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CRUSHING AND GRINDING

BY
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FOREWORD

This is the third of a series of bulletins published during the past year which contain the results of comprehensive investigations by the Metallurgical Division of the Bureau of Mines.

It is intended that this series will be augmented from time to time as material becomes available.

R. S. DEAN,
Chief Engineer, Metallurgical Division.

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CRUSHING AND GRINDING ¹

By JOHN GROSS ²

INTRODUCTION

From the time when man first pounded one rock upon another or used heat and sudden chilling with water to disintegrate rock, the art of crushing advanced but slowly to the mule-driven arrastre and the Chilean mill stage until the advent of power-driven appliances made material progress possible. The combined efforts of manufacturers and operators have made modern machines highly efficient mechanically.

Although marked progress has been made along mechanical lines, the theory and conception of underlying principles have not advanced so rapidly. This lack of advance in theory may be attributed to the unfortunate situation resulting from the controversy as to whether the Rittinger or the Kick law is applicable to crushing. Many pages of the technical literature are burdened with theoretical discussions in favor of one or the other of these laws, which have tended to cloud rather than to clear the atmosphere.

Realizing the situation, the Milling Committee of the American Institute of Mining and Metallurgical Engineers, through its secretary, E. A. Hersam, studied the milling industry and, because of the apparently hopeless lack of agreement in the Kick versus Rittinger dispute (5, 13, 14, 16, 18, 19, 21, 22, 23, 24, 30, 34),³ concluded that a foundation for the theory of crushing was of primary importance. Since then the importance of such a foundation has been emphasized by further contributions (39, 43, 46, 50, 54). This study was made cooperatively by the Bureau of Mines, the Massachusetts Institute of Technology, and the University of California, and the report to the Milling Committee (35) presented the consensus of opinion that, in view of the chaotic state of the knowledge of crushing, a study of the subject, especially in relation to the law governing it, was of the highest importance.

As a result, the Bureau of Mines inaugurated study on crushing at its Intermountain Experiment Station at Salt Lake City in the fall of 1924. The results of the earlier part of this work, which was carried out by Stuart R. Zimmerley, S. J. Swainson, and the writer, have been published at various periods (53, 74, 79, 80, 81, 95) but will be reviewed in this paper with additional comments warranted by subsequent developments.

The purpose of this bulletin is to discuss the theory of crushing and grinding in relation to present-day discoveries and developments

¹ Work on manuscript completed April 1936.

² Senior metallurgist, Bureau of Mines.

³ Figures in parentheses refer to bibliography at end of bulletin.

and to consider the various mechanical appliances from a theoretical basis.

CRUSHING AND GRINDING AS COMMUNUTING OPERATIONS

The term "comminution" may be applied to either crushing or grinding, and although comminution may more logically cover both operations crushing and grinding will probably continue to be used in spite of the fact that no acceptable definition for either can be formulated.

Crushing generally applies to the coarser breaking by jaw crushers, gyratories, disk machines, and rolls, whereas grinding applies to the finer comminution by ball, rod, and tube mills. Crushing generally implies an impact or compression, whereas grinding implies abrasion or attrition, but both actions take place in each operation. In fact, no sharp line can be drawn between them.

PURPOSES OF CRUSHING

The purpose of crushing or grinding would seem to be self-evident, but the requirements for the finished products vary so much that a satisfactory comminution for one material might be unsatisfactory for another. The degree of crushing and grinding will necessarily depend on whether a finished product is desired or whether the comminution is primarily to free constituent minerals or to increase or expose surface.

COMMUNITION TO PRODUCE A FINISHED PRODUCT

This heading includes the crushing of rock for road metal, the preparation of concrete aggregate, and the grinding of cement, coal, or paint products, in all of which the main purpose is to yield a product of the desired fineness.

Road metal and concrete aggregate require primarily a relatively coarse crushing to a fixed maximum size. It would seem that after they had been so crushed economically, no further thought would need to be given the products. However, the various sizes that constitute these products may have a very important bearing on their value.

In road material the property of being able to pack well depends, partly at least, on the gradation of size, and in these days of wide-spread oiling the quantity of surface exposed has a direct bearing on the quantity of oil required for thorough agglomeration without an excess of the oil being exuded.

In concrete aggregate the distribution of sizes should be such that a minimum of voids would be present and that fine material or dust would not be present in excessive quantities.

Even in these simple examples of comparatively coarse crushing a study of the crushing operation that produces the most desirable product apparently is of importance.

In grinding cement the main object is to produce a minimum of plus 200-mesh material, but it is questionable whether extreme fineness is necessary or desirable.

In grinding coal a product that gives the best combustion is desired, and this depends not only on the size of the particles but on the surface exposed.

In grinding materials for paint the production of surface in the ground product is the prime objective. Here it would appear that the production of excessively fine particles is desirable rather than detrimental, therefore the grinding of paint differs from the grinding of other materials.

That producing a finished material in fine grinding is not a simple operation of grinding to extreme fineness is evident by the extensive amount of research work on this subject, especially the grinding of paint.

COMMINATION TO FREE CONSTITUENT MINERALS

This heading includes all crushing or grinding to produce a material suitable for subsequent separation of the constituent minerals. In general, the raw material is an ore and may have either metallic or nonmetallic minerals as its valuable constituents. The bulk of the crushing and grinding of ores is done for this purpose. With the crushing and grinding methods now available the object is attained by comminution of the ore to the size at which the smallest mineral grain becomes free, but frequently it may not be economically advisable to carry comminution to the ultimate point of freeing the smallest grain.

The operation is accompanied by considerable comminution beyond the size at which the smallest mineral particles are freed. This results in a loss of crushing energy and in pulverizing some of the minerals so finely that loss would occur in the subsequent treatment.

The ideal method of comminution would be one that simply fractures along the boundary of the minerals without breaking the minerals themselves. This highly desirable result may never be accomplished, but the nearer it can be approached the more satisfactory the comminution. New principles of comminution are needed, and explosive shattering, which is discussed later, is a step in this direction.

COMMINATION TO INCREASE OR EXPOSE SURFACE

Aside from paint grinding, this heading includes the grinding of an ore for flash roasting or for hydrometallurgy.

In flash roasting a rapid oxidation of the surface is desired. The extent of oxidation in a given time is a direct factor of the surface exposed to the oxidizing atmosphere. Although an increase in surface is desirable for rapid and efficient oxidation, it does not follow that it is best obtained by very fine grinding.

There is always a size that will give enough surface for the oxidation within a reasonable time limit. All finer material may be considered to represent not only a waste of energy but a source of loss in dusting.

In leaching an ore for the extraction of gold, silver, and copper, it is unnecessary for the valuable minerals to be separated, but if the ore is broken so as to expose one boundary of the metal the result may be looked upon as satisfactory, and any further crushing represents wasted energy, possibly with greater difficulty in leaching.

ECONOMIC IMPORTANCE OF GRINDING

A very large tonnage of material is subjected to comminution for a variety of purposes. Any increase in efficiency or decrease in cost of crushing or grinding or any improvement in the product would be of economic importance to many industries.

A decrease in crushing or grinding costs will result from the use of new and more efficient appliances rather than from improvements in present machines or their operation.

Major improvements will include the elimination of useless grinding or overgrinding. From an economic aspect the elimination of useless grinding is highly desirable in reducing power costs and may result further in making a comparatively coarser product of greater value as a direct marketable product or more desirable for subsequent metallurgical treatment.

When it is considered that about 80 percent of the energy used in grinding an ore for flotation is useless, it is perhaps safe to assume that one-half of every horsepower expended in crushing and grinding is wasted. Whether or not this waste may ever be entirely avoided seems doubtful, but the saving in energy and the production of a more valuable or desirable product by elimination of overgrinding call for serious and intelligent research. An excellent survey of the research work on crushing prior to 1929 has been made by Shaw (92).

ACKNOWLEDGMENTS

The earlier investigations were carried on at Salt Lake City in cooperation with the University of Utah. During this period the work of S. R. Zimmerley and S. J. Swainson was of great help.

The work on explosive shattering and nozzle crushing was done at the Minneapolis station in cooperation with the University of Minnesota. It is especially gratifying to acknowledge the valuable assistance of C. E. Wood in this work.

Grateful acknowledgment is due R. S. Dean for his untiring interest and constructive criticism.

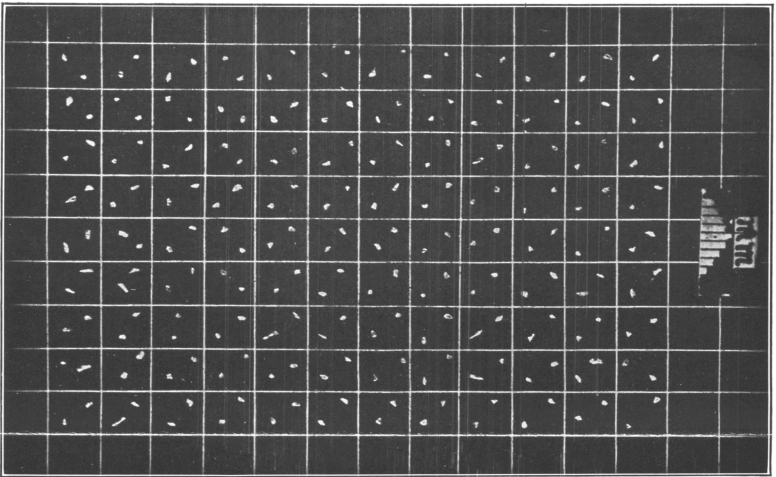
MECHANISM OF CRUSHING

There is still considerable confusion as to what actually takes place when a solid is fractured. This may be attributed to an apparently complicated set of conditions as to application of the crushing force and its results. Acceptance of the Rittinger law that crushing energy is proportional to new surface produced simplifies the conception of the operation, and it is now thought possible to visualize the actual mechanism of rupture.

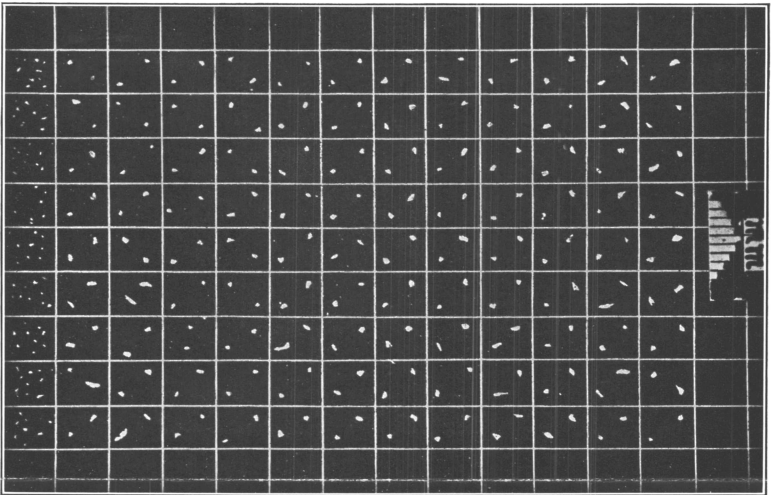
NATURE OF FRACTURES IN SOLIDS

During crushing, the crystal lattice must be elastically deformed. When deformation reaches a certain point new surface is produced, the volume of strain released is proportional to this surface, and the energy of deformation appears as heat. When the deformation falls short of that necessary for the formation of new surface the strain is released as mechanical energy to be returned to the crushing appliance. The condition is analogous to breaking a rubber band, and the energy given out at rupture is proportional to the cross section of the band and therefore to the surface formed.

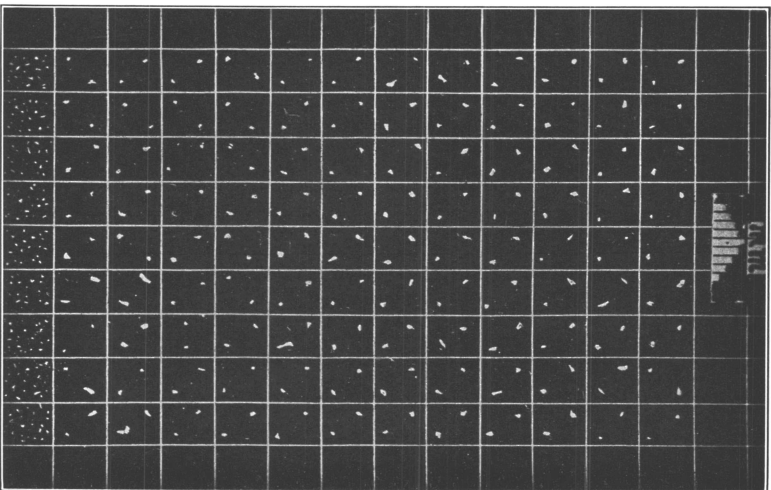
Disintegration by attrition or abrasion is subject to the same analysis. If friction exists the surfaces are under strain, and if new surface is not formed friction must cause the particles to move when the energy is released. For the energy to be released as heat new



A

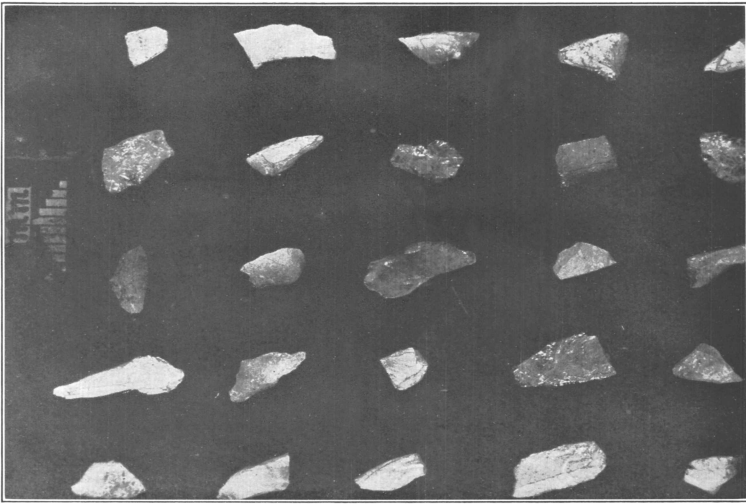


B

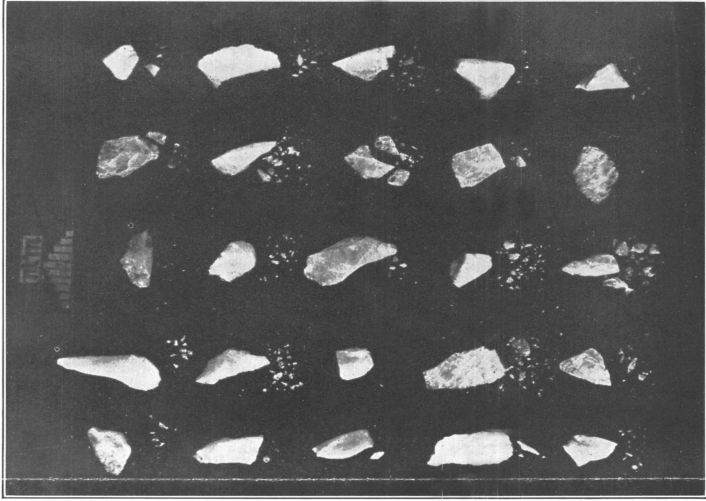


C

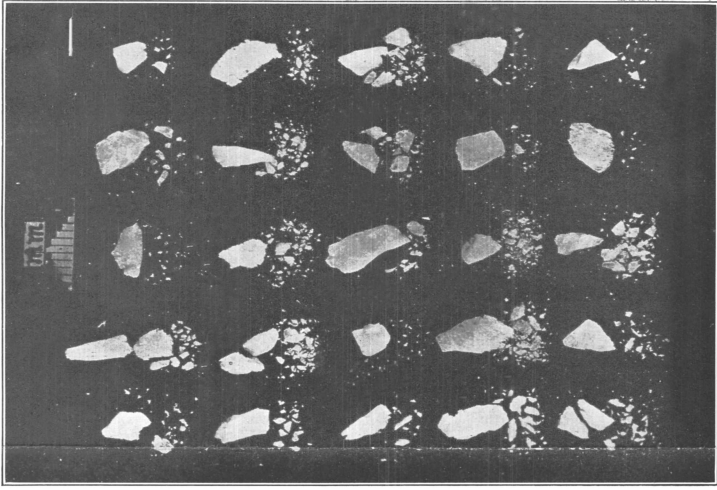
FIGURE 11.—3 μ to 10 μ Spanish quartz: A, Before hydrofluoric treatment; B, after 2 hours' hydrofluoric treatment; C, after 3 hours' hydrofluoric treatment.



A



B



C

FIGURE 10.—8- to 8-mesh quartz: A, Before hydrofluoric treatment; B, after 2 hours' hydrofluoric treatment; C, after 3 hours' hydrofluoric treatment.

surface must be formed. Even frictional heat is proportional to the roughening or deformation of the surface.

For brittle materials all the new surface formed is fracture surface, but for more malleable materials less of the surface formed is fracture surface and more is internal surface, such as glide planes or grain boundaries. Dean and Koster developed the concept of internal surface at length by a paper given before the Institute of Metals Division, American Institute of Mining and Metallurgical Engineers, February 1936.

No matter how the force is applied the resultant efficiency expressed in total surface formation probably remains the same, but the kind of crushing may vary greatly. A badly applied force may result in material that is too coarse or too fine, and the greatest useful efficiency results when a properly applied force is selected for the purpose required.

Repeated blows with a small hammer upon a large rock would certainly not be an efficient method of crushing, and it would not be considered good practice to use a pile driver on small particles and shatter them to dust, yet a measurement of the product—extremely fine dust in both cases—would probably disclose the Rittinger relationship between surface and work actually done in the crushing operation, although the useful efficiency would be extremely low.

Whether or not the crushing force is applied to good advantage depends on the nature of the appliance, and the question arises as to whether a relatively large force, acting slowly as in a jaw crusher, or a relatively small force, acting quickly as in the Hadsel mill, is the better. Experimentation on this subject would be profitable.

TIME-LOAD-DEFORMATION SURFACE

Virtually no data are available on the time-load-deformation relations of rocks and minerals. Extensive work has been done, however, on metals, and the recent work of Maier entitled "The Energy Relations in Crystal Aggregates", presented at the American Institute of Mining and Metallurgical Engineers in February 1936, indicates that, even when fracture does not occur, the Rittinger law applies over a considerable range of deformation. That is, the internal surface as measured by coercive force, which is discussed later, is proportional to the energy used in deforming the metal as indicated by its density.

Apparently, therefore, the energetics of metal deformation and mineral crushing are similar except that in metal deformation the surface formed is not a fracture surface but an internal surface (glide planes or grain boundaries).

The ability of minerals to deform like metals has also been shown repeatedly. Joffé (133a) discusses the mechanical behavior of crystals.

Koster (134) from a mathematical study of deformation concluded that the time-load-deformation relation of all solids could be represented as a surface and that the formation of a fracture surface or an internal surface depended on the rate of deformation. If the rate of deformation exceeded the rate of welding a fracture surface resulted.

For a given temperature every substance has a critical rate of deformation which produces fracture but is not a true breaking load.

Extension of the Rittinger law, which appears possible from Maier's work, indicates that the energy required to produce internal surface is at least roughly the same as that required to produce fracture surface.

Because in mineral crushing only a fracture surface is desired the most efficient conditions are those that give a maximum rate of deformation. As a small force acting for a long time tends to produce internal surface and a large force acting for a short time tends to produce fracture with a minimum of internal surface, the latter is more efficient for crushing.

The general form of the relationship of time, load, and deformation has not been determined. For lead alloys used in cable sheath, Dean and Ryjord (101) have shown that the load varies inversely as the logarithm of the time of rupture; that is, time is a negative exponential function of the force. The same relationship has been found for other metals and if extended to minerals would indicate that the efficiency of a crushing process increases exponentially with the crushing load up to the limit of effective application.

The confirmation of the Rittinger law for quartz and magnetite in the drop-weight machine, which is discussed later, indicates that for brittle materials a force suddenly applied results in a negligible production of internal surface. For less-brittle materials increased forces efficiently applied by special processes may greatly increase grinding efficiency.

IMPACT CRUSHING

Impact crushing may be considered to be the result of a force applied suddenly, which acts for a short period and, presumably, through a short distance. Impact crushing is exemplified in hammer mills, the Hadsel mill, stamps, and the fall of balls in a ball mill. This type of crushing is accompanied by a small amount of plastic deformation.

COMPRESSION CRUSHING

Compression crushing may be considered as the result of a force applied slowly which acts for a longer period and probably through a longer distance than in impact crushing. Compression crushing is exemplified in jaw, gyratory, and disk crushers, rolls, Chilean-mill-type grinders, tube mills, and the rolling action of balls in ball mills. This type of crushing is accompanied by a relatively large amount of deformation.

WAVE SHATTERING

The breaking down of the bonds that hold particles together by wave motion has possibilities that have not as yet been utilized (97). Rupture by this method can be looked upon as a heaving force acting within the particle.

There is some evidence that explosive shattering by the sudden evaporation of superheated water is due, partly at least, to wave motion.

The possibility of shattering with very short ultrasonic waves is a promising field for investigation.

ENERGY OF CRUSHING

CRUSHING LAWS

The two laws that have been identified with crushing may be defined as follows:

Rittinger law.—"The energy required for crushing is proportional to the new surface formed" (1) or "the energy required is proportional to the reduction in diameter" (57).

Kick law.—"The energy required for producing analogous changes of configuration of geometrically similar bodies of equal technological state varies as the volumes or weights of the bodies" (3, 18) or "for any unit weight of ore particles the energy required to produce any desired reduction in volume of all the particles in the mass is constant no matter what may be the original size of the particle" (57).

The energy required in successive reduction steps increases arithmetically with the Kick law and geometrically with the Rittinger law, from which it would seem to follow that the work represented in the finer sizes is lower according to the Kick law. That this is fallacious has been shown by efficiency calculations based on the Kick law, which show that reduction would be accomplished with less energy than that represented by the surface energy (98).

The Rittinger law has perhaps had more adherents than the Kick law. Many simple and obvious experiments may be cited to prove the applicability of the Rittinger law, and the earlier experimental evidence seemed to uphold it. Experimental proof, however, lacked finality owing to the difficulty or impossibility of correctly apportioning the material finer than available sieve sizes. The work done by the Bureau of Mines at Salt Lake City and by Martin (53, 58), in which surface determinations of the crushed materials confirmed the Rittinger law, would seem to be final.

THERMODYNAMICS OF CRUSHING

The work required to overcome the forces that hold a solid together and thus produce new surface is very small.

Attempts to measure the heat of solution of fine and coarse calcite have not shown measurable differences; the accuracy of the determinations was such, however, that it can be concluded that not over 10 percent of the energy input appears in the ground material.

Several investigators have determined the absolute efficiency of crushing on the assumption that the energy required in crushing is only that necessary to create the new surface energy. Such calculations are of academic interest only and have no practicable significance, as they ignore the distance through which the crushing force must act before new surface is produced.

Gaudin (67) found that 1.3 percent of the energy input in roll crushing was represented by new surface, while Martin (62) calculated the efficiency in ball milling as one-sixteenth of 1 percent. Gaudin used Edser's figure of 920 ergs per square centimeter as the surface energy of quartz, while Martin used approximately 310 ergs per square centimeter. In experiments with the drop-weight machine it was found that, based on Edser's value, an efficiency of 3 percent was obtained.

Data have been obtained by Lipsett, Johnson, and Maass on the heat of solution of different sizes of rock salt, from which the surface energy is calculated as 367 ergs per square centimeter (75). This value was used to determine efficiency by crushing 4- to 6-mesh salt in the drop-weight machine, and the surface in the crushed product was determined by comparison with the known surface of quartz particles of sieve and elutriation sizes. The total work expended was 175.5×10^7 ergs, and that represented by the new surface was 1.62×10^7 ergs, hence the increased surface energy was less than 1 percent of the energy input.

Calculations based on the surface energy of solids are of doubtful validity, but they do show that the energy necessary to create new surface is very small and that over 90 percent of the work expended in crushing does not appear as surface energy.

SIGNIFICANCE OF RITTINGER LAW

It has been shown (81, 138, 142) that the surface produced in crushing quartz and magnetite in a frictionless machine has a linear relationship to the work input, which confirms the Rittinger law. Rittinger, in stating the law, was evidently guided by the idea that all the energy of crushing went into new surface. The law is capable of another explanation, however, and it does not follow that all of the energy input is represented as new surface and exists as surface energy. The full significance of this fact has perhaps not been realized. The direct relationship that exists between work input and surface in a frictionless drop-weight machine does not mean that the energy is expended solely in producing surface and that the surface energy of the crushed product is equal to the energy input. It must be pointed out that the machine is frictionless only insofar as the mechanism is concerned and not insofar as the movement of ore particles over one another is concerned. Besides simple crushing there must have been considerable attrition due to particles rubbing together, yet the Rittinger law is accurate within 5 percent.

ENERGY MEASUREMENTS

As the actual energy used in producing new surface is so small that measurement is virtually impossible, it must be concluded that the problem of measuring the energy used in crushing and grinding is one of mechanics rather than of physical chemistry.

The energy used in breaking a substance either by compression or tension is the product of the breaking load and the distance through which it acts. The actual energy expended in producing new surface obviously is proportional to the new surface, and since the energy expended in crushing is also proportional to the new surface produced the breaking load is related to the area of new surface by a proportionality factor characteristic of the substance.

This proportionality factor of the Rittinger law is determined by the distance through which the force acts in breaking and the nature of the fracture surface. This is a constant for any given material and is designated as the Rittinger constant. It is an index of grindability and will be discussed later.

The measurement of the energy represented in crushing or grinding depends upon and is directly related to the amount of surface produced, therefore energy figures must be derived from surface determinations.

SURFACE MEASUREMENT

The determination of the surface of fine material is extremely important in many industries where the question of grinding efficiency may be of secondary importance. All products used for covering, such as paint and ground mica, depend on their inherent surface for covering power. Knowledge of surface measurement may therefore be valuable in fields other than crushing and grinding.

Methods that have been used to determine surface may be classed as direct and indirect.

Direct methods depend upon the direct measurement of some property of the surface.

Indirect methods depend on microscopic measurement, sieve sizing, and elutriation or subsidence, in which surface is determined indirectly from particle size.

DIRECT METHODS OF SURFACE MEASUREMENT

To determine surface satisfactorily the surface reaction that is to be utilized must be large enough to give measurable quantities. The properties of surfaces that have been used successfully so far are the rate of solubility (of quartz) and the coercive force (of magnetic materials). Other surface properties that might be utilized are adsorption (9, 48, 49, 97, 107), heat of wetting (103), and the formation of chemical or mechanical coatings, but methods based on these phenomena have not yet proved satisfactory.

DISSOLUTION METHOD OF SURFACE MEASUREMENT

This method is founded on Wenzel's law which states that the velocity of reactions between solids and liquids is proportional to the area of contact.

Several methods of determining surface, based on Wenzel's law, have been published. Wolff (36, 49) used a mixture of sodium carbonate and sodium hydroxide as the solvent for powdered glass, and Kreige (65) used a dissolution method to determine surfaces of limestone and calcite.

As a solvent probably does not exist for a heterogeneous mixture of particles in which the dissolution rate of all particles is equal, the method is applicable to a single mineral only.

So far the dissolution method has been developed for quartz only. This method will be discussed at some length. Two methods were developed simultaneously and independently for measuring the surface of ground quartz by dissolution. One was developed by Martin (58, 70) in England and one by the Bureau of Mines in the United States (53, 74).

Martin's method is based on the quantity of quartz dissolved in hydrofluoric acid in 1 hour compared to that dissolved in the same time from a cuboid quartz crystal of known surface. This method

ignores changes in the concentration of the solvent or changes in the surface during dissolution. The effect of these changes is small for coarse material but increases with finer particles.

The method developed by the Bureau of Mines is based on the initial rate of dissolution of quartz in hydrofluoric acid. According to Wenzel the reaction velocity, although proportional to the area of contact, is a constant only when conditions remain constant. Owing to a reduction in the concentration of the solvent, the accumulation of products of reaction, and a change in the amount of surface in contact with the solvent, the rate of dissolution decreases as dissolution proceeds. If, however, the dissolution rate is determined for zero time, this initial rate is that which proceeds before any change in the liquid or solid has taken place and may be considered as a true measure of surface. It is obtained by mathematical extrapolation of a percentage-rate curve based on data from several dissolution tests.

CONDITIONS REQUISITE TO DISSOLUTION METHOD

In general, any material treated with a suitable solvent will give a dissolution rate from which the initial rate may be obtained. A suitable solvent must have properties such that the rate curve is reasonably flat to assure greater accuracy in extrapolation of the curve. A suitable solvent should have the following properties:

1. The rate of diffusion should be greater than the rate of dissolution. If the rate of diffusion is slower than the rate of dissolution, the rate of diffusion is measured instead of the rate of dissolution. Although correction factors may be obtained experimentally on the diffusion rate with particles of different sizes, this procedure introduces possible errors.

2. The reaction should not result in a gaseous product. The production of a gas during dissolution leads to mechanical difficulties, and unless the gas pressure is controlled results are apt to be erratic.

3. The reaction should not be autocatalytic. An autocatalytic reaction would result in a rate curve that would probably be impracticable to extrapolate.

4. Equilibrium should not be reached within the time allotted to the dissolution tests. If the reaction comes to equilibrium or approaches equilibrium during dissolution, a rate curve results that has an excessive curvature during the earlier part of the dissolution period. Such a curve cannot be accurately extrapolated.

5. The products of the reaction should not mask the surface of the solid. If they do, a rate curve may result that is similar to that in which equilibrium is approached.

6. The solvent should have a high enough concentration that the change in dissolution rate does not diminish too rapidly. A solvent of low concentration, due to its rapid depletion, would give a rate curve with too great a curvature for satisfactory extrapolation.

7. The solvent should not dissolve too large or too small a quantity of the solid. If too large a quantity of the solid is dissolved, the rate curve becomes undesirably steep, with excessive curvature. The dissolution of large quantities of the solid results in complete solution of the finer particles, a condition that adds to the difficulty and accuracy of the extrapolation. If the solvent dissolves only a little of

the solid, the possibility of errors in mechanical manipulation becomes too great.

These requirements show that selection of a suitable solvent demands considerable care and patience. A suitable solvent for many minerals may not even exist. When, however, such a solvent is found, it remains only to determine the best concentration of solution, the temperature during dissolution, the ratio of solid to solvent, and the time necessary for dissolution.

The nearer the rate curve approaches a straight line the smaller is the danger of error in extrapolation. The ideal curve may be approached by the use of a lower concentration of solvent, lower temperature, and a lower ratio of solid to solvent. These conditions are adjusted so that the quantity dissolved is the smallest possible consistent with accuracy. The time periods for the rate figures are selected so that the first will be as near zero as accuracy will allow and that extrapolation will represent as short a part of the curve as possible.

DISSOLUTION TEST METHODS

Many methods for making dissolution tests suggest themselves, and any method that can be kept under control may be employed; control of temperature and the time period for the dissolution are the important factors. The method developed by the Bureau of Mines for the dissolution of quartz in hydrofluoric acid is given as an example.

A bakelite tube stoppered at both ends is used; this is convenient for loading, handling, and discharging. The tube and solvent are brought to the required temperature of 25° C., and the accurately weighed solid, at the same temperature, is introduced. The tube and contents are placed in a thermostatically controlled water bath, which is revolved to mix the contents. Just before the allotted time of dissolution is completed the tube is removed and the contents are rapidly filtered and washed. The time of dissolution is that from introduction of the solid to application of the wash water.

The standard hydrofluoric solution adopted and used in all tests was 3.66 normal, or 73.2 grams HF per liter.

Dissolutions are made for ½-, 1-, 1½-, and 2-hour periods, and the quantity dissolved is determined for each period. As there is no satisfactory method for determining silica in the presence of free hydrofluoric acid, the quantity of quartz dissolved is determined by the weight of the quartz after the filter paper is burned off. A small ignition correction, determined for each sample, is applied.

The rate figure is obtained from the formula:

$$\text{Percent per hour} = \frac{w \times 100}{W \times T},$$

in which w = weight dissolved, in grams;

W = weight of original sample, in grams; and

T = time, in hours.

The rate curve may be extrapolated graphically, but this becomes increasingly dangerous and more liable to error with increased slope and curvature.

The rate curves and the cumulative curves plotted in figure 1 are taken from the following experimental data on minus 400-mesh quartz:

Milligrams dissolved, hours:	1-gram charge	2-gram charge
½-----	130.6	241.2
1-----	210.5	369.9
1½-----	268.7	455.2
2-----	315.5	519.0
Percent rate, hours:		
½-----	26.12	24.12
1-----	21.05	18.495
1½-----	17.913	15.173
2-----	15.775	12.975

The rate and cumulative curves (which are constructed from the same dissolution data) have opposite curvatures; however, the cumulative curve requires no extrapolation to zero time. These two curves suggest the possibility of combination into a straight line.

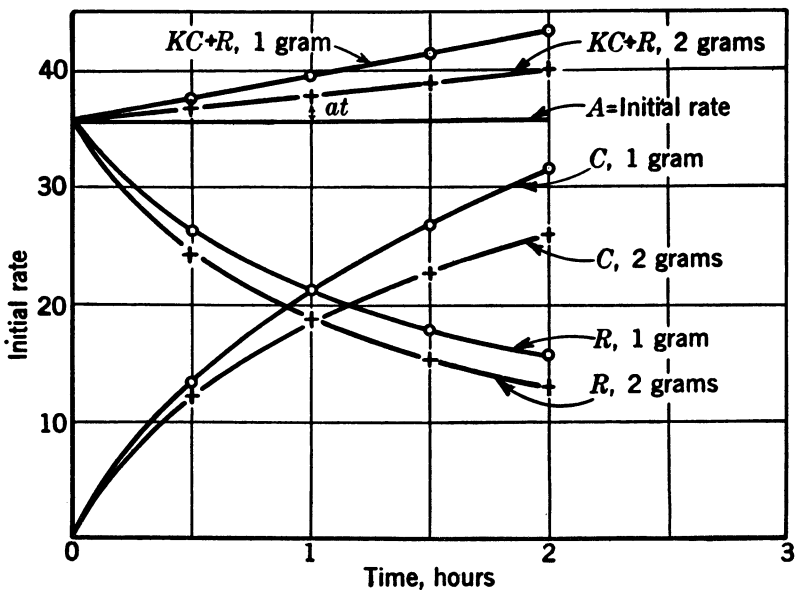


FIGURE 1.—Curves for dissolution test showing determination of initial rate.

From the data obtained on quartz, a mathematical expression for the rate curve has been derived from the two curves. This expression has been found to fit all of the experimental data up to a dissolution period of 9 hours and hundreds of surface determinations on quartz from 3-mesh to the finest elutriation product averaging 1 micron. The derivation of the expression is given below, and it is represented graphically in figure 1.

- Let A = initial rate;
 t = time, in hours;
 C = values for cumulative curve (percent dissolved);
 R = values for rate curve (percent dissolved per hour); and
 K and a are constants.

If cumulative-curve values C multiplied by a constant, K , are combined with rate-curve values R , these curves may be expressed as a straight line, $KC+R$. To satisfy the straight-line condition,

$$KC+R=A+at, \quad (1)$$

$$\frac{c}{t}=R. \quad (2)$$

From 1 and 2, the values for R and C become:

$$R=\frac{A+at}{1+Kt}, \quad (3)$$

$$C=\frac{At+at^2}{1+Kt}. \quad (4)$$

In equation 1 the three unknowns (K , A , and a) may be determined from three sets of values for C and R , and the value of A , the initial rate, becomes known. As only three dissolution values are needed to obtain A , the fourth is used in equation 3 or 4 as a check on the data.

DIFFERENT RATIOS OF SOLID TO SOLVENT AS A CHECK

The initial rate depends upon the amount of surface per unit weight of the solid, irrespective of the ratio of solid to solvent. With different ratios different curves are obtained which, however, must extrapolate to the same initial rate. Two sets of data from two different ratios of solid to solvent may therefore be used as a further check on the accuracy of the data. Figure 1 shows the results of such a check. The accuracy of the data used in figure 1, which were taken from one of many surface determinations, may be judged from the following figures:

	<i>Solid, 1 gram; solvent, 25 grams</i>	<i>Solid, 2 grams; solvent, 25 grams</i>
Initial rate.....	35. 5700	35. 5794
Difference for initial rate..... percent.....	-----	. 03
½-hour point from data..... percent per hour.....	26. 12	24. 12
½-hour point from data..... by calculation.....	26. 10	24. 10
Difference for ½-hour point..... percent.....	. 07	. 08

CALIBRATION OF INITIAL RATE

The initial-rate figure, while a true figure of surface, is not in ordinary units. Although this figure is entirely adequate for comparing data on the same material, it is desirable that the initial rate be translated into specific surface units.

Naturally the simplest method of calibration would be to determine the initial rate on a piece of similar material of known surface. Unfortunately this method cannot be used with quartz, as quartz does not dissolve equally in all directions. The initial rate of dissolution for a quartz crystal is zero, as dissolution starts along certain definite lines. The initial rate value of quartz was calibrated indirectly by comparing silver coatings on crushed quartz of definite size with those on quartz crystals of known area. A high degree of accuracy is not claimed for this method. The initial rates, however, are considered to be accurate, and any calibration applied to the initial rate does not affect the true

comparison of surface. It was found that the initial rate multiplied by 170 gave the surface in square centimeters per gram.

ACCURACY OF DISSOLUTION METHOD

It is impossible at present to give rigid proof of the accuracy of the dissolution method. The possibilities of error in manipulation are discussed in the following paragraphs.

Sample.—An accurate sample is obviously a prime requisite. With sized material it is easy to obtain an accurate sample, but with unsized material a sample probably will not be representative. Error may be avoided by sizing the entire lot and weighing proportionate amounts of each size. As the quantity of silica dissolved is determined by difference, the sample must be accurately weighed.

Handling sample.—As any loss in handling the sample will be counted as silica dissolved, such losses must be minimized by using the greatest care during the entire operation. Experiments indicate that with reasonable care the loss from this source represents a maximum of 2.0 percent.

Timing dissolution.—As the time can be controlled rather accurately, no great source of error is likely. A difference of 10 seconds, which can easily be controlled, results in a maximum error of 0.6 percent.

Agitation.—If the rate of diffusion is slower than the rate of dissolution, agitation has a decided effect on the amount of material dissolved. Although diffusion is slower than dissolution in most materials, in some the reverse is true (63, 68). As the diffusion rate of quartz in hydrofluoric acid is faster than the dissolution rate, increased agitation or movement of the particles in the solvent has no measurable effect. Agitation, however, prevents packing of the quartz and possibly masking of some surface.

Abrasion.—Abrasion may result in more material being dissolved, but experiments showed that the quantity so dissolved was negligible with quartz.

Temperature.—Experiments at various temperatures on dissolution of quartz showed that a variation in temperature of 0.1° C. results in an error of 0.5 percent.

Concentration of solvent.—Experiments with hydrofluoric acid of various strengths showed that a variation of 0.25 percent (limit of accuracy in standardization) resulted in an error of less than 0.4 percent.

Ignition correction.—The correction for ignition losses is small, and when the amount of quartz dissolved is not unduly low any error from this source is negligible.

In summing up these possible sources of error it is seen that they are largely compensating. While losses due to handling tend to give a higher initial rate, losses due to ignition act in the opposite direction. The general compensatory nature of these possible errors is borne out by the reproducibility of results. Hundreds of duplicate dissolutions show a probable variation of less than 1 percent from the average. An appreciable error in the data can be detected readily upon construction of the curves.

Two possibilities for error inherent in the solubility reaction itself may exist.

1. It is known that silica dissolves only in the direction of its vertical axis. What effect this may have on the dissolution rate of crushed quartz is not known, but from the law of averages it may be assumed that in an ordinary sample consisting of thousands of particles the relation between dissolving surface and total surface is a constant. As the initial rate was the same on similar crushed and sized quartz from a solid crystal, from massive quartz, and from Ottawa sand it may be concluded that the rate of dissolution is directly proportional to the surface.

2. The solubility of extremely fine particles may affect the results. In the dissolution of quartz under standard conditions, a particle 0.5 micron in diameter is entirely dissolved in 1 hour. What effect this may have on the final results is not known. Probably, however, there is a regular gradation of sizes in a crushed or ground product, and complete dissolution of particles is a gradual process which is taken into account by the rate curve. In extreme cases where all or most of the sample comprises particles less than 0.5 micron in diameter, special manipulation and the use of a solvent of lower concentration would be necessary.

Opinions differ as to whether these fine particles tend to give a high or a low value to the initial rate. Taggart (74) thinks that particles below the limit of microscopic visibility dissolve faster and give an initial rate that is too high. Tartaron (81) also believes that the greater activity of the finer particles would give a high initial rate. On the other hand, Gaudin (81) believes from the results of his experimental work that complete dissolution of the finest particles would give a low initial rate. Hancock (119) thinks the Wenzel law does not apply to very fine particles, as these would be more active. He concludes, from a mathematical calculation based on an average size of 37 microns for minus 200-mesh material, that the initial rate is abnormally high. This assumption as to the average size of minus 200-mesh material contradicts experimental evidence and may be seriously questioned.

Possibly the dissolution method may be inaccurate for extremely fine particles, but there is no direct evidence that it is. Theoretical deductions as to its accuracy are not conclusive; the method must stand or fall on the strength of experimental work. Meanwhile it gives a more reliable surface figure than one based on assumptions and not upon experiment.

When absolute proof is lacking reasonable evidence may support the contention that a method is accurate. Such evidence is presented herewith for the dissolution method.

1. The reproducibility of the determinations on different samples of quartz of the same size is evidence of an equal rate of dissolution, although this proves nothing regarding the action of fine particles.

2. When the ratios of solvent to quartz are different, the two curves extrapolate to the same initial rate. Although this may not be admitted as proof it encourages belief in the accuracy of the method.

3. Perhaps the most reasonable evidence that the method is not seriously inaccurate is the relationship of particle size to surface. The curves for the logarithmic relation between surface and size,

given later, may be extended to the unit crystal continuously and regularly. The trend of these curves encourages the belief that the dissolution method has considerable claim to accuracy, but here again it may be contended that this furnishes no evidence as to its accuracy for very fine material. Within the limits of sizes under consideration (6,680 to 37 microns), however, there is no indication of increased activity in the finer sizes; in fact, the reverse is true.

4. As final evidence of the belief that the dissolution method gives surface figures with a fair degree of accuracy, the results that corroborate the Rittinger law may be presented. As these results, which depend on the surface figures of crushed products that contain the finest material, give such a uniform relation to energy input, they strengthen the claim for accuracy and may be looked upon as confirming evidence.

The dissolution method may be simplified somewhat by determining the amount dissolved in 1 hour enough times to establish a curve.

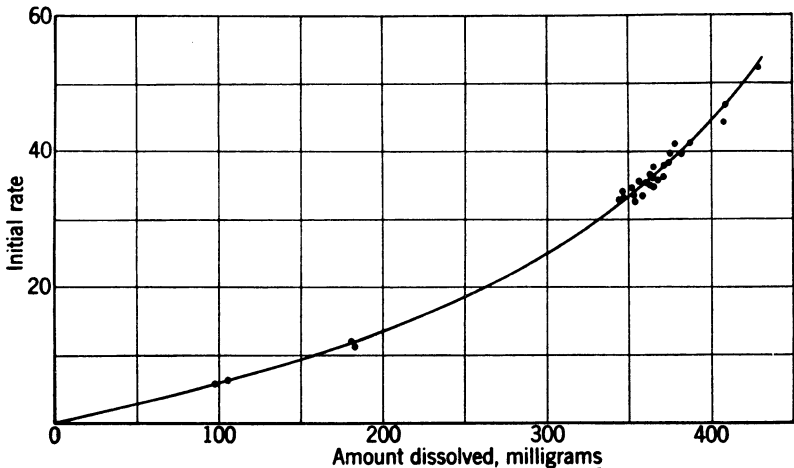


FIGURE 2.—Relation of initial-rate values to amount of silica dissolved from 2-gram charges in 1 hour.

The most consistent results were obtained with a 2-gram charge, and the relation of the initial rate to the amount dissolved in 1 hour from a 2-gram charge is plotted in figure 2 for 30 determinations. The initial rate for various amounts dissolved was calculated from smoothed curves to obtain the values of K and a in the equation,

$$R = \frac{A + at}{1 + Kt}$$

and these values gave the curve of figure 2.

Table 1 gives the variations between the determined initial-rate values and those taken from the curve. To establish such a curve as that in figure 2 a greater variation of determinations is desirable, but the work was not done for this purpose, and the curve has been constructed from all available data. It has been used to simplify the work and under the conditions of the tests may be considered to give fairly reliable results.

TABLE 1.—Initial rates as determined and as taken from figure 2; 2-gram charge, 1-hour dissolution

Milligrams dissolved	Initial rates		Difference	Difference (percent)	Milligrams dissolved	Initial rates		Difference	Difference (percent)
	As determined	By curve				As determined	By curve		
98.....	5.89	5.60	-0.29	4.9	363.....	37.48	35.70	-1.78	4.9
106.....	6.42	6.20	- .22	3.4	364.....	36.35	35.90	- .45	1.2
181.....	12.06	11.60	- .46	3.7	365.....	34.71	36.10	+1.39	4.0
183.....	11.13	11.80	+ .67	6.0	365.....	37.76	36.10	-1.66	4.4
344.....	33.02	32.08	- .94	2.8	368.....	35.84	36.80	+ .96	2.7
346.....	33.12	32.50	- .62	1.9	371.....	36.25	37.40	+1.15	3.2
346.....	34.17	32.50	-1.67	4.9	372.....	37.99	37.60	- .39	1.0
347.....	33.30	32.72	- .58	1.7	374.....	38.47	38.00	- .47	1.2
352.....	34.64	33.52	-1.12	3.2	375.....	39.74	38.20	-1.54	3.9
353.....	33.40	33.60	+ .20	.6	378.....	41.13	39.12	-2.01	4.9
354.....	32.50	33.70	+1.20	3.7	382.....	39.66	39.80	+ .14	.4
356.....	35.66	34.40	-1.26	3.5	387.....	41.29	41.00	- .29	.7
358.....	33.48	34.64	+1.16	3.5	408.....	44.16	46.40	+2.24	5.1
361.....	35.36	35.30	- .06	.2	409.....	46.00	46.60	+ .60	1.3
363.....	35.00	35.70	+ .70	2.0	429.....	52.44	52.40	- .04	.1

MEASUREMENT OF SURFACE BY COERCIVE FORCE

In a study of the magnetic properties of magnetite Gottschalk (137, 138) found that the coercive force of magnetite was directly proportional to the area of the surface regardless of the distribution of sizes.

The method of determining coercive force is very simple, and a determination may be made in a few minutes. The apparatus and method of operation have been described by Davis and Hartenheim (140). A brief description follows.

The apparatus comprises a long, primary solenoid connected to a source of current. The central part of the primary solenoid is surrounded by a secondary solenoid connected to a ballistic galvanometer. Means are provided for moving a sample along the axis of the primary solenoid.

The current for the primary is obtained from storage batteries with suitable variable resistances to supply a steady current.

The secondary comprises a Ford spark-plug coil connected to a Leeds & Northrup, type HS, high-sensibility ballistic galvanometer.

A diagram of the apparatus is shown in figure 3. *A* is a tube of bakelite, glass, or other material on which the primary solenoid *B* is wound. *C* is the secondary solenoid at the center of *B*. *D* is a holder for *A*, *E*₁ and *E*₂ are the terminals for the primary, and *F*₁ and *F*₂ are the terminals for the secondary. The apparatus is mounted on a board *G*. The lever *L*, pivoted at *M*, operates the rod or piston *K*.

The sample *S* is placed in a tube closed at both ends with a rubber stopper. The stoppers hold the material firmly in place and act as cushions to prevent breakage through piston impact. The sample is magnetized and placed inside the primary solenoid so as to be symmetrical with respect to the secondary solenoid. A sharp push on the lever sends the sample out of the coil and results in a deflection of the galvanometer connected to *F*₁ and *F*₂, unless a balancing field is present in the primary solenoid.

The point of balance is determined by regulating the primary current to give one or two galvanometer deflections in each direction. These deflections, if not too great, plotted against the demagnetizing current, give a straight line, of which the intercept on the horizontal axis multiplied by a constant, characteristic of the primary, gives the magnetization-coercive force of the sample in oersteds.

