
 - constants of integration
- c2)

Cp specific heat at constant pressure of coolant

- d diameter of tubes
- D non-dimensional equivalent diameter = Deh

De equivalent diameter of coolant channel

h heat transfer coefficient

k as defined in equation 4

K themal conductivity of fuel element material

1 axial length

m mass flow of coolant per unit area

n number of channel or fuel element

Nu Nusselt number

Pr Prandtl number

q power density in fuel element material

r radius

R non-dimensional radius = th

Re Reynolds number

T temperature

X ratio of inner to outer tube radii

fuel element tube thickness .

Subscripts

\$

- f corresponding to coolant
- i value at inner radius of fuel element tube
- m value at radius defined in equation 4
- n corresponding to the n-th fuel element tube or channel

-7-

- o value at outer radius of fuel element tube
- p value at radius where $\frac{dT}{dr} = o$ within fuel material.

APPENDIX 1

DERIVATION OF THE GENERAL EQUATION

The longitudinal temperature equation is given by equating the heat generation in the fuel element to that gained by the coolant.

Thus

. .

.

*

•.

$$q \pi (r_{pn}^2 - r_{in}^2 + r_{on}^2 - r_{p(n-1)}^2) = m_n \pi (r_{in}^2 - r_{o(n-1)}^2) C_p \frac{d T_{fn}}{dl} .$$
 (i)

Therefore

$$\frac{d T_{fn}}{dl} = \frac{q}{m_n C_{pn}} \left[\frac{r_{pn}^2 - r_{pn}^2(n-1)}{r_{in}^2 - r_{onn}^2(n-1)} - 1 \right] .$$
(ii)

The steady state radial temperature distribution for the axially symmetric case is given by the solution of the relevant Poisson's equation as follows:

$$T = \frac{-q}{4K} r^2 + C_1 \ln r + C_2 , \qquad (iii)$$

where for the n-th tube,

$$C_{in} = \frac{T_{on} - T_{in} + \frac{q}{4K}(r_{on} - r_{in}^2)}{\ln \frac{r_{on}}{r_{in}}}$$

and C2 is given in terms of temperature and radius at either surface.

Now at rp

an

$$\frac{d1}{dr} = 0 ,$$

and therefore

$$r_{pn}^{2} = \frac{2K}{q} C_{1n} \qquad (iv)$$

Equating heat transfer and heat generation at the surface,

$$2r_{on}h_{on}(T_{on} - T_{f(n+1)}) = q(r_{on}^{2} - r_{pn}^{2}) , \qquad (v)$$

and

$$2 r_{in} h_{in} (T_{in} - T_{fn}) = q (r_{pn}^{2} - r_{in}^{2}) .$$
 (vi)

Eliminating the surface temperatures from equations (iii) to (vi),

$$r_{pn}^2 = a_n \frac{2K}{q} (T_{f(n+1)} - T_{fn}) + r_{mn}^2$$

where

and

$$\ln\left(\frac{r_{on}}{r_{in}}\right) + K\left(\frac{1}{r_{on}h_{on}} + \frac{1}{r_{in}h_{in}}\right)$$

$$r_{mn}^{2} = a_{n} K \left[\left(\frac{r_{0n}}{h_{0n}} \right)^{4} \left(\frac{r_{in}}{h_{in}} \right)^{2} - \frac{\left(r_{0n}^{2} - r_{in}^{2} \right) \left(\frac{1}{r_{on}h_{on}} + \frac{1}{r_{in}h_{in}} \right)}{2 \ln \frac{r_{on}}{r_{in}}} \right] + \frac{r_{on}^{2} - r_{in}^{2}}{2 \ln \left(\frac{r_{on}}{r_{in}} \right)}$$



APPENDIX 1 (continued)

1

.

