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THE REGENERATION FACTOR AS A FUNCTION OF TIME IN A Th^{232}-U^{235} THERMAL REACTOR

by

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ABSTRACT

This report is concerned with a theoretical investigation of the variation of the regeneration factor η in a Th^{232}-U^{235} thermal reactor. The abundances of the significant isotopes in the thorium-uranium cycle have been derived as a function of irradiation time at constant reactor power. The change in η as a function of irradiation time at constant power was calculated for combinations of enrichment and resonance escape probability considered likely to exist in a thermal reactor. The effect upon η of the absorption cross section of ${}_{91}Pa^{233}$ and of the fission products has been shown.

I. INTRODUCTION

The regeneration factor η is defined as the number of neutrons produced per thermal neutron absorbed in the fuel:

$$\eta = \frac{\nu\Sigma_f^{235} + \nu\Sigma_f^{233}}{{}_{92}\Sigma_a^{233} + {}_{92}\Sigma_a^{234} + {}_{92}\Sigma_a^{235} + {}_{91}\Sigma_a^{233} + {}_{91}\Sigma_a^{232}}$$

where Σ indicates the macroscopic cross section and ν represents the number of fast neutrons per thermal neutron fission. The left subscript denotes the atomic number and the superscript denotes the mass number.

In this investigation the product of the two variables Σ_f and ϕ_t is held constant. The variation of Σ_a, Σ_f, ϕ_t and η as a function of irradiation time is determined from the simultaneous solution of the following set of linear differential equations:

$$\frac{d[Th^{232}]}{dt} = -[Th^{232}]\,\sigma_a\phi_t - \mu([U^{235}]\,\sigma_a\eta + [U^{233}]\,\sigma_a\eta)\phi_t$$

$$\frac{d[Pa^{233}]}{dt} = [Th^{232}]\,\sigma_a\phi_t + \mu([U^{235}]\,\sigma_a\eta + [U^{233}]\,\sigma_a\eta)\phi_t - [Pa^{233}]\,\lambda - [Pa^{233}]\sigma_a\phi_t$$

$$\frac{d[U^{233}]}{dt} = [Pa^{233}]\lambda - [U^{233}]\sigma_a\phi_t$$

$$\frac{d[U^{234}]}{dt} = [U^{233}]\sigma_c\phi_t - [U^{234}]\sigma_a\phi_t + [Pa^{233}]\sigma_a\phi_t$$

$$\frac{d[U^{235}]}{dt} = [U^{234}]\sigma_a\phi_t - [U^{235}]\sigma_a\phi_t$$

where λ = the radioactive decay constant of Pa^{233}

$$\phi_t = \frac{K}{\Sigma_f^{235} + \Sigma_f^{233}}$$

$\mu = \epsilon(1-p)\,e^{-B^2\tau}$

σ = the nuclear cross section

p = the resonance escape probability

ϵ = fast fission factor

$e^{-B^2\tau} = L$ = leakage

B = the buckling

K = constant

τ = the Fermi age of the neutrons

The nuclear constants used with the above equations are given in Table I.

Table I

CROSS SECTIONS AND NUCLEAR CONSTANTS

	U^{233}	U^{234}	U^{235}	Pa^{233}	Th^{232}
$\sigma_a \times 10^{24}$	564	89	680	150	7
$\sigma_f \times 10^{24}$	514	0	575	0	0
$\sigma_c \times 10^{24}$	50	89	105	150	7
η	2.32		2.09		
ν	2.55		2.48		
λ				0.294×10^{-6}	

In the thorium-uranium system, U^{235} is the primary source and Th^{232} is the secondary source of neutrons. Reactors using this combination are known as converters, and the term "conversion ratio" is used as a measure of the efficiency with which fissionable atoms undergo transmutations. As the U^{235} is depleted it is replaced to a varying extent by the fissionable isotope U^{233}. At any time after the start, the fissionable material consists of a mixture of U^{235} and U^{233}. The U^{235} decreases at first and then slowly increases; the U^{233} increases at first and then decreases. The inflection is shown on Figure 3.

The initial conversion ratio formula consists of two components: one due to thermal absorption and one due to resonance absorption.

<u>Thermal absorption</u>

Neutrons absorbed in Th^{232}

<u>Resonance absorption</u>

Σ^{235}_{a} α neutron absorbed in U^{235}.

$\Sigma^{235}_{\eta\epsilon}$ α fast neutrons produced per thermal neutron absorbed.

$\Sigma^{235}_{a}\eta\epsilon\,(1-p)$ α resonance neutrons captured per thermal neutron absorbed.

<u>Total initial conversion ratio (ICR)</u>

$$ICR = \frac{\Sigma^{232}_{a}}{\Sigma^{235}_{a}} + \eta\epsilon\,(1-p)e^{-B^2\tau}$$

As irradiation proceeds, the conversion ratio is defined by the following expressions:

1. The instantaneous conversion ratio at any time is the ratio of the rate of production of new fissionable isotopes to the rate of destruction of old fissionable isotopes.
2. The cumulative conversion ratio at any time is defined as the ratio of the number of fissionable atoms existing at the beginning of the irradiation.

The solutions cannot be considered general since μ depends upon the design characteristics of a reactor. The size, geometry, fuel arrangement and moderator characteristics all affect μ ; therefore, it is a parameter in this investigation.

II. CALCULATION METHODS

The equations of isotopic abundance were solved simultaneously and the dependent variable plotted against irradiation time by means of a Reeves Electronic Analog Computer. Thus a family of curves of each isotope versus irradiation time with μ as a parameter were obtained for a range of enrichments. From these isotopic abundance curves the flux, Σ_f, η and Σ_a of the fission products were readily calculated.

Since some uncertainty exists as to the value of resonance integral of thorium, values of $\mu = c\ (1-p)L$ were chosen to cover an area considered to be of current interest. It is quite easy to cover a wider range.

The set of differential equations on p. 5-6 is considered to apply to a point in the reactor at which the fuel and moderator are homogeneous and the neutron temperature equals the moderator temperature (293K).

The power is held constant with time; therefore, the flux and Σ_f vary with time.

Since the cross section of the aggregate fission products is questionable, the term "Σ_a fission products" is transferred from the expression for η to that for thermal utilization "f." Thus an uncertainty is removed from the expression for η and is added to the expression for thermal utilization, the latter depending upon the specific reactor design.

The variation of Th^{232} due to thermal and resonance absorption is taken into account.

III. RESULTS

The thorium-uranium reaction is shown diagrammatically on Figure 1 and it is defined mathematically by the set of differential equations on p. 5-6.

The isotopes which are considered to be significant in the reaction are plotted in Figures 2 and 3 against irradiation time. These isotopes are Th^{232}, Pa^{233}, U^{233}, U^{234}, and U^{235}. There are many others, but they either have such a short half life or low cross section that they have little effect upon η.

The area of investigation is indicated by the range of enrichments and values of μ.

Atomic Ratio $\frac{Th^{232}}{U^{235}}$	μ
80, 70, 60, 50	0.05, 0.10, 0.15, 0.20, 0.25

Certain combinations of enrichment and μ shown on the graphs of Figures 2 and 3 are incompatible in a critical core; however, they may occur in a thorium blanket. The combinations which are considered to be most likely in a critical core vary from a combination of $\frac{Th^{232}}{U^{235}} = 70$ and $\mu = 0.15$ to a combination of $\frac{Th^{232}}{U^{235}} = 50$ and $\mu = 0.05$.

The curves of isotopic abundances versus irradiation time are shown in Figures 2 a-e. Figure 3 shows the characteristics of the curves if the power be increased by a factor of 10.

The variation of η with μ as a parameter is shown on Figures 4 a-c.

The variation of η with enrichment as a parameter is shown on Figure 5.

There are an infinite number of combinations of μ and enrichment within the area considered. Figures 4 and 5 serve to show the variation of η in a hypothetical thorium-uranium reactor.

Since $\epsilon = 1$ and since p will vary only slightly during the operation of a reactor $\frac{\eta f}{\eta_o f_o}$ is a measure of the reactivity variation.

The effect of ${}_{91}\sigma_a^{233}$ is shown in Figures 6 and 7. For a flux of the magnitude 10^{13} this cross section has little effect; at higher fluxes the effect will be more pronounced.

Figure 8 shows the effect of including fission products in the denominator of the expression for η.