The relation between work input and coercive force was obtained on a sample of Ural Mountain magnetite. This mineral, composed

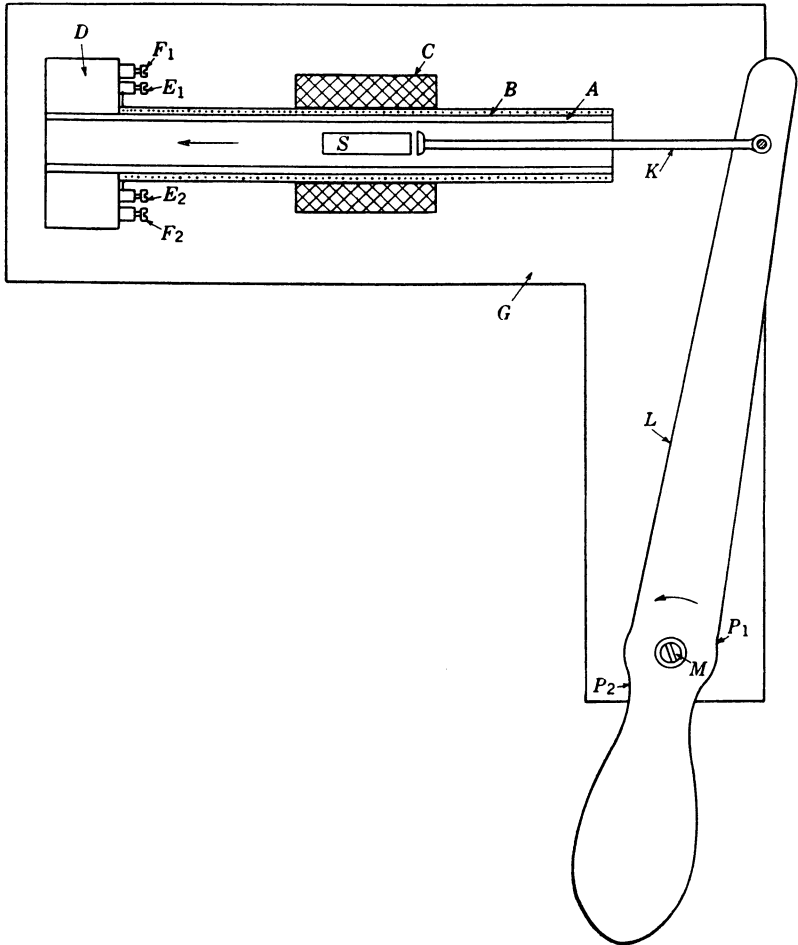


FIGURE 3.—Apparatus for determining coercive force.

of definite octahedra, was crushed through 10-mesh (the minus 400-mesh being discarded) and divided into 16 samples. Of these samples, 15 were crushed in the drop-weight machine described later, the work input being increased. The 16 samples were measured for coercive force. The results are given in table 2 and plotted in figure 4. The linear relationship shown not only confirms the Rittinger law but demonstrates clearly that coercive-force measurements may be utilized to measure surface.

TABLE 2.—Relation of coercive force to work input for Ural Mountain magnetite

Sample	Coercive force, oersteds	Work input per gram, kg-cm	Sample	Coercive force, oersteds	Work input per gram, kg-cm
1.....	5.12	0.0	9.....	13.34	85.2
2.....	6.10	11.4	10.....	13.98	95.4
3.....	7.06	25.9	11.....	15.89	108.6
4.....	8.22	37.6	12.....	16.02	115.2
5.....	9.74	48.3	13.....	16.30	123.9
6.....	10.94	58.5	14.....	17.21	134.8
7.....	11.77	68.5	15.....	18.38	144.4
8.....	12.18	75.7	16.....	18.94	150.3

As there is some coercive force in the original sample before drop-weight crushing, the net coercive force due to work input is the final coercive force diminished by the coercive force of the original material.

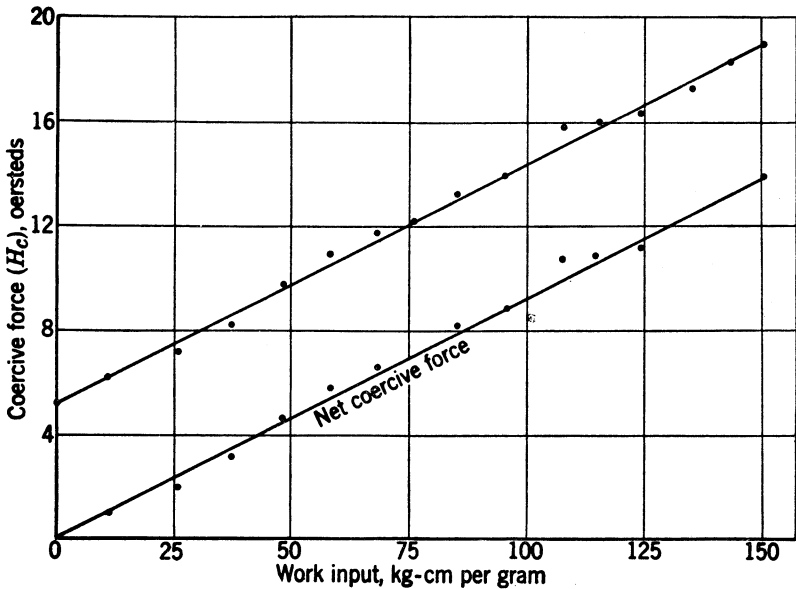


FIGURE 4.—Relation of coercive force to work input for crushing Ural magnetite.

This net coercive force is shown in figure 4 and forms, of course, a straight line that parallels the total coercive-force figures.

As magnetite may be obtained in quantity, it may be utilized to determine crushing and grinding efficiencies.

The graph for net coercive force in figure 4 may be used for determining the work expended in a crushing or grinding operation upon determining the coercive force of the magnetite before and after crushing.

While the data given are all that are needed to determine comparative crushing efficiencies, they do not give actual surface figures in terms of ordinary surface units. Approximate figures may be obtained by making certain assumptions. To determine the surface of the magnetite before drop-weight crushing the method described under "Surface determinations on a heterogeneous material" is utilized.

The theoretical surface of 1 gram of magnetite, in either spheres or cubes, with a density of 5.1, is $\frac{11,765}{D}$, where D is the average diameter in microns. The actual surface is this figure multiplied by a factor that depends on the relation between actual and theoretical surface. This factor is obtained by deliberately assuming a curve in figure 8 on the basis that crushed magnetite is slightly more irregular than pyrite. From the ratios obtained from the assumed curve the figures for surface given in table 3 are calculated.

To determine the surface produced by the drop-weight machine recourse is had to the crushing resistance of magnetite compared to that of quartz. Under "Relative grindability of rocks" experiments and data are given for relative grindability of several minerals, but as no figure is available for magnetite it is assumed that its grindability about equals that of pyrite, or that the surface produced per unit of work is 1.3 times that of quartz. As the quartz surface produced per kilogram-centimeter in the drop-weight machine is 17.56 cm², the magnetite surface produced per kilogram-centimeter would be 22.8 cm². The surface figures in table 4 are based on this assumption

TABLE 3.—Surface calculations for magnetite before drop-weight crushing

Size, mesh	Theoretical surface per gram, cm ²	Ratio of calculated to theoretical surface	Calculated actual surface per gram, cm ²	Weight of each size, grams	Calculated total surface, cm ²
10-14	8.34	3.46	28.9	15.5	448
14-20	11.75	3.19	37.5	39.4	1,478
20-28	16.55	2.90	48.0	17.4	835
28-35	23.39	2.69	62.9	11.7	736
35-48	33.05	2.50	82.6	5.1	421
48-65	46.69	2.34	109.3	4.7	514
65-100	66.10	2.21	146.1	3.9	570
100-150	93.37	2.10	196.1	1.95	382
150-200	132.2	2.01	265.7	1.15	306
200-270	186.7	1.94	362.2	.95	344
270-400	261.4	1.88	491.4	.45	221
			61.2	102.20	6,255

TABLE 4.—Surface figures calculated for drop-weight-crushed magnetite

Sample	Work input per gram, kilogram-centimeter	Surface produced, based on 22.8 cm ² per kilogram-centimeter	Total surface based on 22.8 cm ² per kilogram-centimeter	Sample	Work input per gram, kilogram-centimeter	Surface produced, based on 22.8 cm ² per kilogram-centimeter	Total surface based on 22.8 cm ² per kilogram-centimeter
1	0.0	0	61	10	95.4	2,175	2,236
2	11.4	260	321	11	108.6	2,476	2,537
3	25.9	591	652	12	115.2	2,627	2,688
4	37.6	857	918	13	123.9	2,825	2,886
5	48.3	1,101	1,162	14	134.8	3,073	3,134
6	58.5	1,334	1,395	15	144.4	3,292	3,353
7	68.5	1,562	1,623	16	150.3	3,427	3,488
8	75.7	1,726	1,787				
9	85.2	1,941	2,002				

In figure 5 the surface produced is plotted against the work input. This results in a straight line. Although the assumption on which this graph is constructed may be open to criticism, it is correct for

comparison and has the advantage of yielding surface figures in terms of known units.

INDIRECT METHODS OF SURFACE MEASUREMENT

Indirect methods of measuring surface may be classed as those that depend on determination of particle size. It may be said that all methods of determining surface, except those just discussed, are based on particle size. Such methods are liable to error owing to the difficulty of determining the size of the finer particles and to the use of an incorrect ratio of surface to size.

The main difficulty in determining surface from size is measurement of size. The determination of particle size increases in difficulty as the size decreases, and measurement is impossible at about 1 micron. As the smaller particles usually account for the larger proportion of

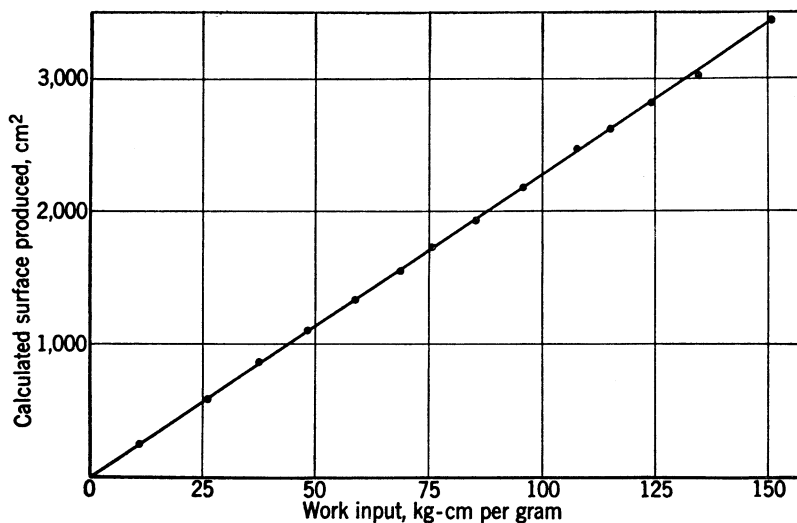


FIGURE 5.—Relation of calculated surface produced on magnetite to work input.

the surface, the liability to error becomes greater as the material becomes finer.

An incorrect factor for the ratio of surface to size may not lead to serious error, but such a ratio is at best only an approximation.

The methods that have been utilized or proposed for surface determinations, based on particle size, are: (1) Microscopic measurement; (2) sieve sizing; (3) elutriation or subsidence; (4) the sedimentation balance; (5) light transmission and other methods; and (6) distribution of particle sizes.

MICROSCOPIC MEASUREMENT

Microscopic measurement has long been utilized to determine particle size, and numerous methods have been devised to simplify technique and increase accuracy (28, 31, 38, 41, 51, 55, 77, 106, 114). Although these methods have their place, certain inherent factors in microscopic measurement eliminate it as an accurate method for determination of surface.

It is difficult to obtain a representative sample of the material for microscopic work, although this objection to the method is of minor importance.

Incomplete dispersion of the sample may result in measurement of aggregates instead of individual particles.

Difficulties in measurement increase as the limit of visibility is approached, and between this limit and the unit crystal is a range entirely outside of microscopic possibilities.

Particles spread upon a slide are oriented with the shortest dimension in the line of vision. A measurement of the visible area therefore gives a value for diameter that is too high.

The difficulties encountered near the limit of visibility and the absence of measurements on particles of submicroscopic size lead to errors that have an especially important bearing on the data when they are used for extrapolation to calculate the invisible portion of the sample. Although the quantity of fine material may be small, its large specific surface makes it important.

The determination of surface, therefore, by microscopic measurement of subsieve sizes cannot be considered accurate. Aside from the inherent difficulties in determining particle size, the estimation of the invisible portion must depend on extrapolation.

SIEVE SIZING

Because of the convenience and availability of standard sieves they have been used universally in analyzing crushed products. However, because of the limited range of sieve sizes this method fails entirely in separating material finer than 37 microns, but sieve sizing is important in a study of the coarser material. In any attempt to determine surface by the use of sieves only a value must be assumed for subsieve sizes, and the results so obtained are likely to be very inaccurate.

As crushed mineral particles are irregular in shape and as larger particles have more cracks than smaller ones, a variable factor is necessary to convert size to surface; this may not be a serious objection in an ordinary grinding operation, but any attempt to ignore the subsieve product or to assign an arbitrary value thereto cannot be considered by any means as giving surface figures.

ELUTRIATION OR SUBSIDENCE

Sizing by elutriation or subsidence may be considered an extension of sieve sizing and therefore only an indirect attempt to measure surface. It is not as accurate as sieves for sizing, but it is used in the study of grinding, especially in the study of size distribution and the indirect method of determining the surface of a heterogeneous material, which is discussed later.

Many methods have been used that depend upon the settling rate of particles in air or liquids (33, 61, 69, 90, 102, 104, 115, 123).

The determination of particle size usually is based on Stokes' law, although other laws of settling have been proposed. All of the equations for these laws, given herewith, have been converted to the common basis of velocity in millimeters per second and particle size in

millimeters as applied to quartz, with a density of 2.65, settling in water.

Rittinger's formula (57), also known as Newton's (91), does not take into account the temperature or viscosity of the liquid. The formulas for different-shaped grains are:

$$V = 111\sqrt{D} \text{ for roundish grains.}$$

$$V = 99\sqrt{D} \text{ for average grains.}$$

$$V = 78\sqrt{D} \text{ for flattish grains.}$$

Wagoner's formula is based on the settling rates of quartz as determined by Richards (57); it also fails to take into account the temperature or the viscosity. The formula is:

$$V = K \frac{D^{3/2}}{\sqrt{0.903D^2 + 0.5195}}$$

Where K , for slowest settling grains = 100, and K , for fastest settling grains = 400.

As the formula apparently gives settling rates that are too high when the factor for the fastest grains is used and too low when the factor for the slowest grains is used, an average factor, $K=250$, would more nearly represent actual settling conditions. The term $0.903D^2$ has no significance in the smaller sizes, as the settling rate for minus 200- plus 270-mesh material varies less than 0.3 percent whether or not this term is used; therefore it may be eliminated for all subsieve sizes. The Wagoner formulas then become:

$$V = 139D^{3/2} \text{ for slowest grains.}$$

$$V = 347D^{3/2} \text{ for average grains.}$$

$$V = 555D^{3/2} \text{ for fastest grains.}$$

Allen's formula for spheres as given by Lunnun (91) is:

$$V = 173D \text{ for } 25^\circ \text{ C. (coefficient of viscosity} = 0.00893).$$

Stokes' formula is:

$$V = 1010D^2 \text{ for } 25^\circ \text{ C.}$$

Table 5 gives the settling rates for quartz according to these formulas. The rates are continuations of the sieve series and are designated by ordinal numbers according to Weinig (130). No settling rates are figured for sizes finer than ordinal 28, as colloidal material begins to appear at this size.

These settling rates and the finer sizes of Richards' settling rates (57) have been plotted in figure 6. Unfortunately, Richards' settling rates do not go into subsieve sizes, but as Wagoner's formulas are based on Richards' figures Richards' curves may be united with those of Wagoner.

The curves show great variation, and an attempt was made to compare the results of each formula with sedimentation analyses of the minus 400-mesh product from two of the ball-mill experiments discussed later, on which surface determinations had been made. As the results showed several unexplained discrepancies they will not be discussed at this time, but it may be concluded that the Rittinger, Allen, and Wagoner fastest-grain formulas do not represent true settling conditions, as their settling rates are too high. It may also be concluded that the Wagoner slowest-grain formula gives a settling rate that is too low.

TABLE 5.—Settling rates for quartz according to various formulas

Ordinal no.	Size (millimeter)	Settling rate (millimeters per second) according to—					
		Wagoner slowest grains	Wagoner fastest grains	Wagoner average grains	Rittinger roundish grains	Allen spheres (25° C.)	Stokes spheres (25° C.)
28	{ 0.00081	0.0032	0.0128	0.0080	3.164	0.140	0.00066
	{ .00114	.0054	.0214	.0134	3.752	.197	.00131
29	{ .00162	.0091	.0362	.0226	4.468	.280	.00265
30	{ .00229	.0152	.0608	.0380	5.311	.396	.00530
31	{ .00324	.0256	.1024	.0640	6.316	.561	.0106
32	{ .00458	.0431	.1721	.1076	7.515	.792	.0212
33	{ .00647	.0723	.2889	.1806	8.924	1.119	.0423
34	{ .00915	.121	.486	.304	10.622	1.583	.0846
35	{ .0130	.206	.823	.514	12.654	2.249	.1707
36	{ .0183	.344	1.374	.859	15.018	3.166	.3382
37	{ .0259	.580	2.314	1.447	17.860	4.481	.6775
38	{ .0366	.973	3.885	2.429	21.301	6.332	1.353
39	{ .0518	1.639	6.543	4.091	25.264	8.961	2.710
40	{ .074	2.797	11.167	6.982	30.192	12.802	5.531

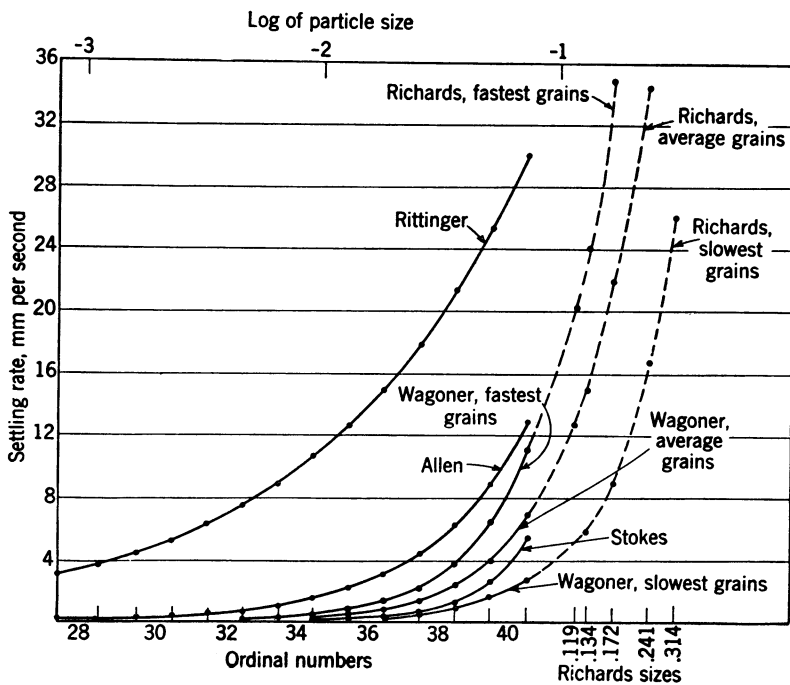


FIGURE 6.—Settling rates of quartz according to different formulas.

Stokes' formula gives settling rates that are too low, and a factor must be applied to give true settling conditions. This factor is as yet unknown. Weing (130) has given a correction curve for Stokes' formula based on the observed settling rates of ordinal groups of three ores with a density of 2.7. This curve implies that the correction factor varies within the range from sieve size to colloidal, with the highest factor at ordinal 31 and a small correction at either end.

The settling rate obtained from Weing's curve gives figures that are remarkably near those for Wagoner's formula for average-size grains.

In an investigation now under way in the Bureau of Mines the correction factor for Stokes' formula has been determined tentatively as 1.7 at the coarser end of the subsieve sizes, but the work has not progressed far enough to accept this figure. It is not known whether or not a straight factor is applicable, but it is believed that the factor should be somewhat larger for the finer than for the coarser sizes.

Of the formulas advanced for determination of settling rates only the Wagoner formula for average grains and the Stokes' formula, with a correction factor, seem to hold promise of giving true settling figures. Before either of these can be adopted they must be subjected to rigorous investigation, and Wagoner's formula must take temperature into account, as a change of 1° C. with water causes a change of about 2 percent in the results.

Although a reliable settling law is lacking, sedimentation or elutriation is the only method available for sizing large quantities of material finer than sieve size. Elutriation is preferable to sedimentation, as it is more rapid and accurate, and Stokes' law must be used as a basis until definite data are at hand to establish modification of the law if required.

It is necessary, however, in elutriation to eliminate as much as possible sources of error due to temperature, flocculation, and mechanical entanglement. Temperature can be controlled readily; flocculation can generally be prevented by the use of deflocculating agents; and mechanical entanglement can be prevented by proper construction of the apparatus.

Description of elutriator.—Many ingenious devices for size separation by sedimentation or elutriation have been developed and described (29, 33, 42, 72, 104, 105, 112, 115, 122, 128). The elutriator developed by the Bureau of Mines at Salt Lake City (94) and used to determine surface on a heterogeneous material is shown in figure 7. A description of the apparatus and method of operation follows. The best proportion of diameter and height was not investigated, but the machine as constructed gave satisfactory results. The glass tube, 2 inches in diameter and 2 feet in height, has a conical bottom and an annular overflow at the top. The rotor is a three-eighths-inch glass tube with a rubber impeller at the lower end; the upper end is fastened into a pipe which has a bearing and a drive sheave. The rotor is revolved at about 80 r. p. m. and is kept in line by the upper bearing and by the impeller in the conical bottom.

The feed water enters the top of the rotor from a constant-level tank controlled by a circular opening valve, discharges at the lowest point of the cone, and passes upward through the charge, which is kept in motion by the revolution of the impeller. The upward current rises evenly and without eddies.

Operation of elutriator.—The sample, which usually weighs 25 grams, is mixed with 200 cm³ of 0.1-percent gum-arabic solution, which was found to be a good disperser, and allowed to stand overnight for

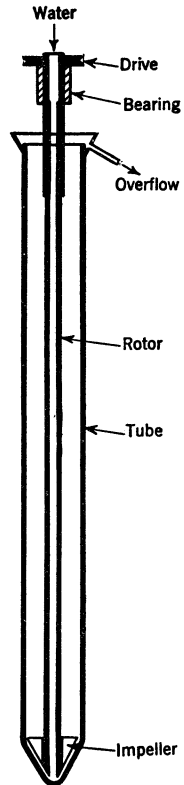


FIGURE 7.—Cross section of elutriator.

dispersion. After good dispersion is attained there is little danger of flocculation. At the start the constant-level tank is filled with 0.05-percent gum-arabic solution. As elutriation proceeds the constant-level tank is replenished with water, and the gum arabic is gradually replaced, as its presence is no longer necessary.

To operate the elutriator the upward flow is started for the lowest speed, and the dispersed sample is transferred to the tube with just enough 0.1-percent gum arabic at 21° C. to bring the level in the tube within about 6 inches of the overflow. The rotor is then started. If the upward stream is started before the charge is added the rotor tube is prevented from choking. The temperature of 21° at the start compensates for the slightly greater viscosity of the gum arabic. The temperature is lowered gradually as the gum arabic is displaced by water, after which it is maintained as near 20° C. as possible. While the tube is being filled, the upward current is regulated approximately. When the overflow begins it is measured and carefully regulated. Regulation of the overflow is checked at intervals during elutriation.

When the overflow for the lowest current speed is completed, that for the next speed is started. This is measured and regulated as before. Naturally, as many products can be made as desired; each product represents a particular size, which depends on the rate of overflow. In the work by the Bureau of Mines only four products (three overflows and residue) were made.

Quartz elutriation results.—The settling rate of quartz in water at 20° C., according to Stokes' law, is:

$$V = 900 D^2,$$

in which V = settling rate, in millimeters per second, and D = diameter of particle in millimeters.

If V represents the velocity of the rising current, then D is the diameter of a quartz particle that will, theoretically, remain suspended. To cause this particle to overflow, a somewhat stronger upward current will be required. The velocities of the upward current necessary barely to overflow the quartz have been calculated in table 6 for a continuation of the $\sqrt{2}$ sieve scale.

Actual results of quartz elutriation were made only on the four products in braces in table 6. Surface determinations were made on the resultant products, from which the average size was taken from the curve in figure 12.

TABLE 6.—*Rising current velocities for quartz*

Size desired (mm)			Upward current velocity (mm per sec. for largest size)	
From—	To—	Average		
0.104	0.074	0.089	Last sieve size	
.074	.052	.063		
.052	.037	.0445		
.037	.026	.0315		
.026	.0185	.0223		
.0185	.013	.0158		
.013	.0092	.0111		
.0092	.0065	.0079		
.0065	.0046	.0056		
.0046	.0000	.0023		
				4.93}
				2.43}
				1.23}
				.61}
			.31}	
			.15}	
			.08}	
			.04}	
			.02}	

The results of elutriation determinations on quartz given in table 7 indicate that the upward current was not strong enough to overflow the desired size and that the settling rate is faster than that derived from Stokes' formula. Microscopic measurements of the elutriated products gave a higher "average size" than that obtained from surface measurements. Doubtless this was due to the measurement of thin, flat particles on their longest diameter.

In elutriation, particles continue to overflow for an indefinite period, and as in sieving there is no point at which the production of undersize actually ceases. It becomes necessary, therefore, to establish a time period for each overflow. Experiments in which the amount of overflow was determined for each displacement (a volume equal to that of the tube) showed that approximately 90 percent of the possible overflow was obtained with 4, 6, and 8 displacements for the E4, E3, and E2 products, respectively. This corresponds to 320, 120, and 40 minutes of overflow time for the three products. These periods were adopted as the end point in all elutriation experiments.

TABLE 7.—*Elutriation results on quartz*

Size desired (mm)			Produced by—	Settling rate of coarsest size (mm per sec.)	Rising current used (mm per sec.)	Surface per gram (cm ²)	Average size as determined (mm)	Designation of product
From—	To—	Average						
0.147	0.074	0.110	Sieves.....					100-200
.074	.037	.056	Residue.....	4.93		1,150	0.038	E1
.037	.0185	.028	Third overflow.....	1.23	1.92	2,100	.020	E2
.0185	.0092	.014	Second overflow.....	.31	.48	5,400	.007	E3
.0092	.0000	.0046	First overflow.....	.08	.12	24,000	.0015	E4

Elutriation charges.—As results vary somewhat with different-size charges, a 25-gram charge was adopted as a convenient standard. A dried sample is preferable for elutriation as it is more convenient and assures accuracy in the original weight of material treated. The elutriation of a wet sample gives the same results as one that has been dried if both samples are properly dispersed.

Since this work was done a more extensive investigation of elutriation has been undertaken, from which it is hoped that a method may materialize based on theoretical accuracy and having a definite end point for each size.

SEDIMENTATION BALANCE

The distribution of particle size in subsieve material may be determined by sedimentation balance. This consists simply in having one balance pan suspended in the pulp and weighing at intervals as the particles settle in the pan. The method and operation are described by Weinig (130). A similar method, in which the sedimentation is weighed on a suspended plate by electrical means, is given by Odén (56). This method does not yield a sized product but is convenient for checking results obtained by other methods.

LIGHT-TRANSMISSION AND OTHER METHODS

Wagner (128) has devised a method for the determination of surface that depends upon the amount of light transmitted through a suspension of the material in kerosene. Although this method was devised for

cement it may also be used to determine the particle size of ore if a correction is made for the density of the various constituents of the ore. This correction would be difficult, as the proportions of the constituents cannot be determined accurately.

Gooskov (87) has proposed the use of moving pictures to determine the size of falling particles in water, and Aborn and Davidson (96) have proposed the measurement of particles by X-ray.

It is questionable, however, whether these methods are any more accurate than simple elutriation.

DISTRIBUTION OF PARTICLE SIZE

The distribution of particle size in a crushed product has been given considerable attention. Martin (47, 60, 64, 76, 78), Kuhn (55), Gaudin (67), Andreason (84), Work (85, 120), Hatch and Choate (88), Aborn and Davidson (96), Roller (114, 123), and Weinig (130) have investigated the subject and proposed methods or laws to establish the frequency of size distribution.

When this method is firmly established, it will be possible to determine surface on all sizes with fair accuracy. If the distribution of sizes in any material is known over an adequate field, an accurate estimate can presumably be made for the finest part.

Establishing the method on a firm basis depends on (1) accurate sizing, by whatever method, and (2) an established and accepted law of size distribution.

As sieve sizing usually does not give enough data to determine the distribution of sizes, particularly on a ball-mill product, sizing of subsieve sizes must be used for accurate results.

The distribution of sizes can be calculated by the various proposed formulas, but apparently the results vary so and are so sensitive to even small variations in the sizing analysis that no method or procedure can yet be accepted as final.

SURFACE DETERMINATIONS ON HETEROGENEOUS MATERIAL

Although the dissolution method is at present applicable only to quartz, the surface figures so obtained have been made the basis for an indirect method for determining the surface of ore. The method hardly lends itself to routine mill operations but may be of considerable value in special studies. It is not scientifically accurate, as several assumptions are made, but it opens a field of study that would otherwise be closed.

This method (95), which has been used satisfactorily in the study of plant milling operations, consists essentially in sieving to 200-mesh, elutriating the minus-200-mesh material, and determining the predominant minerals in each size. Surface figures may be calculated from the data in table 8.

The figures for size relationship are not used in a routine calculation of surface, but their derivations are given as the intermediate step in establishing the surface figures in table 9.

TABLE 8.—*Size relationship of elutriation products*

	Rising current (mm per sec.)	Quartz	Galena	Sphalerite	Pyrite	Calcite
Correction factor for density, <i>A</i>		1.00	0.50	0.74	0.64	0.97
Correction factor for shape, <i>B</i>		1.00	.82	.90	.96	.82
Final correction factor, <i>A</i> × <i>B</i>		1.00	.41	.67	.61	.80
Elutriation products, average size, microns:						
Residue, E1		38	16	25	23	30
Third overflow, E2	1.92	20	8.2	13.4	12.2	16.0
Second overflow, E348	7	2.9	4.7	4.3	5.6
First overflow, E412	1.5	.62	1.00	.92	1.20

DETERMINATION OF SIZE

In sieve sizing the density and shape of the particles do not greatly affect the separation of the various sizes, but in elutriation these two factors must be taken into account for each mineral.

The effect of density on elutriation sizes, as calculated from Stokes' law, is given as density factors in table 8.

The effect of shape can be approximated from the ratio 100 : 82, which Richards and Locke (57) give as the relation of particle size of quartz to that of galena. As this correction factor is small, a reasonable factor may be assumed for any mineral without greatly affecting the accuracy of the results. As crushed quartz is one of the most irregular and galena one of the most regular minerals, the shape factor for other minerals usually will lie between those for quartz and galena. Table 8 gives the shape factors that have been used.

The average sizes of quartz for E1 to E4 products are those given in table 7 for the products obtained by elutriation. The final correction factor for each mineral was applied to that of quartz and the respective size of each elutriation product thus obtained.

Other minerals may be added to the table if a shape factor is assumed. As no data are available the assumption of a shape factor must be based on judgment.

DETERMINATION OF SURFACE

To determine the surface of minerals other than quartz, a factor must be applied to the theoretical surface to correct for the irregularity of the particles. This factor is the ratio of the actual to the theoretical surface.

The theoretical surface figures given in table 9 assume all particles to be cubes or spheres and equal:

$$\frac{60,000}{dD} \text{ cm}^2 \text{ per gram,}$$

in which *d*=density of mineral, and *D*=average diameter in spherical particles or average edge of cubical particles, in microns.

The ratio of actual to theoretical surface of quartz, as determined for sieve sizes and elutriation products, is plotted in figure 8. The theoretical ratio is a straight line.

The irregularity of the particles affects the ratio for all sieve and elutriation sizes. No figures are available on the irregularity of minerals other than quartz, but the minerals considered are less irregular than crushed quartz and more irregular than perfect cubes or spheres.

The ratio must therefore lie between the curve for quartz and the theoretical curves. In quartz the increasing ratio for coarser sizes is due largely to cracks or internal surface. To what extent such cracks exist in other minerals is not known; but presumably they do exist, although possibly not to such an extent as in quartz. The curves in figure 8 for the other minerals have been drawn with these considerations in mind and are based largely on judgment. As both galena and calcite have perfect cleavage, the curves are considered

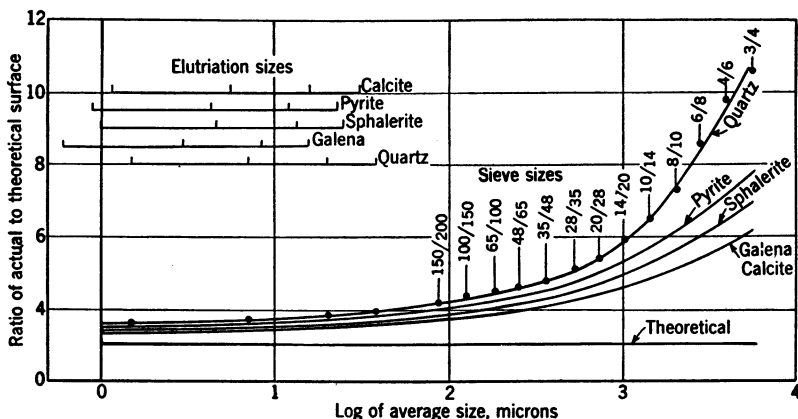


FIGURE 8.—Ratio of actual to theoretical surface for various minerals; quartz curve based on experimental results.

to coincide, which also assumes the same proportion of cracks in each. The ratio of actual to theoretical surface in table 9 has been obtained from these curves, and the actual surface figures have been calculated therefrom.

This table covers sieve sizes from 3- to 200-mesh in the $\sqrt{2}$ series and the four elutriation sizes. The elutriation figures are used only for minus 200-mesh size, but the 200- to 270-mesh and 270- to 400-mesh figures are appended to complete the sieve series.

Ordinarily, in fine grinding, the surface of the material coarser than 200-mesh is not a large percentage of the total surface, and alternate sieve sizes can be taken with considerable saving of time and not much sacrifice in accuracy. Table 9, however, is given as it is more complete and would be applicable in a study of coarse crushing.

TABLE 9.—Surface figures for various minerals

Size, mesh, and elutriation products	3-4	4-6	6-8	8-10	10-14	14-20	20-28	28-35	35-48	48-65
Quartz (specific gravity 2.65):										
Average diameter.....microns..	5,690	4,013	2,845	2,007	1,410	1,001	711	503	356	252
Theoretical surface.....cm ² /g..	3.98	5.64	7.96	11.3	16.1	22.6	31.8	45.0	63.6	89.8
Ratio, actual to theoretical surface.....	8.59	7.78	6.56	5.28	4.49	3.95	3.38	3.11	2.77	2.64
Actual surface.....cm ² /g..	34.2	43.9	52.2	59.7	72.3	89.3	107.4	139.9	176.5	236.8
Galena (specific gravity 7.43):										
Average diameter.....microns..	5,690	4,013	2,845	2,007	1,410	1,001	711	503	356	252
Theoretical surface.....cm ² /g..	1.42	2.01	2.84	4.02	5.73	8.07	11.4	16.1	22.7	32.0
Ratio, actual to theoretical surface.....	4.10	3.80	3.47	3.10	2.87	2.62	2.36	2.22	2.08	1.97
Actual surface.....cm ² /g..	5.8	7.6	9.9	12.5	16.4	21.1	26.9	35.7	47.2	63.0
Sphalerite (specific gravity 4.00):										
Average diameter.....microns..	5,690	4,013	2,845	2,007	1,410	1,001	711	503	356	252
Theoretical surface.....cm ² /g..	2.64	3.74	5.27	7.47	10.6	15.0	21.1	29.8	42.1	59.5
Ratio, actual to theoretical surface.....	4.95	4.50	4.09	3.65	3.30	3.00	2.69	2.50	2.32	2.20
Actual surface.....cm ² /g..	13.1	16.8	21.6	27.3	35.0	45.0	56.8	74.5	97.7	130.9
Pyrite (specific gravity 5.00):										
Average diameter.....microns..	5,690	4,013	2,845	2,007	1,410	1,001	711	503	356	252
Theoretical surface.....cm ² /g..	2.11	2.99	4.22	5.98	8.51	12.0	16.9	23.9	33.7	47.6
Ratio, actual to theoretical surface.....	5.75	5.22	4.66	4.14	3.69	3.32	3.00	2.75	2.54	2.39
Actual surface.....cm ² /g..	12.1	15.6	19.7	24.8	31.4	39.8	50.7	65.7	85.6	113.8
Calcite (specific gravity 2.71):										
Average diameter.....microns..	5,690	4,013	2,845	2,007	1,410	1,001	711	503	356	252
Theoretical surface.....cm ² /g..	3.89	5.52	7.78	11.0	15.7	22.1	31.1	44.0	62.2	87.9
Ratio, actual to theoretical surface.....	4.10	3.80	3.47	3.10	2.87	2.62	2.36	2.22	2.08	1.97
Actual surface.....cm ² /g..	15.9	21.0	27.0	34.1	45.1	57.9	73.4	97.7	129.4	173.2

Size, mesh, and elutriation products	65-100	100-150	150-200	E1	E2	E3	E4	200-270	270-400
Quartz (specific gravity 2.65):									
Average diameter.....microns..	178	126	89	38	20	7	1.5	63	45
Theoretical surface.....cm ² /g..	127	180	254	596	1,130	3,240	15,100	359	503
Ratio, actual to theoretical surface.....	2.48	2.37	2.15	1.93	1.86	1.66	1.59	2.03	1.99
Actual surface.....cm ² /g..	314.7	425.7	547.2	1,150	2,100	5,400	24,000	728.8	1,000.0
Galena (specific gravity 7.43):									
Average diameter.....microns..	178	126	89	16	8.2	2.9	.62	63	45
Theoretical surface.....cm ² /g..	45.4	64.1	90.7	505	985	2,780	13,000	128	180
Ratio, actual to theoretical surface.....	1.89	1.80	1.71	1.45	1.40	1.35	1.30	1.65	1.60
Actual surface.....cm ² /g..	85.8	115.4	155.1	732.3	1,380	3,750	16,900	211.2	288.0
Sphalerite (specific gravity 4.00):									
Average diameter.....microns..	178	126	89	25	13.4	4.7	1.01	63	45
Theoretical surface.....cm ² /g..	84.3	119	169	600	1,120	3,190	14,900	238	333
Ratio, actual to theoretical surface.....	2.09	1.99	1.86	1.62	1.56	1.51	1.44	1.77	1.70
Actual surface.....cm ² /g..	176.2	236.8	314.3	972	1,750	4,820	21,500	421.3	566.1
Pyrite (specific gravity, 5.00):									
Average diameter.....microns..	178	126	89	23	12.2	4.3	.92	63	45
Theoretical surface.....cm ² /g..	67.4	95.2	135	522	985	2,790	13,000	190	267
Ratio, actual to theoretical surface.....	2.27	2.15	2.03	1.75	1.68	1.60	1.53	1.93	1.85
Actual surface.....cm ² /g..	153.0	204.7	274.1	913.5	1,650	4,460	19,900	366.7	494.0
Calcite (specific gravity, 2.71):									
Average diameter.....microns..	178	126	89	30	16.0	5.6	1.20	63	45
Theoretical surface.....cm ² /g..	124	176	249	738	1,380	3,950	18,500	351	492
Ratio, actual to theoretical surface.....	1.89	1.80	1.71	1.54	1.46	1.38	1.32	1.65	1.60
Actual surface.....cm ² /g..	234.4	316.8	425.8	1,140	2,010	5,450	24,400	579.2	787.2

The figures calculated for alternate sieve sizes are given in table 10. The average diameter is again the arithmetical mean, and the theoretical surface is obtained as in table 9. The ratio of actual to theoretical surface is taken from figure 8 as before, and the figures for the elutriation products are the same as in table 9.

When a sample has been sieve-sized and elutriated with the same upward currents as those given in table 7 and the predominating minerals have been determined, the surface figures may be obtained from these data. A definite figure is thus obtained for the finer part of the sample without which a satisfactory figure for efficiency cannot be obtained.

RELATION OF SURFACE TO PARTICLE SIZE

AVERAGE PARTICLE DIAMETER

The average diameter of sieved particles is usually taken as the arithmetical mean between the two limiting mesh apertures. This average is not strictly correct, as usually the particles weigh more at either the coarse or fine end, depending on the trend of the particle-size distribution. As sieve analyses generally reach a peak at some intermediate size, the coarser sizes have a smaller average diameter and the finer sizes a larger average diameter than the arithmetical mean.

TABLE 10.—Surface figures for various minerals

Size, mesh, and elutriation products	4-8	8-14	14-28	28-48	48-100	100-200	E1	E2	E3	E4
Quartz (specific gravity, 2.65):										
Average diameter.....microns...	3,530	1,765	879	442	221	110	38	20	7	1.5
Theoretical surface.....cm ² /g....	6.41	12.8	25.8	51.2	102	206	596	1,130	3,240	15,100
Ratio, actual to theoretical surface..	7.25	4.95	3.72	2.98	2.60	2.29	1.93	1.86	1.66	1.59
Actual surface.....cm ² /g....	46.5	63.4	96.0	152.6	265.2	471.7	1,150	2,100	5,400	24,000
Galena (specific gravity, 7.43):										
Average diameter.....microns...	3,530	1,765	879	442	221	110	16	8.2	2.9	.62
Theoretical surface.....cm ² /g....	2.29	4.58	9.19	18.3	36.5	73.4	505	985	2,780	13,000
Ratio, actual to theoretical surface..	3.69	3.03	2.52	2.18	1.96	1.78	1.45	1.40	1.35	1.30
Actual surface.....cm ² /g....	8.5	13.9	23.2	39.9	71.5	130.7	732.3	1,380	3,750	16,900
Sphalerite (specific gravity, 4.00):										
Average diameter.....microns...	3,530	1,765	879	442	221	110	25	13.4	4.7	1.01
Theoretical surface.....cm ² /g....	4.25	8.50	17.1	33.9	67.9	136	600	1,120	3,190	14,900
Ratio, actual to theoretical surface..	4.35	3.52	2.89	2.44	2.18	1.95	1.62	1.56	1.51	1.44
Actual surface.....cm ² /g....	18.5	29.9	49.4	82.7	148.0	265.2	972.0	1,750	4,820	21,500
Pyrite (specific gravity, 5.00):										
Average diameter.....microns...	3,530	1,765	879	442	221	110	23	12.2	4.3	.92
Theoretical surface.....cm ² /g....	3.40	6.80	13.7	27.1	54.3	109	522	985	2,790	13,000
Ratio, actual to theoretical surface..	4.98	3.95	3.19	2.67	2.35	2.11	1.75	1.68	1.60	1.53
Actual surface.....cm ² /g....	16.9	26.9	43.7	72.4	127.6	230.0	913.5	1,650	4,460	19,900
Calcite (specific gravity, 2.71):										
Average diameter.....microns...	3,530	1,765	879	442	221	110	30	16.0	5.6	1.20
Theoretical surface.....cm ² /g....	6.27	12.5	25.2	50.1	100	201	738	1,380	3,950	18,500
Ratio, actual to theoretical surface..	3.69	3.03	2.52	2.18	1.96	1.78	1.54	1.46	1.38	1.32
Actual surface.....cm ² /g....	23.1	37.9	63.5	109.2	196.0	357.8	1,140	2,010	5,450	24,400

The methods for determining average size given by Perrott and Kinney (38) and Weigel (51) are based on a count of the different sizes, and the average size varies greatly with the method used. This variation becomes less serious as the limiting sizes in a sample approach each other. In a sieve-sized product, where the limiting sizes are not far apart, the average size as calculated from the number and diameter of the particles, the number and surface of the particles, or the number and volume of the particles does not vary greatly from the arithmetical mean.

The average diameter of 28- to 35-mesh crushed quartz containing 4,350 particles per gram was calculated according to Perrott and Kinney's formulas, as follows:

	<i>Average diameter, microns</i>
Based on number of particles.....	495
Based on size of particles.....	499
Based on surface of particles.....	504
Based on volume of particles.....	508
Based on arithmetical mean.....	503

It would seem, therefore, that no serious error would result from adopting the arithmetical mean as the average particle size in a sieve-sized product.

THEORETICAL SURFACE FIGURES

Table 11 gives data for sieve sizes of quartz. The table is based on the Tyler standard sieve scale, except for the 270- to 400-mesh material. It is interesting to note that the actual count of crushed-quartz particles lies between the figures for cubes and spheres, which gives further assurance that the arithmetical mean, upon which the number of cubes and spheres is based, is not seriously in error.

The theoretical surface figures given in table 11 have no value in surface calculations, unless a factor is applied. Such a factor is possible for crushed quartz, since the surface figures have been determined. The factor is not a constant but increases with the coarser sizes owing largely to cracks or internal surface.

TABLE 11.—Theoretical surface data for sieve-sized quartz

[Diameter of cube is measured along one edge. Number of particles in 1 gram of quartz = $\frac{377.36}{D^3}$ for cubes, $\frac{720.72}{D^3}$ for spheres. Surface in square centimeters per gram for quartz = $\frac{22.64}{D}$ for cubes or spheres]

Mesh	Size of particle (mm)			Number of particles in 1 gram of quartz		Theoretical quartz surface (cubes or spheres), cm ² per gram
	Maximum	Minimum	Average	Cubes	Spheres	
	26.67	18.85	22.76	0.032	0.061	0.99
	18.85	13.33	16.09	.091	.173	1.41
	13.33	9.423	11.377	.256	.489	1.99
	9.423	6.680	8.052	.72	1.38	2.81
3-4-----	6.680	4.699	5.690	2.05	3.91	3.98
4-6-----	4.699	3.327	4.013	5.84	11.15	5.64
6-8-----	3.327	2.362	2.845	16.4	31.3	7.96
8-10-----	2.362	1.651	2.007	48.7	89.1	11.3
10-14-----	1.651	1.168	1.410	135	257	16.1
14-20-----	1.168	.833	1.001	376	718	22.6
20-28-----	.833	.589	.711	1,050	2,005	31.8
28-35-----	.589	.417	.503	2,966	5,663	45.0
35-48-----	.417	.295	.356	8,361	15,975	63.6
48-65-----	.295	.208	.252	23,585	45,046	89.8
65-100-----	.208	.147	.178	67,114	128,205	127
100-150-----	.147	.104	.126	188,679	359,712	180
150-200-----	.104	.074	.089	534,760	1,020,410	254
200-270-----	.074	.052	.063	1,508,300	2,881,840	359
270-400-----	.052	.037	.045	4,141,600	7,909,100	503

MEASURED SURFACE FIGURES

Surface figures were obtained on crushed quartz, Ottawa sand, and outer surface of crushed quartz.

The surface figures for sieve-sized crushed quartz are given in table 12. The large increase in the ratio of measured to theoretical surface with coarser sizes is due largely to cracks rather than to change of configuration.

TABLE 12.—*Measured surface of sieve-sized crushed quartz*

Mesh	Average size of particle (mm)	Theoretical surface per gram of quartz (cubes or spheres), cm ²	Measured surface per gram, cm ²	Ratio of measured to theoretical surface
3-4	5.690	3.98	34.2	8.59
4-6	4.103	5.64	43.9	7.78
6-8	2.845	7.96	52.2	6.56
8-10	2.007	11.3	59.7	5.28
10-14	1.410	16.1	72.3	4.49
14-20	1.001	22.6	89.3	3.95
20-28	.711	31.8	107.4	3.38
28-35	.503	45.0	139.9	3.11
35-48	.356	63.6	176.5	2.77
48-65	.252	89.8	236.8	2.64
65-100	.178	127	314.7	2.48
100-150	.126	180	425.7	2.37
150-200	.089	254	547.2	2.15
200-270	.063	359	728.8	2.03
270-400	.045	503	1,000.0	1.99

Table 13 gives the surface figures for Ottawa sand. The fairly constant ratio of measured to theoretical surface for the Ottawa sand is due to regular spherical shape and lack of cracks.

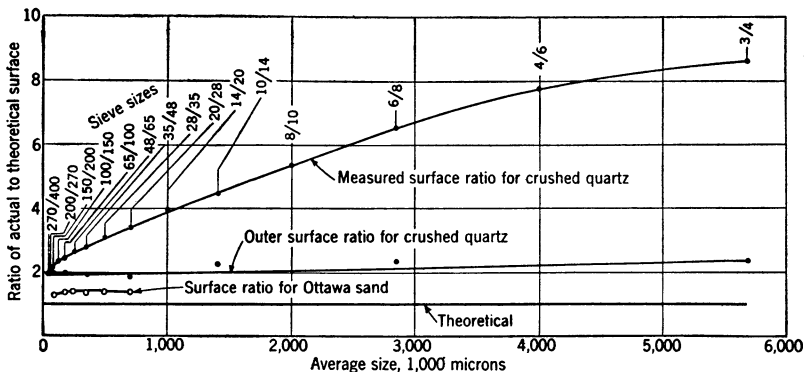


FIGURE 9.—Ratio of actual to theoretical surface for crushed quartz, Ottawa sand, and outer surface of crushed quartz.

In silver coating to convert the initial rate figure to definite surface units only the outer surface of the quartz particle is coated. This method gave the outer surface figures for the crushed-quartz sizes in table 14.

The surface figures for crushed quartz, Ottawa sand, and outer surface of crushed quartz are shown in figure 9, in which the ratio of actual to theoretical surface has been plotted.

That the high ratio for crushed quartz is due mainly to cracks has been demonstrated by disintegration in hydrofluoric acid. Cracks, which are normally invisible, rapidly become visible by the penetration of hydrofluoric acid whose action is so effective as to cause the particles to fall apart. Owing to the rapidity of this phenomenon it is believed that a rather large percentage of the interior surface represented by cracks is measured in the dissolution method.

TABLE 13.—*Measured surface of sieve-sized Ottawa sand*

Mesh	Average size of particle (mm)	Theoretical surface per gram of quartz (cubes or spheres), cm ²	Measured surface per gram, cm ²	Ratio of measured to theoretical surface
20-28	0.711	31.8	43.4	1.37
28-35	.503	45.0	60.9	1.35
35-48	.356	63.6	84.5	1.33
48-65	.252	89.8	124.4	1.39
65-100	.178	127	173.4	1.37
100-150	.126	180	235.6	1.31
150-200	.089	254	326.9	1.29

TABLE 14.—*Comparison of total and outer surface of crushed quartz*

Mesh	Theoretical surface per gram (cubes or spheres), cm ²	Total surface		Outer surface		Ratio of interior to outer surface
		Cm ² per gram	Ratio to theoretical surface	Cm ² per gram	Ratio to theoretical surface	
3-4	3.98	34.2	8.59	9.4	2.35	2.64
6-8	7.96	52.2	6.56	18.6	2.33	1.81
10-14	16.1	72.3	4.49	36.0	2.24	1.01
20-28	31.8	107.4	3.38	59.1	1.86	.82
35-48	63.6	176.5	2.77	122.1	1.92	.45
65-100	127	314.7	2.48	252.9	1.99	.24
150-200	254	547.2	2.15	493.0	1.94	.11

Figures 10 and 11 show the disintegration of quartz particles. The coarse particles in figure 10, *A*, are 6- to 8-mesh, and the disintegrating effect after 2 or 3 hours' contact with hydrofluoric acid is shown in figure 10, *B* and *C*. The finer particles, 35- to 48-mesh, are shown in figure 11, *A*, and the small amount of disintegration after 2 or 3 hours contact with hydrofluoric acid is represented by the particles in the bottom horizontal rows in figure 11, *B* and *C*.