Therefore

$$\frac{d T_{fn}}{dl} = \frac{q}{m_n C_{pn}} \left[\frac{r_{mn}^2 - r_m^2(n-1)}{A_{n/\pi}} - 1 \right] + \frac{2K}{m_n C_{pn}} \frac{1}{A_{n/\pi}} \left[a_{(n-1)} (T_{f(n-1)} - T_{fn}) + a_n (T_{f(n+1)} - T_{fn}) \right]$$

APPENDIX 2

DERIVATION OF EQUATION 5

From equation 4 of the text when
$$\frac{d T_{fn}}{dl} = k$$
,
 $r_{mn}^{s} = r_{m}^{s}(n-1) + \frac{1}{\pi} \left[\frac{m_{n}C_{pn}}{q} + 1 \right] \Lambda_{n}$

and by successive substitution

$$r_{mn}^{a} = r_{mo}^{a} + \frac{1}{\pi} \sum_{i=1}^{m} \left[\frac{m_{i}C_{pi}}{q} + i + 1 \right] A_{i} ,$$

10

$$r_{mn}^{3} = r_{mN}^{3} - \frac{1}{\pi} \sum_{i=n+1}^{N} \left[\frac{m_{i}C_{pi}}{q} k + 1 \right] A_{i}$$

With equal mass flow per unit area and noting that Cp will also be equal for equal temperatures,

$$r_{mn}^{i} = r_{mo}^{i} + \frac{1}{\pi} \left[\frac{m C_p}{q} k + 1 \right] \sum_{i=1}^{n} A_i$$
$$= r_{mN}^{i} - \frac{1}{\pi} \left[\frac{m C_p}{q} k + 1 \right] \sum_{i=n+1}^{N} A_i$$

Therefore

$$\left[\frac{m}{q}C_{p}k+1\right] = \frac{r_{mN}^{2} - r_{mo}^{2}}{\frac{1}{\pi}\sum\limits_{i=1}^{N}A_{i}} = \frac{1}{C}$$

and also

$$\left[\frac{m C_p}{q} k + 1\right] = \frac{t_{mn} - t_{m(n-1)}}{\Lambda_{n/\pi}} = \frac{1}{C}$$

where C is the voidage defined as the flow cross-section area divided by the sum of the flow and fuel cross-section areas.

APPENDIX 3

LIMITING VALUES

1. Limiting value of tmn at large radii

When ton and rin become large, their ratio X approaches unity. The expression $\frac{X \ln X}{1 \cdot X}$ then approaches the value $\frac{X-1}{2}$.

Substituting in equation 7 of the text

It is apparent also that as h/k - 0 , the same result holds.

2. Limiting Tube Thickness

To obtain an approximate analysis of the tube thickness substituting in equation 9 gives

$$\tau_n = \frac{D_e}{2} \frac{1-C}{C} = -\left(1 - \frac{D_e}{2\tau_{in}}\right) \left(\tau_{n-1} - \frac{D_e}{2} \frac{1-C}{C}\right).$$

that is for any positive difference between tube thickness and $\frac{De}{2} \cdot \frac{1-C}{C}$. there will be a smaller but negative difference corresponding to the thickness of the adjacent tube.

3. Temperature Limits

Using the limiting value of tmn . we have for large radii

$$\ln \left(\frac{r_{\min}}{r_{\min}}\right) - \ln X$$

Substituting in equations 11, 12, and 13 of the text,

$$\frac{T_{mn} - T_{fn}}{\frac{Deq}{h}} \cdot \frac{R_{on} - R_{ie}}{2D} \left[1 \cdot \frac{R_{on} - R_{in}}{4} \right]$$
$$\frac{T_{in} - T_{fn}}{\frac{Deq}{h}} \cdot \frac{T_{on} - T_{fn}}{\frac{Deq}{h}} \cdot \frac{1}{2} \frac{R_{on} - R_{in}}{D}$$



FIGURE I. TEMPERATURES AND RADII AS USED





.









CONDITIONS FOR TUBE CLUSTERS OF UNIFORM TUBE THICKNESS AND COOLANT TEMPERATURES.