In the event that the neutron flux is other than that shown in the parametric presentation, the irradiation time scale may be varied directly as the flux ratio. This procedure is not strictly correct because the variation with time of two different initial fluxes is not necessarily in phase. For small variations in flux, however, the discrepancy is considered negligible.

Interpolation between enrichments gives satisfactory results.

IV. CONCLUSIONS

It is concluded that:

1. A Th^{232}-U^{235} fuel is attractive from the point of view of the variation of η with irradiation time provided enough excess reactivity can be designed into the reactor to overcome the initial depression of η.

2. If any economic advantage is to be realized, thorium reactors should be designed for long operation (10,000 mwd per ton or greater). The initial fuel must be left in the reactor at least until η has recovered from the initial depression; incremental loading is then feasible.

3. It is desirable to have as low an enrichment and as high a value of μ as is consistent with good design.

4. The characteristics of the η variations in the Th^{232}-U^{235} fuel are almost a mirror image of the η variations in the natural and slightly enriched uranium fuels. Therefore, it is concluded that a reactor which is loaded with a mixture of natural uranium and thorium may have a η variation such that the reactivity is held nearly constant during the lifetime of the reactor.

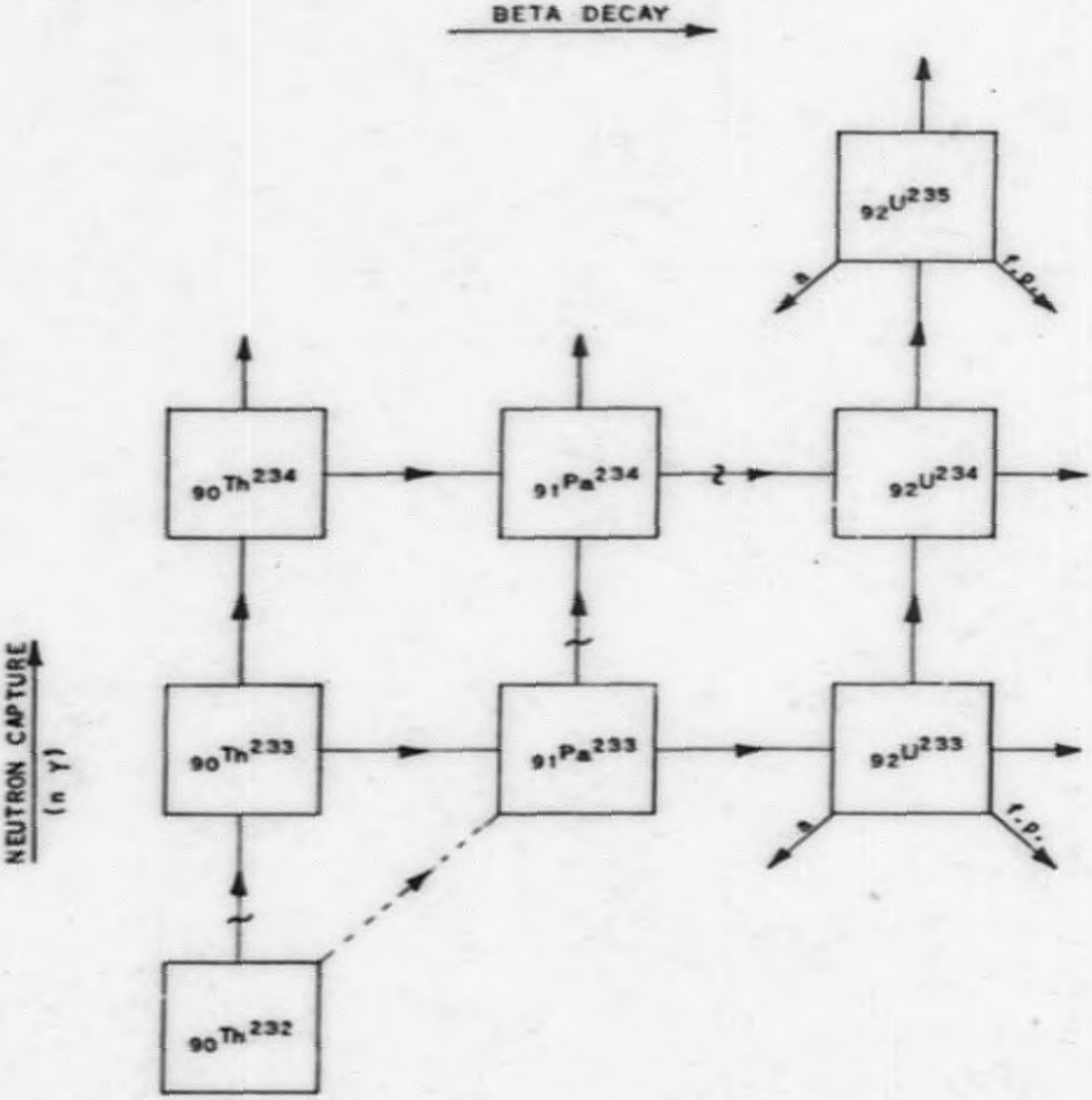


FIG. 1
THORIUM-URANIUM REACTION

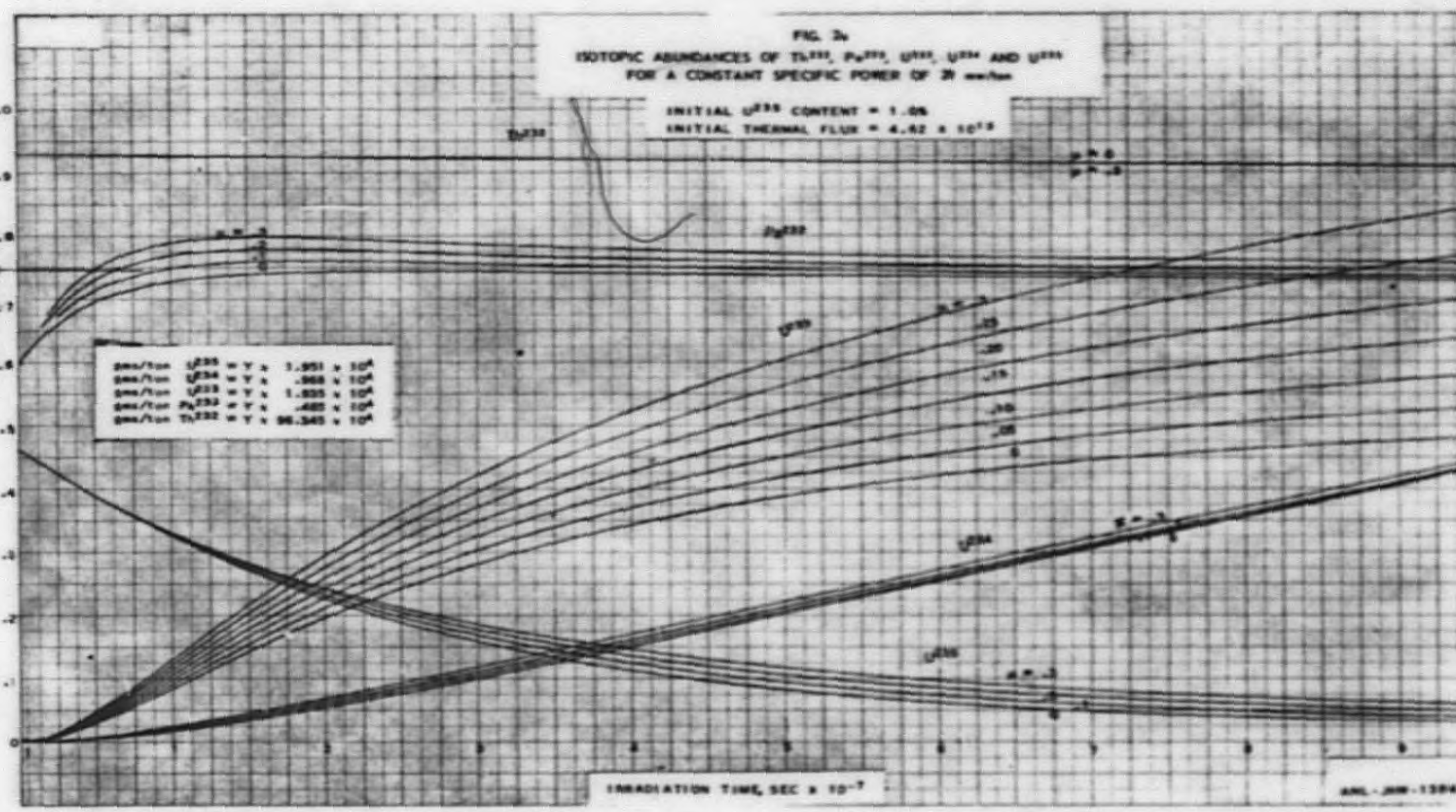


FIG. 2a
ISOTOPIC ABUNDANCES OF Th^{232}, Pa^{233}, U^{233}, U^{234} AND U^{235}
FOR A CONSTANT SPECIFIC POWER OF [illegible] mw/ton