If the logarithm of particle size is plotted against the logarithm of theoretical surface, as in figure 12, a straight line is obtained which may be extended as a continuation of the straight line to a similarly plotted point for the unit crystal of quartz. The unit-crystal structure of quartz is taken as a triangular prism with edges of 5.4 and 4.9 Angstrom. These unit crystals have 66,500,000 cm² of surface per gram, based on an average diameter of 0.0005 micron.

For crushed quartz this relationship gives a flat curve (because the number of cracks and internal surface are increasing in the coarser sizes), which may be extended directly to the unit crystal.

It may be assumed that in crushed quartz all sizes are present down to the unit crystal, at which size crushed quartz would cease to be irregular in shape; that, therefore, as the particles become smaller they approach perfect shapes; and that the crushed-quartz curve becomes flatter as it approaches the unit crystal.

Surface measurements of Ottawa sand give essentially a straight line, as shown in figure 12, which can likewise be extended to the unit crystal. The outer surface values, also plotted in figure 12, extend as a straight line to the unit crystal.

The fact that all three curves extend to the unit crystal point suggests that the dissolution method gives accurate sieve sizes.

There is a large range of unexplored territory between the 270- to 400-mesh size (47 microns) and the unit crystal, but there is no reason to expect the curves to change within this range.

The curve for crushed quartz may be used to determine the average size of a crushed-quartz sample, such as an elutriated product. The average size will be that of particles of which 1 gram has the same surface as 1 gram of the sample.

MINUS 200-MESH SIZE

An average size for minus 200-mesh material cannot be chosen arbitrarily or determined accurately by elutriation or subsidence. Microscopic methods fail, largely because material finer than approximately 1 micron is invisible. Only a direct determination of surface remains, then, from which the average size may be obtained from the curve in figure 12. This procedure may be questioned, as it assumes

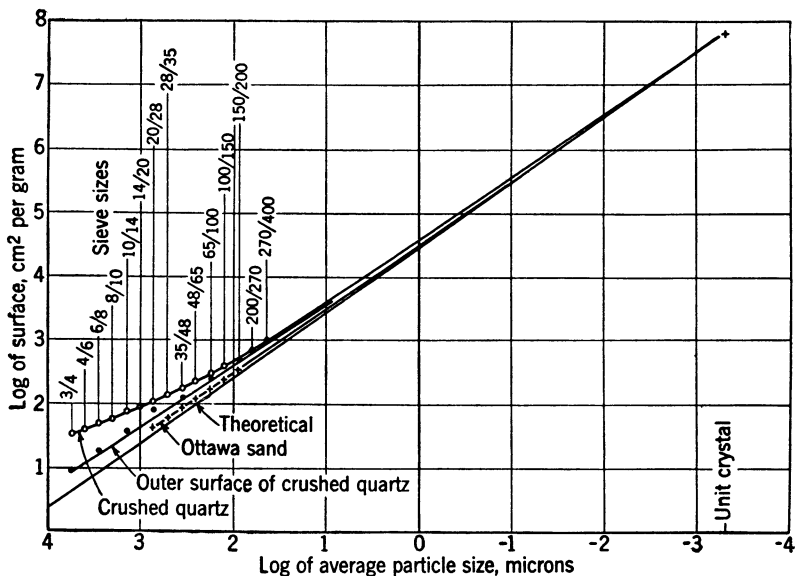


FIGURE 12.—Relation of surface to size of quartz particles.

accuracy for the dissolution method and an unsubstantiated average value. It may be said, however, that for crushing purposes the average size is less important than the surface, and the determination of surface has considerable claim to reliability.

Minus 200-mesh material can vary greatly in fineness, therefore the average size also varies greatly. The inaccuracy of arbitrarily designating 37 microns or any other size as average is evident. The importance of the fine material in the determination of efficiency has been generally recognized (8, 16, 23, 24, 67).

In 96 quartz-grinding experiments the minus 200-mesh material ranged from 4,593 to 18,710 cm² per gram, which corresponds to an average diameter of 12.4 to 2.2 microns on the curve. No general relationship exists between the average size or fineness of the minus 200-mesh material and the percentage produced. The observed limits could be deliberately separated further, but they are suffi-

ciently illuminating to show the fallacy of adopting an arbitrary figure for minus 200-mesh material or using any factor based on the percentage of the material produced.

RELATION OF SURFACE TO WORK INPUT

By utilizing the dissolution method for surface determinations the relationship of surface produced to work input may be ascertained directly for crushed quartz.

As most of the surface measurements in the crushing experiments to determine this relationship were to be made on the crushed product without sizing, it was thought advisable to show that surface measurements made directly on an unsized sample were identical with those on the sized material. Two composite samples were prepared from sizes whose surface had been determined; one sample consisted of sieve sizes from 28- to 270-mesh and one of four elutriation sizes finer than 200-mesh.

The surface determinations on these two samples gave the following results:

Surface determinations on samples of crushed quartz

	Sample within sieve scale		Sample of minus 200-mesh elutriation products	
	Initial rate	Surface, cm ² per gram	Initial rate	Surface, cm ² per gram
Calculated from separate determinations.....	1.487	252.75	21.882	3,720
Determination on composite sample.....	1.479	251.4	21.229	3,609
Do.....	1.477	251.1	21.829	3,711
Do.....			22.082	3,754
Average.....	1.478	251.25	21.713	3,691
Difference.....percent.....		0.6		0.8

These results indicate that surface measurements on the crushed material may be made directly with confidence.

The work-input measurements must be as accurate as possible, the applied crushing force transmitted directly, and all possible frictional losses eliminated. These requirements preclude the use of the ordinary types of crushers, rolls, and ball mills.

CRUSHING APPLIANCE

In the crushing appliance adopted for this work a free-falling ball was used, the energy of which was applied to the quartz through a small plunger operating in a diamond mortar (81). This appliance was simple and entirely free from loss due to friction. A view of the drop-weight machine and a sectional view of the crushing chamber are shown in figures 13 and 14.

The steel mortar *M*, 3 inches in diameter by 3 inches high, has a crushing chamber *C* in which a steel plunger *P* fits neatly to hold the quartz in place. The mortar is centered by means of the three screws *A*. Two mortars were used; one with a crushing chamber 1 inch in diameter by ½ inch deep having a capacity of 4 grams of

quartz and the other with a crushing chamber $1\frac{1}{2}$ inches in diameter by $\frac{3}{4}$ inch deep having a capacity of 12 grams of quartz.

Plungers of different lengths gave the same results, and the lengths adopted were $\frac{1}{2}$ and $\frac{3}{4}$ inch, respectively, for the two chambers.

Three standard-size aluminum wires *W* are placed between the mortar and base *B*. The base, made of steel, is 6 inches in diameter by 8 inches in height and is set on a concrete column.

The steel ball is suspended by a cord which rests in an adjustable crossbar. This suspension allows accurate centering and prevents swinging of the ball. When the cord is cut just above the crossbar, the ball drops perfectly straight without danger of being thrown out of line.

The machine had a maximum drop of 60 centimeters, and 1.360-, 2.050-, and 2.935-kilogram balls were used.

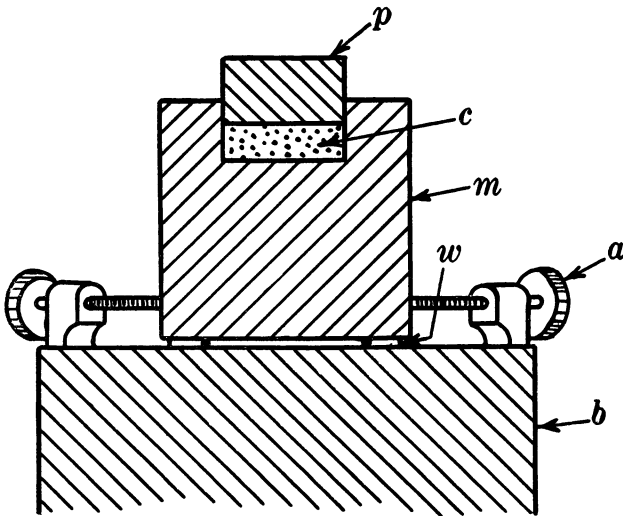


FIGURE 13.—Sectional view of crushing chamber.

WORK USED IN CRUSHING

With a definite weight and drop a definite amount of energy is delivered upon impact if no rebound of the ball occurs. Rebound is prevented by the three aluminum wires whose deformation or flattening is a measure of the work not utilized in crushing.

It is more satisfactory to eliminate rebound than to measure the rebound to determine the energy delivered to the plunger. Several attempts were made to measure rebound. The most satisfactory result was obtained by taking a photograph with an exposure covering the time of rebound. The resulting photograph showed the path of the ball, from which the highest point of the rebound could be measured. This method, however, is rather cumbersome and not well adapted where many experiments are to be made.

Elimination of the rebound, if possible, was therefore decided upon. It was found that if an inelastic substance that will deform easily is placed between the mortar and the base, rebound is eliminated and the deformation is a measure of the force transmitted through the anvil.

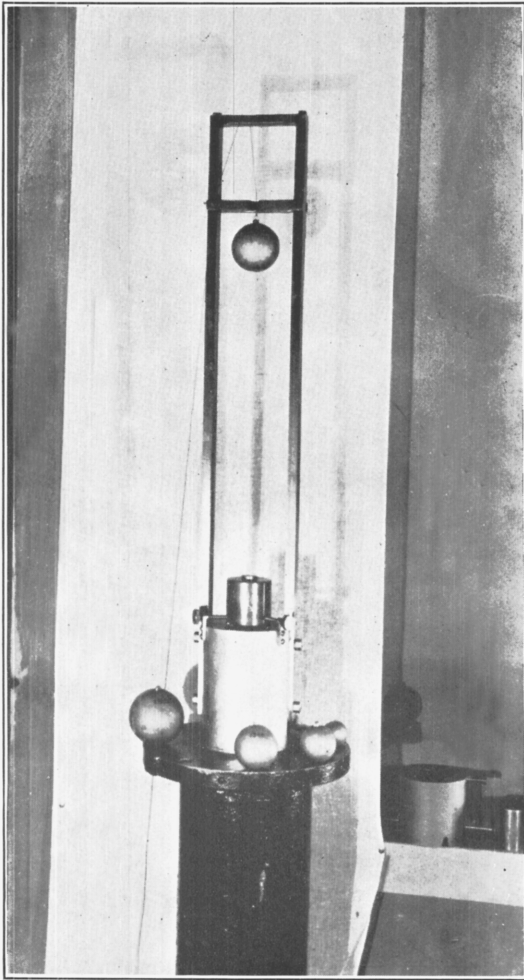


FIGURE 14.—Drop-weight machine.

The difference between the work of impact and that represented by the deformation gives the amount of work used in crushing.

A satisfactory deformable substance is one that yields or deforms enough to prevent rebound but has sufficient resistance to allow a maximum of the impact to be used in crushing. Lead shot, fuse wire, aluminum wire, and copper wire were tried; of these, aluminum wire was adopted, as it best fulfilled the requirements and dependably uniform wire could be obtained. Three aluminum wires, 3.232 millimeters in diameter by 1 centimeter in length, were used and were

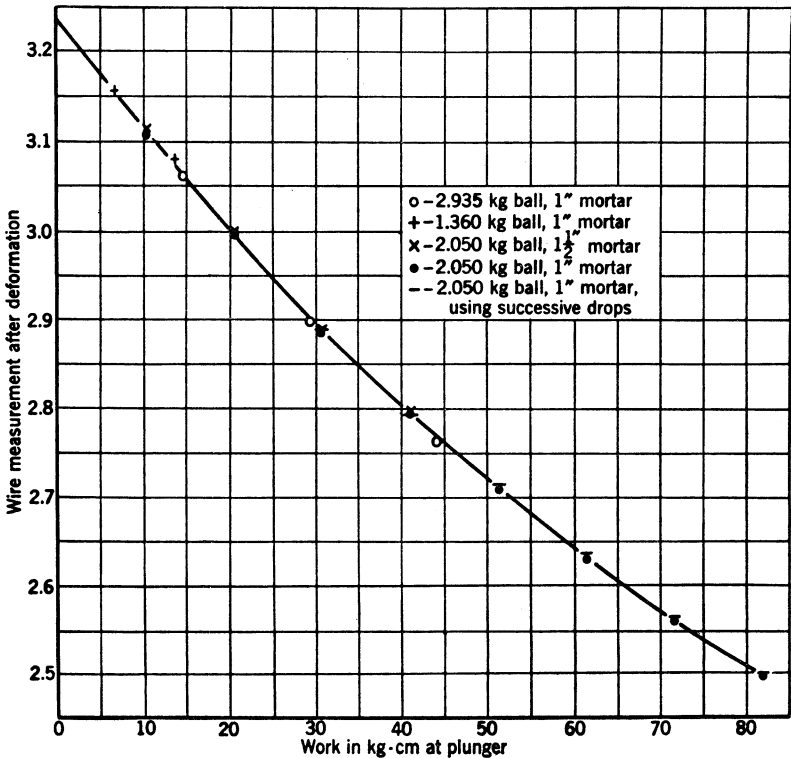


FIGURE 15.—Work in kilogram-centimeters at plunger for deformation of three aluminum wires 1 cm long and 3.232 mm in diameter.

placed radially 120° apart. When no crushing is done—that is, when the plunger rests on the bottom of the crushing chamber—the deformation represents the total energy applied to the plunger by the falling ball. The deformation curve for three wires is given in figure 15. The work represented by the deformation is obtained from the curve and from the measurement of the wires after a crushing experiment. In the preparation of this curve the three different balls and both mortars were used. All points shown in the curve agree closely, as do those obtained from repeated short drops instead of one long drop.

PROCEDURE IN CRUSHING TESTS

The accurately weighted sample is placed in the crushing chamber, leveled off, and moistened with absolute alcohol; then the plunger is put in place. The alcohol prevents loss of dust, and as alcohol evapo-

rates rapidly the crushed material may be removed easily. As the loss of dust would seriously affect the results of surface measurement, moistening with alcohol is quite important; when alcohol is used, the loss is less than 0.10 percent by weight.

The mortar, which has been centered with the adjusting screws, is placed on the three wires. As further prevention against loss a cylindrical apron is placed around the mortar to retain any material that may accidentally be thrown outward—an emergency protection usually having no function to perform.

The ball is then suspended at the desired height and the suspension cord firmly anchored. When the ball is perfectly quiet the cord above the suspension bar is cut and the ball allowed to drop.

The average deformation of the three wires is found with a micrometer. The deformation of all three wires is virtually the same with a well-centered drop. Unequal deformation means a poorly centered drop, with attendant binding of the plunger, and a test that gives this result is discarded. The work represented by the deformation, taken from the curve, is deducted from the work of the ball to ascertain the work used in crushing.

As numerous crushings were required for the dissolution tests there was ample evidence of the reproducibility of the crushing tests; the figures for the work used in crushing checked with remarkable accuracy.

RESULTS OF CRUSHING TESTS

Crushing tests were made under varying conditions in which the input of work ranged from 3.02 to 46.45 kg-cm. The results, given in table 15, show that in this machine an average of 17.56 cm² of quartz surface is produced per kilogram-centimeter of work used in crushing. The figures for the different experiments, from which this average is obtained, fall within the allowable error by the Chauvenet criterion, and if the wide variation in the nature of the experiments is considered the results would appear to be reliable.

TABLE 15.—Data on quartz crushing tests

Experiment	Amount crushed each crushing and total, grams	Sieve size of quartz before crushing	Crushing chamber	Ball weight; kg drop, cm	Kg-cm, gram; total net	Surface, cm ² per gram; after crushing, before crushing, and net to crushing	Surface produced, cm ² per kg-cm
A.....	{ 12 72 }	10-14	1½ by ¾.....	{ 2,050 20 }	3.42 3.02	{ 129.7 72.3 57.4 }	19.0
B.....	{ 4 64 }	10-14	1 by ½.....	{ 2,050 20 }	10.25 9.21	{ 227.1 72.3 154.8 }	16.8
C.....	{ 4 20 }	10-14	1 by ½.....	{ 2,050 20 }	10.25 9.37	{ 249.6 72.3 177.3 }	18.9
D.....	{ 4 80 }	35-48	1 by ½.....	{ 2,050 30 }	15.37 10.64	{ 393.4 200.4 193.0 }	18.1

TABLE 15.—Data on quartz crushing tests—Continued

Experiment	Amount crushed each crushing and total, grams	Sieve size of quartz before crushing	Crushing chamber	Ball weight; kg drop, cm	Kg-cm, gram; total net	Surface, cm ² per gram; after crushing, before crushing, and net to crushing	Surface produced, cm ² per kg-cm
E.....	{ 4 40 }	35-48	1 by ½.....	{ 2, 935 20. 95 }	15. 37 10. 64	400. 0 200. 4 199. 6	18. 8
F.....	{ 4 72 }	35-48	1 by ½.....	{ 1, 360 45. 22 }	15. 37 10. 82	406. 5 200. 4 206. 1	19. 0
G.....	{ 4 48 }	10-14	1 by ½.....	{ 2, 050 20+20 }	20. 50 14. 77	328. 1 72. 3 255. 8	17. 3
H.....	{ 4 32 }	20-28	1 by ½.....	{ 2, 050 40 }	20. 50 14. 79	349. 2 97. 4 251. 8	17. 0
I.....	{ 4 296 Sized }	20-28	1 by ½.....	{ 2, 050 40 }	20. 50 15. 04	340. 3 97. 4 242. 9	16. 2
J.....	{ 4 56 }	10-14	1 by ½.....	{ 2, 050 40 }	20. 50 16. 12	343. 1 72. 3 270. 8	16. 8
K.....	{ 3 108 Sized }	65-100	1 by ½.....	{ 2, 050 50 }	34. 17 19. 85	690. 2 369. 8 320. 4	16. 1
L.....	{ 4 96 }	10-14	1 by ½.....	{ 2, 050 40+40 }	41. 00 25. 43	518. 2 72. 3 445. 9	17. 5
M.....	{ 2 16 }	3-4	½ by ¼.....	{ 2, 935 40 }	58. 70 46. 45	878. 4 34. 2 844. 2	18. 2
Total kg-cm per gram to crushing.....					206. 15		
Total net surface per gram due to crushing.....						3, 620. 0	
Average surface produced, cm ² per kg-cm.....							17. 56

When the work expended in crushing is plotted against the surface produced, a straight line results which goes through zero, as shown in figure 16. The work of crushing, then, is directly proportional to the surface produced and confirms the Rittinger law.

The 17.56 cm² of quartz surface produced per kilogram-centimeter of work is equivalent to 243 cm² per foot-pound or 0.25 ft.² per foot-pound. The production of 1 ft.² of surface, then, requires about 4 foot-pounds compared with 60.9 foot-pounds obtained by Martin (59) in the ball milling of quartz.

In all experiments except I and K the surface determinations were made directly on the crushed quartz; any loss in weight during crushing was calculated as dissolved silica in the dissolution test. In experiments I and K the crushed quartz from all crushings was com-

bined and sieved, and separate surface determinations were made on the sized portions. Any loss in crushing would be equivalent to the removal of fine material with high specific surface before the surface determinations were made. This indicates that the surface determinations for these two experiments would be somewhat low, and although they were low they were not low enough to warrant discarding.

CRUSHING EFFICIENCY

SAMPLING

As no calculation on a crushing or grinding operation can be more accurate than the sample upon which the calculation is based, obviously care must be taken to obtain samples that represent the material satisfactorily.

The theory and practice of sampling have received so much attention that they have become standardized, and there is little excuse for poor samples.

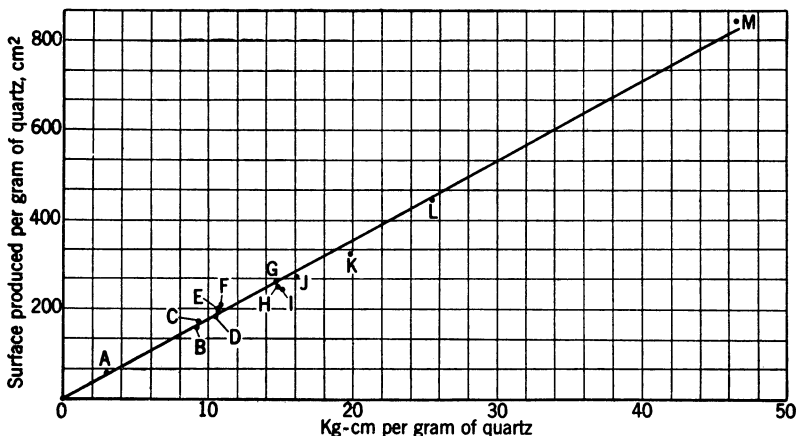


FIGURE 16.—Relation of work input to surface produced in crushing quartz.

It may be pertinent, however, to remember that in sampling a mill circuit a reliable check upon the work comprises a balance of the tonnage flow and of the various properties of the material. With a timed sample of the whole stream at all points in a mill circuit such a check gives assurance that the samples may be considered accurate. Checking depends simply on the fact that the feed to any unit in the circuit must equal the discharge of that unit.

Examples of such checks are given in the discussion on "Use of the indirect method on heterogeneous material." Checks are given on tonnage, mineral contents, and sizing actually obtained in the sampling of a mill grinding circuit.

SIEVING

Sieving is the means most generally adopted to determine the effectiveness of crushing or grinding, and although sieving leaves much to be desired owing to its limitations in finer sizes it is of enough value to warrant care in making the sieve analysis.

If material could be sized on sieves to a definite oversize and undersize, sieving would be an exact operation. Unfortunately, it cannot. Certain particles will be undersize or oversize, depending on their orientation to the sieve. These so-called "difficult grains" usually constitute a percentage large enough to produce undersize for an indefinite period of sieving. If the amount of undersize produced in sieving for 1-minute intervals is plotted, the resulting curve approaches zero asymptotically, therefore sieving may be conceived as an operation in which all of the undersize can never be separated in a sample. Some definite end point is therefore needed at which the operation may arbitrarily be called complete.

In the proposed sieving standard approved by the American Standards Association (*52, 124*) a preliminary wet separation of the finer material is recommended; this disintegrates agglomerated lumps, removes dust particles from the coarser grains, and prevents loss of dust. The end point recommended is that at which less than 0.05 percent of the original sample passes the sieve after 1 minute of sieving. The wet split and the end point are the two most important factors for accuracy.

The adopted standard may not be suited to ordinary comparative work and plant operation owing to the time required to carry the analysis to completion. For careful work and special investigations, however, it is desirable. In any sieving operation the wet separation should be adopted, and an approach at least should be made to the end point. Obviously, some materials do not lend themselves to this procedure, such as those that undergo change on wetting or those subject to undue abrasion. The general classes of ore do not come under this head, with the possible exception of galena, in which abrasion may affect the results. A good workable method of sieving is proposed and used by Probert (*108*); this takes due account of these two important points.

EARLIER ATTEMPTS TO MEASURE EFFICIENCY

As the amount of work expended in any crushing operation is directly proportional to the amount of surface produced, the efficiency of a crushing or grinding operation must be the relation between the input of work and the surface produced.

In measuring the work input in a crushing operation, that actually used in crushing, irrespective of dead load and transmission and friction losses, must be the basis for crushing-efficiency measurements. The dead load is the power required to operate the grinding unit when not grinding. In ball, rod, or pebble mills the dead load is determined by the power required to run the mill with the load of balls (or equivalent weight) centrally fixed in the mill (*121*). It is justifiable, however, for comparing mill operations to take the work input to the motor, but the results so obtained are not strictly comparable with those from another machine.

Haultain (*43*), in measuring the work used in rolls, employed an ingenious differential integrator in which the work input was automatically recorded. This device was later employed satisfactorily by the Bureau of Mines at Salt Lake City (*109*). Coghill (*100*),

Martin (62), and Fahrenwald (110) based work input on electrical measurements of power.

Efficiency figures based on any measure applied to a crushed product other than total surface produced are not true efficiency figures, and while such figures may be valuable for comparisons they cannot be considered as having any scientific value.

No definite basis is available for the calculation of crushing efficiencies. The theoretical energy required to produce a unit of surface in a crushing operation is not known. To base efficiencies on the surface energy of solids is unsatisfactory, as the figures of surface energy for solids are extremely unreliable; moreover, this energy represents only a minor part of the total energy required.

METHODS OF MEASURING CRUSHING EFFICIENCY

All earlier methods for measuring the efficiency of comminution have been based on sieve sizing, and usually some value has been apportioned to each size. This value has had some relation to either the Rittinger law or the Kick law, depending on the convictions of the calculator. Although usually the Rittinger law was adopted, the results were far from conclusive, as it was not known which law really applied, and the work represented by the finer sizes was an unknown but extremely large factor.

The pioneers in the study of crushing efficiencies were Rittinger (1) and Von Reytt (5). Because their knowledge of fine material was so meager, efficiency figures could be related only to sieve sizes.

Subsequent studies of efficiency by Pearce and Caldecott (8), Chapman (11, 12), Stadler (13), Warwick (14), and Del Mar (15) simply accentuated the absolute lack of knowledge regarding the finer material.

In 1913 Gates (16) introduced his diagram of crushing surface and later (17, 22) applied this diagram to the study of the crushing laws. This diagram is based on the Rittinger law and, if carried beyond sieve sizes by utilizing a sizing method such as elutriation, could by extrapolation give a surface figure of reasonable accuracy.

The work of Bell (23) in studying the efficiency of crushing machines demonstrated the weakness of any calculations of efficiency while the surface of the minus 200-mesh material was unknown.

Taggart (25), Davis (27), and Delano (32) determined the efficiency of crushing for various machines but advanced no new methods for this purpose. Herman (39) and Hersam (40) stressed the need of an accepted and accurate method for determining efficiency.

Haultain (43) in 1923 proposed a new formula for crushing efficiency, as neither the Rittinger law nor the Kick law checked with the energy used in a long series of carefully made experiments. As Haultain used an average size of 37 microns for minus 200-mesh material, his formula cannot hold in the light of present knowledge.

Rose (66) in 1926 proposed a method for determining efficiency and developed a formula based on sieve sizing, tonnage treated, and power input and not on either the Kick law or the Rittinger law. This

formula, however, conforms to the Rittinger law. Rose ignores all sizes finer than the required one, the amount of material ground to the required size being a measure of the efficiency irrespective of the fineness of the material; he claims that the grinding mill should receive no credit for useless work. This method obviously fails to give any information as to true efficiency or useless work.

Coghill (82) uses a method based on sieve sizes only to determine the work in grinding. Surface figures are taken as the reciprocal of the size, and the mill is credited with grinding the ore to a limiting size. The relationship of horsepower-hours is taken as the efficiency. The method is simple and is of value for comparing mills but does not give a true measure of work input.

Fahrenwald (111) has attempted to measure input of work by the amount of heat produced in ball-mill grinding. As the heat accompanying the production of new surface must be very small it would seem that any thermal measurements in ball milling would measure mainly heat of friction, which would have no relation to work done (20).

Attempts to measure efficiency have been handicapped by lack of an accepted crushing law and by inefficient knowledge of the characteristics of subsieve sizes. With the establishment of the Rittinger law by the dissolution of quartz and measurement of the coercive force of magnetite true efficiency figures are obtainable for either quartz or magnetite. Two standard materials are available, therefore, for crushing and grinding investigations and for the study of machine efficiency. For materials other than these two minerals it is only possible to measure efficiencies indirectly.

The method developed by the Bureau of Mines, in which the indirect measurement of surface previously described (95) has been utilized, is given herewith in considerable detail.

USE OF INDIRECT METHOD ON HETEROGENEOUS MATERIALS

An example of the results obtained by applying the indirect method of surface measurement to actual mill operation has been published (95). A time sample was taken of the entire pulp stream to and from each machine in the grinding circuit. It is difficult to get a satisfactory check on tonnage, and for this reason the samples were taken to cover as long a period as possible.

Figure 17 shows a flow sheet of the circuit which indicates the samples taken and the points at which water is introduced. This is a rather general type of grinding circuit and requires only seven samples to tell the whole story.

In the final tonnage calculations some adjustment is necessary both for the ore and for the water. The tonnage and adjusted figures in table 16 give an idea of the adjustment that ordinarily may be necessary.

TABLE 16.—*Tonnage adjustment, tons per 24 hours*

Product	Sample	Measured tonnage		Adjusted tonnage	
		Ore	Water	Ore	Water
Primary mill:					
Initial feed	1	1,037	40	1,100	43
Classifier return	2	634	116	640	115
Water			533		534
Mill feed		1,671	689	1,740	692
Mill discharge	3	1,673	659	1,743	692
Primary classifier:					
Primary mill discharge	3	1,673	659	1,740	692
Water			845		804
Classifier feed		1,673	1,504	1,740	1,496
Sand	2	634	116	640	115
Overflow	4	1,092	1,394	1,100	1,381
Classifier discharge		1,726	1,510	1,740	1,496
Secondary mill:					
Classifier return	5	1,261	303	1,270	304
Water			353		353
Mill feed		1,261	656	1,270	657
Mill discharge	6	1,177	654	1,270	657
Secondary classifier:					
Primary classifier overflow	4	1,092	1,394	1,100	1,381
Secondary mill discharge	6	1,177	654	1,270	657
Water			1,654		1,654
Classifier feed		2,269	3,702	2,370	3,692
Sand	5	1,261	303	1,270	304
Overflow	7	1,096	3,359	1,100	3,388
Classifier discharge		2,357	3,662	2,370	3,692
Total tonnage streams:					
Initial feed	1	1,037	40	1,100	43
Primary classifier overflow	4	1,092	1,394	1,100	1,381
Secondary classifier overflow	7	1,096	3,359	1,100	3,388

The samples were wet-sieved on 200-mesh; the dried oversize was hand-sieved and the minus 200-mesh material elutriated according to the conditions in table 7. As the ore comprised essentially galena, sphalerite, pyrite, and a siliceous gangue, the products were analyzed for lead, zinc, and iron, and the remainder was considered as quartz. Table 17 gives the tonnage of mineral of each size throughout the circuit calculated from these data.

Not only must the tonnage to and from each machine be equal, but at several points the entire tonnage passes. These equalities, together with the known capacity of the mill, allow an intelligent adjustment to be made. It is surprising that a greater adjustment is not required, considering the brief period possible for some samples. In the adjusting the tonnages represented by the longer sampling periods should be given more weight.

The data in table 17 are the basis for all subsequent calculations. Their accuracy may be checked by a mineral balance of the feed and discharge of each machine, as is done in table 18, which includes a balance for the three points at which the total tonnage passes. The mineral balance may be carried still farther by balancing the mineral contents for the various sizes going to and coming from the classifiers, as no change in particle size takes place in these machines. This bal-

ance is given in table 19. These two balance sheets indicate possible errors that may have occurred in the preparation of table 17.

The balances in table 18 are quite satisfactory; the greatest difference is 7.2 percent in the galena for the secondary mill. Several larger discrepancies are found in table 19, especially in the secondary classifier and the plus 48-mesh size. As the analyses of these samples were checked the discrepancies are probably due to erratic samples.

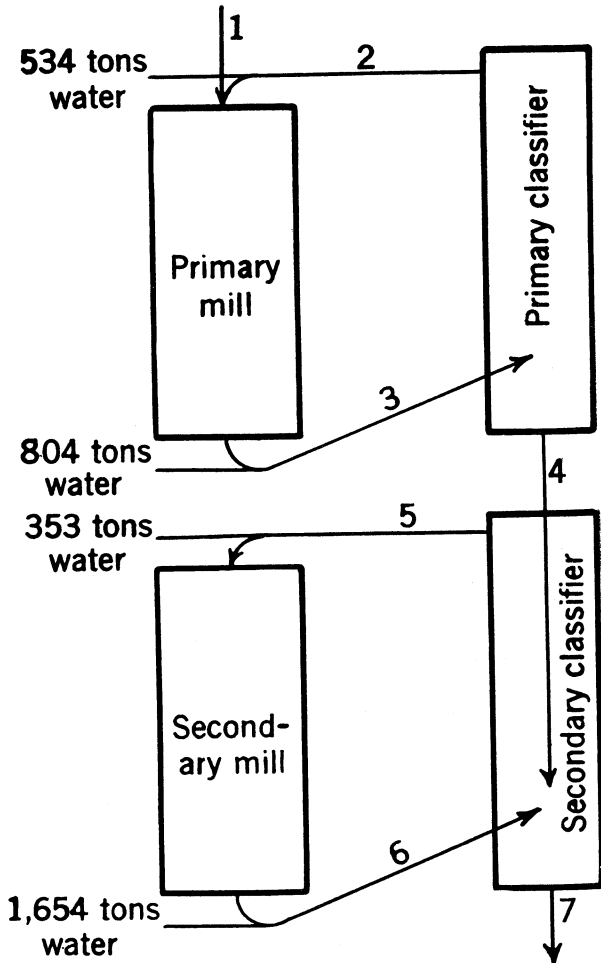


FIGURE 17.—Flow sheet of grinding circuit; numbers are sampling points.

The surface figures for the different minerals may now be obtained from the tonnage figures in table 17 and the surface figures in table 10. These surface figures are given in table 20 and should be multiplied by 1,000,000 to give square centimeters. Although it is not possible to verify the specific accuracy of these figures two methods of checking are available. One method depends on the fact that the total surface of the classifier feed must equal that of the classifier products, and the other is based on two different methods of calculating the amount of

surface produced in the ball mills. These methods, which utilize the surface figures of all samples, give an excellent idea of the reliability of the figures.

TABLE 17.—*Tonnage of mineral sizes, tons per 24 hours*

INITIAL FEED, 1,100 TONS: SAMPLE 1

	Sieve sizes, mesh			Elutriation products				Total
	+48	48-100	100-200	E1	E2	E3	E4	
Galena.....	96.24	16.01	9.39	11.47	2.12	1.36	2.88	139.47
Sphalerite.....	109.22	14.33	9.91	8.94	3.09	1.98	2.44	149.91
Pyrite.....	220.80	35.84	21.09	21.16	5.41	3.96	6.27	314.53
Gangue.....	361.75	22.75	19.93	18.43	15.08	17.48	40.60	496.02
Total.....	788.01	88.93	60.32	60.00	25.70	24.78	52.19	1,099.93

PRIMARY CLASSIFIER SAND, 640 TONS: SAMPLE 2

Galena.....	28.34	63.20	26.48	12.05	1.46	0.65	1.33	133.51
Sphalerite.....	34.49	24.59	7.35	3.67	1.19	.67	1.09	73.08
Pyrite.....	108.28	111.86	23.53	10.59	1.88	.99	1.87	259.00
Gangue.....	124.60	18.84	9.55	6.59	4.06	3.09	7.82	174.45
Total.....	295.61	218.49	66.91	32.90	8.59	5.40	12.11	640.01

PRIMARY MILL DISCHARGE, 1,740 TONS: SAMPLE 3

Galena.....	33.07	87.01	57.98	71.79	9.35	5.12	15.79	280.11
Sphalerite.....	45.32	61.22	38.71	41.44	11.81	7.04	12.54	218.08
Pyrite.....	134.63	197.17	96.57	87.24	17.91	10.40	23.70	567.62
Gangue.....	205.69	113.91	84.45	75.04	49.14	37.83	108.09	516.53
Total.....	418.71	459.31	277.71	275.51	88.21	60.39	160.12	1,739.96

PRIMARY CLASSIFIER OVERFLOW, 1,100 TONS: SAMPLE 4

Galena.....	2.83	18.73	31.01	59.27	7.24	4.11	12.59	135.78
Sphalerite.....	11.69	39.65	30.59	36.32	11.30	6.40	11.41	147.36
Pyrite.....	15.72	90.49	72.00	77.14	16.02	8.81	20.33	300.51
Gangue.....	92.98	100.55	69.61	70.79	46.86	35.38	100.36	516.53
Total.....	123.22	249.42	203.21	243.52	81.42	54.70	144.69	1,100.18

SECONDARY CLASSIFIER SAND, 1,270 TONS: SAMPLE 5

Galena.....	3.56	38.40	88.65	74.96	1.53	0.68	2.12	209.90
Sphalerite.....	9.13	85.06	62.77	20.09	2.13	.95	1.60	181.73
Pyrite.....	17.01	213.41	247.44	72.81	3.38	1.56	3.15	558.76
Gangue.....	104.29	134.33	45.54	17.05	6.66	3.81	8.12	319.40
Total.....	133.99	471.20	444.40	184.91	13.30	7.00	14.99	1,269.79

SECONDARY MILL DISCHARGE, 1,270 TONS: SAMPLE 6

Galena.....	1.42	17.35	58.99	94.55	6.65	3.83	12.02	194.81
Sphalerite.....	3.51	44.45	59.38	45.83	10.77	6.43	10.18	180.55
Pyrite.....	7.22	126.60	226.33	142.08	21.77	11.42	20.71	556.13
Gangue.....	21.65	85.44	73.01	63.14	34.71	21.23	39.68	338.76
Total.....	33.80	273.84	417.71	345.60	73.90	42.91	82.49	1,270.25

TABLE 17.—Tonnage of mineral sizes, tons per 24 hours—Continued
SECONDARY CLASSIFIER OVERFLOW (FLOTATION FEED), 1,100 TONS: SAMPLE 7

	Sieve sizes, mesh			Elutriation products				Total
	+48	48-100	100-200	E1	E2	E3	E4	
Galena.....		1.16	6.11	72.50	15.10	8.35	21.96	125.18
Sphalerite.....		4.55	27.70	59.83	19.90	12.02	19.72	143.72
Pyrite.....		5.94	50.09	136.37	34.43	18.70	38.79	284.32
Gangue.....		65.66	103.20	114.19	75.67	52.03	135.91	546.66
Total.....		77.31	187.10	382.89	145.10	91.10	216.38	1,099.88

TABLE 18.—Mineral balance, tons per 24 hours

Machine	Galena	Sphalerite	Pyrite	Gangue	Total
Primary mill feed.....	272.98	222.96	573.53	670.47	1,739.94
Primary mill discharge.....	280.11	218.08	567.62	674.15	1,739.96
Primary classifier feed.....	280.11	218.08	567.62	674.15	1,739.96
Primary classifier discharge.....	269.29	220.41	559.51	690.98	1,740.19
Secondary mill feed.....	209.90	181.73	558.76	319.40	1,269.79
Secondary mill discharge.....	194.81	180.55	556.13	338.76	1,270.25
Secondary classifier feed.....	330.59	327.91	856.64	855.29	2,370.43
Secondary classifier discharge.....	335.08	325.45	843.08	866.06	2,369.67

TOTAL TONNAGE STREAM

Initial feed.....	139.47	149.91	314.53	496.02	1,099.93
Primary classifier overflow.....	135.78	147.36	300.51	516.53	1,100.18
Secondary classifier overflow.....	125.18	143.72	284.32	546.66	1,099.88
Average.....	133.48	147.00	299.79	519.74	1,100.00

TABLE 19.—Sized mineral balance, tons per 24 hours

Tons
 Primary classifier: Feed..... 1,740 primary mill discharge
 Discharge..... 640 sand
 1,100 overflow
 1,740 total

	Sieve sizes, mesh			Elutriation products				Total
	+48	48-100	100-200	E1	E2	E3	E4	
Galena:								
Feed.....	33.07	87.01	57.98	71.79	9.35	5.12	15.79	280.11
Discharge.....	31.17	81.93	57.49	71.32	8.70	4.76	13.92	269.29
Sphalerite:								
Feed.....	45.32	61.22	38.71	41.44	11.81	7.04	12.54	218.08
Discharge.....	46.18	64.24	37.94	39.99	12.49	7.07	12.50	220.41
Pyrite:								
Feed.....	134.63	197.17	96.57	87.24	17.91	10.40	23.70	567.62
Discharge.....	124.00	202.35	95.53	87.73	17.90	9.80	22.20	559.51
Gangue:								
Feed.....	205.69	113.91	84.45	75.04	49.14	37.83	108.09	674.15
Discharge.....	217.48	119.39	79.16	77.38	50.92	38.47	108.18	690.98
Total:								
Feed.....	418.71	459.31	277.71	275.51	88.21	60.39	160.12	1,739.96
Discharge.....	418.83	467.91	270.12	276.42	90.01	60.10	156.80	1,740.19

TABLE 19.—*Sized mineral balance, tons per 24 hours—Continued*

Secondary classifier: Feed.....	Tons	1,100 primary classifier overflow
		1,270 secondary mill discharge
		<u>2,370 total</u>
Discharge.....	Tons	1,270 sand
		1,100 overflow
		<u>2,370 total</u>

	Sieve sizes, mesh			Elutriation products				Total
	+48	48-100	100-200	E1	E2	E3	E4	
Galena:								
Feed.....	4.25	36.08	90.00	153.82	13.89	7.94	24.61	330.59
Discharge.....	3.56	39.56	94.76	147.46	16.63	9.03	24.08	335.08
Sphalerite:								
Feed.....	15.20	84.10	89.97	82.15	22.07	12.83	21.59	327.91
Discharge.....	9.13	89.61	90.47	79.92	22.03	12.97	21.32	325.45
Pyrite:								
Feed.....	22.94	217.09	298.33	219.22	37.79	20.23	41.04	856.64
Discharge.....	17.01	219.35	297.53	209.18	37.81	20.26	41.94	843.08
Gangue:								
Feed.....	114.63	185.99	142.62	133.93	81.57	56.61	139.94	855.29
Discharge.....	104.29	199.99	148.74	131.24	81.93	55.84	144.03	866.06
Total:								
Feed.....	157.02	523.26	620.92	589.12	155.32	97.61	227.18	2,370.43
Discharge.....	133.99	548.51	631.50	567.80	158.40	98.10	231.37	2,369.67

TABLE 20.—*Surface figures for all sizes of minerals*

INITIAL FEED: SAMPLE 1

	Surface produced per 24 hours: cm ² times 10 ⁶							Total
	+48	48-100	100-200	E1	E2	E3	E4	
Galena.....	2,502	1,038	1,113	7,618	2,654	4,626	44,146	63,697
Sphalerite.....	5,898	1,924	2,384	7,882	4,905	8,656	47,581	79,230
Pyrite.....	10,157	4,148	4,400	17,532	8,096	16,019	113,169	173,521
Gangue.....	34,366	5,472	8,527	19,223	28,723	85,614	883,781	1,065,706
Total.....	52,923	12,582	16,424	52,255	44,378	114,915	1,088,677	1,382,154

PRIMARY CLASSIFIER SAND: SAMPLE 2

Galena.....	737	4,099	3,139	8,004	1,827	2,211	20,387	40,404
Sphalerite.....	1,862	3,301	1,768	3,235	1,889	2,929	21,256	36,240
Pyrite.....	4,981	12,946	4,909	8,774	2,814	4,005	33,752	72,181
Gangue.....	11,828	4,532	4,086	6,874	7,733	15,134	170,226	220,413
Total.....	19,408	24,878	13,902	26,887	14,263	24,279	245,621	369,238

PRIMARY MILL DISCHARGE: SAMPLE 3

Galena.....	860	5,643	6,873	47,683	11,703	17,414	242,034	332,210
Sphalerite.....	2,447	8,218	9,311	36,534	18,745	30,777	244,536	350,568
Pyrite.....	6,193	22,819	20,145	72,282	26,803	42,070	427,768	618,080
Gangue.....	19,541	27,400	36,130	78,270	93,597	185,284	2,352,903	2,793,125
Total.....	29,041	64,080	72,459	234,769	150,848	275,545	3,267,241	4,093,983

TABLE 20.—Surface figures for all sizes of minerals—Continued

PRIMARY CLASSIFIER OVERFLOW: SAMPLE 4

	Surface produced per 24 hours; cm ² times 10 ⁶							
	+48	48-100	100-200	E1	E1	E3	E4	Total
Galena.....	74	1, 215	3, 676	39, 367	9, 062	13, 979	192, 983	260, 356
Sphalerite.....	631	5, 322	7, 358	32, 020	17, 936	27, 979	222, 501	313, 747
Pyrite.....	723	10, 472	15, 020	63, 914	23, 975	35, 638	366, 942	516, 684
Gangue.....	8, 833	24, 186	29, 781	73, 838	89, 254	173, 284	2, 184, 636	2, 583, 812
Total.....	10, 261	41, 195	55, 835	209, 139	140, 227	250, 880	2, 967, 062	3, 674, 599

SECONDARY CLASSIFIER SAND: SAMPLE 5

Galena.....	93	2, 490	10, 509	49, 788	1, 915	2, 313	32, 496	99, 604
Sphalerite.....	493	11, 418	15, 098	17, 711	3, 381	4, 153	31, 201	83, 455
Pyrite.....	782	24, 699	51, 618	60, 326	5, 058	6, 311	56, 851	205, 649
Gangue.....	9, 908	32, 311	19, 484	17, 784	11, 923	18, 661	176, 756	286, 827
Total.....	11, 276	70, 918	96, 709	145, 609	22, 277	31, 438	297, 308	675, 535

SECONDARY MILL DISCHARGE: SAMPLE 6

Galena.....	37	1, 125	6, 993	62, 800	8, 324	13, 027	184, 246	276, 552
Sphalerite.....	190	5, 967	14, 283	40, 404	17, 095	28, 110	198, 515	304, 564
Pyrite.....	332	14, 652	47, 215	117, 720	32, 580	46, 196	373, 801	632, 496
Gangue.....	2, 057	20, 551	31, 236	65, 858	66, 112	103, 980	861, 577	1, 151, 371
Total.....	2, 616	42, 295	99, 727	286, 782	124, 111	191, 313	1, 618, 139	2, 364, 983

SECONDARY CLASSIFIER OVERFLOW (FLOTATION FEED): SAMPLE 7

Galena.....	75	724	48, 154	18, 900	28, 400	336, 609	432, 862
Sphalerite.....	611	6, 063	52, 746	31, 586	52, 548	384, 550	528, 704
Pyrite.....	687	10, 449	112, 989	51, 526	75, 646	700, 132	951, 429
Gangue.....	15, 794	44, 152	119, 106	144, 129	254, 833	2, 958, 489	3, 536, 503
Total.....	17, 167	61, 988	332, 995	246, 141	411, 427	4, 379, 780	5, 449, 498

Table 21 gives a check on the first of the methods by a classifier balance for both primary and secondary classifiers.

The surface produced in the grinding mills is the difference between the surface in the discharge and the feed and between the surface of the entire stream preceding and following each mill. These figures are given in table 22.

Except for the galena in the primary classifier the classifier figures check with reasonable accuracy. The fact that the same difference also occurs in the surface produced in grinding probably indicates that the surface figures for galena in primary classifier sand are too low. This difference may be due to sampling.

GRINDING EFFICIENCY

Both methods of calculating surface produced in the grinding mills should give the same result, and their agreement is a criterion of their reliability. The figures based on the entire stream are used in calculating the power figures in table 23 as they are thought to be more reliable owing to facilities for getting better samples.

TABLE 21.—*Surface balance for classifiers*

	Sample	Surface per day (cm ² × 10 ⁶)				
		Galena	Sphalerite	Pyrite	Gangue	Total
Primary classifier:						
Going to classifier, primary mill discharge.....	3	332, 210	350, 570	618, 080	2, 793, 120	4, 093, 980
Coming from classifier, sand and overflow.....	2-4	300, 760	349, 990	588, 870	2, 804, 220	4, 043, 840
Secondary classifier:						
Going to classifier, primary classifier overflow and secondary mill discharge.....	4-6	536, 910	618, 310	1, 149, 180	3, 735, 180	6, 039, 580
Coming from classifier, sand and overflow.....	5-7	532, 460	612, 160	1, 157, 080	3, 823, 330	6, 125, 030

TABLE 22.—*Surface produced in grinding mills*

	Sample	Surface per day (cm ² × 10 ⁶)				
		Galena	Sphalerite	Pyrite	Gangue	Total
Primary mill:						
Total discharge and feed:						
Coming from mill, primary mill discharge.....	3	332, 210	350, 570	618, 080	2, 793, 120	4, 093, 980
Going to mill, initial feed and primary classifier sand.....	1-2	104, 100	115, 470	245, 700	1, 286, 120	1, 751, 390
Surface produced in primary mill.....		228, 110	235, 100	372, 380	1, 507, 000	2, 342, 590
Entire stream:						
Primary classifier overflow.....	4	260, 360	313, 750	516, 680	2, 583, 810	3, 674, 600
Initial feed.....	1	63, 700	79, 230	173, 520	1, 065, 700	1, 382, 150
Surface produced in primary mill.....		196, 660	234, 520	343, 160	1, 518, 110	2, 292, 450
Secondary mill:						
Total discharge and feed:						
Coming from mill, secondary mill discharge.....	6	276, 550	304, 560	632, 500	1, 151, 370	2, 364, 980
Going to mill, secondary classifier sand.....	5	99, 600	83, 450	205, 650	286, 830	675, 530
Surface produced in secondary mill.....		176, 950	221, 110	426, 850	864, 540	1, 689, 450
Entire stream:						
Secondary classifier overflow.....	7	432, 860	528, 710	951, 430	3, 536, 500	5, 449, 500
Primary classifier overflow.....	4	260, 360	313, 750	516, 680	2, 583, 810	3, 674, 600
Surface produced in secondary mill.....		172, 500	214, 960	434, 750	952, 690	1, 774, 900

TABLE 23.—*Kilowatt-days represented by surface produced in mills*

	Kilowatt-days represented by surface				
	Galena	Sphalerite	Pyrite	Gangue	Total
Primary mill.....	2. 366	4. 732	17. 201	98. 196	122. 495
Secondary mill.....	2. 075	4. 337	21. 792	61. 623	89. 827
Both mills.....	4. 441	9. 069	38. 993	159. 819	212. 322
Work on each mineral in primary mill..... percent.....	1. 93	3. 86	14. 04	80. 17	100. 00
Work on each mineral in secondary mill..... do.....	2. 31	4. 83	24. 26	68. 60	100. 00
Work on each mineral in both mills..... do.....	2. 09	4. 27	18. 37	75. 27	100. 00

Table 22 gives the distribution of surface for the various minerals but cannot be used directly to calculate efficiency. The amount of work required to produce the same degree of fineness or the same amount of surface differs with different minerals, depending on their grinding resistance. Figures have been determined for the minerals involved in the present study (93). The figures on crushing resistance of quartz, based on surface measurement, are: Quartz, 1.00; pyrite, 0.775; sphalerite, 0.312; galena, 0.186.