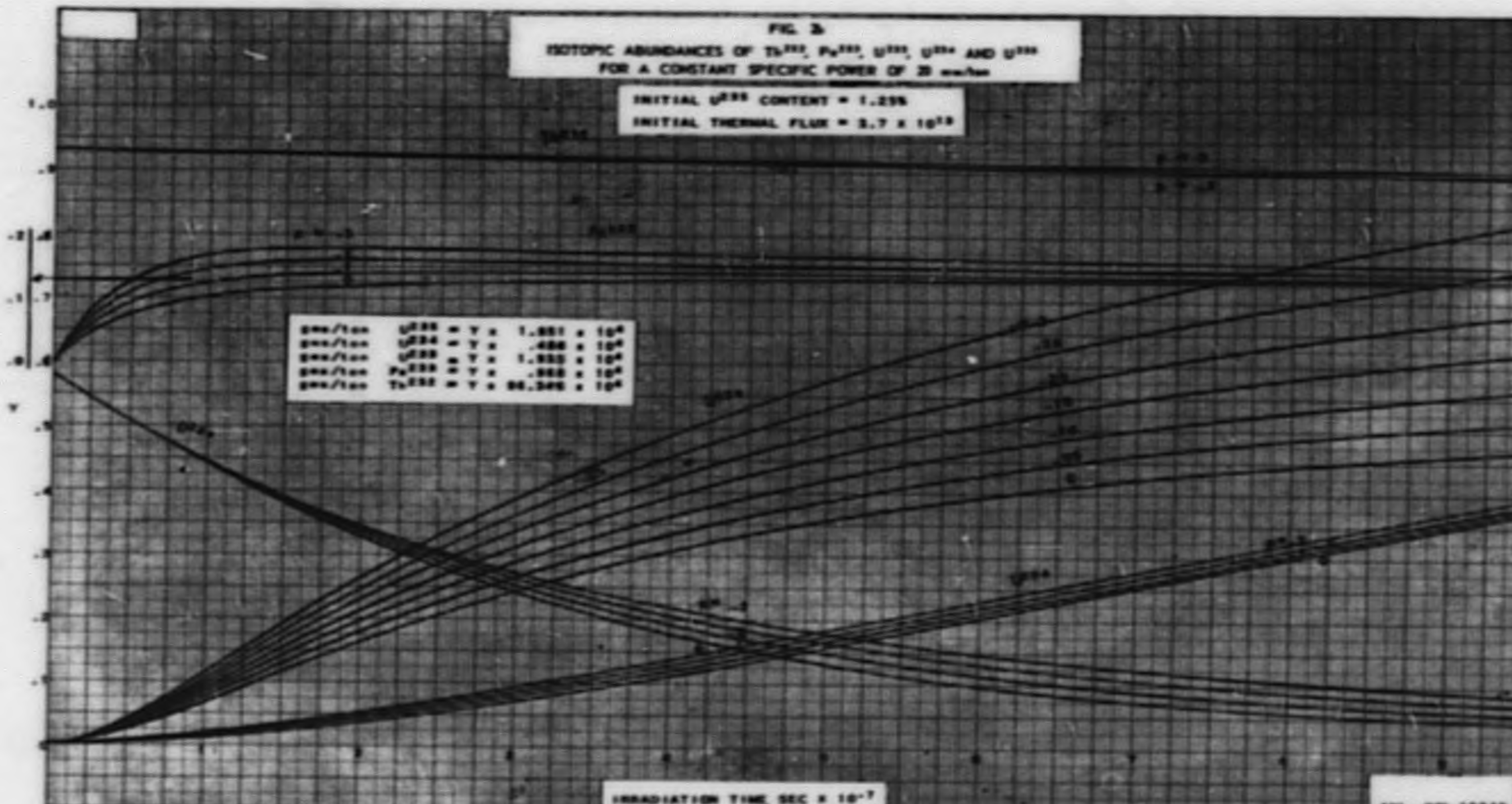
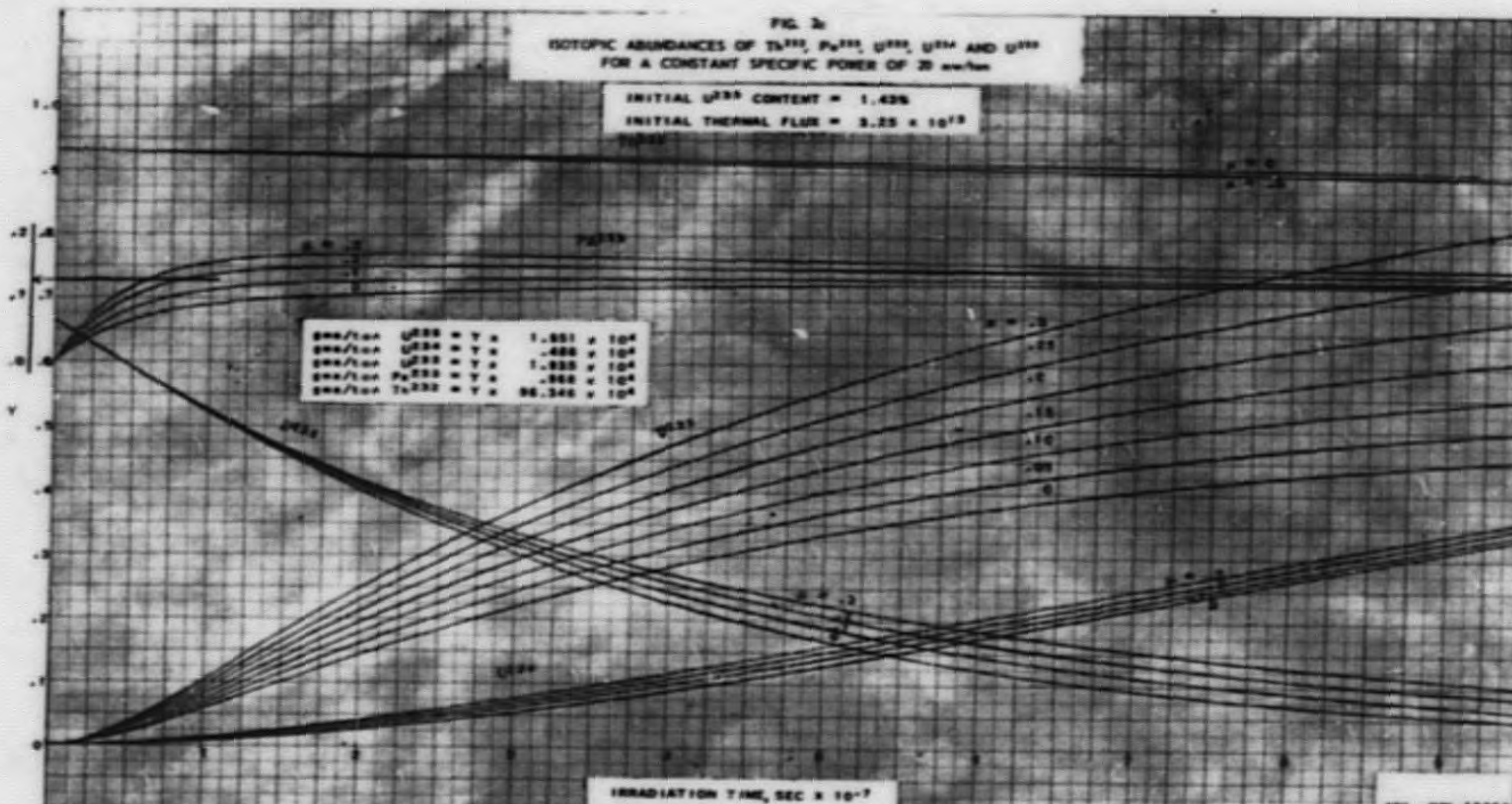
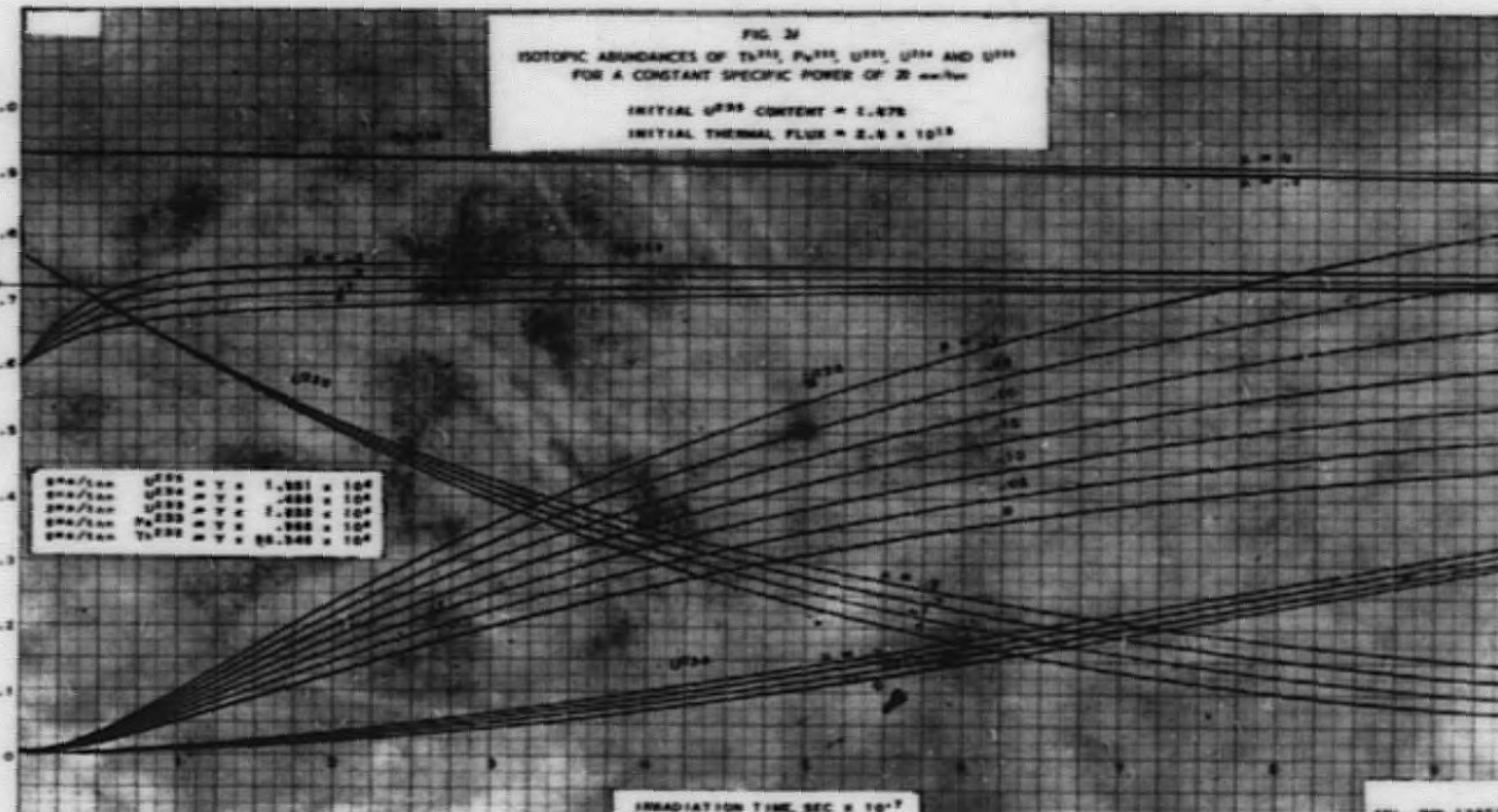
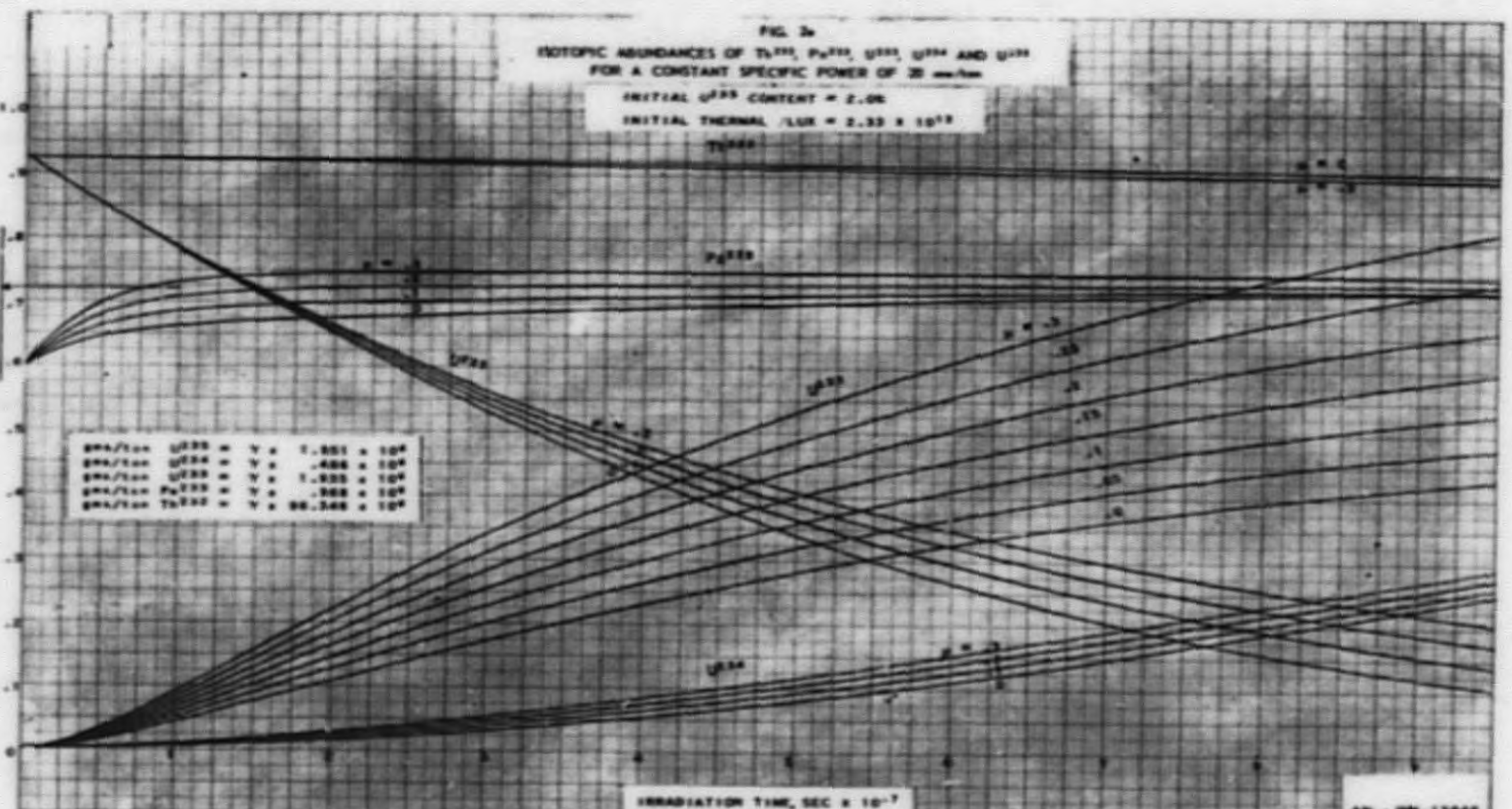