It has been mentioned that when quartz was crushed in the drop-weight machine 1 kg-cm of work was expended to produce 17.56 cm² of surface.

If the factors obtained for crushing resistance are applied and kilogram-centimeters are converted to kilowatt-days, 1 kilowatt-day should produce:

Quartz surface.....	<i>Cm²</i> 15.460 × 10 ⁶
Pyrite surface.....	19.950 × 10 ⁶
Sphalerite surface.....	49,560 × 10 ⁶
Galena surface.....	83,130 × 10 ⁶

In using these figures the efficiency of the results is, of course, compared to that of the drop-weight machine at 100 percent. From these figures the kilowatt-days, represented by the surface figures in table 22, are calculated in table 23.

The power used for the grinding mills was obtained by taking power readings during sampling and is the power input to the motors.

Table 24 gives the final efficiencies. These figures show a high total grinding efficiency which is largely useless, as the material is ground finer than needed. Table 23 shows the large amount of work expended in grinding the gangue. This suggests a possibility of separating gangue at a coarser size to avoid undue grinding, if a clean gangue is separable.

USEFUL GRINDING EFFICIENCIES OF MILLS

The efficiencies obtained indicate little opportunity for improvement in grinding appliances or their operation. In most metallurgical operations there is a size at which recovery is maximum; any further grinding represents waste power and is designated as overgrinding or useless grinding.

If ore is ground to 200-mesh the grinding usually may be looked upon as satisfactory for flotation. Most of the product, however, will be much finer than 200-mesh or 74 microns. By elutriating the minus 200-mesh material the critical size and the amount of useless grinding may be determined.

TABLE 24.—*Mill efficiencies*

	Primary mill	Secondary mill	Both mills
Power input to motor.....kilowatts..	260.4	213.6	474.0
Work represented in crushing.....	122.495	89.827	212.322
Efficiency compared to drop-weight figure of 17.56 cm. per kg-cm, percent..	47.04	42.05	44.80

The critical size for optimum recovery on this ore varied somewhat for the different minerals, but for all practical purposes the highest grade and best recoveries were made in the E1 elutriation size. Any grinding beyond this size may be looked upon as not only useless but detrimental. To demonstrate the calculation on useless work the critical size for all minerals is taken as that corresponding to the size of the E1 product.

From the expression for useless work, $\frac{S_1 - S_2}{S} 100$, in which S_1 is the actual surface produced in E1 and finer sizes, S_2 is the actual surface produced in E1 if all were just E1 size, and S is the total actual surface produced, the results in table 25 are obtained.

These figures show that a large percentage of the work used in grinding is useless. A larger percentage of useless work is done in the secondary than in the primary mills. This is to be expected, as the secondary mill is essentially the fine-grinding unit. The figures on useless grinding do not represent consumption of power as the more easily crushed minerals require much less power.

TABLE 25.—Useless work in grinding mills

	Surface per day (cm ² × 10 ⁶)				
	Galena	Sphalerite	Pyrite	Gangue	Total
Primary mill:					
Surface on E1 and finer..... S_1	196,350	231,410	335,650	1,503,670	2,267,080
Surface on E1 and finer if all were E1 size. S_2	47,880	47,610	78,100	186,070	359,660
Useless surface..... $S_1 - S_2$	148,470	183,800	257,550	1,317,600	1,907,420
Total surface produced..... S	196,660	234,520	343,160	1,518,110	2,292,450
Useless surface..... percent.	75.5	78.4	75.1	86.8	83.2
Secondary mill:					
Surface on E1 and finer..... S_1	176,670	220,990	449,820	955,550	1,803,030
Surface on E1 and finer if all were E1 size. S_2	25,410	44,750	96,820	143,070	310,050
Useless surface..... $S_1 - S_2$	151,260	176,240	353,000	812,480	1,492,980
Total surface produced..... S	172,500	214,960	434,750	952,690	1,774,900
Useless surface..... percent.	87.7	82.0	81.2	85.3	84.1
Both mills:					
Surface on E1 and finer..... S_1	373,020	452,400	785,470	2,459,220	4,070,110
Surface on E1 and finer, if all were E1 size. S_2	73,290	92,360	174,920	329,140	669,710
Useless surface..... $S_1 - S_2$	299,730	360,040	610,550	2,130,080	3,400,400
Total surface produced..... S	369,160	449,480	777,910	2,470,800	4,067,350
Useless surface..... percent.	81.2	80.1	78.5	86.2	83.6

From the factors already given for the production of surface per kilowatt-day the actual energy expended in useless work for each mineral may be calculated as in table 26.

The figures on power expended in useless grinding indicate that the greatest economy in grinding is possible through the reduction of useless grinding. This is especially true of the gangue and pyrite and shows the desirability of removing them from the grinding circuit before the ore is ground finer than necessary.

Although the grinding efficiency of the mills in table 24 appears to be rather high, the useful efficiency leaves much to be desired.

FINENESS OF MINUS 200-MESH MATERIAL

Sieve analyses give no information on overgrinding or useless work, but elutriation does. Considerable data as to the minus 200-mesh material may be obtained from table 17. Table 27 shows the effect of primary and secondary grinding on the fineness of the minus 200-mesh material. The minus 200-mesh material in the initial feed is compared with that produced in the two mills.

TABLE 26.—Power consumed in useless work

	Energy represented by useless work, kilowatt-days			Useless work, percent of total work input		
	Primary, 122.40	Secondary, 89.91	Both mills, 212.31	Primary	Secondary	Both mills
Galena.....	1.79	1.82	3.61	1.46	2.02	1.70
Sphalerite.....	3.71	3.56	7.27	3.03	3.96	3.42
Pyrite.....	12.91	17.69	30.60	10.55	19.68	14.41
Gangue.....	85.23	52.55	137.78	69.63	58.45	64.90
Total.....	103.64	75.62	179.26	84.67	84.11	84.43

TABLE 27.—Minus 200-mesh material produced in mills

	Elutriation sizes					Percent of size in product			
	E1	E2	E3	E4	Total	E1	E2	E3	E4
Galena:									
Initial feed.....	11.47	2.12	1.36	2.88	17.83	64.3	11.9	7.6	16.2
Produced in primary mill.....	47.80	5.12	2.75	9.71	65.38	73.1	7.8	4.2	14.9
Produced in secondary mill.....	13.23	7.86	4.24	9.37	34.70	38.1	22.7	12.2	27.0
Sphalerite:									
Initial feed.....	8.94	3.09	1.98	2.44	16.45	54.4	18.8	12.0	14.8
Produced in primary mill.....	27.38	8.21	4.42	8.97	48.98	55.9	16.8	9.0	18.3
Produced in secondary mill.....	23.51	8.60	5.62	8.31	46.04	51.1	18.7	12.2	18.0
Pyrite:									
Initial feed.....	21.16	5.41	3.96	6.27	36.80	57.5	14.7	10.8	17.0
Produced in primary mill.....	55.98	10.61	4.85	14.06	85.50	65.5	12.4	5.7	16.4
Produced in secondary mill.....	59.23	18.41	9.89	18.46	105.99	55.9	17.4	9.3	17.4
Gangue:									
Initial feed.....	18.43	15.08	17.48	40.60	91.59	20.1	16.5	19.1	44.3
Produced in primary mill.....	52.36	31.78	17.90	59.76	161.80	32.4	19.6	11.1	36.9
Produced in secondary mill.....	43.40	28.81	16.65	35.55	124.41	34.9	23.1	13.4	28.6
Total:									
Initial feed.....	60.00	25.70	24.78	52.19	162.67	36.9	15.8	15.2	32.1
Produced in primary mill.....	183.52	55.72	29.92	92.50	361.66	50.7	15.4	8.3	25.6
Produced in secondary mill.....	139.37	63.68	36.40	71.69	311.14	44.8	20.5	11.7	23.0

The minus 200-mesh sulphides produced by the primary mill are coarser than those in the initial feed, whereas those produced by the secondary mill are finer. This indicates more useless grinding in the secondary mill, especially on galena, due to its low resistance to grinding and its presence in excessive amounts in the circulating load. The minus 200-mesh gangue becomes coarser as it passes through the grinding circuit, partly because of its higher resistance to grinding and partly because of its more rapid removal by the classifiers.

Other tables may be prepared from the data on minus 200-mesh material to view the results from different angles, but all show excessively fine grinding, especially on galena, and great variation in fineness of the minus 200-mesh material.

EFFECT OF SIZE ON RECOVERY

While the amount of the various sizes recovered is not a part of the grinding problem it does enter into the question of desirable size of grinding. The results obtained in sampling the flotation products while the grinding circuit was being sampled are therefore given.

When the bulk of the ore is ground finer than 200-mesh the recovery of the various sizes can be studied by recourse to elutriation. The flotation samples were treated in the same manner as those from the grinding circuit, and the results are given in table 28. This table lists the distribution of the minerals by sizes in the different products. These data show that the best recovery for galena and sphalerite is obtained on the E1 size and the poorest on the E4 size. Thus the E1 size is the critical size for best recovery; material coarser than this gives poorer recovery because of unfreed particles, and material finer than this is not so amenable to flotation and represents useless grinding. The lower recovery on the sizes finer than the E1 may be due, partly at least, to oxidized sulphides. A determination showed that the E4 tailing contained much oxidized lead.

Various curves and tables may be prepared from these data and the effect of size on recovery studied therefrom, but the following table will suffice as an example.

TABLE 28.—*Distribution of minerals in flotation products*

	Plus 200-mesh size		E1 size		E2 size		E3 size		E4 size		Total	
	Tons	Weight (percent)	Tons	Weight (percent)	Tons	Weight (percent)	Tons	Weight (percent)	Tons	Weight (percent)	Tons	Weight (percent)
Galena in—												
Lead concentrates	5.64	77.6	67.97	93.8	13.94	92.3	7.04	84.3	12.52	57.0	107.11	85.6
Zinc concentrates29	4.0	1.44	2.0	.51	3.4	.43	5.2	1.62	7.4	4.29	3.4
Iron concentrates65	8.9	1.12	1.5	.32	2.1	.41	4.9	2.25	10.2	4.75	3.8
Tailing69	9.5	1.97	2.7	.33	2.2	.47	5.6	5.57	25.4	9.03	7.2
Total	7.27		72.50		15.10		8.35		21.96		125.18	
Sphalerite in—												
Zinc concentrates	26.32	81.6	51.75	86.5	16.48	82.8	9.63	80.1	13.61	69.0	117.79	82.0
Lead concentrates	1.55	4.8	6.32	10.6	2.55	12.8	1.36	11.3	1.74	8.8	13.52	9.4
Iron concentrates	2.93	9.1	1.39	2.3	.50	2.5	.56	4.7	1.85	9.4	7.23	5.0
Tailing	1.45	4.5	.37	.6	.37	1.9	.47	3.9	2.52	12.8	5.18	3.6
Total	32.25		59.83		19.90		12.02		19.72		143.72	
Pyrite in—												
Iron concentrates	31.21	55.7	73.99	54.3	17.53	50.9	5.47	29.3	4.89	12.6	133.09	46.8
Lead concentrates	1.52	2.7	6.91	5.1	1.60	4.7	.83	4.4	1.73	4.5	12.59	4.4
Zinc concentrates	2.85	5.1	7.14	5.2	1.46	4.2	.66	3.5	.97	2.5	13.08	4.6
Tailing	20.45	36.5	48.33	35.4	13.84	40.2	11.74	62.8	31.20	80.4	125.56	44.2
Total	56.03		136.37		34.43		18.70		38.79		284.32	
Gangue in—												
Tailing	164.10	97.2	102.21	89.5	67.28	88.9	45.17	86.8	116.76	85.9	495.52	90.6
Lead concentrates76	.4	3.77	3.3	1.80	2.4	1.81	3.5	6.43	4.7	14.57	2.7
Zinc concentrates	1.35	.8	2.36	2.1	.80	1.1	.42	.8	1.48	1.1	6.41	1.2
Iron concentrates	2.65	1.6	5.85	5.1	5.79	7.6	4.63	8.9	11.24	8.3	30.16	5.5
Total	168.86		114.19		75.67		52.03		135.91		546.66	

CLASSIFIER EFFICIENCY

Although a study of classifier operation may not come strictly within the scope of a grinding study the efficiency of the classifier has a pronounced effect on the results (83, 86, 99). It is interesting to note that the data in table 17 may be used to calculate classifier efficiency, and a brief discussion thereon follows. Table 29 gives the amount and percentage of mineral of each size in the overflow.

The data show clearly how classification differs from true sizing and results in excessively large gangue particles going to the overflow and excessively fine sulphides failing to overflow. The former is not a serious defect, but the latter may well be. The fact that the E2, E3, and E4 sizes do not overflow completely may be attributed largely to their suspension in the water accompanying the sand. This suggests the use of a wash water.

Table 30 shows that much of the heavier minerals, already ground fine enough, failed to overflow. The minus 200-mesh size should not be ground further, yet the tonnage of this sent back for regrinding is large. The figures in table 30 also indicate how improper sizing may result in useless work. The heavier the mineral the lower the efficiency and the less the circulating load is loaded with the sulphide minerals. This suggests their removal by table concentration or by flotation from the circulating load.

TABLE 29.—Classifier efficiency, tons per day

	Sieve sizes, mesh			Elutriation sizes			
	+48	48-100	100-200	E1	E2	E3	E4
Primary classifier; 54.5 percent pulp density:							
Galena:							
Sand..... tons..	28.34	63.20	26.48	12.05	1.46	0.65	1.33
Overflow..... do.....	2.83	18.73	31.01	59.27	7.24	4.11	12.59
Overflow..... percent..	9.1	22.9	53.9	83.1	83.2	86.3	90.4
Sphalerite:							
Sand..... tons..	34.49	24.59	7.35	3.67	1.19	.67	1.09
Overflow..... do.....	11.69	39.65	30.59	36.32	11.30	6.40	11.41
Overflow..... percent..	25.3	61.7	80.6	90.8	90.5	90.5	91.3
Pyrite:							
Sand..... tons..	108.28	111.86	23.53	10.59	1.88	.99	1.87
Overflow..... do.....	15.72	90.49	72.00	77.14	16.02	8.81	20.33
Overflow..... percent..	12.7	44.7	75.4	87.9	89.5	89.9	91.6
Gangue:							
Sand..... tons..	124.50	18.84	9.55	6.59	4.06	3.09	7.82
Overflow..... do.....	92.98	100.55	69.61	70.79	46.86	35.38	100.36
Overflow..... percent..	42.8	84.2	87.9	91.5	92.0	92.0	92.8
Secondary classifier; 38.8 percent pulp density:							
Galena:							
Sand..... tons..	3.56	38.40	88.65	74.96	1.53	.68	2.12
Overflow..... do.....		1.16	6.11	72.50	15.10	8.35	21.96
Overflow..... percent..		2.9	6.4	49.2	90.8	92.5	91.2
Sphalerite:							
Sand..... tons..	9.13	85.06	62.77	20.09	2.13	.95	1.60
Overflow..... do.....		4.55	27.70	59.83	19.90	12.02	19.72
Overflow..... percent..		5.1	30.6	74.9	90.3	92.7	92.5
Pyrite:							
Sand..... tons..	17.01	213.41	247.44	72.81	3.38	1.56	3.15
Overflow..... do.....		5.94	50.09	136.37	34.43	18.70	38.79
Overflow..... percent..		2.7	16.8	65.2	91.1	92.3	92.5
Gangue:							
Sand..... tons..	104.29	134.33	45.54	17.05	6.26	3.81	8.12
Overflow..... do.....		65.66	103.20	114.19	75.67	52.03	135.91
Overflow..... percent..		32.8	69.4	87.0	92.4	93.2	94.4

TABLE 30.—Classification on minus 200-mesh minerals, tons per 24 hours

Mineral	Primary classifier			Secondary classifier		
	Minus 200-mesh, tons		Minus 200-mesh, percent	Minus 200-mesh, tons		Minus 200-mesh, percent
	Going to classifier	Going to sand	Going to sand	Going to classifier	Going to sand	Going to sand
Galena	98.70	15.49	15.7	197.20	79.29	40.2
Sphalerite	72.05	6.62	9.2	136.24	24.77	18.2
Pyrite	137.63	15.33	11.1	309.19	80.90	26.2
Gangue	274.95	21.56	7.8	413.04	35.24	8.5
Total	583.33	59.00	10.1	1,055.67	220.20	20.9

USELESS WORK

In grinding for metallurgical work the desired product often consists largely of minus 200-mesh material. Sieve analyses are of little help in determining efficiencies in such operations, and surface figures only will yield this information. An example of the use of surface figures to determine useless work has been given for an actual mill operation.

The energy used in making the fine material indicates that much of the work expended in crushing is dissipated in overgrinding or in producing material finer than required. This is useless work and may be determined when the critical size is known and a sizing analysis made to determine the percentage of undesirable size. Having determined the surface for the total crushed product (S), for the undesirable portion (S_1), and for the material of desired fineness (S_2) the useless work, expressed as a percentage of the total work input, is $\frac{S_1 - S_2}{S} 100$.

S and S_1 must be determined, while S_2 may be taken from the curve in figure 12.

Results for laboratory experiments in ball milling of quartz are now considered on the basis of useless grinding. Three experiments have been chosen from a long series. One represents coarse grinding with the smallest production of minus 400-mesh material, one intermediate grinding, and one very fine grinding with the largest production of minus 400-mesh material. The feed for these experiments was composed of 52 percent of ½-inch to 8-mesh, 24 percent 8- to 14-mesh, 15 percent 14- to 28-mesh, and 9 percent 28- to 48-mesh material.

The importance of surface in the minus 200-mesh size is shown in table 31, in which surface calculations are given for the three experiments.

In these results it is assumed that material finer than 37 microns (1,170 cm² per gram from fig. 12) is not desirable, and the useless work has been calculated on this basis.

In these examples a large part of the work was expended on the minus 200-mesh material; even in the coarsest grinding 9.1 percent by weight represents 85.1 percent of the work. The results of these experiments, which are typical, show the importance of determining the extent to which the fine material affects the efficiency.

TABLE 31.—*Surface data on quartz grinding*

PRODUCING SMALL AMOUNT OF MINUS 400-MESH

Mesh	Percent by weight	Surface in product			Work represented (percent)	Useless work (percent)
		Total	Old	New		
+200.....	90.9	105	50	55	14.9	} 62.2
-200+400.....	2.7	23	4	19	5.2	
-400.....	6.4	304	10	294	79.9	
	100.0	432	64	368	100.0	

PRODUCING MEDIUM AMOUNT OF MINUS 400-MESH

+200.....	52.8	179	21	158	7.3	} 71.2
-200+400.....	13.1	111	8	103	4.8	
-400.....	34.1	1,936	35	1,901	87.9	
	100.0	2,226	64	2,162	100.0	

PRODUCING LARGE AMOUNT OF MINUS 400-MESH

+200.....	3.6	18	2	16	0.2	} 85.4
-200+400.....	15.0	131	6	125	1.8	
-400.....	81.4	7,016	56	6,960	98.0	
	100.0	7,165	64	7,101	100.0	

In the entire series of experiments on ball milling of quartz the useless work, based on 37 microns as the critical size, ranged from 62 to 94 percent. This would indicate that the elimination of over-grinding might increase grinding efficiency.

RELATIVE GRINDABILITY OF ROCKS

The term "grindability" or "crushing resistance" refers to the new surface produced in relation to the energy expended. It may be expressed as the Rittinger constant in terms of square centimeters of surface produced per foot-pound. As this figure depends on the elastic constants of the material and their relation to the ultimate strength, it is theoretically a constant for a given substance but only when crushing takes place in a specified way. For practical purposes it is desirable to know what this constant is for various substances and how it may vary with speed and method of breaking.

Koster (134), from a mathematical analysis of fracture, finds that fracture is not determined by a definite load but by a definite rate of deformation. Even the most brittle substances will adjust themselves to a slowly applied load, and fracture will not occur until the rate of load application or deformation has reached a certain amount. The Rittinger constant will depend therefore on the rate at which the load is applied, and it may be predicted that the energy required to produce fracture will be less with rapidly applied loads.

Koster has given formulas for calculating the Rittinger constant, but the numerical data necessary for quantitative use of the formulas are not available or easy to obtain. Table 32 gives the "Rittinger constant" calculated in terms of square centimeters of surface pro-

duced per foot-pound and shows that for the same material this "constant" may vary greatly under different conditions of crushing or grinding. Thus, while the constant is 243 for the drop-weight machine, it is only 36.1 for the most inefficient ball-mill experiment.

These figures show, in comparison to the drop-weight machine, 41.1 percent efficiency for the best ball-mill experiment and 14.8 percent efficiency for the poorest.

TABLE 32.—"Rittinger constant" for various conditions

Machine:	Material crushed	"Rittinger constant", surface produced per ft.-lb., cm ³
Drop weight.....	quartz	243. 0
Laboratory ball mill:		
40 r. p. m.....	do	77. 1
60 r. p. m.....	do	82. 1
80 r. p. m.....	do	79. 6
5 percent pulp density by volume.....	do	42. 6
10 percent pulp density by volume.....	do	60. 3
20 percent pulp density by volume.....	do	72. 5
30 percent pulp density by volume.....	do	78. 0
40 percent pulp density by volume.....	do	81. 7
50 percent pulp density by volume.....	do	82. 1
60 percent pulp density by volume.....	do	73. 9
1½-inch balls.....	do	83. 2
1¼-inch balls.....	do	85. 4
1-inch balls.....	do	85. 4
¾-inch balls.....	do	88. 0
½-inch balls.....	do	80. 9
36-pound ball charge.....	do	36. 1
71-pound ball charge.....	do	65. 7
142-pound ball charge.....	do	82. 1
178-pound ball charge.....	do	91. 7
249-pound ball charge.....	do	78. 6
4-pound quartz charge.....	do	99. 9
8-pound quartz charge.....	do	92. 0
16-pound quartz charge.....	do	81. 1
28-pound quartz charge.....	do	73. 8
Galena.....	galena	1, 324
Sphalerite.....	sphalerite	789
Pyrite.....	pyrite	317
Calcite.....	calcite	1, 068
Drop weight.....	ore ¹	507
Do.....	do. ²	525
Plant primary ball mill.....	do. ¹	138
Plant secondary ball mill.....	do. ²	131

¹ Ore contained 15.8 percent galena, 12.7 percent sphalerite, 32.8 percent pyrite, 38.7 percent gangue.

² Ore contained 15.9 percent galena, 14.3 percent sphalerite, 43.9 percent pyrite, 25.9 percent gangue.

The plant primary ball mills show an efficiency of 27.2 percent and the secondary mills of 25 percent. This Rittinger constant is a measure of the over-all efficiency of crushing. The low value for the ball mill compared to the drop-weight machine is doubtless due to a high percentage of ineffective blows.

METHODS OF DETERMINATION

Numerous methods have been used to determine the crushing accomplished per unit input of work. As most of these methods are based only on sieve analyses, they are of use only for comparative purposes and give no specific results.

USE OF SURFACE DIAGRAM

The crushing-surface diagram of Gates (16) has been used to compare grindability of ores, but as it depends on sieve sizing only it has limited application.

Lennox (26) made a series of grinding tests in a ball mill on a variety of ores, using the Gates crushing-surface diagram. The curves for feed and product were plotted from sieve sizes and extended into the minus 200-mesh zone.

Certain assumptions were made in calculating the area between the two curves, which is the measure of grindability. Rigid accuracy was not claimed for the treatment of minus 200-mesh material; the surface on an equal amount of minus 200-mesh material in the product was assumed to be the same as the minus 200-mesh ore in the feed. This is not exactly true, and as the amount of minus 200-mesh ore in the feed decreases the method becomes more difficult of solution; finally, when no minus 200-mesh ore is present in the feed, it becomes hopeless.

Hardgrove (129) has developed a ball grinding machine for determining grindability. The sieved product is compared to a standard material pulverized under identical conditions in the same machine.

USE OF DROP-WEIGHT MACHINE

The drop-weight machine employed by the Bureau of Mines (81) in determining the work expended in crushing was used to measure the crushing resistance of several minerals. With this apparatus the input of work was determined in crushing quartz, pyrite, sphalerite, calcite, and two varieties of galena. With the method of calculating surface that is based on a comparison of sieve and elutriation sizes with similar sizes of quartz (94) surface figures were obtained for the various minerals (93).

These figures are revised in table 33, in which all data are compared to quartz as unity. The two samples of galena 1 were duplicates. Galena 1 was a coarsely crystalline variety and galena 2 a finely crystalline (steel) variety. The weight of material crushed per unit of work, given in the last column, is the grindability.

USE OF SCLEROSCOPE

The possibility of using the scleroscope to measure grindability or crushing resistance has been investigated (133).

If a very hard body is allowed to fall on the smooth surface of a much larger fragile mass the energy expended will be partly absorbed, and the balance will appear as rebound. With extremely fragile material the energy will be completely absorbed, and there will be no rebound. With extremely tough material, such as rubber, no energy will be absorbed, and the rebound will substantially equal the drop. With material between these two limits part of the energy is absorbed and part appears as rebound. With malleable materials, such as metals and alloys, the absorbed energy is used to produce a zone of permanent deformation, and the percentage of rebound under certain specified conditions is defined as the scleroscope hardness.

TABLE 33.—Comparative crushing resistance or grindability, quartz taken as unity

Mineral	Surface produced per unit of work	Work required per unit of surface	Work required per unit of weight	Weight of material crushed per unit of work
Quartz.....	1.000	1.000	1.000	1.000
Pyrite.....	1.290	.775	.411	2.433
Sphalerite.....	3.205	.312	.207	4.831
Calcite.....	4.338	.231	.226	4.425
Galena 1.....	5.309	.188	.067	14.925
Galena 1.....	5.175	.193	.069	14.493
Galena 2.....	5.653	.177	.063	15.873
Galena average.....	5.379	.186	.0663	15.097

In brittle materials the zone of deformation becomes a zone of fracture, and to produce fracture it is necessary to move some of the material through a distance determined by its elastic constants. The energy absorbed will be proportional therefore to the weight of material fractured and may be used as a measure of resistance to crushing.

The exact pattern of the crushed area will depend upon the shape of the hammer and, to a smaller extent, on cleavages and other lines of weakness in the material. This resistance to grinding obviously is not a constant, as extremely heavy blows will tend to make materials more nearly equal in crushability. For the range of blows struck in ball milling, however, it was thought that the determination of energy absorbed by allowing a scleroscope hammer to fall on the surface of a block of mineral might be a reliable index of resistance to crushing.

Investigations.—Preliminary experiments were made by dropping steel balls on different minerals and noting the rebound. This simple method may be employed when a scleroscope is not available but is not as convenient or accurate as it is difficult to obtain a fair hit and to determine the amount of rebound. Roller-bearing balls were used and dropped through a 2-inch glass tube onto the specimen.

A ball that weighed 1.046 grams was found to be satisfactory; heavier balls caused undue damage to the specimen.

As the crushing resistance of quartz, galena, sphalerite, pyrite, and calcite had already been determined, these minerals were the only ones subjected to scleroscopic determinations.

Experiments with the scleroscope indicated that a hammer with a sharp end, such as a diamond-tipped or diamond-pointed steel hammer, was not satisfactory, owing to excessive local crushing of the material. A special, round-pointed, tungsten-carbide-tipped hammer that weighed 1.945 grams gave a minimum breakage of the specimen at the point of impact.

It was found that in experiments with the falling ball and the scleroscope three conditions affect the rebound:

1. *Size of specimen.*—Experiments indicated that with a specimen smaller than 1 inch in thickness an undue proportion of the energy passes into the anvil, thus giving low rebounds. With specimens of sufficient thickness (over 1 inch) the nature of the anvil appears to have no effect, and good rebound is obtained by simply holding the specimen in the hand.

2. *Degree of polish given specimen.*—In general, the higher the polish the higher the rebound, although it is unnecessary to carry the polishing to extremes. In some experiments rebounds were as good with fairly rough surfaces as with polished surfaces, although with rough or irregular surfaces rebounds are likely to be low.

3. *Angle of hammer contact.*—The falling hammer must hit the specimen at right angles to the surface. Even slight tilting of the specimen causes a glancing blow and a resultant low rebound.

In a scleroscope determination the specimen should not be clamped too tightly. It should, however, be held firmly enough to prevent it from shifting.

Repeated hits on the same spot give unreliable results. This is especially true of minerals, such as galena, which become compressed under successive blows and give increasing rebounds.

As imperfect hits (those near the edge of the specimen or in proximity to cracks) or hits on irregular surfaces give low rebounds, the highest rebound obtainable may be considered as that for a perfect crystal. The average rebound may be considered to represent the state of the particular specimen and would therefore indicate crushing resistance.

The standard scleroscope used had a 10-inch drop. To obtain smaller drops spare tubes were cut to one-third and two-thirds the length of the scleroscope. These were mounted in a frame in the standard scleroscope and fitted to the top-tripping mechanism and the lower end of the scleroscope. This arrangement assured equal release of the hammer and equal air vent for all three drops. Measurements could thus be made for 10-inch, 6.37-inch, and 2.75-inch drops.

Table 34 gives the results of the average rebounds. As the crushing resistances obtained with the drop-weight machine were obtained on minerals that could hardly have been perfect crystals the average rebounds would be better for comparison than the highest rebounds. However, the highest rebounds show the same relation to crushing resistance as the average rebounds, which indicates that a fairly definite weakness exists in all of the minerals and tends to lower resistance to crushing.

There is no direct relationship between specific gravity or hardness of the mineral and rebound, and as the scleroscope hammer actually performs work in crushing no such relationship would be expected. As the absorption of some of the drop is due to the work performed in crushing a definite volume of material, the straight-line relationship shown in figure 18 follows. These straight lines, however, do not pass through the origin but intersect the energy-absorbed axis at values approximately proportional to the height of drop. These intercepts represent the energy absorbed by an ungrindable material, therefore they are the elastic losses in the hammer and holding system.

TABLE 34.—Data on crushing resistance

Mineral	Specific gravity	Hardness	Weight of material crushed per unit of work, by drop-weight determination	Scleroscope data for minerals 1.2 inches thick—energy absorbed (inches of drop absorbed).		
				10-inch drop	6.7-inch drop	2.75-inch drop
Quartz.....	2.65	7.0	1.000	2.73	1.32	0.45
Pyrite.....	5.00	6.2	2.433	3.36	1.58	.62
Sphalerite.....	4.00	3.7	4.831	3.43	1.59	.53
Calcite.....	2.71	3.0	4.425	3.72	1.83	.69
Galena.....	7.43	2.6	15.097	7.96	4.85	2.11

CORRECTED FOR ELASTIC LOSSES

Elastic loss.....				2.10	0.90	0.30
Quartz.....				.63	.42	.15
Pyrite.....				1.26	.68	.32
Sphalerite.....				1.33	.69	.23
Calcite.....				1.62	.93	.39
Galena.....				5.86	3.95	1.81

If the absorbed-energy values for each mineral are plotted against the height of drop the curves in figure 19 are obtained. The fact that these curves are definitely concave upward indicates that the absorp-

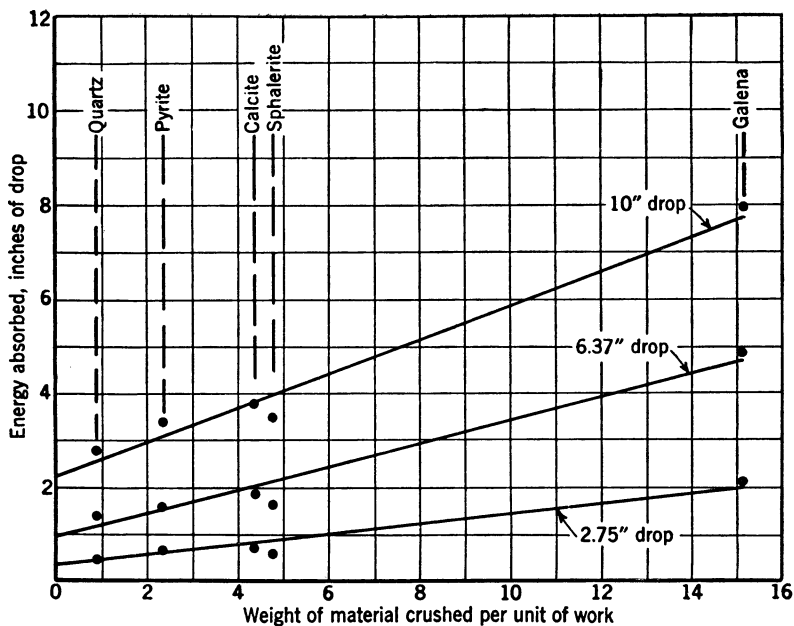


FIGURE 18.—Relation between crushing resistance and rebound.

tion is made up of an elastic loss and a crushing loss. If the values in figure 19 are corrected for the elastic losses given in figure 18 and table 34 the straight lines in figure 20 result. This shows that the energy

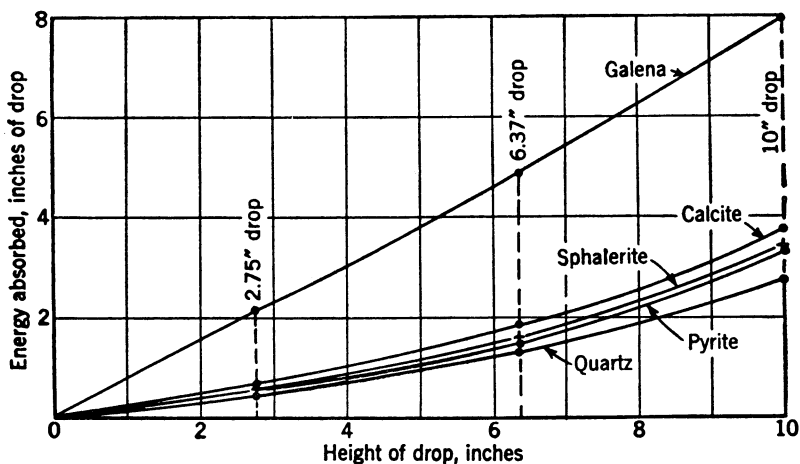


FIGURE 19.—Scleroscope determinations for quartz, pyrite, sphalerite, calcite, and galena.

absorbed due to crushing is proportional to the height of drop in the range studied and that grindability is independent of the height of drop. The grindability scale, in terms of percentage of energy absorbed, can therefore be established for any scleroscope.

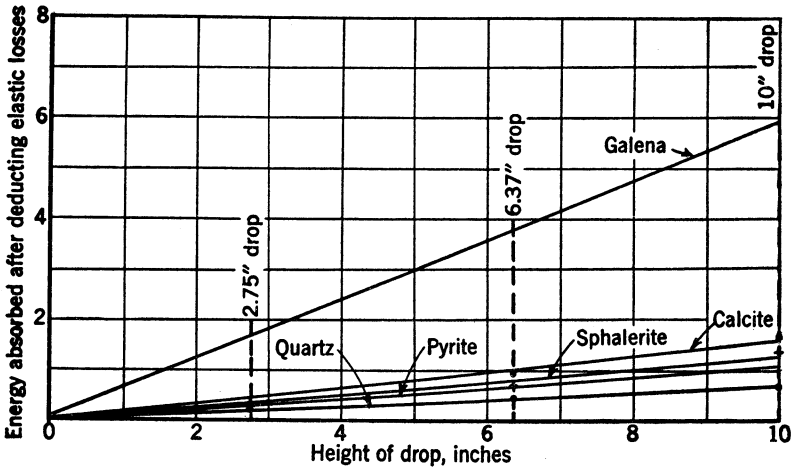


FIGURE 20.—Relation between height of drop and net energy absorbed.

CRUSHING AND GRINDING MACHINES

The history of crushing and grinding machines is not within the scope of this paper. Many mining men can still remember when virtually all coarse crushing was done by jaw crushers, intermediate crushing by rolls, and fine crushing or grinding by some type of roller mills. The principles employed in these machines of the last century are still inherent in modern practice. A description of the various machines that have been employed in crushing and grinding is not undertaken here. Naske (127) has recently covered this field rather completely, and Coghill (89) has given detailed historical data on ball milling.

CRUSHERS

Crushers used for the coarsest breaking may be subdivided into jaw, gyratory, and disk machines.

Jaw crushers are represented by the Blake and Dodge types. The former has the movable jaw pivoted at the top, which gives a variable discharge, while the latter is pivoted at the bottom, which gives a fixed discharge. This difference results in a greater capacity for the Blake crusher and a more uniform product for the Dodge (37.) A new crusher, the Eimco-Fahrenwald, has lately been put on the market (135). This machine has a double jaw of the Blake type operated by a lever arm instead of toggles. Specially shaped jaws tend to equalize crushing from the mouth to the throat of the jaws. A spring-suspended stationary jaw protects the crusher from tramp iron. Uniformity of product and large capacity appear reasonable as claims for this machine.

The action in both Blake- and Dodge-type jaw crushers comprises largely direct compression in which the force acts slowly. If a large body of the material is present between the jaws, crushing is also done by shearing and flexure.

The gyratory crusher is essentially a Blake-type crusher in which the jaw has been bent into a circle. The movable jaw, with a lateral movement in relation to the fixed jaw, as well as the movement at right angles, results in greater capacity and less danger of choking.

In crushers of this type direct compression is accompanied by shearing due to oblique compression as a result of lateral movement; this probably accounts for most of the crushing. The gyratory is a continuous crusher, whereas the jaw machine is intermittent. The type to be placed first in a crushing plant is much debated.

The disk crusher is based on the principle of the "coffee mill", modified to meet the requirements of coarse crushing. This machine, which has lately come into favor, has a large capacity and gives a uniform product. Crushing is due largely to shearing, although some compression results from the use of corrugated faces.

INTERMEDIATE MACHINES

This heading includes rolls, hammer mills, and disk grinders. Crushing in rolls is due to compression modified more or less by the roll movement which results in shearing.

A hammer mill, which may also be considered as a fine-grinding machine, compresses the material by sudden blows; and, while abrasion does occur in machines of this class, a quick-acting compressive force largely accounts for the breakage.

The action of a disk grinder in the field of intermediate grinding is due largely to shearing and abrasion.

FINE-GRINDING MACHINES

These may be listed as follows: Arrastre, Chilean mill, Huntington mill, disk pulverizers, tube, rod, and ball mills, Hadsel mill, nozzle crushing, and explosive shattering.

The evolution of fine-grinding machines from the original mule-driven arrastre, through the Chilean and other types of roller mills to the tube, ball, and rod mill has apparently ended with almost universal adoption of the ball-mill type of machine.

Grinding in an arrastre is simply shearing and abrasion. This historic apparatus, now extinct, helped to pave the way for introduction of the Chilean mill.

The Chilean mill, which includes the various types of roller mills such as the Huntington, acts similarly to rolls, although the compressive action is somewhat modified by the roll movement. This type of mill and the arrastre may be considered to have been displaced in modern practice.

In disk pulverizers with rough or corrugated faces the action is similar to that of a gyratory crusher, although abrasion, especially with smooth-faced disks, undoubtedly accounts for much of the grinding.

In tube, rod, and ball mills grinding results from similar forces. The ball mill, which is simply a shortened tube mill with metallic balls instead of flint pebbles, is the most generally used of these three types. The Hardinge conical mill may be included with ball mills, as the resultant grinding force is the same although it is applied differently. Many ball mills of other designs have been introduced in an attempt to improve efficiency, but these have made little headway due, doubtless, to the simplicity of the regular ball mill.

Direct compression is probably the major force that causes disintegration, although to some extent, especially in the rod mill, a roller action implies some compression and shearing. Abrasion and attri-

tion probably are important in grinding, but their quantitative effect is still a matter of dispute (6, 7, 45, 67, 73, 131).

The Hardinge-Hadsel mill, which has been recently developed and improved (139), may also be classed among coarse and intermediate crushers, as it takes run-of-mine material and yields a finished product in one machine. As the ore is crushed by falling upon impact plates the machine may be considered as acting like the hammer mill; however the impact is largely that of ore on ore. As the ore is simply raised to a certain level (about 16 feet net) and allowed to drop, this means of imparting energy is very economical. The crushing force comprises sudden blows, therefore comminution is due largely to compression accompanied by abrasion.

Other methods of fine grinding have been used, of which some find application in the nonmetallic field. Pulverizers of the ring-roller or ball type, such as the B. & W. pulverizer (141), may be mentioned in this connection. Such machines are very efficient where dry pulverization of a material softer than the general run of ore is desired. The crushing force in such machines is compression accompanied by shearing and abrasion.

In this brief discussion of the forces acting in crushing and grinding only the more generally used appliances have been considered, and it is questionable whether a lengthy discussion would be of benefit. This discussion, however, brings out the fact that the crushing force is almost universally compression, shearing, or abrasion.

It should also be noted that all of these machines depend on the use of power indirectly, that is, the application of steam or electrical energy to some movable part.

The possibility of utilizing the direct energy of steam or electricity without mechanical appliances may be the next advance in comminution efficiency. Two such methods which have been studied by the Bureau of Mines are nozzle crushing and explosive shattering.

NOZZLE CRUSHING

The use of an air or steam jet for comminution is not new, but no material progress has been made along this line. A steam jet is reported to be used successfully in grinding anthracite by blowing it against the sides of a drum. This should encourage further study of similar apparatus or of the nozzle machine.

By ejecting material at high velocity from a nozzle against an impact plate the material is considerably shattered. This may seem to be essentially a "sand-blasting" operation, yet it is so only mechanically. The nozzle machine is rather an outcome of an attempt to make explosive shattering a continuous operation that depends entirely on impact for its results.

The sand-blast nozzle is not well adapted to nozzle crushing, as it lacks a constricted section and allows the charge to blow backward, thus necessitating the use of a closed-feed hopper. This results in a churning of the charge in the feed hopper, a waste of energy, and a hindrance to efficient feeding.

The sand-blast nozzle was tried, however, with steam as the motive force, and while the amount of crushing was encouraging the steam consumption was high. The crushing accomplishment should be considered rather than the cost, as a proper nozzle operated under proper conditions would materially lower this cost.

The nozzle used was $\frac{3}{4}$ inch in inside diameter by 3 inches in length. Steam inlets of $\frac{1}{4}$, $\frac{3}{8}$, and $\frac{1}{2}$ inch were used. The nozzle discharge was received on an impact plate at distances of 3, 5, and 9 inches. The impact distance of 5 inches invariably gave the best results, and only these are reported. With the short impact distance the charge evidently did not spread enough for good impact, and with the long impact distance some of the force had no doubt been lost.

Table 35 gives the results of only those steam-nozzle experiments that show the effect of the variables and are comparable to results of explosive shattering in capacity and efficiency. Nozzle crushing compares favorably with explosive shattering in amount of crushing, but the steam cost is higher.

TABLE 35.—Results of crushing with steam nozzle, compared with those of steam shattering

Type of nozzle	Steam pressure (lb.)	Steam inlet (inch)	Wt. of minus 48-mesh produced (percent)	Weight of minus 48-mesh material (percent)			Capacity (pounds per hour)		Steam consumption, lb. per ton charge	Cost of steam (cents)	
				48-to 100-mesh	100-to 200-mesh	Minus 200-mesh	Feed	Minus 48-mesh		Per ton charge	Per ton of minus 48-mesh material
Straight.....	250	$\frac{1}{2}$	20.36	22.8	16.7	60.5	1,280	261	960	12.24	60.1
Do.....	250	$\frac{3}{8}$	17.52	25.1	20.5	54.4	1,070	187	940	11.99	68.4
Do.....	250	$\frac{1}{4}$	12.15	34.2	16.7	49.1	910	111	760	9.69	79.8
Flaring.....	250	$\frac{1}{2}$	14.42	28.1	19.9	52.0	2,560	369	630	8.03	55.7
Do.....	250	$\frac{3}{8}$	14.42	28.1	19.9	52.0	2,560	369	630	8.03	55.7
Do.....	200	$\frac{1}{2}$	12.60	30.6	21.0	48.4	1,600	202	690	8.80	69.8
Do.....	150	$\frac{1}{2}$	10.15	33.7	21.2	45.1	1,280	130	630	8.03	79.1
Do.....	100	$\frac{1}{2}$	7.35	38.8	17.0	44.2	1,070	79	500	6.37	86.7
Do.....	50	$\frac{1}{2}$	3.27	45.0	16.0	39.0	1,070	35	240	3.06	93.6
Explosive shattering, best capacity.....	250	-----	19.20	29.2	21.9	48.9	-----	-----	277.5	3.54	18.4
Explosive shattering, best efficiency.....	250	-----	8.60	37.8	19.2	43.0	-----	-----	19	0.24	2.8

A larger steam inlet gives greater capacity and better efficiency and produces a finer product. Increased capacity, greater efficiency, and a finer product accompany increased pressure. The flaring nozzle gives greater capacity and better efficiency, but the product is coarser.

The nozzle machine gives a finer product than explosive shattering, and as the fineness of its product increases materially with pressure a high-pressure nozzle has possibilities for very fine grinding.

The latest high-pressure nozzle is shown in figures 21 and 22. The restriction in the nozzle is necessary to give the required suction for feeding rapidly and to prevent the charge from being blown back.

A pressure of approximately 1,500 pounds was obtained by using a fresh tank of nitrogen. Although nitrogen was not a very satisfactory substitute for compressed air its use made possible the series of experiments given in table 36. The consumption of free air was determined to be approximately 1 cubic foot per second with an air jet of 1 mm, $1\frac{1}{2}$ cubic feet with a jet of $1\frac{1}{2}$ mm, and 2 cubic feet with one of 2 mm.

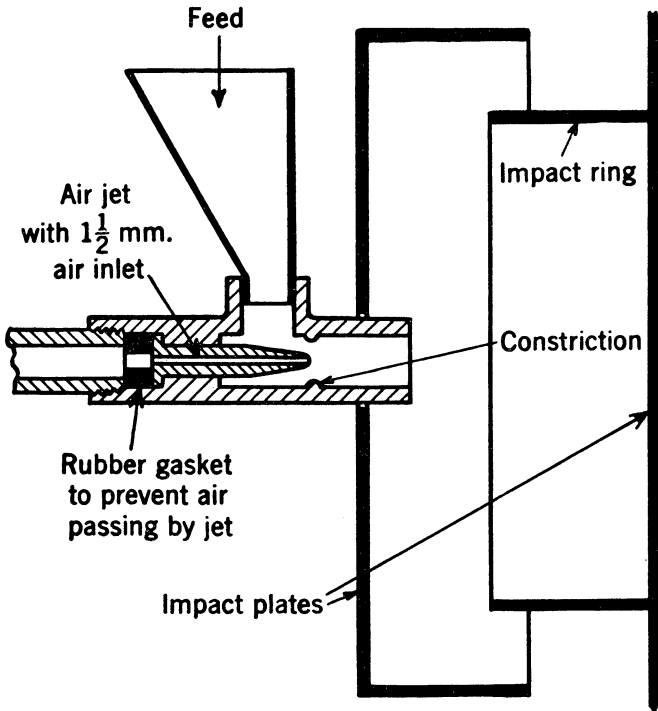


FIGURE 21.—Details of 5/8-inch nozzle crusher.

TABLE 36.—Crushing results with high-pressure nozzle

Size of nozzle (inch)	Feed inlet (inch)	Air pressure (pounds)	Air jet (mm)	Impact distance (inches)	Weight of minus 400-mesh material produced (percent)	Capacity (pounds per hour)		Cost per ton of minus 400-mesh material
						Feed	Minus 400-mesh material	
3/8	1/4	1,400	1	5	20.0	200	40	\$14.94
3/8	1/4	1,250	1 1/2	5	37.5	200	75	11.66
3/8	1/4	1,550	2	5	32.3	220	71	17.24
3/8	3/8	1,400	1 1/2	5	23.8	330	111	8.08
3/8	3/8	1,500	1 1/2	3	37.6	330	124	7.31
3/8	3/8	1,550	1 1/2	2	38.0	330	125	7.34
3/8	3/8	1,500	1 1/2	3	28.5	570	162	5.60
2 9/64	2 9/64	1,150	1 1/2	3	17.9	1,000	179	4.76
5/8	5/8	1,000	1 1/2	3	12.8	670	85	9.53
5/8	5/8	750	¹ 1 1/2	3	15.9	800	127	5.70
5/8	5/8	1,550	1 1/2	3	17.9	1,600	286	3.21

¹ Increased free space around air jet.