FIG. 3
ISOTOPIC ABUNDANCES OF Th^{232}, Pa^{233}, U^{233}, U^{234} AND U^{235}
FOR A CONSTANT SPECIFIC POWER OF 20 mw/ton
INITIAL U^{235} CONTENT = 1.25%
INITIAL THERMAL FLUX = 3.7×10^{13}
IRRADIATION TIME, SEC $\times 10^{-7}$


ISOTOPIC ABUNDANCES OF Th232, Pa233, U233, U234 AND U235
FOR A CONSTANT SPECIFIC POWER OF 20 mw/ton
INITIAL U235 CONTENT = 1.43%
INITIAL THERMAL FLUX = 3.25 x 10^13
IRRADIATION TIME, SEC x 10^-7


ISOTOPIC ABUNDANCES OF
FOR A CONSTANT SPECIFIC POWER OF
INITIAL CONTENT
INITIAL THERMAL FLUX
IRRADIATION TIME, SEC


ISOTOPIC ABUNDANCES OF
FOR A CONSTANT SPECIFIC POWER OF 20
INITIAL CONTENT
INITIAL THERMAL
IRRADIATION TIME, SEC x 10

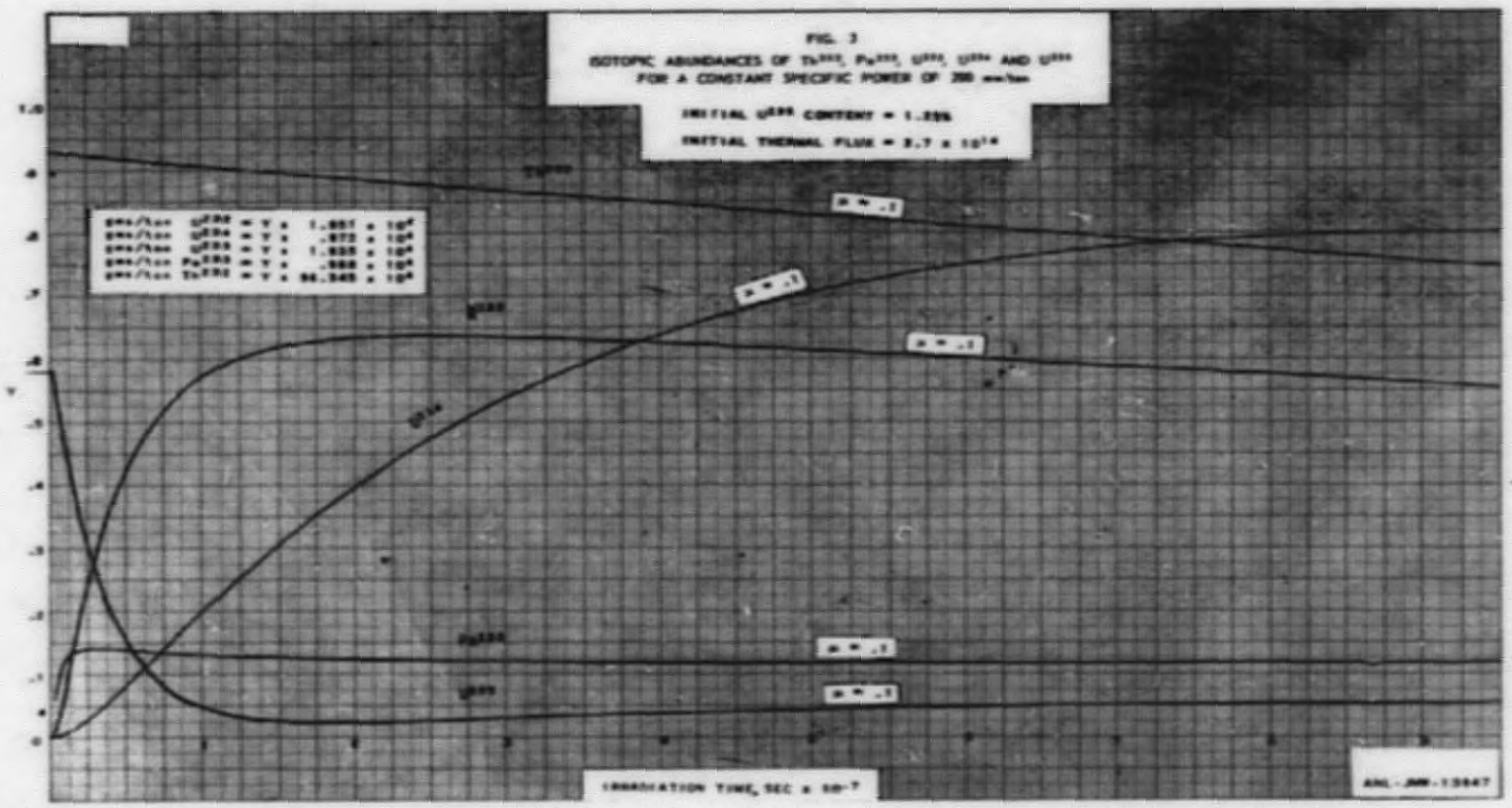