The feed used in these experiments was 100- to 200-mesh hard pyrite. The cost of compressed air is based on a cost of 1 cent per horsepower-hour. The costs given in the table are for compressed air only.

These results show that the 1 1/2-mm air jet is the most economical and that a 3-inch impact distance is more satisfactory than either a 2- or a 5-inch distance. An increase in size of nozzle, size of feed inlet, free space around air jet, and air pressure gives greater capacity and higher efficiency.

The work on nozzle crushing so far has been preliminary only, but the few experiments performed have indicated that the nozzle has possibilities as a simple, high-capacity, fine-grinding appliance. The cost would seem to be prohibitive under some conditions, but further study may reduce the cost materially. It is estimated that if steam were used as the motive force and the same volume of steam were as effective as compressed air the cost per ton would be about one-tenth that of compressed air.

EXPLOSIVE SHATTERING

It is conceivable that new principles will be utilized to increase efficiency in crushing and grinding. Two possible ways of accomplishing this result are recognized: (1) Instantaneous crushing, in which deformation of the material would be virtually eliminated owing to the suddenness of the blow; (2) differential crushing, in which the ore mineral only would be disintegrated. Both possibilities appear to be inherent in explosive shattering, in which the material is subjected to an almost instantaneous force.

MECHANISM OF EXPLOSIVE SHATTERING

The original conception that the disrupting action in explosive crushing was due mainly to the sudden expansion of a substance within the mineral (*116, 126, 132*) has been modified by recent work.

Besides cracks, cleavage planes, pores, and cavities, metallic minerals and crystalline metals are now considered to have a secondary structure, which is essentially a group of planes of discontinuity dividing the mineral into blocks only a few microns across. These blocks may be separated by gas or other impurities or may be merely slip planes that are susceptible to the penetration of gases.

The effect of mechanical structure of crystals on crushing has been discussed by Dean and Gottschalk (*125*), and a study of the secondary structure of minerals, particularly galena, made by Head (*113*) indicated that within the apparently perfect crystal are microscopic voids and small crystals of sodium chloride and other minerals isomorphous with galena. This is substantiated further by the absorption of large amounts of certain gases by sulphide minerals in a way that can be accounted for only by their condensation in the interstices of a honeycomblike structure.

The idea that the expansion of a substance introduced into these spaces within a mineral would cause rupture suggested explosive shattering.

The introduction of an explosive substance into the mineral and its subsequent explosion appeared particularly attractive as a simple application of explosive shattering; however, no suitable substance was available, although it was demonstrated that an explosive substance actually caused shattering. The sudden expansion of superheated water was found to be quite satisfactory and was used in the extensive experiments subsequently performed.

The impregnation of the mineral with superheated water under pressure and the sudden release of this pressure result in an expansive force or wave that tends to break the mineral along planes of weakness, such as cracks, cleavage planes, and planes of the secondary structure. This action tends to break one mineral from its adjacent

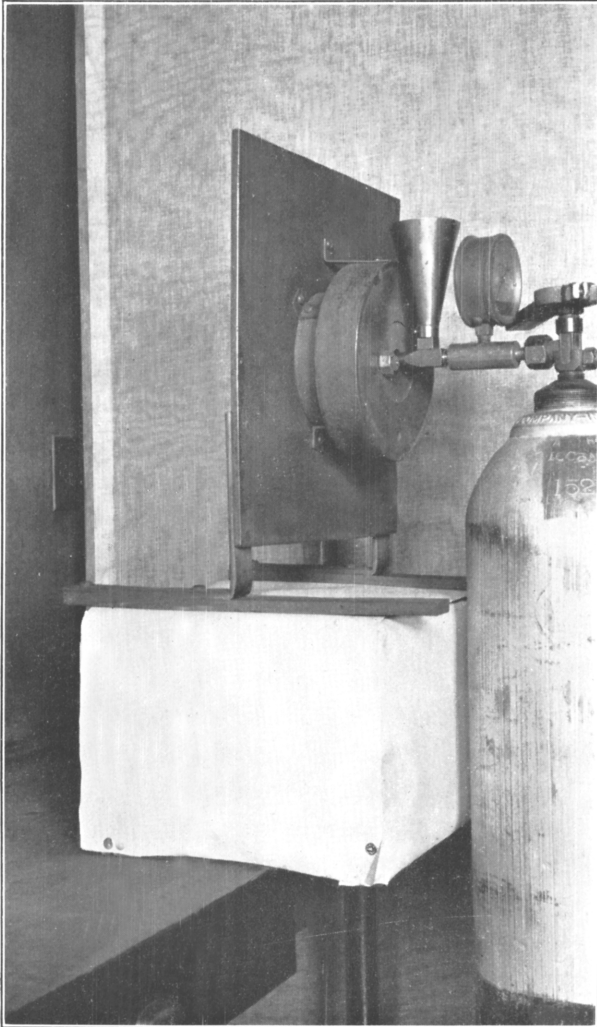


FIGURE 22.—Set-up of $\frac{5}{8}$ -inch nozzle crusher.

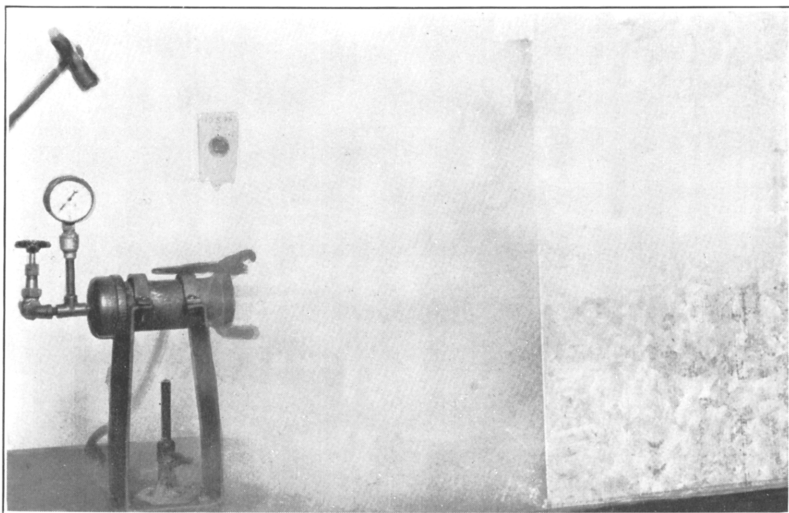


FIGURE 23.—Flame-heated explosive machine 1. Trigger just released and discharge directed into a chamber.

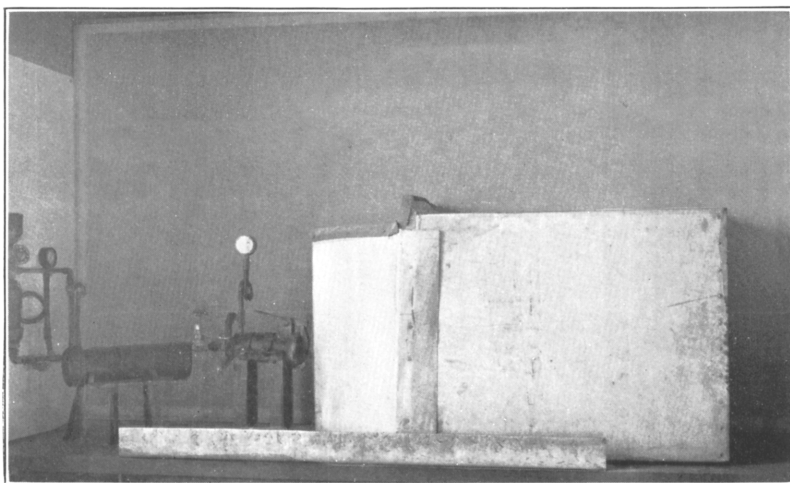


FIGURE 24.—Steam-heated explosive machine 2.

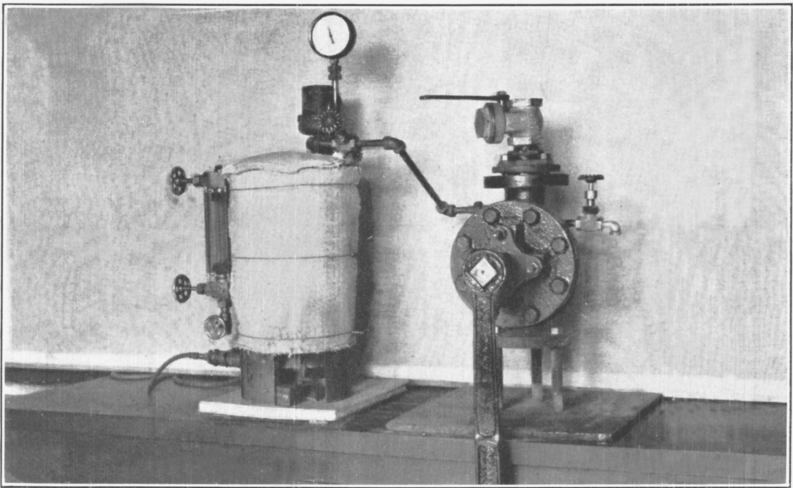


FIGURE 25.—Plug-valve explosive machine 3.

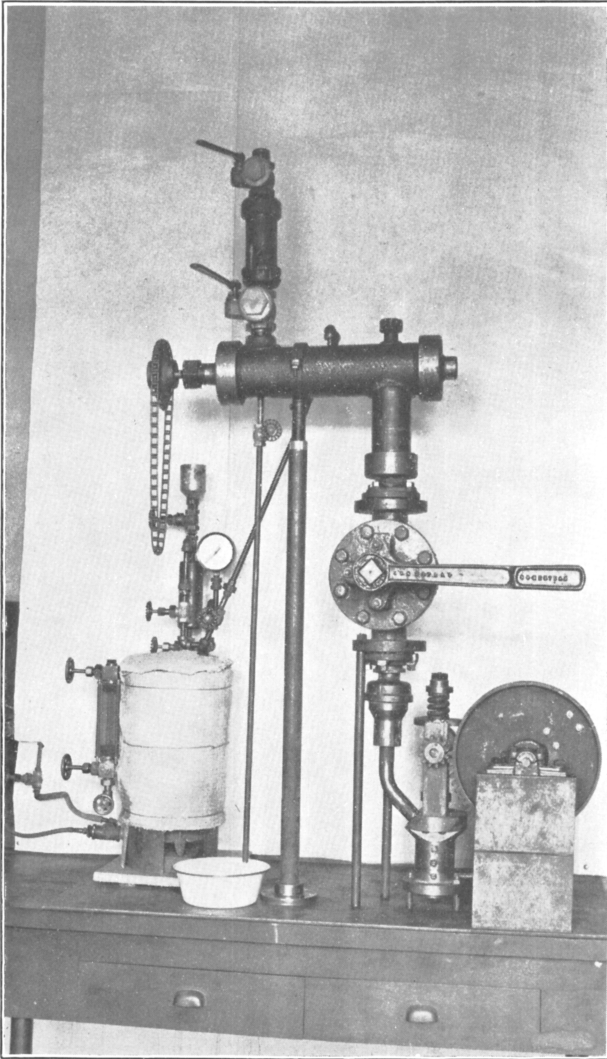


FIGURE 26.—Explosive machine 4 with preparation chamber.

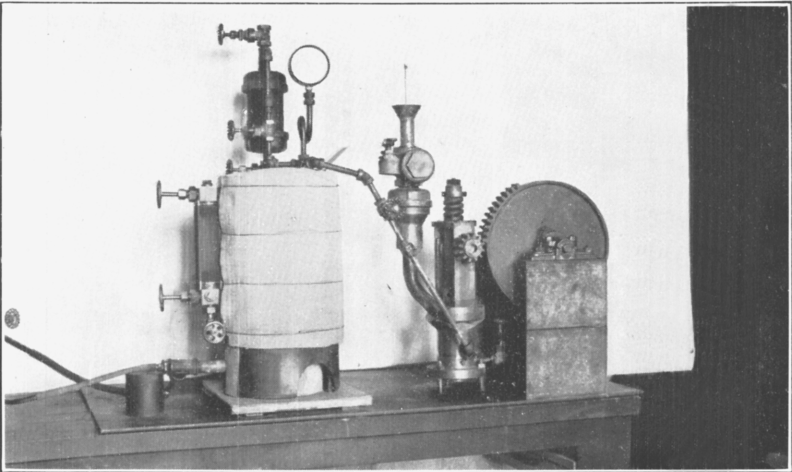


FIGURE 27.—Vertical explosive machine 5.

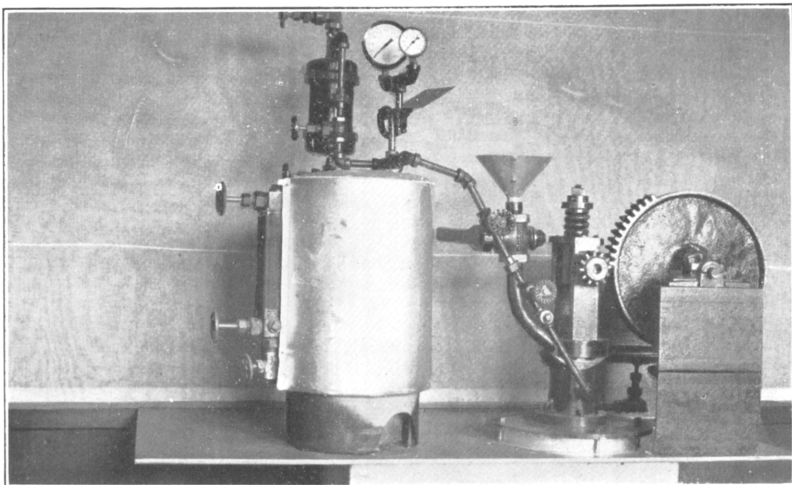


FIGURE 28.—Explosive machine 6.

neighbor, thus resulting in differential crushing; moreover, as the blocks of the secondary structure probably cannot be penetrated they are not ruptured, and no material finer than 1 or 2 microns in size is produced.

This simple conception of explosive shattering does not adequately explain the complete results obtained. If this mechanism were the sum total of what takes place it might be assumed that any procedure tending to cause greater impregnation would increase shattering.

Soaking the material for longer periods before subjecting it to explosion had little effect, and a preliminary soaking was found of doubtful value. Soaking the material in a solution having a different surface tension from that of water to promote better penetration had no effect on shattering. Moreover, evacuating the material before soaking it had no beneficial effect.

In machine 1, with a fixed quantity of water and flame heating, holding the steam pressure from zero to 30 minutes gave such a slight increase in shattering with prolonged steaming as to make it of doubtful benefit.

When steam heating is used, prolonging the steam pressure increases shattering, but the increase is due to the larger volume of water with the charge at the time of the explosion.

As these attempts to increase impregnation were not materially successful it might be concluded either that no impregnation takes place or that the impregnation is so rapid that any acceleration is negligible. The latter assumption is reasonable, as differential crushing and a decided reduction in very fine product accompany explosive shattering.

The explosion from within is now thought to be only part of the mechanism. Goetz (117) has shown by the change of lattice parameter that when aggregate structures are heated the unit blocks expand regularly but that the expansion of the whole structure diverges from this regularity at elevated temperatures. This may be interpreted as meaning that the blocks loosen at high temperatures so that the structure is easier to disintegrate. Any sudden expansion of superheated water along the secondary structure would materially assist its disruption. The effect of temperature on shattering is discussed further under "Effect of pressure and temperature."

Another hypothesis for the mechanism of explosive shattering is that disruption results from the explosion of an envelope or shell of superheated water around each particle, somewhat as in "bulldozing," except that the force is applied at all points of the particle rather than at one point. This is only a hypothesis, as no experimental evidence can be produced. It may be mentioned, however, as possible evidence that more shattering is obtained in a more compacted charge where the explosive effect of the "shell" would be confined and less of it dissipated.

Finally, impact, not merely of particle against particle or particle against the wall of the explosive chamber but more especially of particle against the wall of the receiving chamber, results in considerable shattering. Although impact may be largely eliminated, it may be utilized as an important contribution to shattering.

It may be concluded therefore that explosive shattering does not comprise a single mechanism but may consist of several—shattering from within, shattering from without, and breaking of the structure by high temperature and by impact.

EXPLOSIVE-SHATTERING MACHINE

Machine 1, in which the earlier experiments were made, comprised a cylinder 2 inches in diameter by 4 inches in inside length, fitted at one end with a cover held in place by a trigger that could be released.

This machine, shown in figure 23, was charged with the material and with a definite quantity of water, and the contents were heated until the desired pressure was attained. The explosion was obtained by suddenly releasing the cover, and the exploded charge was received in a suitable chamber. The next step in the development of machine 2 was addition of a steam boiler so that the superheated water was supplied as steam and extraneous heating of the cylinder was avoided. This machine is shown in figure 24.

Many designs were considered in an attempt to pass from the laboratory machines 1 and 2 to one more nearly applicable to actual and continuous operation.

Machine 3 was tried, in which the explosive chamber comprised merely a 2-inch plug valve topped by a quick-opening valve, as shown in figure 25. The material to be exploded was charged above the closed plug and steamed, then it was exploded by opening the plug valve as rapidly as possible. This machine was a failure, as it could not be opened rapidly enough; moreover, the wear on the plug valve was excessive.

In machine 4, shown in figure 26, a preparation chamber was provided on the assumption that preliminary soaking and steaming were required. The results obtained with this machine were so unsatisfactory that the preparation chamber was definitely discarded, and machine 5 (fig. 27) was designed. As this machine gave good results and was used for the efficiency experiments given later

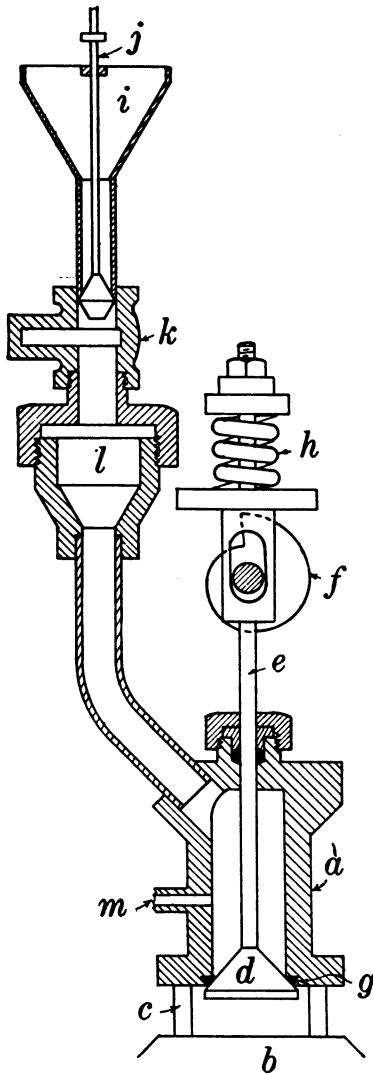


FIGURE 29.—Sectional diagram of explosive machine 5.

it is shown in sectional view (fig. 29) and is described herewith.

The explosive chamber, *a*, is bolted to foundation *b* by bolts *c*, which also hold the chamber the required distance above the foundation to allow the discharge gate to open. Discharge gate *d* is closed by rod *e*, which is lifted by cam *f*. The gate closes against lead seat *g*, and a steam-tight seal is made. Tension on the rod is maintained by spring *h*.

The charge is fed into the chamber by feed funnel *i*, which has a plug bottom held in place by rod *j* and is suspended clear of quick-opening valve *k* until time to feed. When valve *k* opens the feed funnel is released, and both *i* and *j* travel together until *i* is stopped; *j* continues to drop still farther to allow feed to discharge into expanded section *l*, whence it passes to the explosive chamber. This feed mechanism keeps grit out of valve *k*.

When the feed funnel is again raised valve *k* closes and steam enters the explosive chamber through *m*. After a short period of steaming the discharge gate is released by the cam and the explosion occurs.

The cycle of operations, which are synchronized, is as follows: The cam lifts the discharge gate and then stops; the quick-opening valve opens; the feed funnel delivers the charge; the quick-opening valve closes, and steam enters the explosive chamber; the steam is shut off; and the cam again comes into play, and the explosion occurs.

From the experience gained in the use of machine 5 the somewhat simplified and improved machine 6 has been developed. This machine is shown in figures 28 and 30. Its operation is similar to that of machine 5, differing mainly in the impact housing at the discharge end and in the feed mechanism. The feeding is controlled by a plug valve; this is opened, a charge dropped through, and the valve closed.

The crushing done in these machines is compared in table 37.

The results show that the plug-valve discharge is very inefficient, as it cannot be opened rapidly enough to release the pressure suddenly. The vertical machine is seen to be superior to the old horizontal machine, but the preparation chamber is not only unnecessary but detrimental.

CONDITIONS THAT AFFECT SHATTERING

The following conditions affect shattering: The steam pressure or temperature at the time of the explosion, the volume of water with the charge or the length of the steaming period, the size of the charge, and the impact.

TABLE 37.—*Explosive shattering in different machines*

[All steam-heated, using 200-gram charges of dolomite; impact not eliminated]

Machine ¹	Steam pressure, lb.	Cumulative percent in shattered product					
		Minus 200-mesh	Minus 100-mesh	Minus 48-mesh	Minus 14-mesh	Minus 8-mesh	Minus 4-mesh
2. Horizontal.....	150	4.1	6.7	11.0	33.8	54.9	84.5
3. Plug-valve discharge.....	150	2.5	3.3	5.5	23.9	45.2	78.1
4. Vertical, preparation chamber.....	150	4.6	6.8	10.3	30.9	53.2	82.7
5. Vertical, no preparation chamber.....	150	5.6	8.2	12.4	33.9	55.3	81.7
2. Horizontal.....	250	5.7	9.3	15.1	40.3	61.9	88.6
3. Plug-valve discharge.....	250	2.9	4.3	7.0	26.0	46.8	78.7
5. Vertical, no preparation chamber.....	250	9.0	13.1	18.7	42.5	62.9	89.0
6. Plug-valve feed.....	250	10.1	14.4	20.0	44.2	70.5	87.8

¹Machine 4 could not supply enough steam at a pressure of 250 pounds.

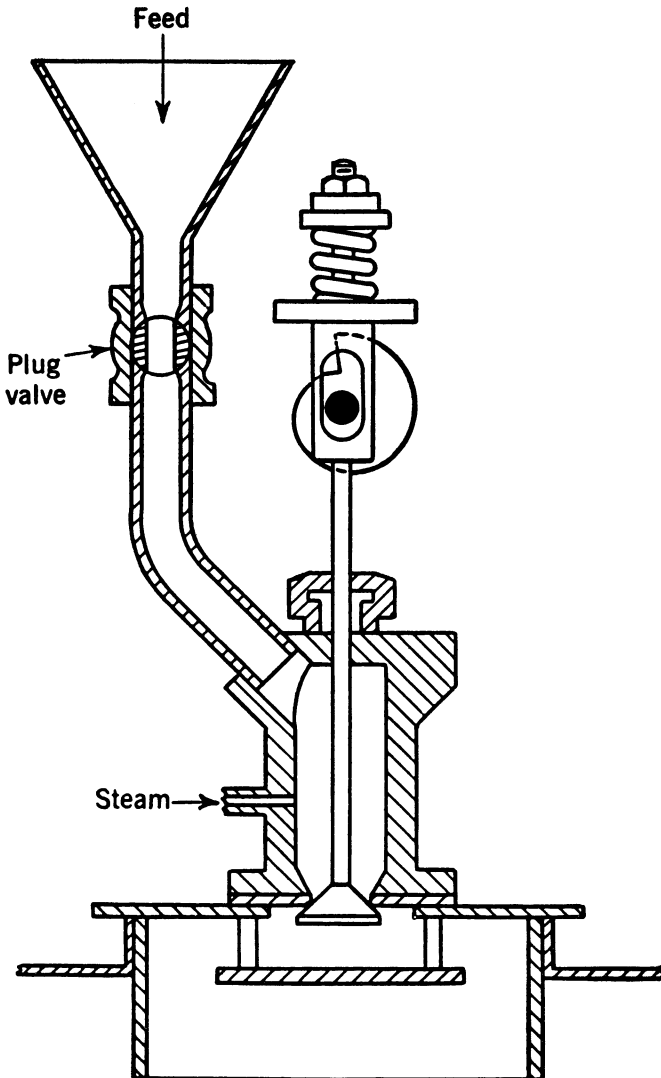


FIGURE 30.—Sectional diagram of explosive machine 6.

EFFECT OF PRESSURE AND TEMPERATURE

Explosive experiments were made in machine 1 on quartz, galena, and sphalerite at various steam pressures from 30 to 240 pounds. In these experiments 100 grams of 3- to 4-mesh material was heated with 50 cm³ of water to the desired pressure and exploded. Table 38 gives the results. The curves for the minus 48-mesh product are plotted in figure 31 for pressures and in figure 32 for temperatures. These curves are shown only for the minus 48-mesh product, as those for coarser sizes are affected by depletion of these sizes. The effect of pressure is not in direct proportion to the shattering, but at the higher temperatures it apparently varies with the temperature.

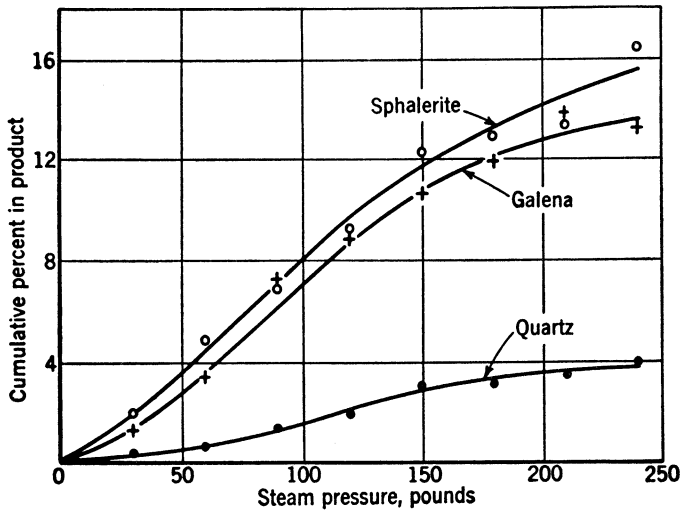


FIGURE 31.—Effect of pressure on explosive shattering of quartz, galena, and sphalerite; minus 48-mesh size, machine 1.

Experiments were made on quartz and sphalerite in machine 2 to note the effect of pressure and temperature when steam was used as the heating agent. The results are given in table 39. Figures 33 and 34 give pressure and temperature curves for the minus 48-mesh product which show that with steam heating shattering appears to be in direct proportion to pressure rather than to temperature.

TABLE 38.—Effect of pressure and temperature in explosive shattering

[100 grams of ¾-mesh material; 50 cm³ of water, exploded in machine 1]

Explosive pressure (pounds)	Explosive temperature (°F.)	Cumulative percent in shattered product					
		Minus 4-mesh	Minus 8-mesh	Minus 14-mesh	Minus 48-mesh	Minus 100-mesh	Minus 200-mesh
Quartz:							
30	250	4.2	1.0	0.7	0.4	0.3	0.2
60	293	13.3	3.3	1.5	.5	.3	.2
90	320	40.9	13.9	5.2	1.3	.7	.4
120	341	59.1	23.7	9.3	1.9	.8	.4
150	358	71.3	34.5	14.6	3.1	1.5	.7
180	374	76.3	37.4	15.8	3.2	1.4	.7
210	385	76.8	40.9	18.5	3.6	1.5	.6
240	395	79.5	44.5	21.0	4.1	1.6	.7
Galena:							
30	250	7.7	4.6	1.7	1.2	.7	.2
60	293	56.0	19.7	9.1	3.5	1.9	.8
90	320	79.8	44.8	24.3	7.3	4.0	2.2
120	341	89.0	57.4	30.4	8.9	4.7	2.7
150	358	91.0	65.2	37.4	10.8	5.4	2.8
180	374	94.0	72.9	44.5	11.9	5.8	3.0
210	385	92.2	70.4	46.9	13.8	6.9	3.4
240	395	96.3	76.2	49.4	13.4	6.1	2.7
Sphalerite:							
30	250	12.7	4.3	2.8	1.9	1.6	1.4
60	293	61.7	28.1	13.8	4.8	3.0	2.1
90	320	74.9	44.6	22.8	6.9	3.9	2.5
120	341	87.1	58.7	32.6	9.3	4.9	2.8
150	358	91.2	65.9	40.1	12.4	6.3	3.4
180	374	89.9	67.4	41.2	13.1	6.8	3.7
210	385	91.8	68.3	42.4	13.4	6.9	3.8
240	395	94.1	76.0	50.1	16.6	8.6	4.7

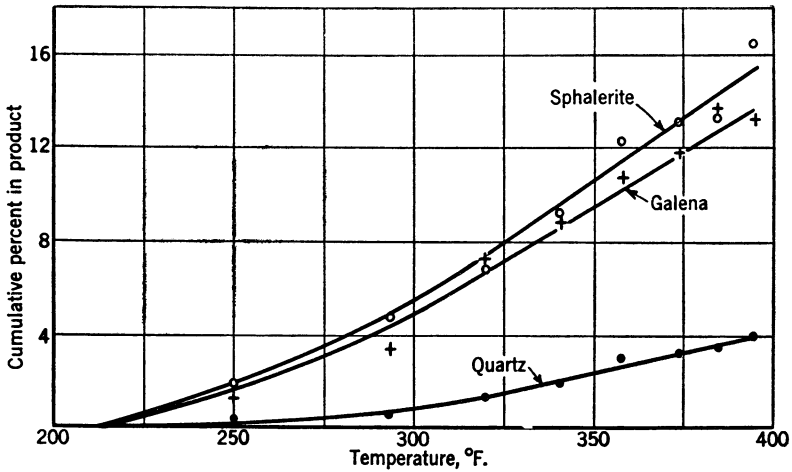


FIGURE 32.—Effect of temperature on explosive shattering of quartz, galena, and sphalerite; minus 48-mesh size, machine 1.

TABLE 39.—Effect of pressure and temperature in explosive shattering

[200 grams of 3/4-mesh material; steam-heated; exploded in machine 2]

Explosive pressure (pounds)	Explosive temperature (°F.)	Cumulative percent in shattered product					
		Minus 4-mesh	Minus 8-mesh	Minus 14-mesh	Minus 48-mesh	Minus 100-mesh	Minus 200-mesh
Quartz:							
30	250	7.7	1.9	1.0	0.5	0.3	0.2
60	293	21.3	5.8	2.5	.7	.4	.2
90	320	34.1	11.7	4.7	1.1	.6	.3
120	341	49.8	17.3	8.7	1.7	.8	.4
150	358	54.1	23.0	10.2	2.2	1.0	.6
180	374	62.5	28.1	12.9	2.6	1.2	.6
Sphalerite:							
30	250	13.2	4.6	3.2	2.1	1.7	1.4
60	293	43.9	16.4	8.7	3.3	2.3	1.7
90	320	61.8	28.7	14.6	4.6	2.8	1.8
120	341	72.9	38.8	21.0	6.0	3.4	2.1
150	358	80.9	50.5	28.3	8.5	4.7	2.7
180	374	85.2	55.6	33.5	9.9	5.3	2.9

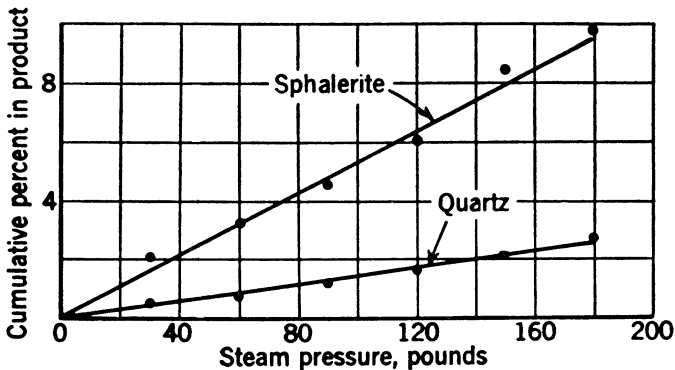


FIGURE 33.—Effect of pressure on explosive shattering of quartz and sphalerite; minus 48-mesh size, machine 2.

A few experiments were made at the higher pressures in machine 5 by exploding dolomite to note the effect of pressure and temperature. Table 40 gives the results.

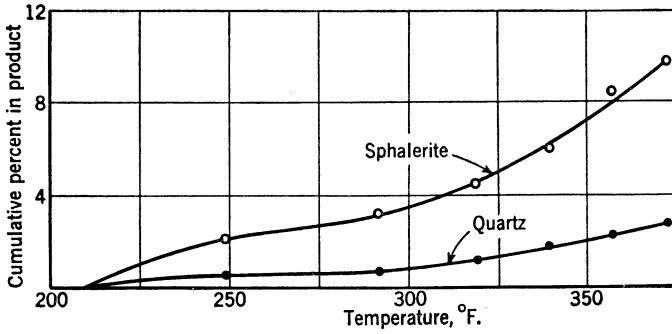


FIGURE 34.—Effect of temperature on explosive shattering of quartz and sphalerite; minus 48-mesh size, machine 2.

The curves for effect of pressure and temperature on the finer sizes (figs. 35 and 36) show that shattering is more nearly in proportion to pressure than to temperature.

A study of the curves for the production of minus 48-mesh material indicates that the production falls off with higher pressures in the flame-heated experiments in machine 1 but that with steam heating the shattering increases regularly with increased pressure.

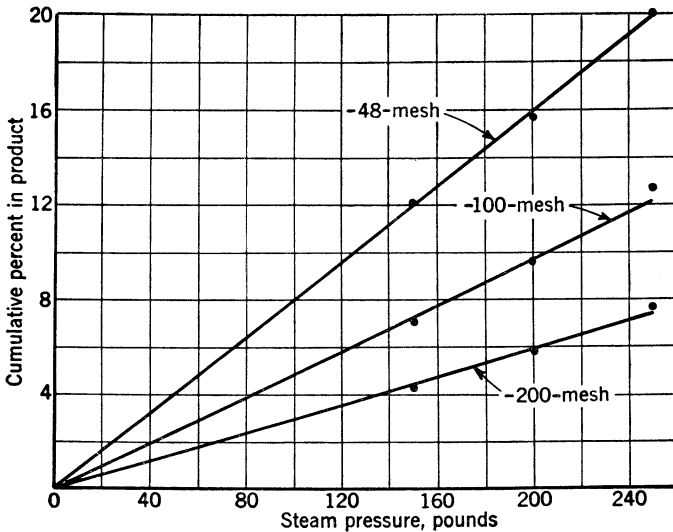


FIGURE 35.—Effect of pressure on explosive shattering of dolomite, machine 5.

High pressures are desirable therefore with steam heating for capacity. As the heat units required to raise the steam pressure increase slowly greater efficiency is to be expected at high pressures.

The effect of steam pressure on the shattering of various materials is compared in table 41. No definite percentage of increased shattering

due to increased pressure is shown, nor would this be expected; however, the figures give a general idea of how steam pressure affects shattering for various substances.

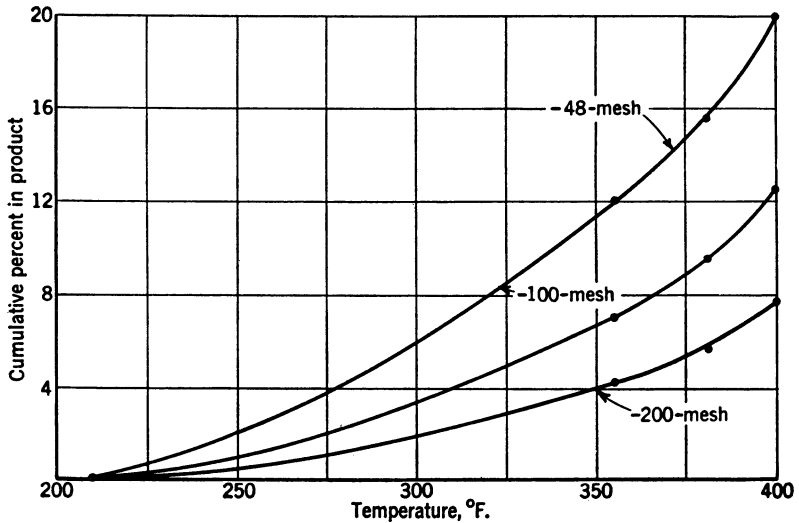


FIGURE 36.—Effect of temperature on explosive shattering of dolomite, machine 5.

TABLE 40.—Effect of pressure and temperature on explosive shattering of dolomite [200 grams of 3- to 48-mesh; steam pressure held 3 minutes; impact not eliminated; exploded in machine 5]

Explosive pressure (pounds)	Explosive temperature (°F.)	Cumulative percent in shattered product					
		Minus 4-mesh	Minus 8-mesh	Minus 14-mesh	Minus 48-mesh	Minus 100-mesh	Minus 200-mesh
150.....	358	85.9	60.7	38.1	12.1	7.1	4.3
200.....	382	92.7	66.7	43.0	15.7	9.6	5.8
250.....	401	92.6	72.9	49.9	20.1	12.6	7.7

TABLE 41.—Effect of pressure on explosive shattering of various materials

Material exploded	Steam pressure (pounds)	Sizes produced (percent)			
		Minus 14-mesh	Minus 48-mesh	Minus 100-mesh	Minus 200-mesh
Graphite ore, soft.....	50	22.6		1.8	
Do.....	100	39.4		2.9	
Do.....	150	51.7		4.2	
Do.....	200	61.3		5.5	
Lead-zinc ore.....	100		16.3		
Do.....	150		21.3		
Graphite ore, hard.....	100	13.6		2.7	
Do.....	150	27.4		5.4	
Do.....	200	44.1		9.4	
Do.....	250	52.4		12.5	
Clay.....	150				18.7
Do.....	200				26.7
Pyrite concentrate.....	200				5.6
Do.....	250				9.3
Scheelite ore.....	200		18.7		
Do.....	250		22.5		
Pyritic ore.....	200		13.2		
Do.....	250		18.5		
Iron ore.....	100		10.2	5.2	
Do.....	175		15.3	7.6	
Do.....	200		20.1	9.9	

EFFECT OF AMOUNT OF WATER

Table 42 gives the results of experiments made on quartz, galena, and sphalerite in machine 1 with varying quantities of water and figure 37 the curves for the minus 48-mesh product. These curves show that shattering of galena and sphalerite continues at a fairly rapid rate with increased water but that an excess of over approximately 50 percent of water results in little additional shattering of quartz.

Experiments in machine 2 with direct application of steam indicate that less water (or steam) is required. The results are given in table 43.

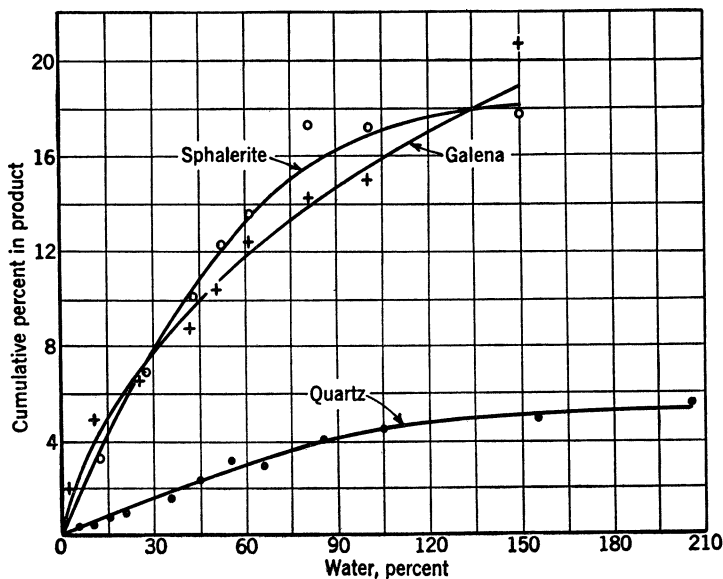


FIGURE 37.—Effect of amount of water on explosive shattering of quartz, galena, and sphalerite; minus 48-mesh size, machine 1.

The amount of water with the charge was determined experimentally for the various steaming periods.

The curves for the minus 48- and minus 100-mesh product of the above experiments are shown in figure 38 and are similar to the curves in figure 37 for flame heating.

With the same amount of water shattering is greater with direct steam heating than with flame heating. Figure 39 compares the curves for the minus 48-mesh product in figures 37 and 38 and shows that for the same amount of water in the lower range of consumption shattering increases approximately 90 percent for quartz, 80 percent for galena, and 60 percent for sphalerite.

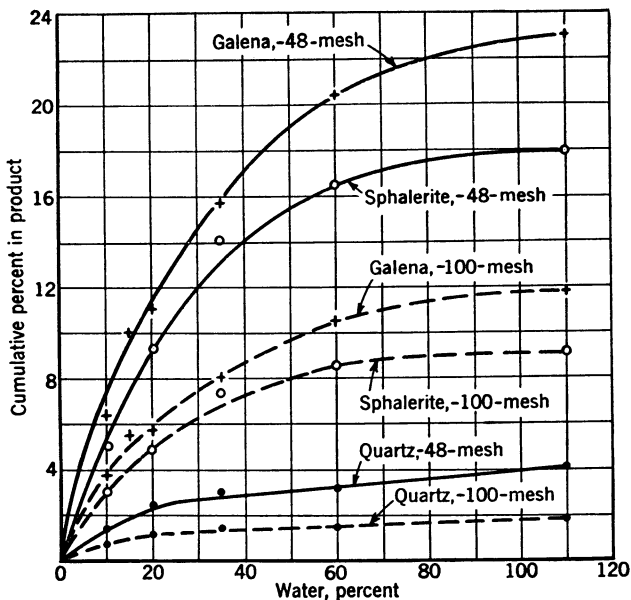


FIGURE 38.—Effect of amount of water (steam) on explosive shattering of quartz, galena, and sphalerite, machine 2.

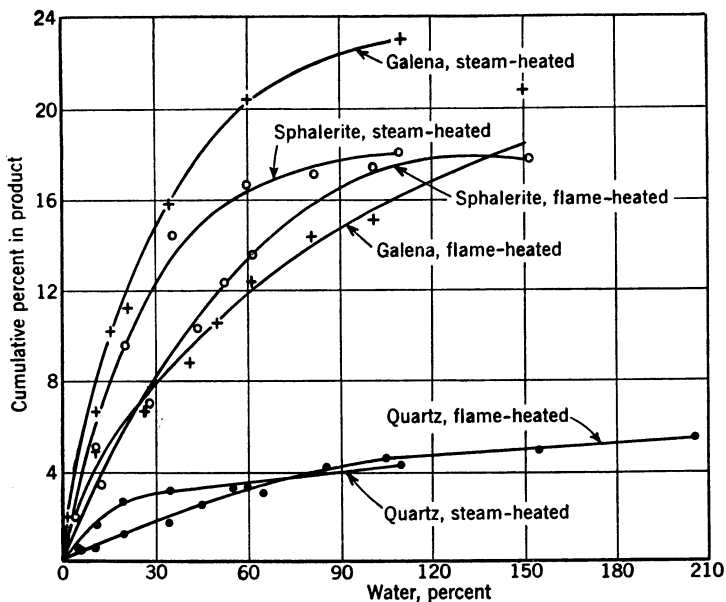


FIGURE 39.—Comparison of effect of amount of water in explosive shattering by flame heating and by steam heating; minus 48-mesh size.

TABLE 42.—*Effect of water in explosive shattering*

[100 grams of 3- to 4-mesh material; exploded at 150 pounds pressure in machine 1]

Water with charge (percent)	Cumulative percent in shattered product					
	Minus 4-mesh	Minus 8-mesh	Minus 14-mesh	Minus 48-mesh	Minus 100-mesh	Minus 200-mesh
Quartz:						
5.....	3.0	0.8	0.6	0.4	0.3	0.2
10.....	8.7	2.2	1.2	.5	.3	.2
15.....	24.2	6.6	2.8	.9	.5	.3
20.....	32.4	10.8	4.2	1.2	.6	.3
35.....	48.4	18.2	7.2	1.6	.7	.4
45.....	61.2	24.2	9.8	2.3	1.1	.6
55.....	71.7	34.6	14.5	3.2	1.4	.7
65.....	71.1	35.7	14.7	3.0	1.3	.6
85.....	77.5	42.6	19.7	4.1	1.7	.8
105.....	81.3	46.3	21.6	4.5	1.9	.8
155.....	79.9	45.6	22.1	4.9	2.1	1.0
205.....	82.0	46.4	22.6	5.5	2.6	1.3
Galena:						
1.....	15.9	3.8	3.2	2.0	1.3	.7
11.....	36.9	14.7	8.9	5.0	3.3	2.1
26.....	66.5	35.3	17.6	6.6	4.0	2.3
41.....	79.5	50.2	28.6	8.8	4.8	2.9
51.....	91.0	65.2	37.4	10.4	5.4	3.4
61.....	92.4	71.8	43.9	12.4	6.4	3.4
81.....	95.4	75.3	47.9	14.3	7.2	3.8
101.....	93.4	77.2	51.5	15.0	7.4	3.9
151.....	98.0	90.6	67.7	20.7	9.9	4.8
Sphalerite:						
2.....	16.0	5.3	2.9	1.9	1.7	1.5
12.....	42.5	16.8	8.6	3.3	2.3	1.7
27.....	70.8	39.5	22.0	6.9	3.8	2.3
42.....	85.7	57.7	34.1	10.3	5.3	2.9
52.....	91.2	65.9	40.1	12.4	6.2	3.4
62.....	94.5	71.9	44.4	13.6	6.8	3.6
82.....	95.8	77.5	51.9	17.4	9.0	4.7
102.....	93.6	77.0	51.6	17.4	9.0	4.7
152.....	97.0	77.7	52.1	17.7	9.2	4.9

TABLE 43.—*Explosive results by direct application of steam*

[100 grams of 3- to 4-mesh material; condensed water drained off at 150 pounds and pressure held at 150 pounds for variable periods of time without further draining. Exploded in machine 2]

Pressure held (minutes)	Water with charge (percent)	Cumulative percent in shattered product					
		Minus 4-mesh	Minus 8-mesh	Minus 14-mesh	Minus 48-mesh	Minus 100-mesh	Minus 200-mesh
Quartz:							
0.....	10	34.7	12.3	5.7	1.4	0.8	0.5
1½.....	20	57.1	26.5	12.0	2.5	1.2	.7
2½.....	35	62.1	29.2	14.2	3.1	1.5	.8
5.....	60	72.3	36.3	17.8	3.2	1.4	.8
10.....	110	75.4	40.1	20.1	4.2	1.9	1.1
Galena:							
0.....	10	79.5	39.8	18.9	6.5	3.8	2.1
¾.....	15	92.3	57.9	32.9	10.1	5.7	3.3
1½.....	20	97.6	70.6	40.7	11.1	5.8	3.1
2½.....	35	98.4	80.9	54.0	15.7	8.1	3.9
5.....	60	100.0	86.8	64.2	20.4	10.3	5.0
10.....	110	98.7	89.9	69.6	23.0	11.8	5.8
Sphalerite:							
0.....	10	62.8	32.1	16.7	5.1	3.1	2.1
1½.....	20	83.3	53.0	31.8	9.4	5.0	2.8
2½.....	35	90.2	65.7	43.1	14.3	7.5	4.0
5.....	60	94.7	73.4	50.3	16.6	8.6	4.5
10.....	110	93.9	75.1	53.0	18.0	9.3	4.9

Possibly heating with a flame, as in machine 1, is nonuniform, and part of the charge might become dry. This would not occur with steam heating and may be the reason for greater shattering with steam heating.

Subsequent work with machine 5 on dolomite showed that good shattering could be obtained with a very low consumption of steam. Some of these results are given in table 44.

These results, which are plotted in figure 40 for the finer sizes, show that a long steaming period does not increase shattering but that with

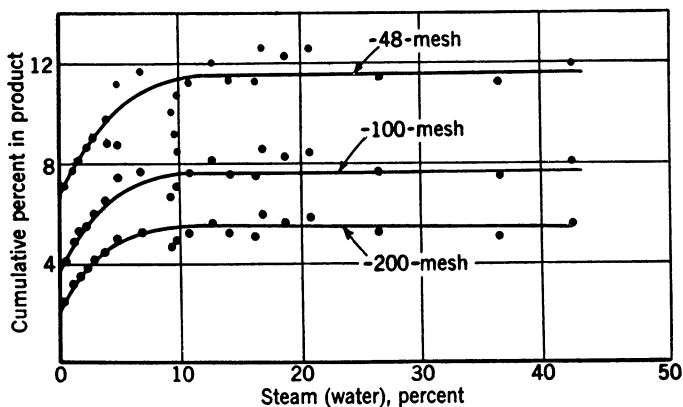


FIGURE 40.—Explosive shattering of dolomite with various amounts of steam, machine 5.

a smaller consumption of steam (below the equivalent of 5 percent of water) shattering falls off rapidly. The efficiency of shattering compared to the expense for steam will be considered later.

These experiments would seem to show that an excess of steam is not beneficial and that it is a question of how the steam is allowed to act rather than its quantity.

TABLE 44.—*Explosive shattering of dolomite with different quantities of steam*

[Exploded at 150 pounds pressure in machine 5]

Steam consumed (percent)	Cumulative percent in shattered product					
	Minus 4-mesh	Minus 8-mesh	Minus 14-mesh	Minus 48-mesh	Minus 100-mesh	Minus 200-mesh
43.....	70.5	48.6	33.5	12.0	8.1	5.6
37.....	82.6	50.6	32.9	11.3	7.5	5.0
27.....	82.1	54.3	33.0	11.6	7.7	5.3
21.....	83.7	57.4	34.1	12.6	8.5	5.9
19.....	81.0	56.3	33.8	12.3	8.3	5.7
17.....	79.8	55.9	33.2	12.7	8.6	6.0
16½.....	79.7	54.6	33.1	11.3	7.5	5.1
14½.....	81.3	58.2	32.5	11.4	7.6	5.3
13.....	79.5	55.4	32.1	12.1	8.2	5.7
11.....	79.9	55.4	32.6	11.2	7.6	5.2
10.....	78.9	52.7	30.9	10.7	7.1	5.0
9½.....	81.4	54.3	31.9	10.1	6.7	4.7
7.....	77.6	52.7	30.9	11.7	7.7	5.3
5.....	76.7	53.7	30.6	11.2	7.5	5.2
4.....	76.0	53.9	29.5	9.8	6.5	4.5
3.....	75.5	49.6	29.4	9.0	5.9	4.1
2½.....	78.1	53.8	30.7	8.7	5.6	3.8
2.....	77.7	52.8	29.9	8.1	5.2	3.6
1½.....	78.1	49.8	28.5	7.7	4.9	3.3

EFFECT OF SIZE OF CHARGE

The larger the charge, within the limits available in the laboratory machines, the greater the production of finer sizes. The results given in table 45 were obtained in machine 2 with steam, and while they show a decreasing percentage of shattering with larger charges the actual production of fines per explosion or per unit of steam is greater with the larger charges. Table 46 gives the weight of each size produced

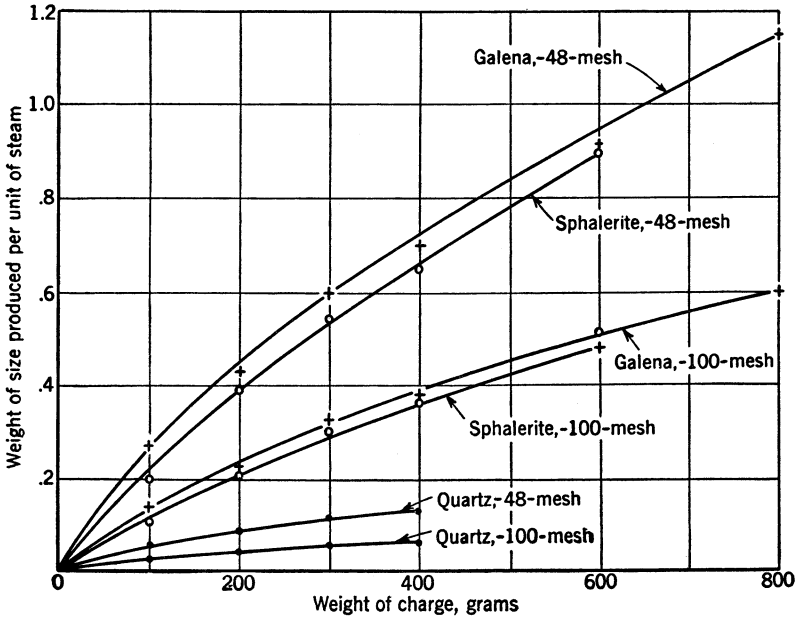


FIGURE 41.—Effect of size of charge on explosive shattering of quartz, galena, and sphalerite, machine 2.

per unit of steam and figure 41 the curves for the minus 48- and minus 100-mesh products.

These results show that better efficiency is to be expected with large charges.

Many dolomite-shattering experiments made in machine 5 with 200- and 400-gram charges also showed that the percentage of shattering is somewhat lower with the 400-gram charges but that the efficiency is much greater. These results are discussed later.

TABLE 45.—Results of explosive shattering with varying weights of charge
[Steam heating at 150 pounds pressure in machine 2]

Material	Weight of charge (grams)	Steam consumption (percent of ore)	Cumulative percent of size produced					
			Minus 4-mesh	Minus 8-mesh	Minus 14-mesh	Minus 48-mesh	Minus 100-mesh	Minus 200-mesh
Quartz.....	100	45	58.9	26.7	12.1	2.6	1.2	0.7
Do.....	200	25	54.1	23.0	10.2	2.2	1.0	.6
Do.....	300	18	53.8	22.6	9.6	2.1	1.0	.6
Do.....	400	15	46.3	19.0	8.2	1.9	.9	.5
Galena.....	100	41	97.6	70.6	40.7	11.1	5.8	3.1
Do.....	200	21	92.4	62.4	34.4	9.1	4.7	2.5
Do.....	300	14	86.1	55.3	29.5	8.4	4.5	2.4
Do.....	400	11	86.7	51.6	28.0	7.7	4.1	2.2
Do.....	600	8	80.8	50.5	26.6	7.3	3.9	1.9
Do.....	800	6	78.2	48.0	24.4	6.0	3.6	1.8
Sphalerite.....	100	42	80.4	47.4	27.6	8.3	4.4	2.6
Do.....	200	22	80.9	50.6	28.3	8.6	4.7	2.7
Do.....	300	15	82.4	49.6	27.8	8.2	4.6	2.7
Do.....	400	12	76.9	46.0	25.4	7.8	4.4	2.7
Do.....	600	8	69.0	39.8	22.1	7.2	4.1	2.6

TABLE 46.—Weight of each size produced per unit of steam in explosive shattering with varying weight of charge

[Steam heating at 150 pounds pressure in machine 2]

Material	Weight of charge (grams)	Weight of each size produced per unit of steam					
		Minus 4-mesh	Minus 8-mesh	Minus 14-mesh	Minus 48-mesh	Minus 100-mesh	Minus 200-mesh
Quartz.....	100	1.309	0.593	0.269	0.058	0.027	0.016
Do.....	200	2.164	.920	.408	.088	.040	.024
Do.....	300	2.989	1.256	.533	.117	.056	.033
Do.....	400	3.687	1.267	.547	.127	.060	.033
Galena.....	100	2.360	1.722	.993	.271	.141	.076
Do.....	200	4.400	2.971	1.638	.433	.224	.119
Do.....	300	6.150	3.950	2.107	.600	.321	.171
Do.....	400	7.882	4.691	2.545	.700	.373	.200
Do.....	600	10.100	6.313	3.325	.913	.488	.238
Do.....	800	13.033	8.000	4.067	1.150	.600	.300
Sphalerite.....	100	1.914	1.129	.657	.198	.105	.062
Do.....	200	3.677	2.300	1.286	.391	.214	.123
Do.....	300	5.493	3.307	1.853	.547	.307	.180
Do.....	400	6.408	3.833	2.117	.650	.367	.225
Do.....	600	8.625	4.975	2.763	.900	.513	.325

EFFECT OF IMPACT

Particles are shattered by impact against each other and especially against obstructions. Many experiments were made to determine the

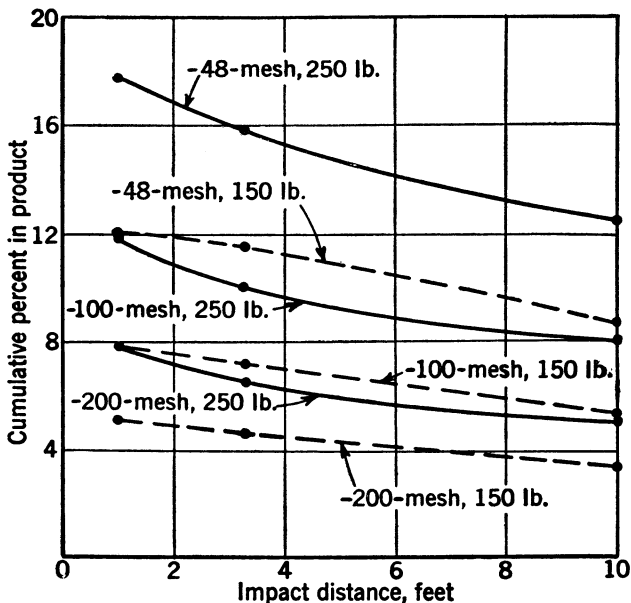


FIGURE 42.—Effect of impact on explosive shattering of dolomite.

amount of shattering caused by impact. The elimination of all impact is impossible, but by providing a rubber buffer at the impact surface much impact shattering is eliminated. This was the method used in the “nonimpact” experiments.

The effect of impact is shown in some results given in table 47. These experiments show that impact has considerable effect on shattering.

Table 48 shows the shattering produced on graphite and on dolomite by impact at 1, 3¼, and 9 feet.

The results of shattering dolomite at different distances of impact are shown in figure 42 for the finer sizes. Shattering appears to be in inverse ratio to the distance of impact; that is, the nearer the impact target the greater the shattering. Cutting the impact distance in half does not, however, double the shattering because the nearer the target the more constricted the impact area.

TABLE 47.—Effect of impact in explosive shattering

Material	Steam pressure (pounds)	Cumulative percent in exploded product											
		Impact					Nonimpact						
		—4 mesh	—8 mesh	—14 mesh	—48 mesh	—100 mesh	—200 mesh	—4 mesh	—8 mesh	—14 mesh	—48 mesh	—100 mesh	—200 mesh
Quartz.....	60	21.3	5.8	2.5	0.7	0.4	0.2	11.5	3.7	1.9	0.7	0.5	0.3
Do.....	120	49.8	17.3	8.7	1.7	.8	.4	29.1	10.0	4.6	1.3	.7	.4
Do.....	180	62.5	28.1	12.9	2.6	1.2	.6	37.1	14.5	7.2	1.7	.9	.5
Large charge.....	150	46.3	19.0	8.2	1.9	.9	.5	28.1	9.8	4.6	1.2	.6	.3
Galena.....	150	91.0	65.2	37.4	10.4	5.4	2.8	55.4	18.8	12.0	5.6	3.5	1.9
Large charge.....	150	78.2	48.0	24.4	6.9	3.6	1.8	44.9	20.1	10.5	4.1	2.7	1.7
Sphalerite.....	60	43.9	16.4	8.7	3.3	2.3	1.7	23.7	9.1	5.3	2.7	2.1	1.7
Do.....	120	72.9	38.8	21.0	6.0	3.4	2.1	47.4	20.8	11.7	4.2	2.7	1.9
Do.....	180	85.2	55.6	33.5	9.9	5.3	2.9	55.6	28.3	16.4	5.3	3.2	2.1
Large charge.....	150	69.0	39.8	22.1	7.2	4.1	2.6	45.8	22.1	12.2	4.3	2.8	2.0
Scheelite ore.....	200	-----	-----	-----	16.0	-----	-----	-----	-----	14.0	-----	-----	-----
Pyritic ore.....	250	-----	-----	-----	19.9	12.2	7.2	-----	-----	9.2	5.7	-----	3.4
Do.....	250	67.1	45.7	33.5	18.5	12.6	7.4	53.2	35.3	23.1	10.9	7.2	4.2
Dolomite.....	150	85.9	60.7	38.1	12.1	7.1	4.3	80.7	51.8	31.7	10.1	6.1	3.8
Do.....	200	92.7	66.7	43.0	15.7	9.6	5.8	83.4	54.5	34.3	12.5	7.6	4.6
Do.....	250	92.6	72.9	49.9	20.1	12.6	7.7	83.1	59.5	37.0	14.3	9.0	5.5

TABLE 48.—Effect of impact in explosive shattering at various distances of impact

Mineral tested	Impact distance (feet)	Steam pressure (pound)	Cumulative percent in exploded product					
			—4 mesh	—8 mesh	—14 mesh	—48 mesh	—100 mesh	—200 mesh
Graphite.....	1	250	-----	-----	-----	90.9	76.9	56.9
Do.....	3¼	250	-----	-----	-----	63.5	42.7	25.3
Dolomite.....	1	150	84.3	60.9	37.0	12.2	7.9	5.3
Do.....	3¼	150	86.1	63.0	38.9	11.6	7.3	4.8
Do.....	9	150	83.0	59.9	32.2	8.8	5.5	3.6
Do.....	1	250	87.7	66.7	44.3	18.0	12.1	8.0
Do.....	3¼	250	88.4	69.4	45.9	15.9	10.2	6.6
Do.....	9	250	86.2	65.3	39.9	12.6	8.1	5.3

EFFECT OF SIZE OF DISCHARGE VALVE

Few experiments have been made to determine the effect of using a discharge valve smaller than the explosive chamber.

An explosive chamber was used with ½- and ¾-inch discharge valves on top. This machine was not well adapted to the work, and only a few experiments were made; the results are given in table 49.

These results are so favorable for small discharges that the investigation is to be continued with variable-size discharge valves in machine 6. With smaller discharges the mechanism for operation is simplified, and a steam-tight joint is more easily maintained in the discharge.

SUMMARY OF CONDITIONS THAT AFFECT EXPLOSIVE SHATTERING

The foregoing results indicate that the following variables affect shattering:

Shattering increases with pressure and temperature.

Shattering increases with the quantity of water present only to a limited degree. It is doubtful whether any benefit is derived from larger quantities of water or longer steaming periods; probably the method of applying the steam has a greater effect on shattering than the quantity of steam.

TABLE 49.—*Effect of size of discharge valve on explosive shattering of dolomite, machine 5*

Discharge-valve diameter (inches)	Charge (grams)	Cumulative percent of product					Fineness of minus 48-mesh material			
		Minus 4-mesh	Minus 8-mesh	Minus 14-mesh	Minus 48-mesh	Minus 100-mesh	Minus 200-mesh	48- to 100-mesh	100- to 200-mesh	Minus 200-mesh
2 $\frac{3}{16}$	200	88.1	68.8	46.0	21.6	15.5	10.7	28.3	22.2	49.5
3 $\frac{1}{4}$	200	87.1	70.1	46.9	19.7	14.0	9.8	28.9	21.3	49.8
1 $\frac{1}{2}$	200	83.1	65.8	42.7	18.4	12.6	8.8	31.5	20.7	47.8
2 $\frac{3}{16}$	400	86.0	66.2	41.9	19.2	13.6	9.4	29.2	21.9	48.9
3 $\frac{1}{4}$	400	86.6	66.0	43.1	16.7	11.4	7.8	31.7	21.6	46.7
1 $\frac{1}{2}$	400	84.1	67.8	43.7	18.2	12.5	8.5	31.3	22.0	46.7

Although when calculated on a percentage basis shattering decreases somewhat with larger charges, the actual amount of shattering per explosion or per unit of steam consumption increases greatly. For capacity and efficiency, large charges are decidedly advantageous.

Shattering is increased by impact, and the nearer the impact target the greater the shattering.

Shattering appears to be slightly decreased when discharge valves are very small, but the mechanical advantages of smaller discharges may well outweigh the slight reduction in shattering.

EFFECT OF SUCCESSIVE EXPLOSIONS ON SHATTERING

Experiments were made on $\frac{3}{4}$ -mesh quartz, galena, and sphalerite in which the plus 4-mesh product of the first explosion was reexploded, as at first. The results, which are given in table 50, show that there was virtually as much shattering in the second explosion as in the first.

TABLE 50.—*Effect of successive explosions on shattering*

	Cumulative percent in shattered product					
	Minus 4-mesh	Minus 8-mesh	Minus 14-mesh	Minus 48-mesh	Minus 100-mesh	Minus 200-mesh
Quartz:						
First explosion on fresh $\frac{3}{4}$ -mesh quartz.	71.3	34.5	14.5	3.1	1.4	0.7
Second explosion on plus 4-mesh material from first explosion.....	67.8	32.3	13.3	2.8	1.2	.6
Galena:						
First explosion on fresh $\frac{3}{4}$ -mesh galena.	91.0	65.2	37.4	10.4	5.4	2.8
Second explosion on plus 4-mesh material from first explosion.....	89.9	71.7	44.5	13.2	7.0	3.7
Sphalerite:						
First explosion on $\frac{3}{4}$ -mesh sphalerite..	91.2	66.0	40.1	12.4	6.3	3.4
Second explosion on plus 4-mesh material from first explosion.....	84.8	59.1	34.6	10.5	5.4	3.1

The shattering effect on a variety of ores of successive explosions is shown in table 51. Ordinarily there is a more easily shattered portion in the material before explosion, but subsequent explosions usually yield about the same percentage of finer sizes.

The quantity of finished product yielded in successive explosions is related closely to the return or circulating load. Experiments were made on plus 48-mesh dolomite in which the minus 48-mesh product was removed after each explosion and replaced with original charge. After eight explosions equilibrium was reached, and the circulating

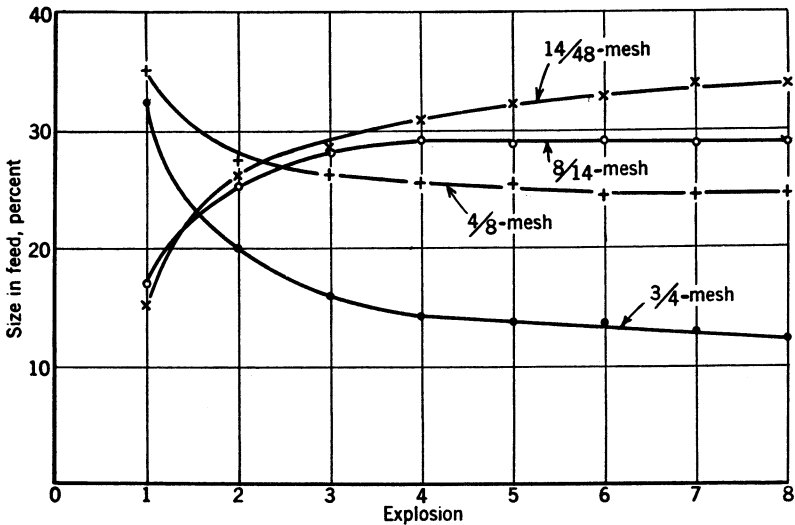


FIGURE 43.—Change in feed for continuous explosions of dolomite.

load was approximately 540 percent. The results, which are given in table 52, show that the production figures remain practically constant after the first explosion; and the curves in figure 43, the data for which are not given herewith, show that the feed is virtually constant at the seventh or eighth explosion.

TABLE 51.—Amount of product obtained from successive explosions

Material	Pressure (pounds)	Size (mesh)	Percent of size produced from explosion							
			1	2	3	4	6	8	10	12
Lead-zinc chert ore.....	100	-48	2.3	3.3	3.8	3.5	3.3	3.2	2.3	2.1
Lead-zinc-iron ore.....	150	48/100	9.7	10.2	9.9
Do.....	100/200	5.3	5.1	4.2
Do.....	-200	6.3	5.7	4.8
Amygdaloid copper ore.....	200	-48	11.9	9.8	9.2	9.1	8.8	9.7	10.3	9.6
Copper ore tailing.....	200	-48	7.0	5.9	5.6	5.5	5.9	5.1	4.9	4.7
Pyritic gold ore.....	250	48/100	6.0	4.8	6.3	7.2	8.0	8.2
Do.....	100/200	5.2	3.6	4.6	5.1	5.7	5.8
Do.....	-200	7.4	4.8	5.8	6.3	7.1	6.9
Pyrite concentrate.....	250	-200	5.2	5.0	5.1	4.7	6.3
Gold ore.....	250	100/200	2.7	2.3	2.2	2.2
Do.....	-200	4.9	4.2	3.8	3.7
Silver ore.....	200	48/100	5.0	3.6	2.9	2.4	2.2
Do.....	100/200	3.7	2.3	1.6	1.3	1.1
Do.....	-200	8.5	5.6	4.1	3.2	2.9
Scheelite ore.....	250	-48	22.5	19.6	17.6	17.8
Taconite.....	200	-100	14.3	13.4	12.0	12.4	12.7	11.3	11.6
Hematite.....	200	-48	14.8	13.9	14.1
Kyanite-biotite ore.....	200	-8	24.2	36.8	29.8
Graphite ore.....	100	14/100	10.9	13.7	12.8	17.7	15.0
Do.....	-100	2.7	3.6	3.0	3.9	3.6
Graphite.....	250	-400	14.2	9.8	10.4
Ilmenite-apatite ore.....	250	-48	4.6	4.5	4.7
Clay.....	200	-200	25.3	23.2	23.3	21.9	21.9	22.1
Mica schist.....	100	-65	7.1	8.6
Limestone.....	250	-270	4.7	3.7	3.8	3.6	3.9
Anthracite.....	200	-100	6.2	4.8	3.9
Bituminous coal.....	200	-100	19.3	9.7

TABLE 52.—Circulating load and product in successive explosions of dolomite to produce minus 48-mesh product

Explosion	New charge (grams)	Return charge (grams)	Product (grams)			Circulating load (percent)
			48- to 100-mesh	100- to 200-mesh	Minus 200-mesh	
1.....	4,000	-----	208	148	341	-----
2.....	697	3,303	192	134	290	474
3.....	616	3,384	191	133	289	549
4.....	613	3,387	193	136	289	553
5.....	618	3,382	190	134	289	547
6.....	613	3,387	195	135	295	553
7.....	625	3,375	194	133	296	540
8.....	623	3,377	194	130	311	542

FINE MATERIAL PRODUCED IN EXPLOSIVE SHATTERING

The conception of explosive shattering would suggest that a smaller amount of fine material would be made than with ordinary grinding. That this actually occurs is shown by various examples in table 53, and while the difference usually is not great any reduction in the amount of excessively fine material is an advantage when subsequent milling is required.

DIFFERENTIAL SHATTERING

The theory that an explosion wave causes preferential rupture along planes of low fatigue strength—meaning the boundaries between unlike minerals or the planes of the secondary structure—indicates that explosive shattering is especially adapted to differential crushing.

Good differential crushing results in clean separation of the different minerals, in which the harder mineral is left comparatively large. Such crushing results in increased recovery and higher grade of product in milling, as well as in a reduction in crushing energy.

Differential crushing is also beneficial in preparing ores for leaching. When the valuable minerals are deposited along the planes of secondary structure they are exposed to the action of the solvent solutions.

TABLE 53.—Comparison of fine material produced in explosive shattering and ordinary grinding

Material	Ordinary grinding				Explosive shattering			
	Fine material produced		Fine-material sizing test		Fine material produced		Fine-material sizing test	
	Mesh	Percent	Mesh	Percent	Mesh	Percent	Mesh	Percent
Ilmenite-apatite ore.....	-20	63.3	20/200	81.5	-20	64.7	20/200	84.7
Do.....	-----	-----	-200	18.5	-----	-----	-200	15.3
Scheelite ore.....	-200	45.2	200/400	33.6	-200	38.1	200/400	40.2
Do.....	-----	-----	37/18 μ	27.6	-----	-----	37/18 μ	33.6
Do.....	-----	-----	18/9 μ	22.8	-----	-----	18/9 μ	14.7
Do.....	-----	-----	9/4.5 μ	10.2	-----	-----	9/4.5 μ	6.8
Do.....	-----	-----	-4.5 μ	5.8	-----	-----	-4.5 μ	4.7
Oxidized graphite ore.....	-14	84.9	14/100	67.4	-14	85.0	14/100	69.8
Do.....	-----	-----	-100	32.6	-----	-----	-100	30.2
Hematite.....	-200	11.4	200/400	17.5	-200	11.2	200/400	19.6
Do.....	-----	-----	37/18 μ	21.1	-----	-----	37/18 μ	76.8
Do.....	-----	-----	-18 μ	61.4	-----	-----	-18 μ	3.6
Dolomite.....	-14	26.9	14/48	49.1	-14	28.9	14/48	70.6
Do.....	-----	-----	-48	50.9	-----	-----	-48	29.4
Gold ore.....	-200	19.5	200/400	25.1	-200	20.2	200/400	36.1
Do.....	-----	-----	-400	74.9	-----	-----	-400	63.9
Pyritic gold ore.....	-48	35.0	48/200	41.4	-48	35.9	48/200	63.0
Do.....	-----	-----	-200	58.6	-----	-----	-200	37.0
Silver ore.....	-48	50.0	48/100	22.0	-48	50.1	48/100	31.7
Do.....	-----	-----	100/200	18.6	-----	-----	100/200	20.0
Do.....	-----	-----	-200	59.4	-----	-----	-200	48.3

Experiments on different ores have shown that in explosive shattering better differential crushing results not only in the separation of minerals for subsequent concentration but also in more completely exposing the minerals for leaching.

Tables 38, 39, 42, 43, and 45 indicate that differential shattering is not affected by the explosive pressure, the amount of water or steam used, or the size of the charge, as the ratio of galena and sphalerite shattered to that of quartz remains approximately constant.

Examples of differential shattering of various ores follow:

Lead-zinc ore.—Earlier experiments showed that explosive shattering concentrated the galena and sphalerite into the minus 48-mesh size (118). Successive explosions are required, however, for a fairly complete separation of the galena and sphalerite into the finer sizes. Tri-State lead-zinc ore, $\frac{3}{8}$ to $\frac{1}{4}$ inch in size, was exploded 12 times at 100 pounds pressure, and the minus 48-mesh material was removed after each explosion. Table 54 gives the results.

TABLE 54.—*Successive explosions on Tri-State lead-zinc ore*

[Analysis of original ore: 0.70 percent Pb, 3.63 percent Zn]

Explosion.	Minus 48-mesh product (cumulative percent)	Analysis (percent)				Contents in minus 48-mesh size (cumulative percent)	
		Plus 48-mesh size		Minus 48-mesh size		Pb	Zn
		Pb	Zn	Pb	Zn		
1.	2.3	0.71	3.13	0.33	24.86	1.1	15.7
2.	5.6	.51	2.51	6.74	21.40	32.2	34.9
3.	9.1	.24	1.91	7.20	17.85	68.5	52.3
4.	12.3	.14	1.40	3.17	15.89	82.7	66.2
5.	15.3	.10	1.07	1.28	10.78	88.2	75.1
6.	18.0	.08	.75	.52	10.59	90.2	83.1
7.	20.1	.08	.55	.31	8.40	91.1	87.9
8.	22.6	.07	.39	.22	5.55	91.9	91.8
9.	24.3	.06	.28	.45	5.27	93.0	94.2
10.	26.0	.05	.17	.74	4.94	94.7	96.5
11.	27.7	.03	.13	1.05	2.11	97.2	97.5
12.	29.1	.00	.13	1.36	1.40	100.0	97.5

¹ Analysis of total minus 48-mesh size: Pb 2.42 percent, Zn 12.17 percent.

The curves for recovery of the minus 48-mesh material and depletion of the plus 48-mesh size given in figure 44 show that a large percentage of the galena and sphalerite is found in the minus 48-mesh size after the fourth or fifth explosion.

Table 55 gives another example of differential shattering of lead-zinc ore. Roll crushing is compared to shattering results of three explosions at 150 pounds pressure. The comparison shows that explosive shattering gives a higher-grade minus 48-mesh size.

Silver ore.—Silver ore, 4- to 48-mesh size, was exploded, and another lot was ball-milled to produce the same percentage of minus 48-mesh material. The silver assays in table 56 show that crushing of the silver minerals was somewhat better with explosive shattering.

Taconite ore.—A hard taconite containing magnetite was exploded at 200 and 250 pounds pressure, and the minus 100-mesh material was taken for magnetic separation. Mechanical grinding with a disk machine was done to parallel explosive shattering in the amount of minus 100-mesh material produced.

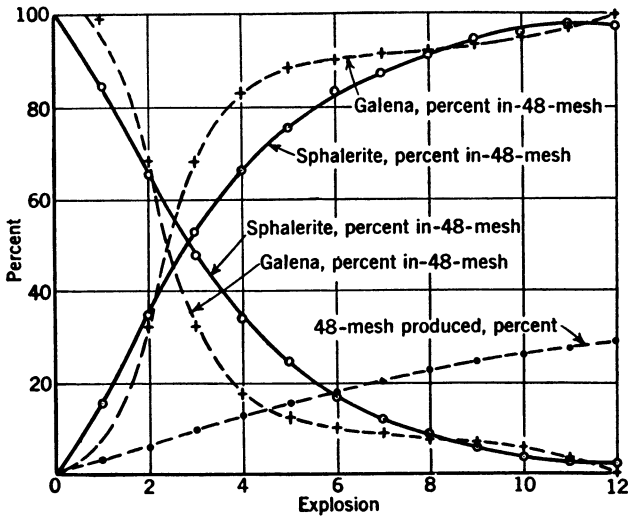


FIGURE 44.—Differential shattering in successive explosions of Tri-State lead-zinc ore.

TABLE 55.—Roll crushing and explosive shattering on a lead-zinc ore

Size (mesh)	Roll crushing				Explosive shattering					
	Weight (per-cent)	Analysis (per-cent)		Contents (per-cent)		Weight (per-cent)	Analysis (per-cent)		Contents (per-cent)	
		Pb	Zn	Pb	Zn		Pb	Zn	Pb	Zn
+48.....	95.2	0.17	4.81	89.9	89.8	91.5	0.15	4.26	69.8	74.0
-48.....	4.8	.38	10.81	10.1	10.2	8.5	.70	16.12	30.2	26.0
	100.0	.18	5.10	100.0	100.0	100.0	.20	5.27	100.0	100.0

TABLE 56.—Explosive shattering and ball-mill grinding of silver ore

Method	Size (mesh)	Weight (percent)	Assay (Ag)	Contents (Ag)	Contents (Ag, per-cent)
Ball-mill grinding.....	+48	54.8	19.63	10.76	46.7
Do.....	48 to 100	9.9	27.69	2.74	
Do.....	100 to 200	8.4	28.72	2.41	
Do.....	-200	26.9	26.42	7.11	
		100.0	23.02	23.02	
Explosive shattering.....	+48	54.8	18.27	10.01	43.2
Do.....	48 to 100	14.5	29.94	4.34	
Do.....	100 to 200	8.9	30.78	2.74	
Do.....	-200	21.8	27.89	6.08	
		100.0	23.17	23.17	

The minus 100-mesh product from each crushing was subjected to magnetic separation in a Davis tube. The results, given in table 57, show that, due to a cleaner differential crushing, recovery in the exploded product was considerably better and the grade of concentrate higher.

Scheelite ore.—A sample of scheelite ore was shattered explosively at 200 and 250 pounds pressure, and another sample was roll-crushed to approximately the same production of minus 48-mesh size.

The minus 48-mesh product from the three crushings was tabled. The results, given in table 58, show that an improved recovery was possible with explosive shattering due to a cleaner differential shattering.

Kyanite-biotite ore.—An intimately intergrown kyanite-biotite ore was shattered explosively, $\frac{1}{4}$ - to $\frac{3}{4}$ -inch pieces being exploded at 200 pounds pressure. As the kyanite and biotite were practically freed at 8-mesh, the minus 8-mesh material was removed after each explosion. As the kyanite resisted shattering more than the biotite the percentage of kyanite in the plus 8-mesh size increased with each explosion. The results are given in table 59.

TABLE 57.—*Explosive shattering and mechanical grinding of taconite*

Crushing	Head analysis (Fe, percent)	Concentrate produced			Iron recovery (percent)
		Weight (percent)	Analysis (percent)		
			Fe	Insoluble	
Mechanical grinding.....	18.1	11.18	66.2	5.52	40.9
Explosive, 200 pounds.....	16.9	12.23	67.4	3.58	48.8
Explosive, 250 pounds.....	18.1	13.18	67.6	3.56	49.2

TABLE 58.—*Roll and explosive crushing on scheelite ore*

Crushing	Concentrate		Tailing		WO ₃ recovery (percent)
	Weight (percent)	WO ₃ (percent)	Weight (percent)	WO ₃ (percent)	
Roll crushing.....	4.5	25.22	95.5	0.46	72.1
Explosive, 200 pounds.....	7.1	26.45	92.9	.42	82.8
Explosive, 250 pounds.....	7.2	23.13	92.8	.26	87.4

TABLE 59.—*Enrichment of kyanite in coarser sizes by explosive shattering*

Explosion	Size (mesh)	Cumulative weight (percent)	Kyanite (percent)	Contained kyanite (percent)	Explosion	Size (mesh)	Cumulative weight (percent)	Kyanite (percent)	Contained kyanite (percent)
1.....	+8	75.8	41.2	97.5	4.....	+8	24.4	79.9	60.8
	-8	24.2	2.6	2.5		-8	75.6	16.6	39.2
2.....	+8	47.8	60.8	90.7	5.....	+8	15.7	86.0	42.1
	-8	52.2	5.7	9.3		-8	84.3	22.0	57.9
3.....	+8	33.6	71.6	75.2					
	-8	66.4	12.0	24.8					

Graphite ore.—A sample of hard graphite ore was treated by ordinary crushing and by explosive shattering at different pressures. The 14- to 100-mesh size was treated electrostatically to recover no. 1 flake graphite. The minus 100-mesh size is considered undesirable for no. 1 flake. The ideal crushing consists therefore in producing a 14- to

100-mesh size that gives the purest no. 1 flake with the highest recovery and the smallest loss in the undesirable size. Explosive shattering, as shown in table 60, produced a higher grade of no. 1 flake and a much higher recovery as a result of the clean separation. The graphite in the undesirable minus 100-mesh product was somewhat greater with explosive crushing.

Coal.—Samples of bituminous coal and anthracite were exploded and ground in a disk machine to produce minus 48-mesh material which was floated to produce a coal froth and sink pyrite. The results, given in table 61, indicate that the pyrite was freed more by explosive shattering and that the ash was removed more completely.

TABLE 60.—*Ordinary and explosive crushing on graphite ore*

Treatment	Production, weight, percent		Graphite analysis, no. 1 flake	Graphite distribution in -14-mesh material			
	14- to 100-mesh	-100-mesh		No. 1 flake	Mid-dling	Tail	-100
Ordinary crushing.....	36.5	2.1	87.6	57.9	20.7	17.4	4.0
Exploded at 75 pounds.....	37.1	3.1	89.7	62.3	16.8	14.1	6.8
Exploded at 100 pounds.....	36.3	2.9	90.0	70.1	9.3	13.6	7.0
Exploded at 150 pounds.....	45.9	4.0	88.9	69.7	8.7	13.0	8.6
Exploded at 200 pounds.....	54.8	5.3	90.5	67.3	13.2	9.6	9.9

TABLE 61.—*Flotation of coal after ordinary and explosive grinding*

	Ordinary grinding			Explosive shattering		
	Weight (percent)	S (percent)	Ash (percent)	Weight (percent)	S (percent)	Ash (percent)
Anthracite float.....	57.4	0.56	2.18	63.4	0.51	1.77
Anthracite residue.....	42.6	.56	3.32	36.6	.65	4.22
Bituminous float.....	85.2	.46	1.53	90.8	.41	1.24
Bituminous residue.....	14.8	.48	4.88	9.2	.94	9.81

Pyritic gold ores.—Explosive shattering of a number of pyritic gold ores has resulted in increased extraction of gold by cyanidation. Some of the results follow:

Pyrite tailing: A pyrite concentrate, made by tabling the tailing from a flotation-cyanide plant, was exploded at 200 pounds pressure with one, two, and three explosions. Table 62 compares the results on the minus 100-mesh size with the original concentrate. Two other lots of the pyrite were exploded at pressures of 200 and 250 pounds; the minus 200-mesh product was cyanided. The higher pressure resulted in a marked increase in extraction.

Pyritic gold ore: A heavy pyritic gold ore was exploded to produce a minus 48-mesh product, and the 48- to 100-, 100- to 200-, and minus 200-mesh sizes were cyanided 24 hours. The results, compared in table 63 with a ball-mill product producing approximately the same percentage of minus 200-mesh material, show that more gold is extracted when the ore is shattered explosively.

Another pyritic gold ore was reduced to minus 48-mesh size by explosive shattering and ball milling and subjected to cyanidation.

The results of the shorter treatment are more favorable with the exploded product, as shown in table 64.

TABLE 62.—Cyanide extraction on pyrite tailing after exploding

Treatment	Minus 100-mesh (percent)	Gold extraction by cyanidation (percent)	
		24 hours	48 hours
Original ore.....	62.5	4.9	8.1
1 explosion.....	66.3	8.7	13.2
2 explosions.....	66.9	10.9	14.8
3 explosions.....	70.0	14.2	19.0
Explosion, 200 pounds, minus 200-mesh.....	19.3	19.3	27.9
Explosion, 250 pounds, minus 200-mesh.....	38.6	38.6	41.2

TABLE 63.—Cyanide extraction on a heavy pyritic ore

Method	48- to 100-mesh, 0.37 ounce Au			100- to 200-mesh, 0.32 ounce Au			Minus 200-mesh, 0.32 ounce Au		
	Weight (per cent)	Tail Au (ounce)	Extraction (per cent)	Weight (per cent)	Tail Au (ounce)	Extraction (per cent)	Weight (per cent)	Tail Au (ounce)	Extraction (per cent)
Ordinary grind, ball mill.....	5.9	0.29	21.6	8.6	0.21	34.4	20.5	0.18	43.8
Exploded at 200 pounds.....	21.8	.24	35.1	14.9	.20	37.5	20.2	.14	56.3
Exploded at 250 pounds.....	24.5	.27	27.0	17.7	.18	43.8	22.8	.125	60.9
Exploded at 250 pounds, nonimpact.....	16.8	.26	29.7	11.6	.16	50.0	15.9	.13	59.4

TABLE 64.—Extraction by cyanidation on a pyritic gold ore

Process	Assay (Au)		Extraction (percent)
	Head	Tailing	
48-hour treatment:			
Ordinary grinding.....	0.22	0.10	54.5
Explosive shattering.....	.49	.20	59.2
96-hour treatment:			
Ordinary grinding.....	.22	.08	63.6
Explosive shattering.....	.49	.18	63.3

Arsenical iron ore.—The results on a low-grade arsenical iron ore, given in table 65, show no particular advantage for explosive shattering over ordinary ball-mill grinding; however, when this ore is heated to incipient arsenic volatilization and then exploded the extraction is increased considerably.

Pyritic concentrates.—A very fine, high-grade pyritic concentrate was subjected to several explosions, another sample was ball-milled to an equivalent fineness, and another was ground extremely fine by ball milling in a cyanide solution followed by 48 hours of cyanide treatment. The gold extractions, given in table 66, were all considerably better on the exploded product; two explosions gave the best results.

Two other pyritic concentrates were exploded and the cyanide extractions compared with the ball-mill product. These results, given in table 67, also show that extraction usually was better with explosive crushing.

TABLE 65.—Cyanidation extraction on a low-grade arsenical iron ore

Method	Mesh	Size treated, percent by weight	48-hour treatment			96-hour treatment		
			Assay (Au)		Extraction (percent)	Assay (Au)		Extraction (percent)
			Head	Tailing		Head	Tailing	
Ordinary grinding.....	—100	49.6	0.23	0.12	47.8	0.23	0.125	45.7
Explosive shattering.....	—100	23.8	.26	.13	50.0	.26	.12	53.3
Ordinary grinding.....	200 to 400	4.9				.30	.14	53.3
Do.....	—400	14.6	.25	.09	64.0	.25	.08	68.0
Explosive shattering.....	200 to 400	7.3				.29	.15	48.3
Do.....	—400	12.9	.27	.09	66.7	.27	.08	70.4
Explosive shattering.....	200 to 400	10.0				.28	.07	75.0
After As ignition.....	—400	16.3	.40	.07	82.5	.40	.07	82.5

TABLE 66.—Cyanide extractions on high-grade fine pyritic concentrates

[Assay, 3.77 ounces Au]

Process	Minus 400-mesh (percent)	24-hour cyanidation		48-hour cyanidation		72-hour cyanidation		96-hour cyanidation	
		Tailing Au (ounces)	Extraction (percent)	Tailing Au (ounces)	Extraction (percent)	Tailing Au (ounces)	Extraction (percent)	Tailing Au (ounces)	Extraction (percent)
		Original material.....	92.5	2.70	28.4	2.06	45.4	2.00	46.9
Ordinary grind, ball milling.....	97.7	2.34	37.9	1.74	53.8	1.90	49.6	1.70	54.9
1 explosion, 250 pounds.....	95.3	1.52	59.7	1.10	70.8	.96	74.5	.90	76.1
2 explosions, 250 pounds.....	97.3	1.12	70.3	.76	79.8	.78	79.3	.74	80.4
4 explosions, 250 pounds.....	97.9	1.70	54.9	1.21	67.9	.90	76.1	.80	78.8
Ball-mill grind, 30 minutes in KCN solution followed by 48-hour treatment.....	99.9			1.30	65.5				

TABLE 67.—Cyanide extractions on pyritic concentrates

Method	48-hour treatment			96-hour treatment		
	Assay (Au)		Extraction (percent)	Assay (Au)		Extraction (percent)
	Head	Tailing		Head	Tailing	
Concentrate A:						
Ordinary grind.....	0.57	0.21	63.2	0.57	0.095	83.3
1 explosion.....	.57	.21	63.2	.57	.11	80.7
2 explosions.....	.57	.20	64.9	.57	.13	77.2
Concentrate B:						
Ordinary grind, 96.2 percent minus 400-mesh.....	2.40	.90	62.5	2.40	.84	65.0
1 explosion, 96.1 percent minus 400-mesh.....	2.40	.76	68.3	2.40	.76	68.3
2 explosions, 97.0 percent minus 400-mesh.....	2.40	.74	69.2	2.40	.76	68.3

EFFICIENCY AND COSTS

Explosive shattering has in its favor the fact that it uses the energy of steam directly without intermediate transmission machinery.

Tables 42, 43, and 44 show the effect of the amount of water with the charge at the time of explosion. Although in experiments with the older machines (1 and 2) increased amounts of water appeared to have a beneficial effect on shattering, experiments on dolomite in machine 5 indicate that more than 5 percent of water has no material benefit.

The quantity of water present with the charge is a function of the steam pressure and the length of the steaming period. The higher the steam pressure the larger should be the amount of water with the charge per unit of time, but with equal quantities of water the higher pressures are more efficient.

Many experiments were made to determine the efficiency of explosive shattering, with varying amounts of steam consumption. These results have been reported in Bureau of Mines Report of Investigations 3268, February 1935 (136), and are reproduced here in part only. As the results of nonimpact and impact shattering parallel each other only the impact results are considered.

Tables 68 and 69 give the production and steaming costs for the finer sizes with 150 and 250 pounds of steam pressure.

The explosive experiments were made on 2,000 grams of hard dolomite; the feed for each experiment consisted of the following sizes:

Mesh:	<i>Weight (percent)</i>
3-4.....	32.6
4-8.....	35.2
8-14.....	16.9
14-48.....	15.3

To determine the consumption of steam the experiment was repeated in an identical manner, but the explosion was not allowed to occur. Instead, the explosion chamber was cooled and the water contained with the charge determined.

The steam-consumption figures represent the amounts of steam actually utilized in the explosion. The steam consumption is decreased (1) by shortening the time between explosions, (2) by shortening the time of steaming, (3) by using steam at lower pressures, and (4) by eliminating radiation as far as possible.

All calculations are based on total consumption of steam, and no allowance is made for possible heat recovery. The thermal units in the steam, expressed in British thermal units, are based on raising water at 25° C. to the temperature of the steam (181° C. for a pressure of 150 pounds and 205° C. for a pressure of 250 pounds).

TABLE 68.—*Efficiency and cost data for exploded dolomite, 150 pounds steam pressure*

Steam consumption (pounds per ton of charge)	Pounds produced per ton of charge			Pounds produced per 1,000 B. t. u.			Cost of steam, cents per ton of—			
	Minus 200-mesh	Minus 100-mesh	Minus 48-mesh	Minus 200-mesh	Minus 100-mesh	Minus 48-mesh	Feed	Minus 200-mesh	Minus 100-mesh	Minus 48-mesh
864.....	112	163	244	0.113	0.164	0.246	11.02	196.8	135.2	90.3
748.....	100	150	226	.116	.175	.265	9.54	190.8	127.2	84.4
546.....	106	154	232	.169	.246	.370	6.96	131.3	90.4	60.0
423.....	118	170	252	.243	.350	.519	5.40	91.5	63.5	42.8
383.....	112	162	242	.255	.369	.550	4.89	87.3	60.4	40.4
330.....	102	150	226	.269	.396	.597	4.21	82.5	56.1	37.3
290.....	106	152	228	.318	.457	.685	3.70	69.8	48.7	32.5
251.....	114	164	242	.396	.569	.840	3.20	56.1	39.0	26.4
225.....	108	158	236	.418	.612	.914	2.87	53.3	36.3	24.3
206.....	100	144	216	.423	.609	.913	2.63	52.6	36.5	24.4
154.....	102	150	228	.577	.848	1.290	1.96	38.4	26.1	17.2
139.....	94	136	216	.589	.852	1.353	1.77	37.7	26.0	16.4
102.....	104	150	224	.888	1.281	1.913	1.30	25.0	17.3	11.6
83.5.....	90	130	196	.938	1.356	2.044	1.07	23.8	16.5	10.9
63.5.....	82	118	180	1.125	1.619	2.469	.81	19.3	13.7	9.0
57.....	84	124	190	1.284	1.869	2.905	.73	17.4	11.8	7.7
51.....	76	112	174	1.299	1.915	2.974	.65	17.1	11.6	7.5
41.....	72	104	162	1.529	2.208	3.440	.52	14.4	10.0	6.4
28.5.....	66	98	154	2.018	2.997	4.709	.36	10.9	7.3	4.7

TABLE 69.—*Efficiency and cost data for exploded dolomite, 250 pounds steam pressure*

Steam consumption (pounds per ton of charge)	Pounds produced per ton of charge			Pounds produced per 1,000 B. t. u.			Cost of steam, cents per ton of—			
	Minus 200-mesh	Minus 100-mesh	Minus 48-mesh	Minus 200-mesh	Minus 100-mesh	Minus 48-mesh	Feed	Minus 200-mesh	Minus 100-mesh	Minus 48-mesh
1,025	185	273	387	0.156	0.230	0.327	13.17	142.4	96.5	68.1
905	170	250	360	.162	.239	.344	11.62	136.7	93.0	64.6
686	180	262	374	.227	.330	.472	8.81	97.9	67.3	47.1
471	190	276	392	.349	.507	.720	6.05	63.7	43.8	30.9
423	184	274	390	.376	.560	.798	5.43	59.0	39.6	27.8
332	198	284	396	.516	.740	1.032	4.26	43.0	30.0	21.5
299	190	270	378	.550	.781	1.094	3.84	40.4	28.4	20.3
277.5	188	272	384	.586	.848	1.197	3.56	37.9	26.2	18.5
255	172	248	354	.583	.841	1.201	3.28	38.1	26.5	18.5
228.5	162	232	334	.613	.878	1.265	2.93	36.2	25.3	17.5
214	180	268	380	.728	1.083	1.536	2.75	30.6	20.5	14.5
160	186	268	376	1.005	1.449	2.032	2.06	22.2	15.4	11.0
127	174	256	364	1.185	1.744	2.480	1.63	18.7	12.7	9.0
117.5	148	214	310	1.090	1.576	2.283	1.51	20.4	14.1	9.7
107	168	244	348	1.358	1.973	2.813	1.37	16.3	11.2	7.9
95	154	222	318	1.403	2.022	2.896	1.22	15.8	11.0	7.7
87	156	234	334	1.551	2.326	3.320	1.12	14.4	9.6	6.7
68.5	138	200	288	1.742	2.525	3.636	.88	12.8	8.8	6.1
53.5	126	186	272	2.039	3.010	4.401	.69	11.0	7.4	5.1
19	74	107	172	3.364	4.864	7.818	.24	6.6	4.6	2.8

The heat units are made up as follows:

	<i>150-pound pressure</i>	<i>250-pound pressure</i>
Heat of liquid (181° and 205° C.)-----	329	374
Heat of liquid (25° C.)-----	45	45
B. t. u. to raise liquid from 25° C. to steam temperature.-----	284	329
Heat of vaporization-----	864	827
Total B. t. u. in 1 lb. of steam-----	1, 148	1, 156

As the energy necessary to produce steam at 250 pounds pressure is only slightly more than that necessary at 150 pounds pressure and as the shattering ability is considerably greater at the higher pressure, the crushing efficiency is higher at 250 pounds. As the energy needed to produce steam increases but slightly with higher pressures it is very probable that higher efficiencies may be obtained with still higher pressures. No machine having a working pressure of more than 250 pounds was available, therefore experiments at higher pressures were impossible.

In the tables only the sizes below 48-mesh have been considered. The "pounds produced per ton of charge" may be considered as a measure of capacity, while the "pounds produced per 1,000 B. t. u." may be considered as a measure of efficiency.

The steam costs are based on a boiler efficiency of 90 percent with coal yielding 10,000 B. t. u. and costing \$2 per ton. On this basis 1 pound of steam at 150 pounds pressure costs 0.01275 cent and at 250 pounds pressure, 0.01285 cent.

The relation of capacity to energy input is shown in figure 45. The curves have been extended to pass through the origin, as no consumption of steam results in no production. These curves show that the higher pressure gives a much greater capacity and that capacity drops off rapidly with very low steaming.

The relation of efficiency to energy input is shown in figure 46. These curves show that efficiency increases rapidly as the steaming is decreased.

In figures 45 and 46 only the data for the minus 48-mesh size are used, and as low steam consumption is of interest mainly in the lower

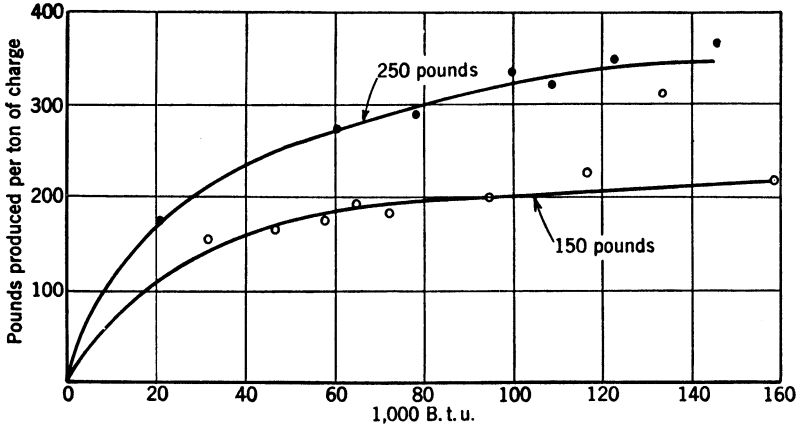


FIGURE 45.—Relationship of minus 48-mesh size produced to energy input.

end, the curves of steam consumption have been carried only to 150 pounds per ton. As the curves are based on input of energy they are comparable, irrespective of the explosive procedure.