FIG. 3

ISOTOPIC ABUNDANCES OF Th^{232}, Pa^{233}, U^{233}, U^{234} AND U^{235}
FOR A CONSTANT SPECIFIC POWER OF 300 mw/ton

INITIAL U^{235} CONTENT = 1.33%

INITIAL THERMAL FLUX = 3.7 x 10^{14}

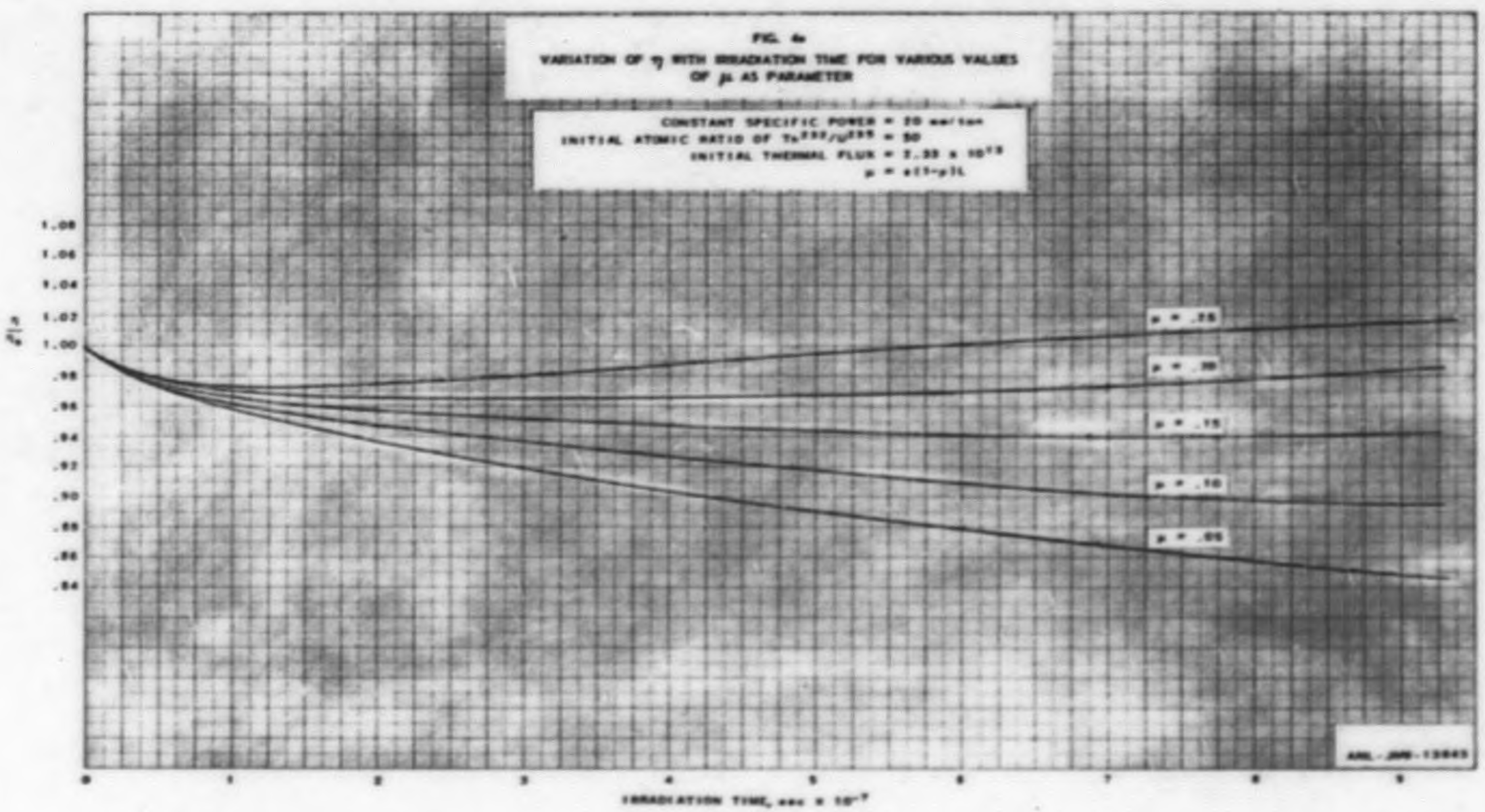


FIG. 4a

VARIATION OF η WITH IRRADIATION TIME FOR VARIOUS VALUES OF μ AS PARAMETER

CONSTANT SPECIFIC POWER = 20 mw/ton
INITIAL ATOMIC RATIO OF Th^{232}/U^{235} = 50

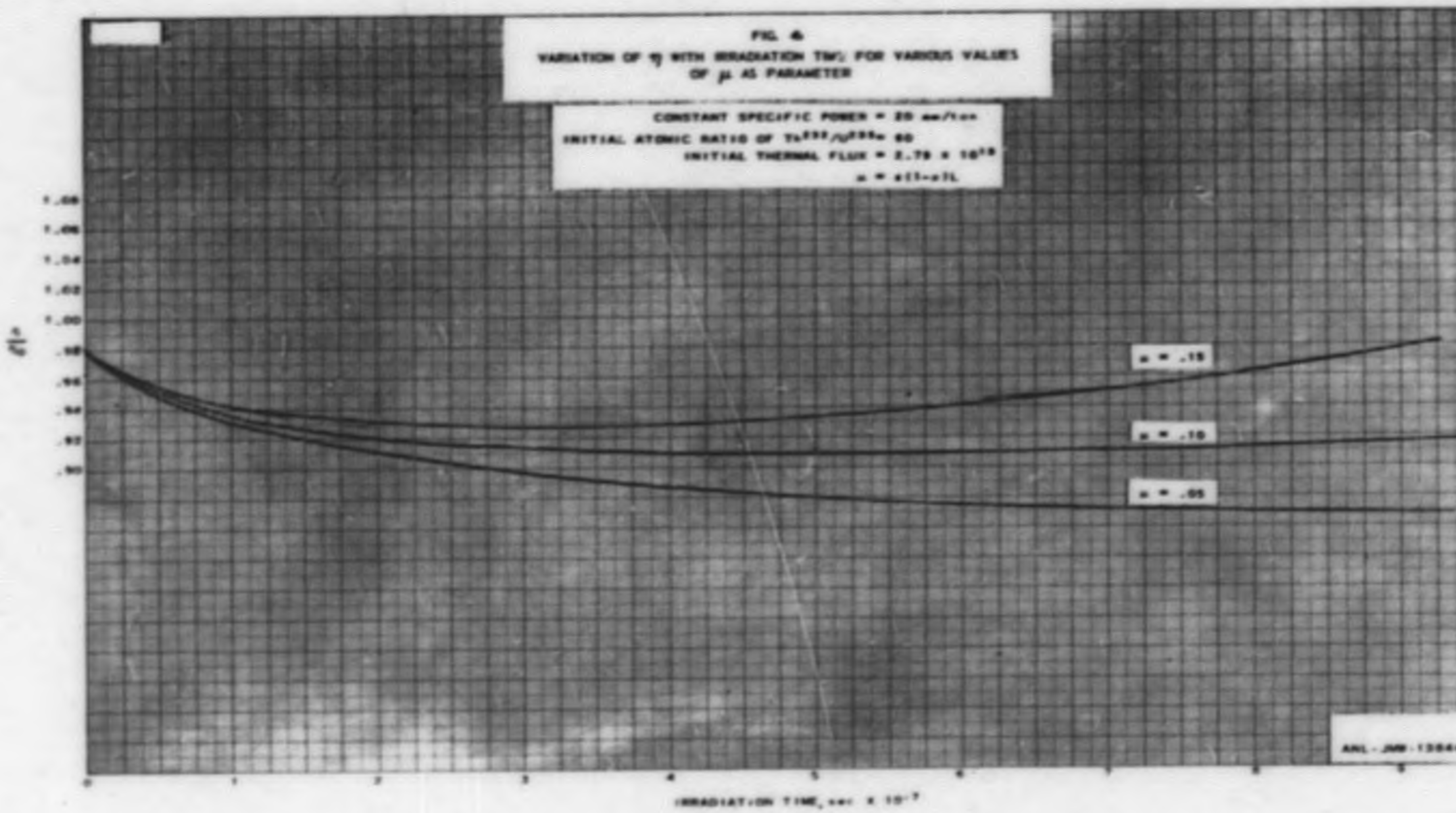


FIG. 6
VARIATION OF η WITH IRRADIATION TIME FOR VARIOUS VALUES OF μ AS PARAMETER
CONSTANT SPECIFIC POWER = 20 mw/ton
IRRADIATION TIME, sec x 10⁻⁷
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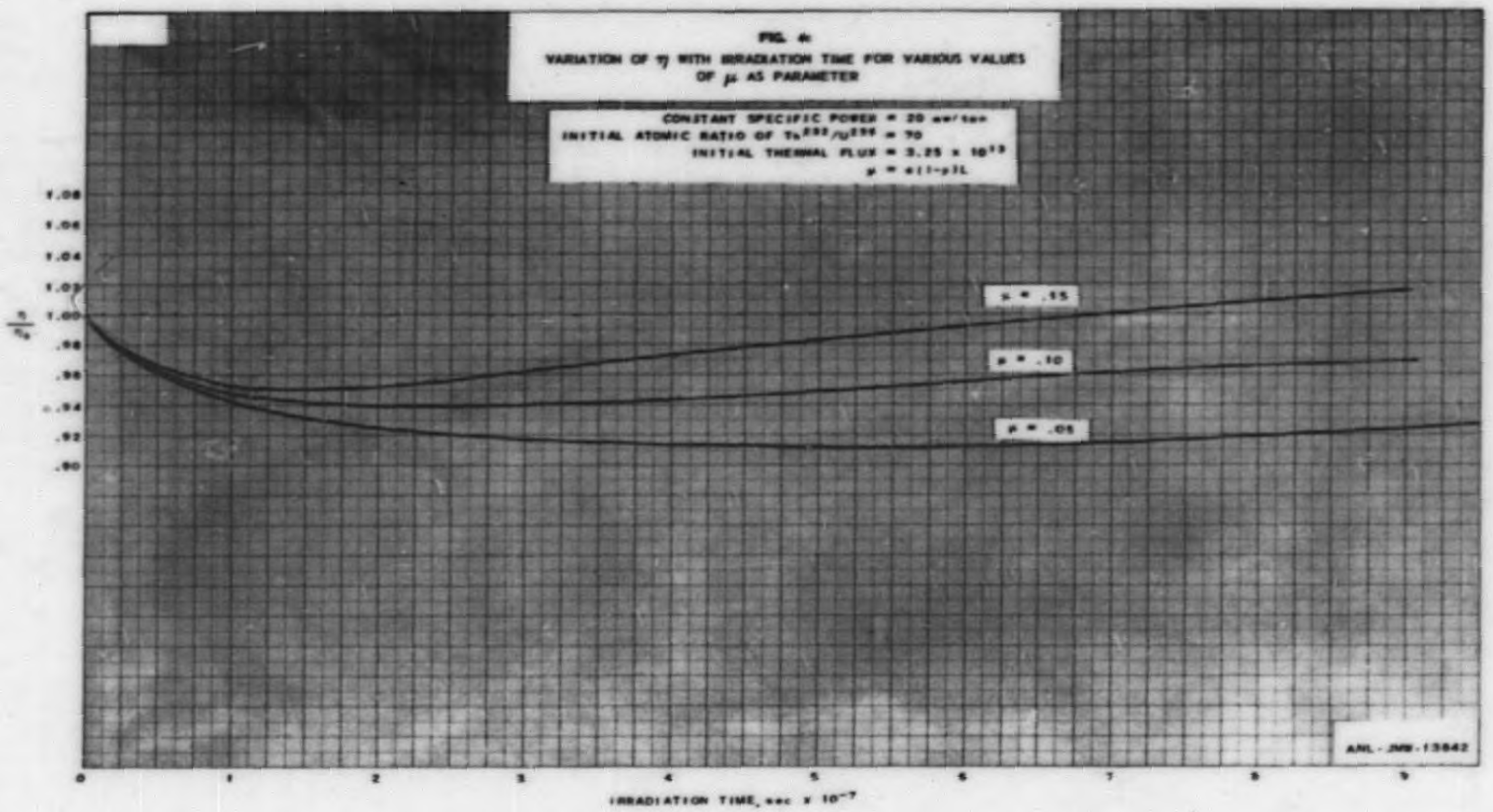