At the higher pressure the efficiency is considerably better, although this relationship is not necessarily a straight line. No experiments

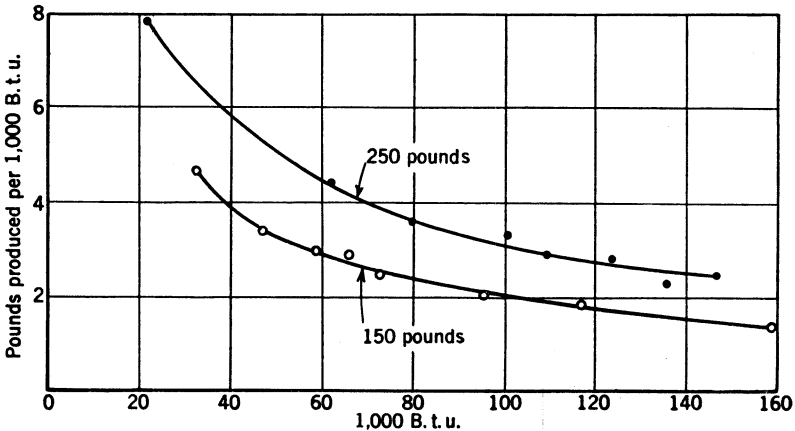


FIGURE 46.—Relationship of efficiency to energy input.

on efficiency have been made at pressures other than 150 and 250 pounds, but a study of the data at these two pressures indicates that efficiency might fall off with higher pressures and that maximum efficiency might be reached at some pressure below the critical pressure of steam.

The relation of steaming cost to energy input is shown in figure 47. The steaming cost does not include cost or depreciation of plant and is only that used in producing the explosion. The points plotted in the lower range of energy input indicate a flat curve, but a plot of the entire range shows a straight-line relationship. These curves show that low steaming costs are to be expected with high pressure, short steaming time, and larger charges.

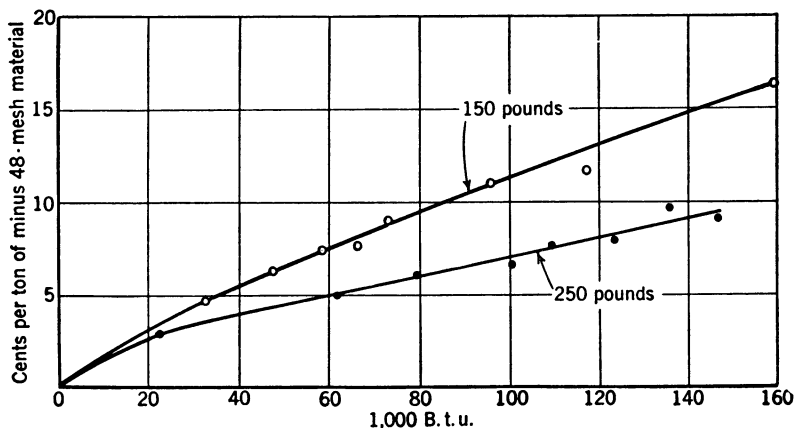


FIGURE 47.—Relationship of steaming cost to energy input.

BALL-MILL EXPERIMENTS

As tube, ball, and rod mills are used generally for fine grinding the ball mill, as exemplifying mills of this type, will be considered in some detail with special reference to the factors contributing to greater efficiency.

The discussion of ball-mill experiments will be considered under the headings: (1) Laboratory experimental work, (2) slippage in the mill, (3) iron wear, and (4) effect of variables.

LABORATORY WORK

The conclusions that may be drawn from laboratory experiments are somewhat doubtful, especially when the experiments are made on batch charges. Nevertheless, such results show definite trends owing to changes in the operation of the mill. Coghill (131) has shown that the operation of small laboratory mills is comparable to that of large mills.

Naturally, laboratory experiments must be made with great care, measurements of power must be reliable, and the nature of the ground product must be determined intelligently. In all experiments made at the Salt Lake station of the Bureau of Mines power readings were obtained with the differential integrating apparatus previously described (109), which is shown in figure 48.

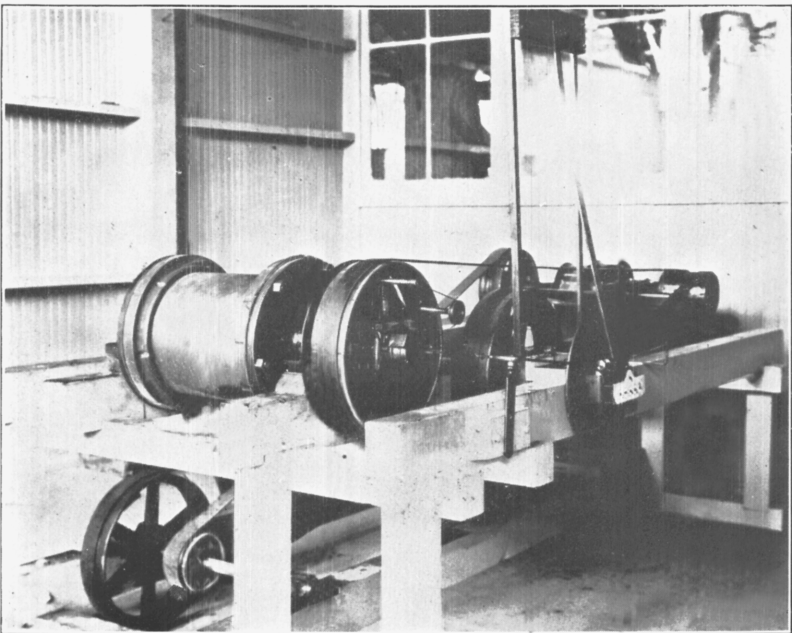


FIGURE 48.—Laboratory ball mill with differential integrating power-recording device.

Power readings were determined for dead loads representing various weights of ball charge and various speeds. The total work delivered to the mill minus the dead load gives the net input of work to the balls. This figure is used in all calculations of efficiency and, compared to the surface determined by the dissolution method, gives the amount of surface produced per foot-pound of work; from this the efficiencies are calculated.

Short-period batch experiments are likely to lead to incorrect conclusions, as considerable time is required to attain equilibrium between the charge and ball load. It was found that equilibrium is delayed by coarse feed, higher speed, and higher pulp density. As the longest time necessary to reach equilibrium was 19 minutes, two ex-

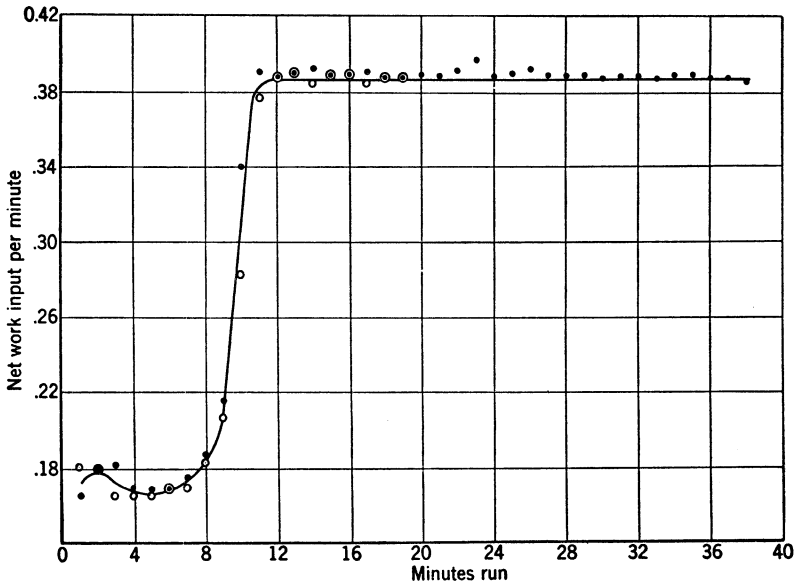


FIGURE 49.—Work input for ball-mill runs of 19 and 30 minutes.

periments were made in which all conditions except time were equal; one charge was ground 19 minutes and the other 38 minutes. Surface determinations were made on the two products. Table 70 gives these results, together with individual power readings and other data. The change in power readings is plotted in figure 49. This curve shows that equilibrium was reached after approximately 12 minutes. The readings taken after 19 minutes may be seen on the curve, so they are not tabulated.

The total efficiency was greater with the second period, although the useless grinding, based on minus 37-micron size as undesirable, increased considerably. These two experiments show a large difference in the work input; 40.6 percent more work went into the second period than the first, and 60.2 percent more work was accomplished in the second period; this work, however, was largely useless grinding, as the useful grinding in the second period was only 64 percent of that in the first period.

TABLE 70.—Comparison between 19- and 38-minute ball-mill runs

[12- by 16-inch iron ball mill; 142 pounds of balls, 200 each of $\frac{1}{2}$, $\frac{3}{4}$, 1, and $1\frac{1}{2}$ inches; 16-pound quartz charge; $\frac{1}{2}$ inch to 6-mesh, 54 percent; 6- to 10-mesh, 20 percent; 10- to 20-mesh, 17 percent; 20- to 35-mesh, 9 percent; 6 pounds of water; pulp density=72.7 percent by weight, 50.0 percent by volume; 80 r. p. m.]

Period (minutes)	Net horsepower per minute		Ground product, mesh	19-minute run			38-minute run			Calculated results for second 19 minutes
	19-minute run	38-minute run		Weight, percent	Surface, each size, cm ² /g	Proportional surface, cm ² /g	Weight, percent	Surface, each size, cm ² /g	Proportional surface, cm ² /g	
1.....	0.1808	0.1656	+28	2.70	-----	1.7	0.05	-----	0.1	-----
2.....	.1784	.1792	28-48	8.21	152.6	12.5	.05	152.6	.1	-----
3.....	.1648	.1816	48-100	29.61	265.2	78.5	1.57	265.2	4.2	-----
4.....	.1648	.1688	100-200	19.44	471.7	91.7	16.79	471.7	79.2	-----
5.....	.1648	.1688	200-400	14.72	820	120.7	23.84	820	195.5	-----
6.....	.1680	.1688	400	25.32	5,906	1,495.4	57.70	7,481	4,316.5	-----
7.....	.1688	.1744	-----	100.00	-----	1,800.5	100.00	-----	4,595.6	-----
8.....	.1832	.1864	Surface in feed, cm ² per g.....	-----	-----	55.1	-----	-----	55.1	-----
9.....	.2064	.2144	Surface produced, cm ² per g.....	-----	-----	1,745.4	-----	-----	4,540.5	2,795.1
10.....	.2824	.3400	Total surface produced, cm ²	-----	-----	12,664,622	-----	32,945,868	20,281,246	-----
11.....	.3768	.3904	Work input, ft.-lb.....	-----	-----	176,062	-----	423,588	247,526	-----
12.....	.3872	.3872	Surface produced.....	-----	-----	-----	-----	-----	-----	-----
13.....	.3888	.3896	Cm ² per ft.-lb.....	-----	-----	71.93	-----	-----	77.78	81.94
14.....	.3848	.3920	Efficiency based on 243 cm ² per ft.-lb.....	-----	-----	29.60	-----	-----	32.01	33.74
15.....	.3888	.3880	-----	-----	-----	-----	-----	-----	-----	-----
16.....	.3888	.3888	Useless work, based on minus 37-micron size as undesirable, percent.....	-----	-----	69.1	-----	-----	80.6	87.7
17.....	.3848	.3904	-----	-----	-----	-----	-----	-----	-----	-----
18.....	.3864	.3888	-----	-----	-----	-----	-----	-----	-----	-----
19.....	.3864	.3888	Iron wear, pounds per ton.....	-----	-----	3.85	-----	-----	7.36	-----

These results show that doubling the time of running does not necessarily double the work input or the work accomplished.

SLIPPAGE IN MILL

It has long been recognized that more or less slippage occurs in ball-mill operation. Laboratory experiments on the effect of variables on slippage, made at the Salt Lake station of the Bureau of Mines, were based upon the centrifuging point of various charges. A certain definite speed of the balls, not the shell, is required for centrifuging, and the difference between shell speed and ball speed is the slippage.

The centrifuging point is

$$n = \frac{54}{\sqrt{r}}$$

where *n* equals the revolutions per minute and *r* the radius in feet.

The centrifuging speed in the mill, 12 inches in diameter by 16 inches long, which was used in the experiments is 82 r. p. m. for the 1½-inch balls.

In the slippage experiments the mill was run at 70 r. p. m. until equilibrium was reached, then the speed was increased at intervals of 1 minute until centrifuging took place.

The work input usually increased with the speed until the centrifuging speed was reached. This resulted in a sharp, sudden increase in the speed.

Slip was figured as the increased revolutions necessary for centrifuging, expressed as a percentage of the theoretical centrifuging speed.

$$\text{Slip} = \frac{\text{Centrifuging speed} - 82}{82}$$

With a high pulp density centrifuging speed cannot be determined definitely, as the stickiness of the charge tends to lower the power re-

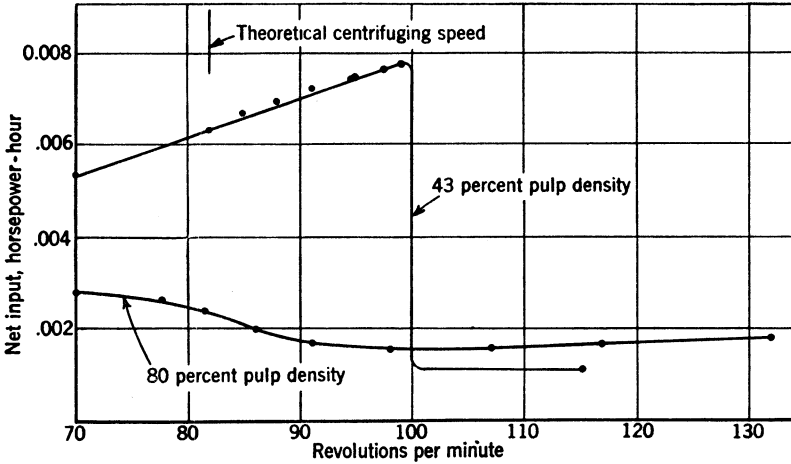


FIGURE 50.—Work-input curves for thin and thick pulps.

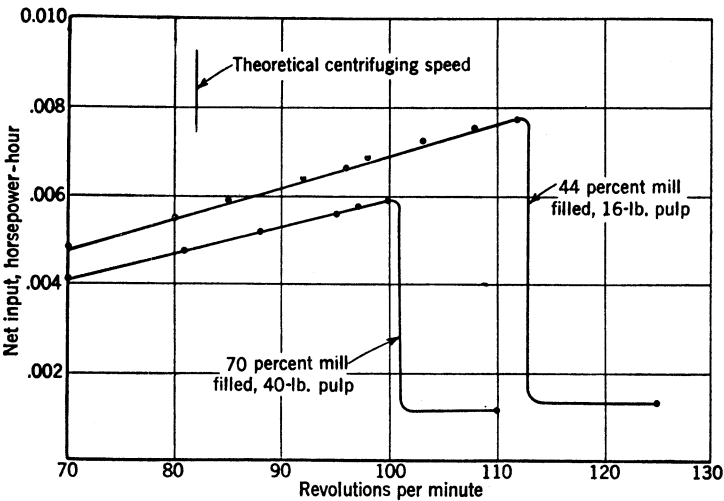


FIGURE 51.—Work-input curves comparing effect of mill feed; 20 percent pulp density by volume.

quirements before the centrifuging point is reached. This is shown by the two curves in figure 50 in which a comparatively thin and a very thick pulp are compared. Obviously, no centrifuging point can be determined for the thick pulp.

When the ball load almost half fills the mill, centrifuging points also become indefinite, as the arrangement of the excess weight of the balls

requires less work with increased speed. This is shown by the curves in figure 54.

The two factors that materially affect slippage are density of pulp and weight of ball charge. The slip increases rapidly with increased dilution of pulp and decreased weight of ball charge. The effect of pulp density may be modified by an increase in mill feed, as shown by the

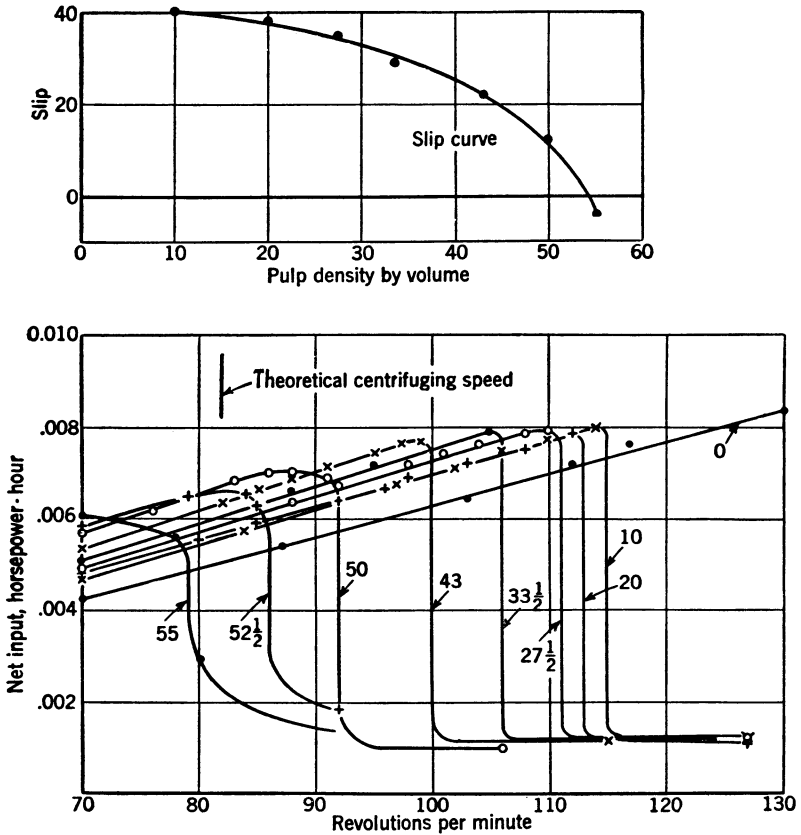


FIGURE 52.—Slip and centrifuging speeds for various pulp densities.

two curves in figure 51. In these two experiments only the weight of the pulp was varied. In one experiment 16 pounds of pulp at a pulp density of 20 percent by volume was used, which filled the mill to 44 percent. In the other, 40 pounds of pulp of the same pulp density was used, which filled the mill to 70 percent of capacity.

Figure 52 shows the effect of pulp density on slippage with a constant volume of pulp. All data for these curves are not given; only the work input for 70 revolutions per minute and for the speeds immediately before and after centrifuging are given in table 71. These

experiments show that slippage is greatly reduced as the pulp becomes thicker. A nonslip charge is one that has a pulp density of 54 percent by volume. When the density of the pulp is increased beyond this the charge begins to stick to the mill lining. These curves also show that for a given speed more input of work is required for thicker

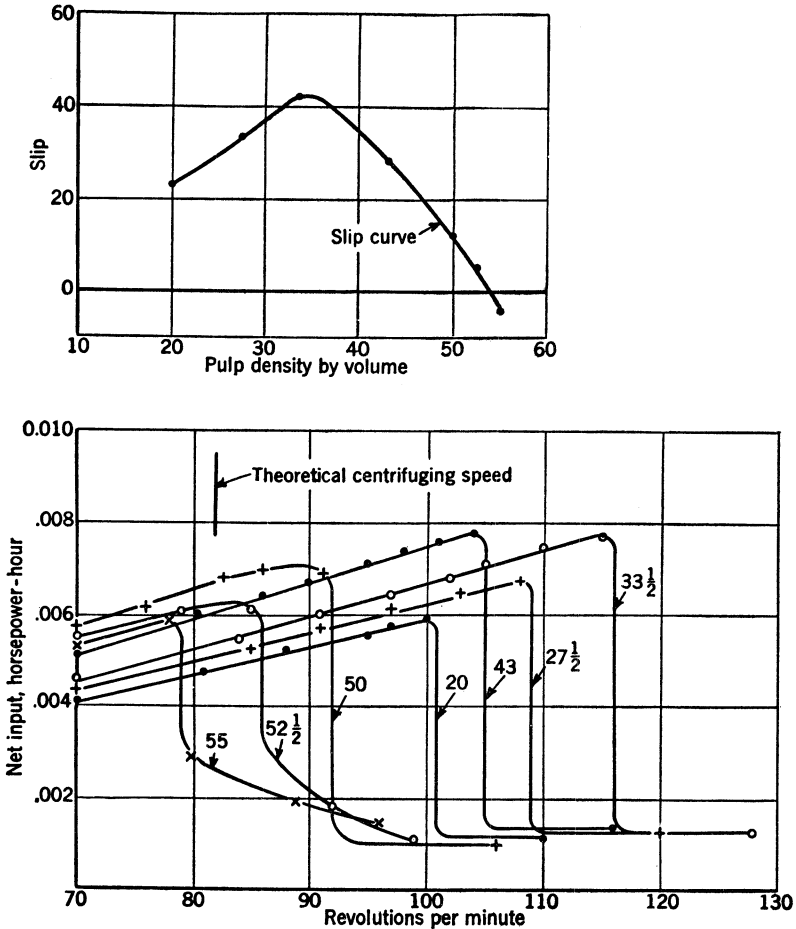


FIGURE 53.—Slip and centrifuging speeds for various pulp densities.

charges, although at speeds just below centrifuging more input is required for thinner charges.

Figure 53 shows the effect of pulp density on slippage when the weight of the quartz was kept constant, and table 72 gives the accompanying data. In these experiments the effect of the thinner pulp is modified by the percentage of mill capacity utilized, although some other modifying cause seems to exist as the slippage reaches a maximum when about 53 percent of the capacity of the mill is utilized.

TABLE 71.—*Slippage due to variable pulp densities*

[142 pounds of balls, 200 each of 1½, 1, ¾, and ½ inch; volume of pulp constant at 0.192 cubic feet; mill filled to 44 percent of capacity]

Quartz (pounds)	Water (pounds)	Pulp density by volume	Ratio of balls to pulp by—		Revolutions per minute	Net input (horse-power-hours)	Slip-page
			Weight	Volume			
0.0	12.0	0.0	11.8	1.46	{ 70 145	{ 0.00425 .00935	77
3.2	10.8	10	10.1	1.46	{ 70 114 127	{ .00470 .00800 .00123	
6.4	9.6	20	8.9	1.46	{ 70 112 127	{ .00478 .00782 .00117	38
8.7	8.7	27.5	8.2	1.46	{ 70 110 127	{ .00497 .00795 .00122	
10.6	8.0	33.5	7.6	1.46	{ 70 105 124	{ .00507 .00785 .00118	29
13.7	6.8	43	6.9	1.46	{ 70 99 115	{ .00532 .00770 .00112	
16.0	6.0	50	6.5	1.46	{ 70 91 106	{ .00572 .00688 .00100	12
16.7	5.7	52.5	6.3	1.46	{ 70 85 92	{ .00583 .00630 .00188	
17.5	5.4	55	6.2	1.46	{ 70 78 80	{ .00600 .00560 .00290	-4

TABLE 72.—*Slippage due to variable pulp densities*

[142 pounds of balls, 200 each of 1½, 1, ¾, and ½ inch; weight of quartz constant]

Quartz (pounds)	Water (pounds)	Pulp density by volume	Ratio of balls to pulp by—		Mill capacity filled (per-cent)	Revolutions per minute	Net input (horse-power-hours)	Slip-page
			Weight	Volume				
16	24	20	3.6	0.58	70	{ 70 100 110	{ 0.00408 .00590 .00113	23
16	16	27.5	4.4	.80	59	{ 70 108 120	{ .00433 .00675 .00123	
16	12	33.5	5.1	.97	53	{ 70 115 128	{ .00458 .00772 .00127	42
16	8	43	5.9	1.25	47	{ 70 104 116	{ .00508 .00777 .00123	
16	6	50	6.5	1.46	44	{ 70 91 106	{ .00572 .00688 .00100	12
16	5.5	52.5	6.6	1.52	43	{ 70 85 92 99	{ .00547 .00610 .00188 .00115	
16	5	55	6.8	1.59	42	{ 70 78 80 89 96	{ .00532 .00583 .00290 .00193 .00143	-4

The effect of the weight of the ball charge on slippage is very great. Figure 54 and table 73 show that with small ball charges the slippage is enormous. The change in the slippage is great for small changes in ball weight, and the slippage curve for these experiments can be only roughly indicated. Apparently a ball charge that occupies 46 percent of the mill volume would assist in establishing nonslip conditions.

Taggart (73) gives the best ball-mill load in pounds as $80 D^2L$, in which D is the diameter of the mill in feet and L the length of the

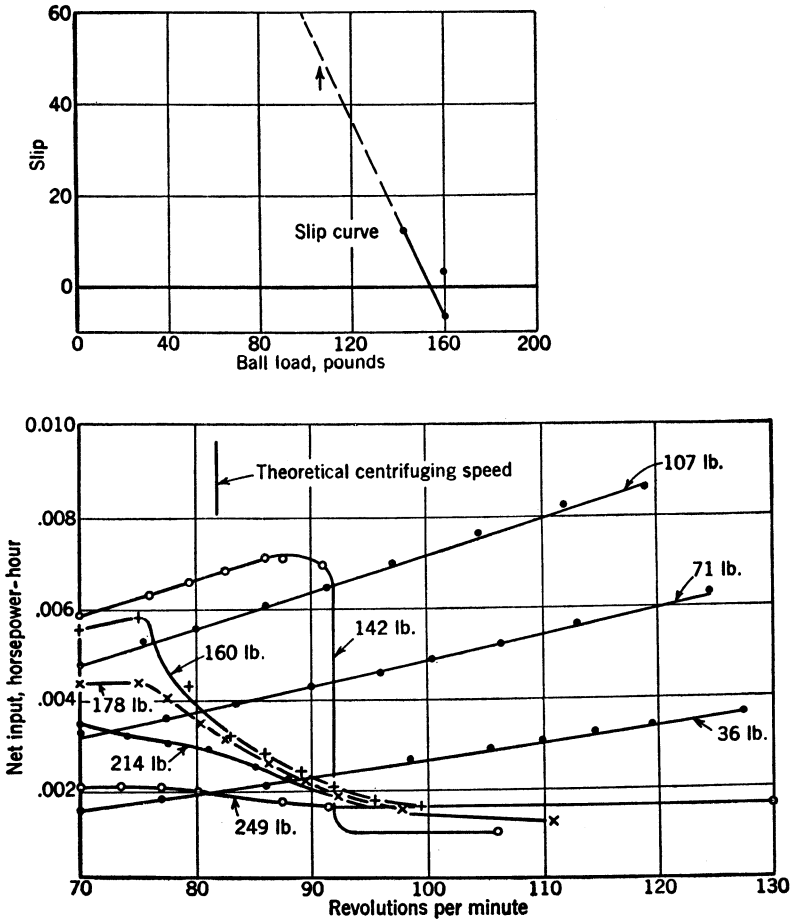


FIGURE 54.—Slip and centrifuging speeds for various ball loads.

mill in feet. According to this formula the best ball load for the 12-by 16-inch mill is 107 pounds. The experiments with such a ball load indicate very high slippage, and the formula to apply to the experimental results given in figure 54 should be $114 D^2L$. It may be concluded that Taggart's formula does not hold for very small mills.

Slippage due to variable weight of pulp is shown in figure 55, and the data are given in table 74. In the experiment with no pulp charge slippage was rather high. This result is included as a matter of interest but is not reliable, as in prolonged running the dry balls

become coated with fine iron and slip more readily. It may be concluded from these experiments that excessive pulp tends to increase slippage, especially when the volume of the pulp amounts to more than the ball voids.

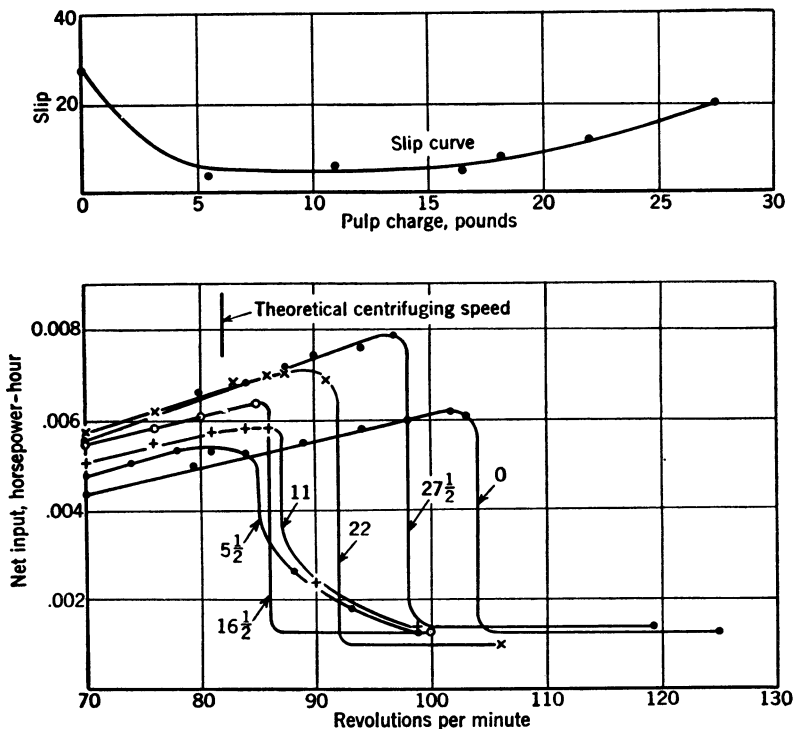


FIGURE 55.—Slip and centrifuging speeds for various pulp charges.

TABLE 73.—Slippage due to variable ball loads

[Equal number of balls (1½, 1, ¾, and ½ inch) in each case. Weight of pulp constant; 16 pounds of quartz, 6 pounds of water, 50 percent pulp density by volume]

Weight of balls (pounds)	Ratio of balls to pulp by—		Mill capacity filled (percent)	Revolutions per minute	Net input (horsepower-hour)	Slippage
	Weight	Volume				
36	1.6	0.37	24	70	0.00148	-----
				127.5	.00365	
71	3.2	.73	31	70	.00311	-----
				124.5	.00628	
107	4.9	1.10	42	70	.00465	} >45
				119	.00858	
142	6.5	1.46	44	70	.00572	} 12
				91	.00688	
				106	.0100	
160	7.3	1.64	48	70	.00545	} -7
				75	.00569	
				132	.00162	
178	8.1	1.83	53	70	.00425	-----
				75	.00425	
				133	.00151	
214	9.7	2.20	64	70	.00333	-----
				74	.00307	
249	11.3	2.56	74	132	.00153	-----
				70	.00199	
				73.5	.00196	
				132	.00169	

TABLE 74.—Slippage due to variable pulp charges

[142 pounds of balls, 200 each of 1½, 1, ¾, and ½ inch; 50 percent pulp density by volume]

Quartz (pounds)	Water (pounds)	Ratio of balls to pulp by—		Mill capacity filled (per cent)	Revolutions per minute	Net input (horsepower-hours)	Slippage
		Weight	Volume				
0	0	-----	-----	42	{ 70 103 125 70	{ 0.00432 .00607 .00122 .00473	{ 27
4	1.5	25.8	5.83	42	{ 84 88 99 70	{ .00525 .00260 .00128 .00502	{ 4
8	3	12.9	2.92	42	{ 86 90 99 70	{ .00580 .00233 .00132 .00542	{ 6
12	4.5	8.6	1.94	42	{ 85 100 70	{ .00632 .00128 .00572	{ 5
16	6	6.5	1.46	44	{ 91 106 70	{ .00688 .00100 .00548	{ 12
20	7.5	5.2	1.17	48	{ 97 120	{ .00782 .00135	{ 20

Maximum input of work.—Figures 52 to 55, inclusive, show the speed accompanied by the maximum input of work. Such a speed is usually reached just before centrifuging. This is the speed for greatest capacity but not necessarily for greatest efficiency.

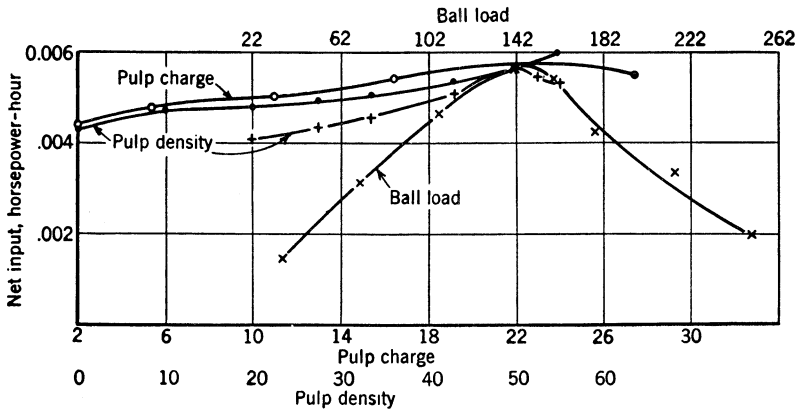


FIGURE 56.—Net work input, horsepower-hour at 60 r. p. m.

Experiments on the efficiency of grinding, made in the same mill used for the experiments just discussed, showed that the greatest total efficiency was obtained at 60 revolutions per minute and was equal to 73 percent of the critical speed. The net input in horsepower-hours at this speed and at 70 revolutions per minute is given in table 75 for the experiments in tables 71 to 74, inclusive. The curves for these data are shown in figures 56 and 57. The curves show that the maximum input of work for this mill is obtainable with:

- A ball load of approximately 42 percent of the mill volume.
- A pulp density by volume of 50 percent or slightly greater.

A pulp weight equal in volume to the voids in the ball load.

The curves for 70 revolutions per minute are higher than those for 60 revolutions per minute, which signifies only that capacity is increased with higher speeds but does not necessarily indicate greater efficiency. In fact, experiments given later show that the efficiency at 70 revolutions per minute is lower than that at 60 revolutions per minute.

TABLE 75.—Net horsepower input at 60 and 70 revolutions per minute under varying operating conditions of ball mill

PULP DENSITY			
Variables	Condi- tion	Net input (horsepower- hours)	
		60 revolu- tions per minute	70 revolu- tions per minute
Pulp density in which volume of pulp density was kept constant (table 32)-----	0.0	0.00354	0.00425
	10.0	.00398	.00470
	20.0	.00407	.00478
	27.5	.00424	.00497
	33.5	.00430	.00507
	43.0	.00452	.00532
	50.0	.00498	.00572
	52.5	.00506	.00583
	55.0	.00597	.00600
	Pulp density in which weight of quartz was kept constant (table 33) ..	20.0	.00347
27.5		.00369	.00433
33.5		.00388	.00458
43.0		.00429	.00508
50.0		.00498	.00572
52.5		.00541	.00547
55.0		.00526	.00532
POUNDS OF BALLS			
Ball load in which weight of pulp was kept constant (table 34)-----	36.0	0.00124	0.00148
	71.0	.00256	.00311
	107.0	.00375	.00465
	142.0	.00498	.00572
	160.0	.00479	.00545
	178.0	.00448	.00425
	214.0	.00339	.00333
	249.0	.00209	.00199
POUNDS OF PULP			
Pulp charge in which weight of balls was kept constant (table 35)-----	0.0	0.00373	0.00432
	5.5	.00412	.00473
	11.0	.00435	.00502
	16.5	.00482	.00542
	22.0	.00498	.00572
	27.5	.00461	.00548

IRON WEAR

Determinations for iron wear were made on the ground product in these experiments. The result was a measure of both ball and shell wear. It was impossible to determine ball wear by the loss of weight of the balls, as the loss was too small.

In determining the iron approximately 250 grams of the ground quartz is heated with dilute hydrochloric acid to which a little nitric acid is added. After the solution is digested and filtered it is boiled with sulphuric acid to fumes. This is diluted and divided into two equal parts for titration. As 0.1 gram or more of iron was usually present the accuracy of the method is high. A small quantity of iron in the quartz called for a correction equivalent to 0.11 pound of iron per ton of quartz.

From grinding experiments, which are given later, the effect of different variables on iron wear is given herewith. In the tables presenting the data for iron wear this is calculated to total iron wear in pounds for the experiment, total iron wear per revolution, iron wear per ton of quartz, and iron wear per ton of quartz per revolution.

Table 76 gives the data for iron wear for variable speeds of mill. The figures show that the wear is not directly proportional to the

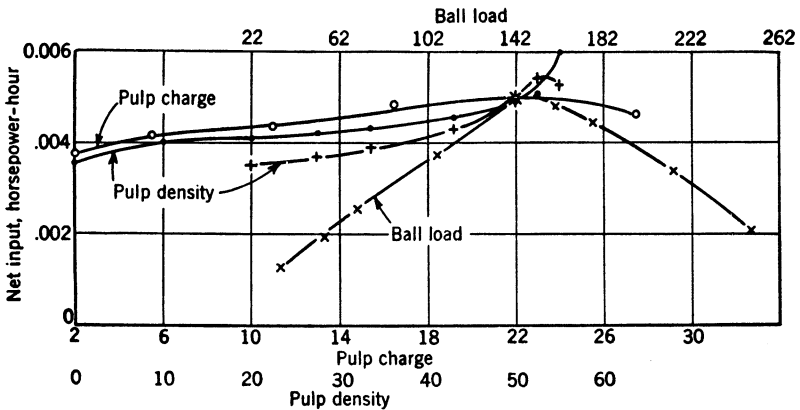


FIGURE 57.—Net work input, horsepower-hour at 70 r. p. m.

time of grinding, as it becomes less as the charge becomes finer. The curves in figure 58 show that, while the iron wear is greater at higher speeds, it is smaller on a per-revolution basis.

The curves for total iron wear and for pounds of iron per ton of quartz are similar, as the quartz charge was constant in the series. The effect of change in speed on iron can therefore be studied from the data and curves either of total iron or pounds per ton of quartz.

The data on iron wear for variations in pulp density are given in table 77. If the total iron wear and the pounds of iron per ton of quartz are plotted against the time of run, as shown in figure 59, the wear is not directly proportional to the time, as was shown for the iron wear at various speeds. There are no experiments for a definite period at all pulp densities, but the 21-minute period, for which there are several determinations, can be determined from the time curves. This 21-minute period was also used for the curves of iron wear at variable speeds in figure 58. These figures, as determined, are also given in table 77, and the resultant curves show that, while

maximum wear took place at a pulp density of 30 to 40 percent, the wear on a basis of pounds per ton of quartz increases very rapidly with thinner pulps. While the total iron wear and the wear per ton of quartz increase with time, the wear per revolution in both cases becomes smaller with time.

Table 78 and figure 60 give the relation of iron wear to a variable weight of charge. The curves for iron wear against time again show

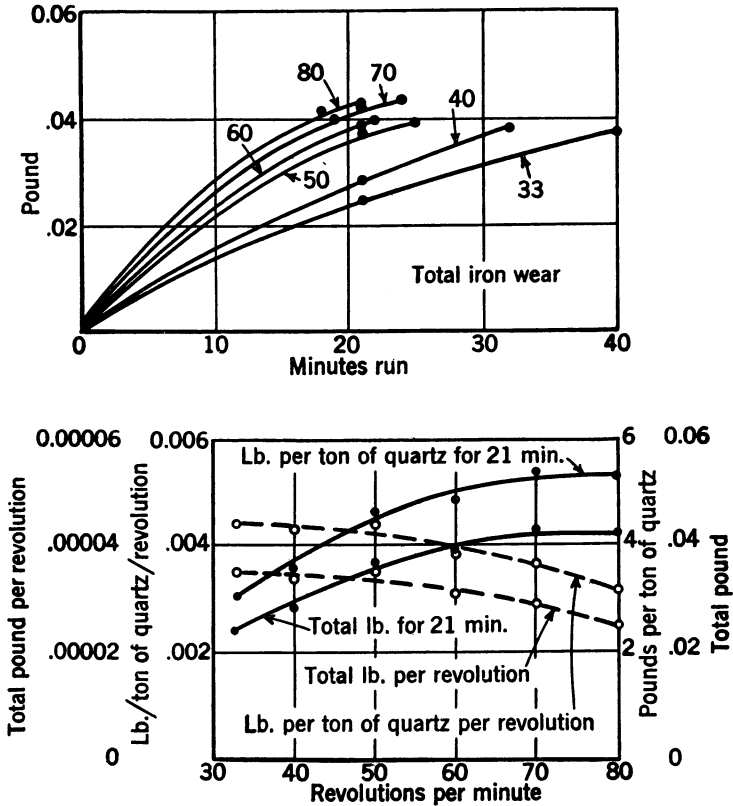


FIGURE 58.—Ball and liner wear for various speeds.

that wear is not directly proportional to time and are used to obtain the wear for the 21-minute period, as was done previously. The resultant curves on the basis of equal time show that the total wear increases with the weight of the charge but that the wear per ton of quartz decreases materially with increased charges.

As in the series previously mentioned the iron wear, both total and per ton of quartz, increases with time of run, but again the wear per revolution decreases with time.

Table 79 and figure 61 give the relation of iron wear to variable weight of ball charges. The curves for wear against time also show that the wear is not directly proportional to the time, although the relationship is nearer to direct proportion than in the previous three series.

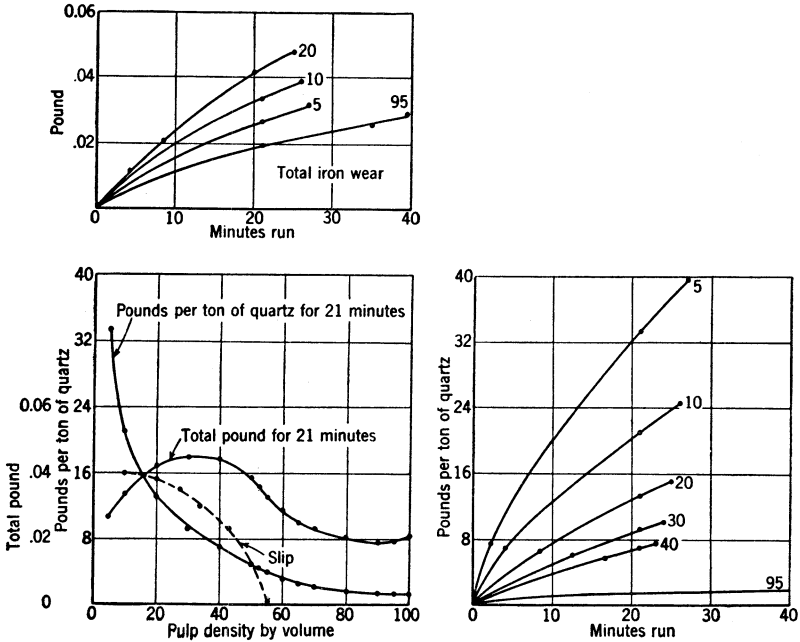


FIGURE 59.—Ball and liner wear for various pulp densities.

TABLE 76.—Ball and liner wear for varying speeds

[142 pounds of balls (200 each of 1½, 1, ¾, and ½ inch); 16 pounds of quartz, 6 pounds of water=50 percent pulp density by volume]

Revolutions per minute	Minutes run	Total revolutions	Total iron wear		Iron wear per ton of quartz	
			Pound	Pound per revolution	Pounds	Pound per revolution
33	21	693	0.0242	0.0000349	3.02	0.00436
33	40	1,320	.0370	.0000280	4.63	.00351
40	21	840	.0284	.0000338	3.55	.00423
40	32	1,280	.0377	.0000295	4.71	.00368
50	21	1,050	.0369	.0000351	4.61	.00439
50	25	1,250	.0385	.0000308	4.81	.00385
60	21	1,260	.0385	.0000306	4.81	.00382
60	22	1,320	.0388	.0000294	4.85	.00367
70	19	1,330	.0392	.0000295	4.90	.00368
70	21	1,470	.0426	.0000290	5.33	.00363
70	24	1,680	.0424	.0000252	5.30	.00315
80	18	1,440	.0410	.0000285	5.12	.00356
80	21	1,680	.0417	.0000248	5.21	.00310

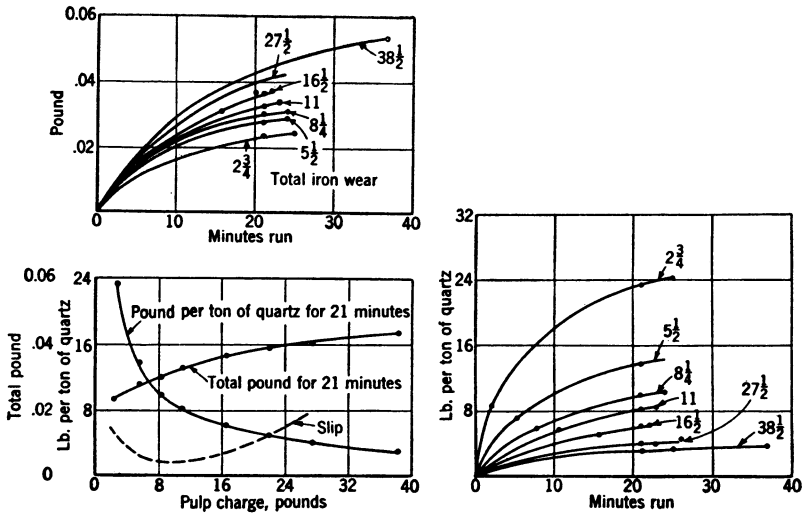


FIGURE 60.—Ball and liner wear for various pulp charges.

TABLE 77.—Ball and liner wear for varying pulp densities

142 pounds of balls (200 each of 1½, 1, ¾, and ½ inch), volume of pulp kept constant at 0.192 cubic feet, 60 r. p. m.]

Pulp density (volumes)	Weight of pulp (pounds)	Minutes run	Total revolutions	Total iron wear		Iron wear per ton of quartz	
				Pound	Pound per revolution	Pounds	Pound per revolution
5	12.99	2:05	125	0.0060	0.0000480	7.61	0.06088
5	12.99	26	1,620	.0315	.0000194	39.68	.02450
		21	1,260	.0265	.0000210	33.30	1.02635
10	13.98	4:11	251	.0111	.0000440	6.99	.02785
10	13.98	26	1,560	.0389	.0000249	24.44	.01567
		21	1,260	.0335	.0000266	21.10	1.01675
20	15.96	8:21	501	.0209	.0000417	6.58	.01313
20	15.96	25	1,500	.0478	.0000319	15.03	1.01002
		21	1,260	.0420	.0000333	13.20	1.01048
30	17.94	12:31	751	.0286	.0000381	6.00	.00799
30	17.94	24	1,440	.0477	.0000331	10.00	.00694
		21	1,260	.0439	.0000348	9.20	1.00730
40	19.90	16:40	1,000	.0363	.0000363	5.71	.00571
40	19.90	23	1,380	.0478	.0000346	7.52	.00545
		21	1,260	.0438	.0000348	6.90	1.00547
50	22.00	21	1,260	.0385	.0000306	4.81	.00382
52½	22.40	21	1,260	.0356	.0000282	4.26	.00338
55	22.90	21	1,260	.0323	.0000256	3.96	.00314
60	23.88	24	1,440	.0318	.0000221	3.33	.00231
60	23.88	25:02	1,502	.0328	.0000218	3.44	.00229
		21	1,260	.0286	.0000227	3.00	1.00238
65	24.87	27:07	1,627	.0310	.000191	3.00	.00184
65	24.87	28	1,680	.0291	.000173	2.82	.00168
		21	1,260	.0248	.000197	2.40	1.00190
70	25.86	29:12	1,753	.0294	.000168	2.64	.00151
70	25.86	30	1,800	.0270	.000150	2.43	.00135
		21	1,260	.0230	.000183	2.07	1.00164
80	27.84	33:23	2,003	.0265	.000132	2.08	.00104
80	27.84	34	2,040	.0300	.000147	2.36	.00116
		21	1,260	.0204	.000162	1.60	1.00127
90	29.82	36	2,160	.0230	.000106	1.61	.00075
90	29.82	37:23	2,253	.0305	.000135	2.13	.00095
		21	1,260	.0185	.000147	1.29	1.00192
95	30.81	35	2,100	.0255	.000121	1.69	.00080
95	30.81	39:38	2,378	.0296	.000124	1.96	.00082
		21	1,260	.0191	.000152	1.26	1.00100
100	31.80	32	1,920	.0277	.000144	1.74	.00091
100	31.80	41:44	2,504	.0377	.000151	2.37	.00095
		21	1,260	.0210	.000167	1.32	1.00105

¹ Calculated.

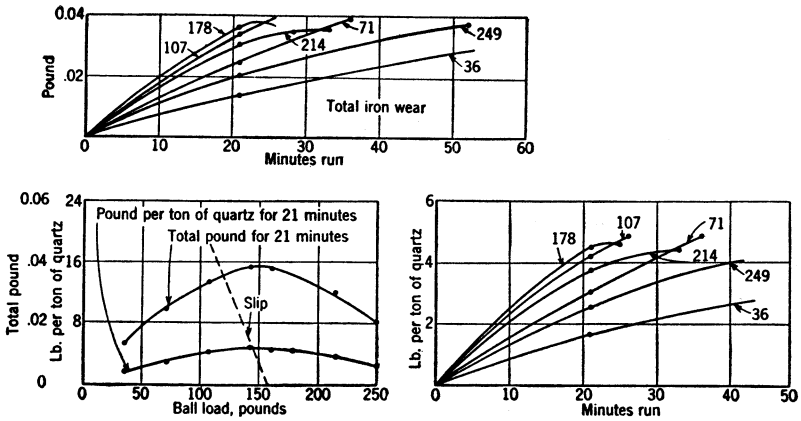


FIGURE 61.—Ball and liner wear for various ball loads.

TABLE 78.—Ball and liner wear for varying pulp charges

[142 pounds of balls (200 each of 1½, 1, ¾, and ½ inch); 50 percent pulp density by volume; 60 r. p. m.]