FIG. 4
VARIATION OF η WITH IRRADIATION TIME FOR VARIOUS VALUES OF μ AS PARAMETER

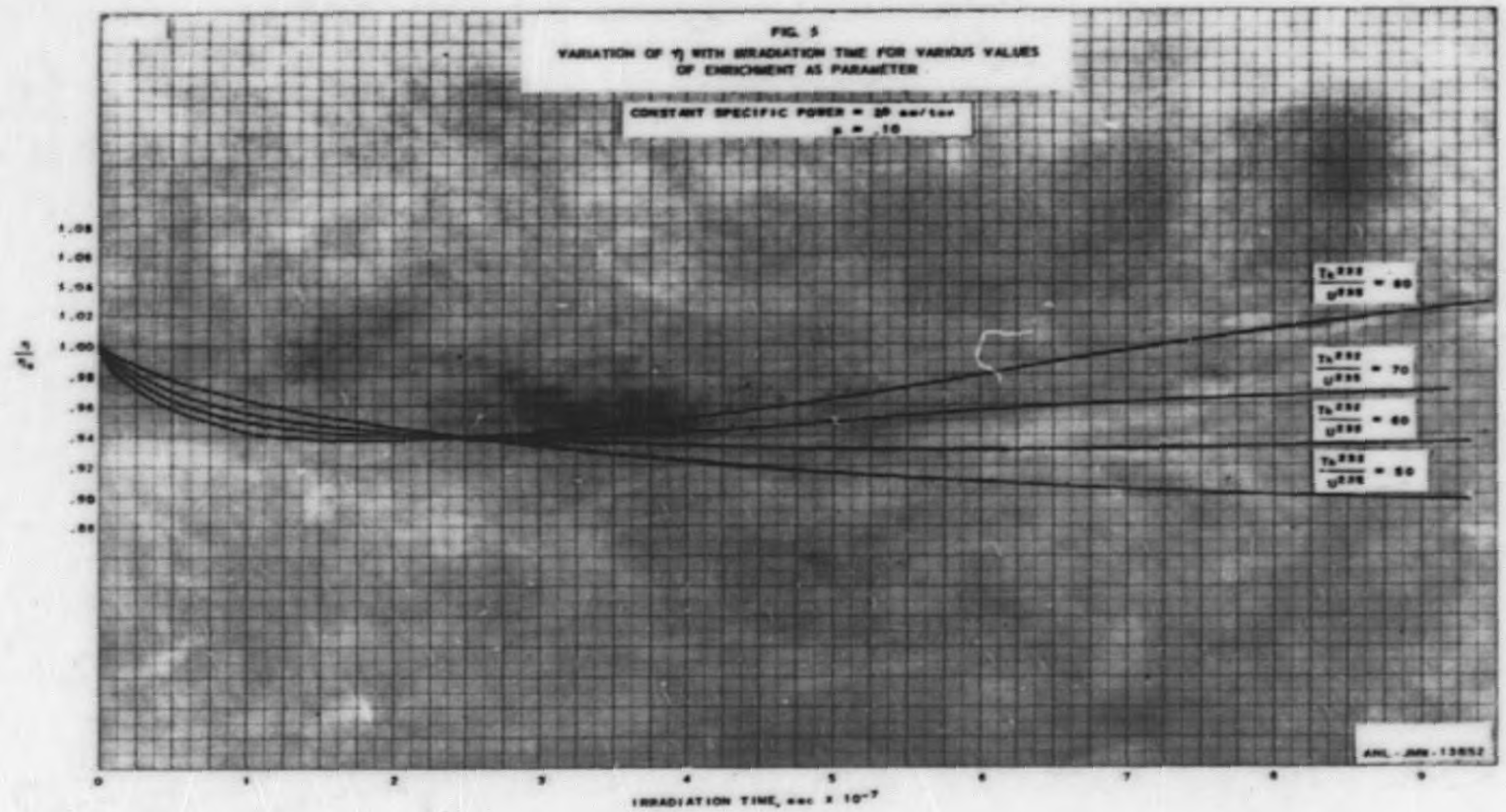

FIG. 5
VARIATION OF η WITH IRRADIATION TIME FOR VARIOUS VALUES OF ENRICHMENT AS PARAMETER

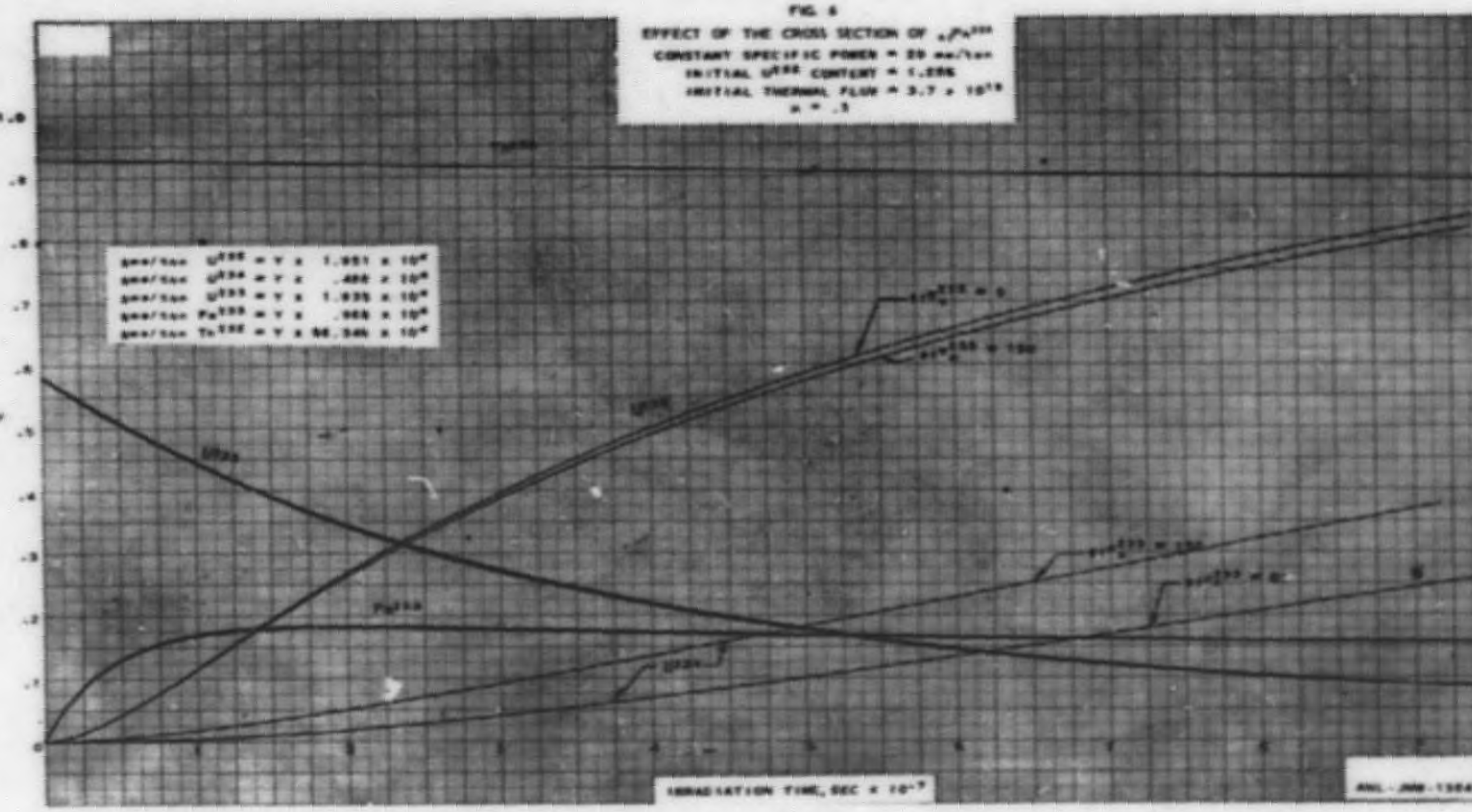

FIG. 6
EFFECT OF THE CROSS SECTION OF $_aPu^{239}$
CONSTANT SPECIFIC POWER = 20 mw/ton
INITIAL U^{235} CONTENT = 1.25%
INITIAL THERMAL FLUX = 3.7 × 10^{13}
α = .1

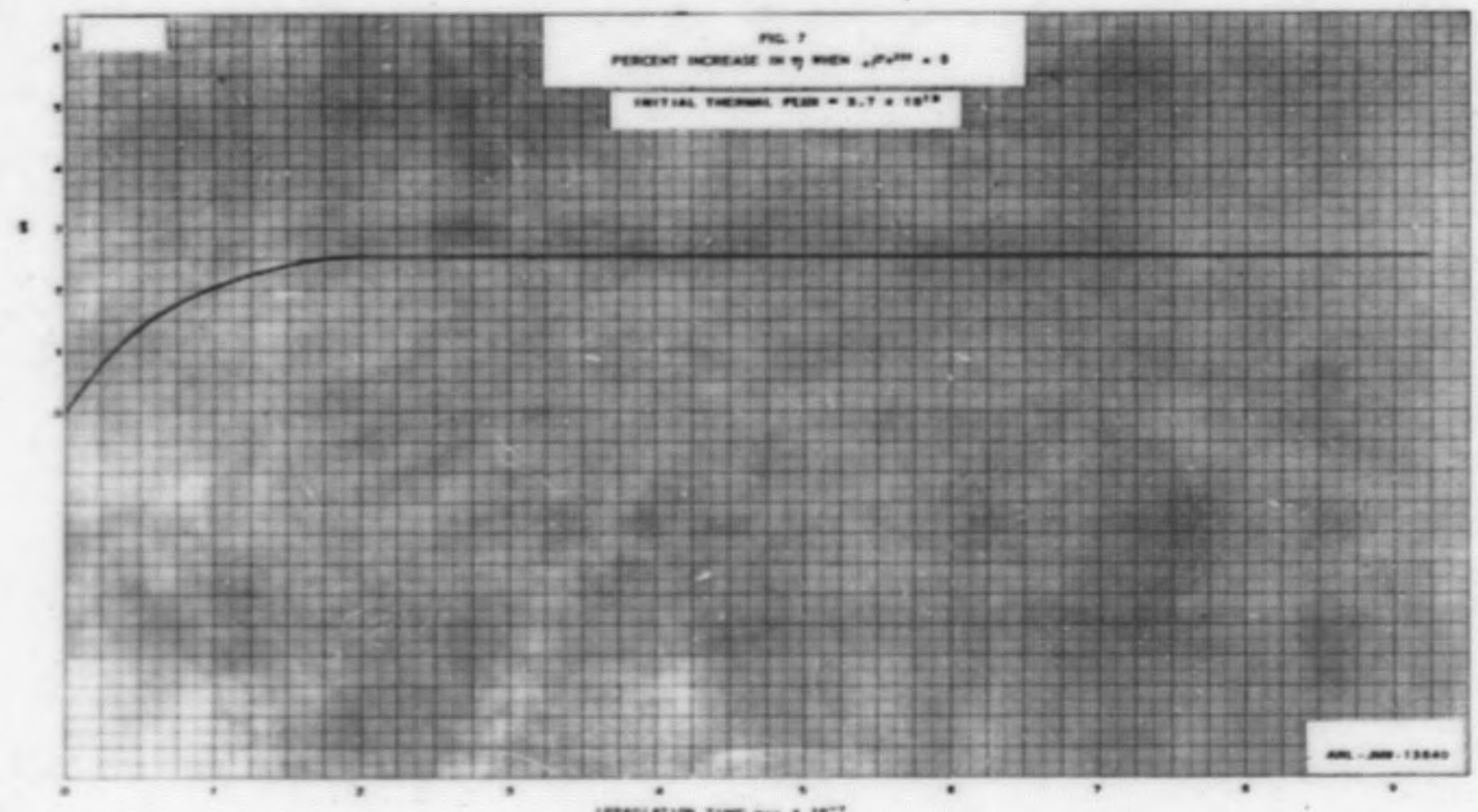

FIG. 7
PERCENT INCREASE IN η WHEN

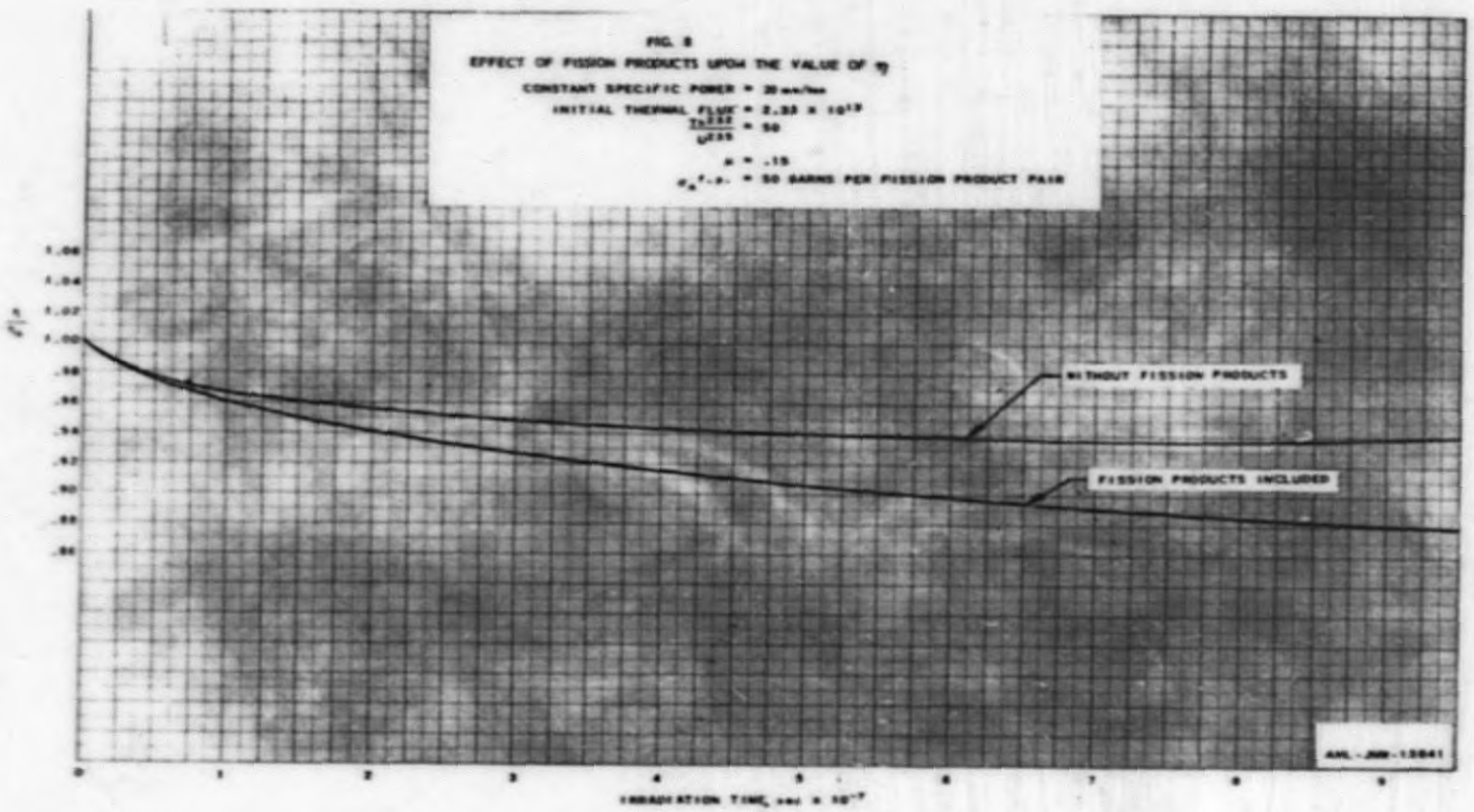

FIG. 8
EFFECT OF FISSION PRODUCTS UPON THE VALUE OF η
CONSTANT SPECIFIC POWER = 20 mw/ton
INITIAL THERMAL FLUX = 2.33 x 10^13
Th232/U235 = 50
= .15
= 50 BARNS PER FISSION PRODUCT PAIR
WITHOUT FISSION PRODUCTS
FISSION PRODUCTS INCLUDED
IRRADIATION TIME, sec x 10^-7