Weight of pulp (pounds)	Minutes run	Total revolutions	Total iron wear		Iron wear per ton of quartz	
			Pound	Pound per revolution	Pounds	Pound per revolution
2.75	2:37	157	0.0086	0.0000548	8.59	0.05471
2.75	25	1,500	.0242	.0000161	24.19	.01613
-----	21	1,260	.0232	.0000184	23.20	¹ .01841
5.5	5:15	315	.0142	.0000451	7.10	.02254
5.5	24	1,440	.0284	.0000197	14.20	.00986
-----	21	1,260	.0274	.0000218	13.70	¹ .01087
8.25	7:52	472	.0177	.0000375	5.89	.01248
8.25	24	1,440	.0310	.0000215	10.34	.00718
-----	21	1,260	.0297	.0000235	9.70	¹ .00770
11	10:30	630	.0222	.0000352	5.55	.00081
11	23	1,380	.0335	.0000243	8.37	.00607
-----	21	1,260	.0324	.0000257	8.10	¹ .00643
16.5	15:45	945	.0306	.0000324	5.10	.00540
16.5	22	1,320	.0370	.0000280	6.17	.00467
-----	21	1,260	.0366	.0000290	6.10	¹ .00484
22	21	1,260	.0385	.0000306	4.81	.00382
47.5	23	1,380	.0397	.0000288	3.97	.00288
27.5	26:15	1,575	.0459	.0000291	4.59	.00291
-----	21	1,260	.0400	.0000317	4.00	¹ .00317
38.5	25	1,500	.0462	.0000308	3.30	.00220
38.5	36:44	2,204	.0533	.0000242	3.81	.00173
-----	21	1,260	.0434	.0000344	3.10	¹ .00246

¹ Calculated.

TABLE 79.—*Ball and liner wear for varying weights of ball charges*

[Equal number of 1½, 1, ¾, and ½ inch balls; 22 pounds of pulp at 50 percent pulp density by volume; 60 r. p. m.]

Ball-charge weight (pounds)	Minutes run	Total revolutions	Total iron wear		Iron wear per ton of quartz	
			Pound	Pound per revolution	Pound	Pound per revolution
36	21	1, 260	0. 0135	0. 0000107	1. 69	0. 00134
36	70	4, 200	. 0369	. 0000088	4. 61	. 00110
71	21	1, 260	. 0244	. 0000194	3. 05	. 00242
71	36	2, 160	. 0392	. 0000182	4. 90	. 00227
107	21	1, 260	. 0337	. 0000268	4. 21	. 00334
107	26	1, 560	. 0392	. 0000251	4. 90	. 00314
142	21	1, 260	. 0385	. 0000306	4. 81	. 00382
160	21	1, 260	. 0378	. 0000300	4. 73	. 00375
178	21	1, 260	. 0361	. 0000287	4. 51	. 00358
178	25	1, 500	. 0368	. 0000245	4. 60	. 00307
214	21	1, 260	. 0303	. 0000240	3. 79	. 00301
214	33	1, 980	. 0354	. 0000179	4. 43	. 00224
249	21	1, 260	. 0203	. 0000161	2. 54	. 00202
249	52	3, 120	. 0379	. 0000121	4. 74	. 00152

The wear for both total iron and for iron per ton of quartz increases rapidly up to a 142-pound ball charge, or 44 percent of mill capacity filled, then falls off rapidly.

The wear per revolution in this series again decreased with the time, although the wear for both total iron and iron per ton of quartz increases with the time.

The general conclusions to be drawn from the iron-wear data are as follows: (1) Iron wear is not directly proportional to time of running on a batch charge, due doubtless to an increasing fineness of the pulp; (2) the total iron wear increases with speed, weight of pulp, pulp density up to approximately 40 percent by volume, and increased ball charge up to 142 pounds, or 44 percent of mill capacity; (3) the iron wear per ton of quartz increases with speed and increased ball charge up to 142 pounds but decreases with increased thickness and weight of pulp; (4) both total iron wear per revolution and wear per ton of quartz per revolution decrease with speed, and wear per revolution decreases with time; (5) the slippage curves, which have been added to figures 59, 60, and 61, show no particular relationship to iron wear except possibly an approximation of increased wear with decreased pulp density and increased weight of pulp.

The method of determining iron wear by analysis of the ground product lends itself admirably to the laboratory study of ball and liner wear with grinding materials that are free from, or nearly free from, iron.

EFFECT OF VARIABLES IN BALL-MILL GRINDING

Numerous experiments were made at the Salt Lake station of the Bureau of Mines on quartz grinding in which the effect of the following variables was investigated: (1) Mill speed, (2) pulp density, (3) ball size, (4) ball load, and (5) pulp charge.

All experiments were made in a 12-inch-diameter ball mill 16 inches long, equipped with a power-recording device that assured accurate determination of work input to the balls.

Two series of experiments were made except for those of varying ball size. In one series the mill capacity was kept a constant, and in the other the work input was approximately the same. The quartz used was of high purity, and all charges were made up of the following sizes:

Mesh:	Percent by weight
¼-inch to 8.....	52
8-14.....	24
14-28.....	15
28-48.....	9

The product from each experiment was carefully sized, and surface determinations were made on the minus 400-mesh material. The surface figures used for all sizes above 400-mesh were those adopted as standard and have already been given.

The net input of work to the balls is that delivered during the latter part of the run and depends on the ground product rather than the feed.

The iron wear in these experiments is much higher than that ordinarily found in practice, owing to the strongly abrasive action of the quartz but more especially to the soft mill shell. The results, however, are comparable.

The amount of finished product per unit of work input is given for three sizes to show the trend as affected by the operating variables.

The total efficiency is calculated as a percentage of the surface produced by the drop-weight machine and although not absolute is a true comparative figure.

The useless work is calculated on the assumption that material finer than 400-mesh (37 microns) is not required for the subsequent metallurgical work. Useless work can be calculated for any other limiting size whose percentage is known.

The curves for equal capacity only are given, as being of greater interest than those for equal input of work. However, the two sets of curves follow each other rather closely, except where there is a large difference in capacity. The effect of capacity will be discussed later.

1. *Varying mill speed.*—Table 80 gives data for the experiments on variable mill speed with equal capacity and table 81 with equal input of work. The data on equal capacity are plotted in figure 62.

The tables and curves show that there is no great difference in efficiency due to change in mill speed. The total efficiency reaches a maximum where the greatest amount of work input per revolution is reached, at 70 revolutions per minute. The useful efficiency and the production of finished sizes per unit of work input decrease with higher speeds. The iron wear per ton of charge increases with higher speeds.

The results of these experiments show that slow speeds are best for production of finer sizes and minimum iron wear. Davis (27) gives the speed for a 12-inch mill as 63.6 revolutions per minute when 40 percent full and 65.1 revolutions per minute when 50 percent full. At this speed the total efficiency is greatest, but it is decidedly not the speed for maximum production or minimum iron wear.

TABLE 80.—Varying mill speed, equal capacity

[Constants: 142 pounds of balls, 200 each 1½, 1, ¾, and ½ inch; quartz, 16 pounds, water, 6 pounds; pulp density, 50 percent by volume, 72.7 percent by weight; 6.5 ratio weight of balls to pulp; 44 percent of mill capacity filled]

	Experiment					
	1	2	3	4	5	6
Revolutions per minute.....	33	40	50	60	70	80
Minutes run.....	21	21	21	21	21	21
Capacity..... pound of quartz per minute..	0.762	0.762	0.762	0.762	0.762	0.762
Net input of work to balls..... ft.-lb.	104,663	129,413	165,033	198,307	223,918	248,926
Net input of work per revolution..... do.	149	151	155	160	164	161
Sizing analyses (mesh):						
4-8.....	32.3	27.9	20.7	6.8	1.0	0.0
8-14.....	4.9	3.5	1.9	1.0	.2	.0
14-28.....	2.5	1.6	1.0	.8	.3	.1
28-48.....	5.0	3.1	1.7	2.0	1.6	.6
48-65.....	5.3	4.4	2.8	3.9	4.7	3.2
65-100.....	8.8	9.1	8.6	11.0	13.5	12.2
100-150.....	7.6	9.0	10.4	12.6	13.4	13.6
150-200.....	5.6	6.7	8.3	9.4	9.8	10.2
200-270.....	4.9	6.1	7.8	8.8	9.2	9.6
270-400.....	3.4	4.0	4.6	6.0	5.8	6.4
400.....	19.7	24.6	32.2	37.7	40.5	44.1
Ball and liner wear..... pounds per ton of quartz..	3.02	3.55	4.61	4.81	5.33	5.21
Finished product (pounds per 100,000 ft.-lb.):						
—48-mesh.....	8.47	7.90	7.24	7.21	6.93	6.38
—100-mesh.....	6.31	6.23	6.14	6.01	5.62	5.39
—200-mesh.....	4.28	4.29	4.32	4.24	3.97	3.86
Surface of —400-mesh..... cm ² per gram..	4,937	4,959	5,100	5,375	5,653	5,658
Total surface produced..... cm ² per ft.-lb..	76.79	77.04	79.91	82.05	81.58	79.45
Total efficiency based on 243 cm ² per ft.-lb.....	31.60	31.70	32.88	33.77	33.57	32.70
Useless work..... percent..	67.07	67.91	69.70	70.78	72.19	72.69
Useful efficiency..... do.....	10.41	10.17	9.96	9.87	9.34	8.93

TABLE 81.—Varying mill speed; equal input of work

[Constants: 142 pounds of balls (200 each of 1½, 1, ¾, and ½ inch); quartz, 16 pounds, water, 6 pounds; pulp density, 50 percent by volume, 72.7 percent by weight; 6.5, ratio weight of balls to pulp, 44 percent of mill capacity filled]

	Experiment					
	7	8	9	10	11	12
Revolutions per minute.....	33	40	50	60	70	80
Minutes run.....	40	32	26	22	19	18
Capacity..... pound of quartz per minute..	0.400	0.500	0.615	0.727	0.842	0.899
Net input of work to balls..... ft.-lb.	189,882	194,792	198,626	203,702	202,534	208,658
Net input of work per revolution..... do.	139	150	149	161	166	161
Sizing analyses (mesh):						
4-8.....	22.0	19.7	13.7	5.6	1.5	0.5
8-14.....	2.2	1.6	1.2	1.0	.5	.2
14-28.....	1.0	.8	.8	.9	.5	.2
28-48.....	1.2	1.1	1.2	2.2	2.5	1.8
48-65.....	1.2	1.6	2.0	3.9	7.0	7.3
65-100.....	5.0	5.4	7.2	11.1	14.9	15.1
100-150.....	8.7	9.0	10.7	12.4	13.2	13.0
150-200.....	8.1	8.4	8.9	9.6	8.9	9.0
200-270.....	8.5	8.7	8.8	9.0	8.5	9.2
270-400.....	5.7	5.7	7.7	5.7	5.6	7.5
400.....	36.4	38.0	37.8	38.6	36.9	36.2
Ball and liner wear..... pounds per ton of quartz..	4.63	4.71	4.81	4.85	4.90	5.12
Finished product (pounds per 100,000 ft.-lb.):						
—48-mesh.....	6.20	6.31	6.70	7.09	7.50	7.46
—100-mesh.....	5.68	5.73	5.95	5.91	5.78	5.74
—200-mesh.....	4.27	4.30	4.37	4.19	4.06	4.05
Surface of —400-mesh..... cm ² per gram..	5,050	5,108	5,250	5,576	5,459	5,500
Total surface produced..... cm ² per ft.-lb..	76.68	78.76	80.08	84.32	80.30	77.93
Total efficiency based on 243 cm ² per ft.-lb.....	31.56	32.41	32.95	34.70	33.05	32.07
Useless work..... percent..	70.46	70.86	70.43	71.92	70.69	70.04
Useful efficiency..... do.....	9.32	9.44	9.74	9.74	9.69	9.61

2. *Varying pulp densities.*—Table 82 gives data for the experiments on variable pulp densities with equal capacity and table 83 with equal input of work. The data on equal capacity are plotted in figure 63.

The data and curves show that variations in pulp density have a decided effect on efficiency. The input of work per revolution up to

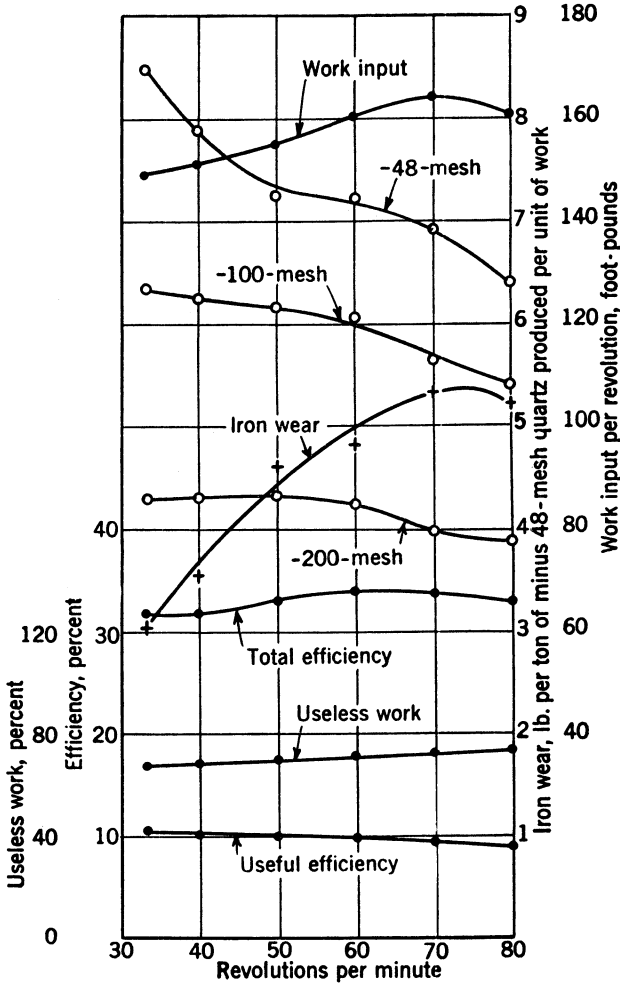


FIGURE 62.—Quartz grinding experiments, varying mill speed.

a pulp density of 50 percent by volume varies with the efficiency and the finished products, but with thicker pulp the total efficiency varies directly with the input of work per revolution.

The greatest total efficiency was obtained at a pulp density of approximately 50 to 55 percent by volume. At this density the charge is virtually nonslip. The useful efficiency is greatest for pulp ranging

from 30 to 50 percent in density, and the maximum amount of finished product is produced at 40 percent. The iron wear decreases rapidly with thicker pulps and reaches a minimum at a density of about 80 percent.

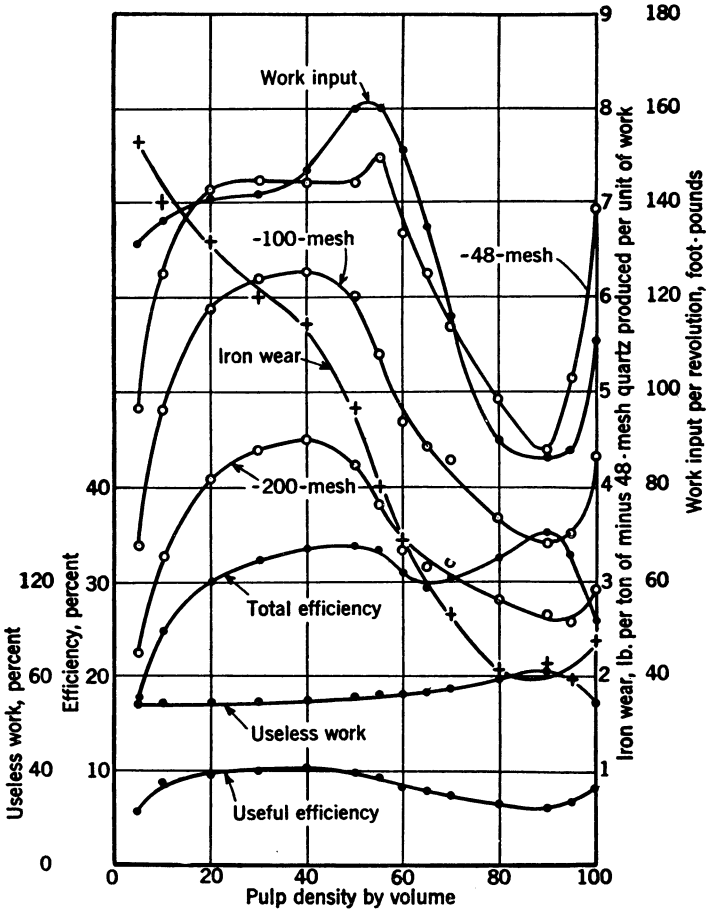


FIGURE 63.—Quartz grinding experiments, varying pulp densities.

On the basis of general efficiency and in the production of a maximum of fine product per unit of work input a pulp density of 40 percent by volume gives the best results. At this density the iron wear is rather high, but the sacrifice in efficiency and production of fines would be too great if the density were adjusted for minimum iron wear.

TABLE 82.—*Varying pulp density: equal capacity*
 [Constants: 142 pounds of balls, 200 each, 1½, 1, ¾, and ½ inch; volume of pulp constant at 0.192 cubic feet; 60 r. p. m.]

		Experiment													
		13	14	15	16	17	18	19	20	21	22	23	24	25	26
Quartz charge.....	pounds.....	1.59	3.18	6.36	9.54	12.7	16.0	17.5	19.08	20.67	22.26	25.44	28.62	30.21	31.8
Water.....	do.....	11.4	10.8	9.6	8.4	7.2	6.0	6.4	4.8	4.2	3.6	2.4	1.2	0.6	0
Pulp density by volume.....	do.....	12.2	10.9	20	30	40	50	55	60	65	70	80	90	95	100
Pulp density by weight.....	do.....	10.9	10.2	38.8	53.2	63.8	72.7	76.4	79.9	83.1	86.3	91.4	96.0	98.1	100
Ratio, weight of balls to pulp.....	do.....	4.4	4.4	8.9	7.9	7.1	6.5	6.2	5.9	5.7	5.5	5.1	4.8	4.6	4.4
Percent of mill filled.....	do.....	2:05	4:11	8:21	12:31	16:40	21:00	22:38	25:02	27:07	29:13	33:23	37:53	39:38	41:44
Minutes run.....	do.....	0.762	0.762	0.762	0.762	0.762	0.762	0.762	0.762	0.762	0.762	0.762	0.762	0.762	0.762
Capacity.....	pounds of quartz per minute.....	16.665	34.485	70.719	108.537	149.160	198.307	212.718	214.995	201.564	190.344	187.466	196.079	213.959	270.032
Net work input to balls.....	foot-pounds.....	151	136	141	142	147	160	160	151	135	116	90	87	88	111
Net input of work per revolution.....	do.....	29.6	4.3	13.5	12.4	10.8	6.8	0.8	2.7	7.6	17.1	27.0	26.3	16.1	2.7
Sizing analyses (mesh):		8-14.....	6.9	2.7	2.0	1.8	1.0	.5	1.9	6.8	13.0	16.3	16.3	13.0	2.8
		14-28.....	4.6	1.5	1.1	1.9	.8	.9	5.4	10.4	10.9	10.1	13.8	17.5	11.5
		28-48.....	8.4	3.0	2.1	2.9	2.0	6.8	14.8	14.3	10.3	10.2	9.9	17.1	23.9
		48-65.....	6.7	4.4	3.3	3.3	3.9	11.0	10.0	8.5	3.7	4.6	3.3	6.0	11.5
		65-100.....	8.5	9.9	9.7	8.8	8.2	11.0	11.7	9.2	6.7	4.7	3.3	3.3	10.8
		100-150.....	6.8	9.3	11.0	11.1	11.1	12.0	8.8	7.1	5.2	3.6	2.9	3.5	7.1
		150-200.....	3.0	6.5	8.7	9.4	9.5	8.0	6.4	5.3	4.0	2.8	2.4	2.8	4.8
		180-250.....	4.1	4.4	8.3	9.1	7.1	7.4	5.0	4.4	3.0	2.6	2.2	2.5	4.2
		200-270.....	3.1	24.4	31.0	34.2	36.4	37.7	33.8	3.5	3.1	1.9	1.2	1.7	2.8
		270-400.....	16.3	7.6	6.99	6.00	3.71	4.81	27.0	22.9	20.6	16.2	14.7	14.0	17.9
Ball and liner wear.....	pounds per ton of quartz.....	7.61	6.99	6.38	6.00	3.71	4.81	4.02	3.44	3.00	2.64	2.08	2.13	1.96	2.37
Finished product (pounds per 100,000 foot-pounds):		48-mesh.....	4.82	6.25	7.13	7.24	7.20	7.21	7.49	6.25	5.67	4.94	4.38	5.12	6.96
		200-mesh.....	3.37	4.80	5.86	6.38	6.26	6.01	6.09	4.43	4.27	3.68	3.42	3.90	4.33
		400-mesh.....	2.24	3.26	4.09	4.38	4.50	4.94	3.35	3.16	2.91	2.91	2.64	2.57	2.93
Surface of -40-mesh.....	cm ² per gram.....	5.239	5.194	5.036	5.022	5.209	5.375	5.824	6.142	6.072	6.286	7.378	8.466	8.663	5.644
Total surface produced.....	cm ² per foot-pound.....	17.51	24.29	52.74	77.80	83.57	83.75	81.54	30.54	30.61	74.05	65.26	81.22	78.99	62.94
Total efficiency based on 243 cm ² per foot-pound.....	percent.....	67.80	68.23	67.69	68.75	69.67	70.75	72.05	72.58	72.67	74.87	82.21	82.31	82.31	25.74
Useless work.....	do.....	5.68	7.88	3.53	10.01	10.18	3.57	9.38	8.39	7.96	7.67	79.17	82.31	79.44	68.46
Useful efficiency.....	do.....	5.68	7.88	3.53	10.01	10.18	3.57	9.38	8.39	7.96	7.67	79.17	82.31	79.44	68.46

TABLE 83.—*Varying pulp density: equal input of work*

[Constants: 142 pounds of balls, 200 each, 1½, 1, ¾, and ½ inch; volume of pulp constant at 0.192 cubic feet; 60 r. p. m.]

		Experiment													
		27	28	29	30	31	32	33	34	35	36	37	38	39	40
Quartz charge.....pound.....	1.59	3.18	6.36	9.54	12.7	16.0	17.5	19.08	20.67	22.26	25.44	28.62	30.21	31.8
Water.....do.....	11.4	10.8	9.6	8.4	7.2	6.0	5.4	4.8	4.2	3.6	2.4	1.2	0.6	0
Pulp density by volume.....	5	10	20	30	40	50	55	60	65	70	80	90	95	100
Pulp density by weight.....	12.2	22.7	39.8	53.2	63.8	72.7	76.4	79.9	83.1	86.3	91.4	96.0	98.1	100
Ratio, weight of balls to pulp.....	10.9	10.2	8.9	7.9	7.1	6.5	6.2	5.9	5.7	5.5	5.1	4.8	4.6	4.4
Percent of mill capacity filled.....	44	44	44	44	44	44	44	44	45	47	49	52	54	55
Minutes run.....	27	26	25	24	23	22	21	24	28	30	34	36	35	32
Capacity.....pound of quartz per minute.....	0.059	0.122	0.254	0.398	0.552	0.727	0.833	0.795	0.738	0.742	0.748	0.795	0.863	0.994
Net input of work to balls.....ft.-lb.....	208,923	214,335	214,599	212,619	214,368	203,702	198,891	206,580	210,738	198,891	190,318	188,872	190,219	201,406
Net input of work per revolution.....do.....	129	136	141	145	152	161	162	149	136	116	90	87	89	99
Sizing analyses (mesh):															
4-8.....	2.3	1.3	1.5	2.2	5.1	5.6	2.7	3.3	7.3	18.4	29.2	28.5	17.7	5.1
8-14.....	3.3	2.2	2.2	3.3	8	1.0	1.3	2.4	6.9	12.8	15.7	18.2	15.6	5.5
14-28.....2	.1	.2	.2	.5	.9	1.9	4.9	11.4	10.3	9.8	15.7	17.6	15.9
28-48.....2	.2	.2	.4	.9	2.2	6.5	12.7	15.0	10.1	9.4	9.2	16.0	24.5
48-65.....1	.2	.2	.5	1.1	3.9	9.3	10.2	8.1	5.5	4.5	3.0	5.6	10.4
65-100.....5	.6	1.0	2.3	4.4	11.1	14.0	12.0	8.9	6.5	4.5	3.3	5.0	9.2
100-150.....	1.3	1.6	3.0	5.6	8.7	12.4	11.1	9.2	6.6	5.2	3.5	2.7	3.5	5.9
150-200.....	1.9	2.7	5.0	7.8	9.6	9.6	8.0	6.5	5.1	4.2	2.7	2.2	2.5	4.1
200-270.....	4.0	6.0	9.2	11.4	11.7	9.0	7.2	6.1	4.7	3.5	2.3	1.9	2.1	3.3
270-400.....	9.0	11.4	14.7	14.7	13.6	5.7	8.3	7.1	3.7	2.5	2.3	1.4	1.8	2.3
400.....	80.2	75.7	65.1	54.6	43.6	38.6	29.7	25.6	22.7	21.0	16.3	13.9	12.6	13.8
Ball and liner wear.....pounds per ton of quartz.....	39.68	24.4	15.03	10.0	7.52	4.85	3.96	3.33	2.82	2.43	2.36	1.61	1.69	1.74
Finished product (pounds per 100,000 ft.-lb.):															
48-mesh.....74	1.46	2.90	4.35	5.49	7.09	7.71	7.08	5.83	5.42	4.80	4.30	5.29	7.74
100-mesh.....73	1.44	2.87	4.22	5.17	5.91	5.66	5.03	4.16	4.07	3.63	3.35	3.57	4.64
200-mesh.....71	1.38	2.63	3.62	4.08	4.10	3.98	3.58	3.01	3.02	2.79	2.61	2.62	3.06
Surface of -400-mesh.....cm ² per gram.....	9,690	8,313	6,756	6,086	5,950	5,576	6,474	6,690	6,163	6,309	7,313	8,242	7,582	5,392
Total surface produced.....cm ² per ft.-lb.....	27,03	43,17	61.71	72.58	76.90	84.32	86.36	80.70	67.84	72.59	87.61	82.62	75.11	63.32
Total efficiency based on 243 cm ² /ft.-lb.....percent.....	11.12	17.77	25.40	29.87	31.65	34.70	35.54	33.21	27.92	29.87	31.61	34.00	30.91	26.06
Useless work.....percent.....	87.13	84.38	79.28	75.34	72.89	71.92	72.87	73.42	73.37	75.54	79.11	81.85	77.92	65.95
Useful efficiency.....do.....	1.44	2.78	5.26	7.37	8.58	9.74	9.64	8.83	7.44	7.31	6.60	6.17	6.86	8.87

3. *Varying ball size.*—The data for the experiments on varying ball size are given in table 84 for equal capacity and equal input of work. These data are plotted in figure 64.

The input of work per revolution varies with the total and useful efficiencies but not with the production of fine material.

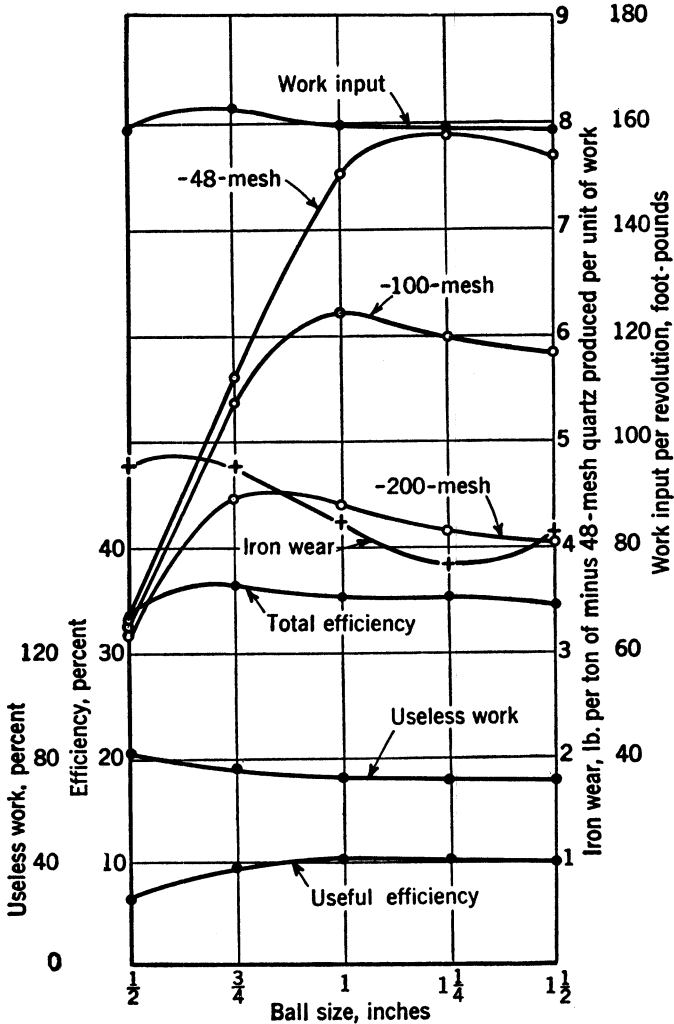


FIGURE 64.—Quartz grinding experiments, varying ball sizes.

The experiments indicate that when the ball size becomes less than three times the diameter of the largest feed the efficiency drops rapidly. The use of excessively large balls tends to give a smaller circulating load, a slightly smaller efficiency, and a less-granular finished product.

The production of fine material per unit of work input increases rapidly to a maximum with 1-inch balls and decreases with larger balls.

Fry (45) has noted an increasing tonnage of fine material when smaller pebbles are used in a pebble mill.

Iron wear decreases with larger balls but increases with the largest balls.

TABLE 84.—*Varying ball size; equal input of work and capacity*

[Constants: 142 pounds of balls, variable size; quartz, 16 pounds, water, 6 pounds; 50 percent pulp density by volume; 72.7 percent pulp density by weight; 6.5 ratio weight of balls to pulp; 44 percent of mill filled; 60 r. p. m.]

	Experiment				
	41	42	43	44	45
Balls:					
Number.....	292	505	986	2,300	7,830
Size..... inches.....	1½	1¼	1	¾	½
Minutes run.....	22	22	22	22	22
Capacity..... lb. of quartz per minute.....	0.727	0.727	0.727	0.727	0.727
Net work input to balls..... ft.-lb.....	200,871	196,977	198,495	200,376	198,957
Net work input per revolution..... do.....	158	159	159	162	158
Sizing analyses (mesh):					
4-8.....	0.7	0.5	3.4	26.0	44.1
8-14.....	.2	.1	.9	2.3	11.4
14-28.....	.3	.2	.9	.9	2.1
28-48.....	2.5	2.3	1.9	.9	.7
48-65.....	8.0	8.2	4.3	.7	.3
65-100.....	15.3	15.5	11.9	2.3	.6
100-150.....	12.9	13.1	12.6	4.7	.8
150-200.....	9.4	9.2	9.9	6.6	1.0
200-270.....	7.8	7.8	8.4	7.7	1.7
270-400.....	6.1	6.3	6.8	6.5	2.6
-400.....	36.8	36.8	39.0	41.4	34.7
Ball and liner wear..... lb. per ton of quartz.....	4.11	3.81	4.21	4.73	4.73
Finished product (pounds per 100,000 ft.-lb.):					
-48-mesh.....	7.67	7.87	7.49	5.58	3.35
-100-mesh.....	5.81	5.95	6.18	5.34	3.28
-200-mesh.....	4.04	4.13	4.37	4.44	3.14
Surface of -400-mesh..... cm ² per gram.....	5,620	5,653	5,403	5,535	6,344
Total surface produced..... cm ² per ft.-lb.....	83.06	85.23	85.29	87.95	80.86
Total efficiency based on 243 cm ² per ft.-lb.....	34.18	35.07	35.10	36.19	33.28
Useless work..... percent.....	71.30	71.38	70.84	74.48	81.07
Useful efficiency..... do.....	9.81	10.04	10.24	9.24	6.30

4. *Varying ball loads.*—Table 85 gives data for experiments on variable ball loads with equal capacity and table 86 with equal input of work. The data on equal capacity are plotted in figure 65.

These experiments indicate that large ball charges give much better results in every respect. The input of work per revolution varies roughly with the efficiency and with the production of finished fines up to approximately a 150-pound ball charge or 50 percent of the mill capacity; with a larger ball load the relationship fails to hold; however, the input of work varies with the iron wear throughout the entire range of ball loads. Although the total efficiency decreases with the largest ball loads the useful efficiency increases steadily as the ball load increases.

The production of fines per unit of work input increases rapidly as the ball load is increased from very small loads, then becomes somewhat erratic to approximately a 180-pound load, after which it again increases more rapidly and reaches a maximum with the greatest load of balls.

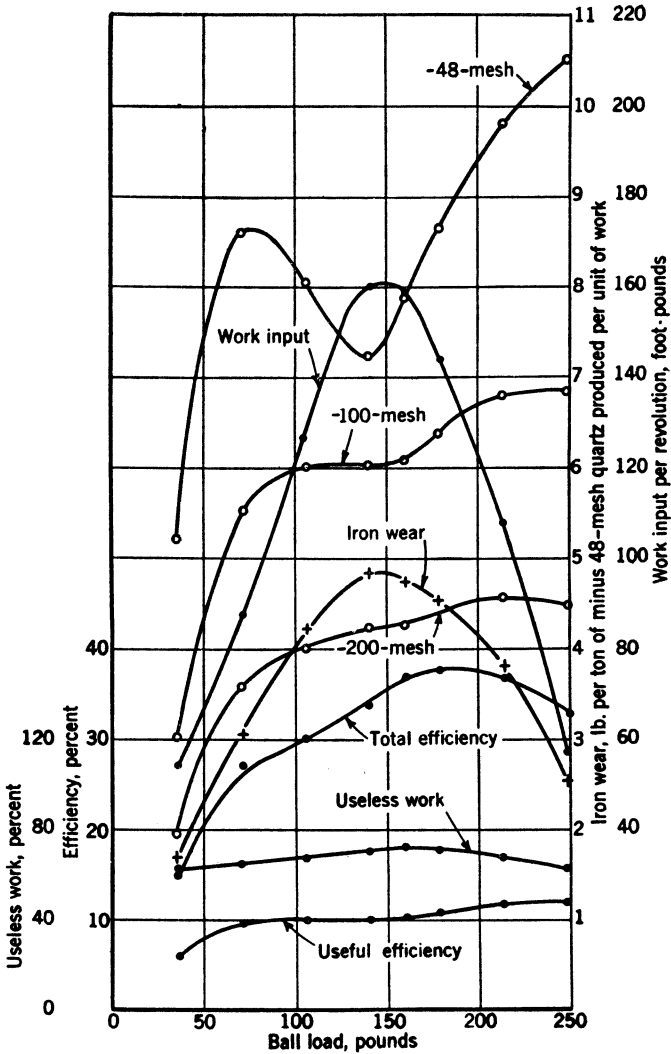


FIGURE 65.—Quartz grinding experiments, varying ball loads.

TABLE 85.—*Varying ball loads; equal capacity*

[Constants: Equal number of 1½-, 1-, ¾-, and ½-inch balls; quartz, 16 pounds, water, 6 pounds; 50 percent pulp density by volume; 72.7 percent pulp density by weight; 60 r. p. m.]

	Experiment							
	46	47	48	49	50	51	52	53
Balls:								
No. of each size.....	50	100	150	200	226	250	300	350
Weight.....lb.	36	71	107	142	160	178	214	249
Ratio, weight of balls to pulp.....	1.6	3.2	4.9	6.5	7.3	8.1	9.7	11.3
Percent of mill filled.....	24	31	42	44	48	53	64	74
Minutes run.....	21	21	21	21	21	21	21	21
Capacity								
pounds of quartz per minute.....	0.762	0.762	0.762	0.762	0.762	0.762	0.762	0.762
Net input of work to balls.....ft.-lb.	74,329	116,206	159,192	198,307	192,515	171,191	124,641	65,498
Net input of work per revolution								
ft.-lb.....	54	87	129	160	159	145	108	57
Sizing analyses (mesh):								
4-8.....	37.1	17.5	12.0	6.8	0.8	0.7	6.6	29.1
8-14.....	13.1	3.5	2.1	1.0	.5	.4	3.0	7.8
14-28.....	12.3	4.1	1.5	.8	.7	.8	3.6	7.9
28-48.....	13.3	12.5	4.3	2.0	3.1	5.4	10.3	12.1
48-65.....	5.4	10.4	7.1	3.9	7.3	9.6	10.4	7.2
65-100.....	4.8	12.0	13.3	11.0	14.3	15.0	13.2	7.9
100-150.....	2.9	8.3	11.6	12.6	13.0	12.3	10.1	5.7
150-200.....	2.0	5.7	8.3	9.4	9.1	8.6	7.3	4.0
200-270.....	1.5	4.7	6.9	8.8	7.9	7.5	5.9	3.1
270-400.....	1.2	3.0	4.9	6.0	5.7	5.6	3.8	2.2
-400.....	6.4	18.3	28.0	37.7	37.6	34.1	25.8	13.0
Ball and liner wear								
pounds per ton of quartz.....	1.69	3.05	4.21	4.81	4.73	4.51	3.79	2.54
Finished product (pounds per 100,000 ft.-lb.):								
-48-mesh.....	5.21	8.59	8.05	7.21	7.89	8.66	9.82	10.53
-100-mesh.....	3.01	5.51	6.00	6.01	6.09	6.37	6.79	6.84
-200-mesh.....	1.96	3.58	4.00	4.24	4.26	4.41	4.56	4.47
Surface of -400-mesh								
cm ² per gram.....	4,752	4,859	4,984	5,375	5,688	5,678	5,175	4,593
Total surface produced								
cm ² per ft.-lb.....	36.01	65.54	72.82	82.05	89.03	91.59	88.71	78.40
Total efficiency based on 243 cm² per ft.-lb.								
ft.-lb.....	14.82	26.79	29.97	33.77	36.64	37.69	36.50	32.26
Useless work.....percent.	62.20	64.39	66.92	70.78	71.99	71.21	67.89	62.95
Useful efficiency.....do.	5.60	9.60	9.91	9.87	10.26	10.85	11.72	11.95

TABLE 86.—*Varying ball loads; equal input of work*

[Constants: Equal number of 1½-, 1-, ¾-, and ½-inch balls with varying weights; quartz, 16 pounds, water, 6 pounds; 50 percent pulp density by volume; 72.7 percent pulp density by weight; 60 r. p. m.]

	Experiment							
	54	55	56	57	58	59	60	61
Balls:								
No. of each size.....	50	100	150	200	225	250	300	350
Weight.....lb.	36	71	107	142	160	178	214	249
Ratio, weight of balls to pulp.....	1.6	3.2	4.9	6.5	7.3	8.1	9.7	11.3
Percent of mill filled.....	24	31	42	44	48	53	64	74
Minutes run.....	70	36	26	22	21	25	33	52
Capacity								
pounds of quartz per minute.....	0.229	0.444	0.615	0.727	0.762	0.640	0.485	0.308
Net input of work to balls.....ft.-lb.	194,555	190,793	203,227	203,702	192,357	202,970	200,416	185,664
Net input of work per revolution								
ft.-lb.....	40	86	129	161	160	145	108	66

TABLE 86.—*Varying ball loads; equal input of work*—Continued

Experiment—Continued

	54	55	56	57	58	59	60	61
Sizing analyses (mesh):								
4-8	4.7	4.8	5.1	5.6	1.5	0	1.0	3.4
8-14	1.5	.7	.7	1.0	.3	.2	.2	.8
14-28	2.9	.7	.6	.9	.5	.3	.3	1.0
28-48	13.0	3.9	1.9	2.2	2.8	2.2	2.5	2.9
48-65	12.1	9.3	5.0	3.9	6.8	6.7	5.8	5.3
65-100	13.6	15.6	12.9	11.1	14.9	15.0	13.0	12.6
100-150	10.2	12.6	13.3	12.4	12.9	12.8	12.5	12.6
150-200	6.7	8.6	9.8	9.6	9.1	9.3	9.4	9.5
200-270	6.1	7.9	8.9	9.0	8.7	8.6	9.1	8.9
270-400	3.6	4.7	5.6	5.7	5.6	5.7	6.1	5.3
-400	25.6	31.2	36.2	38.6	36.9	39.2	40.1	37.7
Ball and liner wear								
pounds per ton of quartz..	4.61	4.90	4.90	4.85	4.73	4.60	4.43	4.74
Finished product (pounds per 100,000 ft.-lb.):								
-48-mesh	6.41	7.54	7.22	7.09	7.89	7.67	7.66	7.92
-100-mesh	4.29	5.45	5.81	5.91	6.08	5.96	6.16	6.38
-200-mesh	2.90	3.67	3.99	4.19	4.26	4.22	4.41	4.47
Surface of -400 mesh								
cm ² per gram..	5.117	5.025	5.126	5.576	5.661	5.785	5.685	5.210
Total surface produced, cm ² per ft.-lb..	56.02	68.01	74.28	84.32	87.40	89.20	90.75	85.30
Total efficiency based on 243 cm ² per ft.-lb.								
Useless work	23.05	27.99	30.57	34.70	35.97	36.71	37.30	35.10
Useful efficiency	67.35	67.34	68.91	71.92	71.60	72.58	72.31	69.86
Useful efficiency	7.53	9.14	9.50	9.74	10.22	10.07	10.33	10.58

These results indicate that a large ball load is decidedly effective, and as the iron wear decreases rapidly after the ball load reaches the half-full stage, maximum useful efficiency, maximum production of fines per unit of work input, and minimum iron wear are attained with ball loads well over 50 percent.

5. *Varying pulp charges.*—Table 87 gives data for the experiments on variable quartz weights with equal capacity and table 88 with equal input of work. The data on equal capacity are plotted in figure 66.

These results show that there is no great difference in efficiency with difference in weight of pulp charges. There is no relationship between the work input per revolution and the efficiency or the production of finished product. The maximum useful efficiency as well as the production of fine material per unit of work input occurs with a pulp charge of approximately 8¼ pounds, although the results with the largest charge are not far below the maximum. The iron wear decreases as the pulp charge increases and, if grinding is carried only to minus 48-mesh, the largest pulp charge would give the best results.

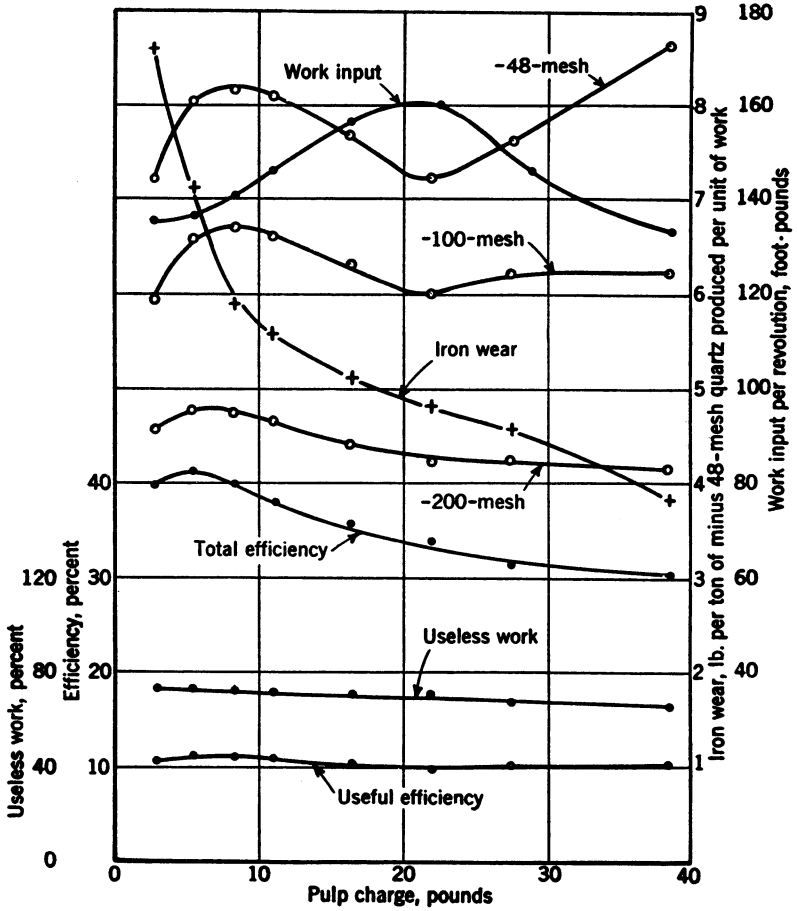


FIGURE 66.—Quartz grinding experiments, varying pulp charges.

TABLE 87.—*Varying pulp charges, equal capacity*

[Constants: 142 pounds of balls (200 each, 1½, 1, ¾, and ½ inch); 50 percent pulp density by volume; 72.7 percent pulp density by weight; 60 r. p. m.]

	Experiment							
	62	63	64	65	66	67	68	69
Pulp.....pounds.....	2¾	5½	8¼	11	16½	22	27½	38½
Ratio, weight of balls to pulp.....	51.6	25.8	17.2	12.9	8.6	6.5	5.2	3.7
Percent of mill filled.....	42	42	42	42	42	44	48	57
Minutes run.....	2:37	5:15	7:52	10:30	15:45	21	26:15	36:44
Capacity, pounds of quartz per minute.....	.762	.762	.762	.762	.762	.762	.762	.762
Net input of work to balls.....ft.-lb.....	20,890	43,164	66,627	91,733	145,807	198,307	231,719	288,506
Net input of work per revolution.....ft.-lb.....	135	136	140	146	156	160	149	133
Sizing analyses, mesh:								
4-8.....	3.3	1.9	3.8	2.9	3.1	6.8	8.2	3.7
8-14.....	2.2	2.1	1.3	.8	.6	1	1	1.3
14-28.....	7	3.4	1.4	.9	.8	.8	.7	1.2
28-48.....	12.4	5.8	3.1	2.7	2.3	2	1.9	5.1
48-65.....	5.8	5.4	4.7	4.8	4.4	3.9	4.2	9.1
65-100.....	7.6	10.3	11.5	12.2	12.3	11	12	15.4
100-150.....	7.6	11.1	12.2	12.6	13	12.6	13	12.5
150-200.....	6.6	8.6	9.3	9.6	9.5	9.4	9.6	8.7
200-270.....	6.6	8.1	8.5	8.6	8.8	8.8	8.4	7.4
270-400.....	5.2	5.5	6.2	6.3	6.1	6	5.9	5.3
400.....	35.7	37.8	38	38.6	39.1	37.7	35.1	30.3
Ball and liner wear.....pounds per ton of quartz.....	8.59	7.10	5.89	5.55	5.10	4.81	4.59	3.81
Finished product (pounds per 100,000 ft.-lb.):								
—48-mesh.....	7.21	8.04	8.14	8.08	7.67	7.21	7.61	8.61
—100-mesh.....	5.92	6.59	6.68	6.60	6.30	6.01	6.21	6.23
—200-mesh.....	4.56	4.76	4.75	4.67	4.44	4.24	4.26	4.17
Surface of —400-mesh, cm ² per gram.....	5.748	5.743	5.653	5.437	5.375	5.375	4.930	4.784
Total surface produced, cm ² per ft.-lb.....	96.75	99.77	96.65	91.85	86.76	81.05	76.29	73.65
Total efficiency based on 243 cm ² per ft.-lb.....	39.81	41.06	39.77	37.80	35.70	33.77	31.40	30.31
Useless work.....percent.....	73.63	72.89	72.07	71.00	70.80	70.78	67.78	65.52
Useful efficiency.....do.....	10.50	11.13	11.11	10.96	10.42	9.87	10.12	10.45

TABLE 88.—*Varying pulp charges, equal input of work*

[Constants: 142 pounds of balls (200 each, 1½, 1, ¾, and ½ inch); 50 percent pulp density by volume; 72.7 percent pulp density by weight; 60 r. p. m.]

	Experiment							
	70	71	72	73	74	75	76	77
Pulp.....pounds.....	2¾	5½	8¼	11	16½	22	27½	38½
Ratio, weight of balls to pulp.....	51.6	25.8	17.2	12.9	8.6	6.5	5.2	3.7
Percent of mill filled.....	42	42	42	42	42	44	48	57
Minutes run.....	25	24	24	23	22	22	23	25
Capacity.....pounds of quartz per minute.....	.080	.167	.250	.340	.545	.727	.870	1.120
Net input of work to balls.....ft.-lb.....	20,168	196,713	202,178	200,831	203,920	203,702	201,821	195,565
Net input of work per revolution.....ft.-lb.....	134	136	139	144	154	161	147	131
Sizing analyses, mesh:								
4-8.....	.2	0	0	0	.3	5.6	9.9	12.6
8-14.....	.1	0	0	0	.2	1	1.6	3.8
14-28.....	0	.1	0	0	.1	.9	.6	3.6
28-48.....	.1	0	0	0	.3	2.2	2.9	11.8
48-65.....	.1	.1	.1	.1	1	3.9	6.1	11.3
65-100.....	0	0	.1	.7	6.4	11.1	14	13
100-150.....	.1	.1	.8	3.7	11.8	12.4	12.4	9.3
150-200.....	.1	.3	2.6	7.3	10.5	9.6	8.6	6.1
200-270.....	.2	2.1	7.1	10.6	10.7	9	7.9	5.2
270-400.....	.5	3.4	7.9	8.9	7.6	5.7	5.4	3.7
400.....	98.6	93.9	81.4	68.7	51.1	38.6	30.6	19.6
Ball and liner wear.....pounds per ton of quartz.....	24.19	14.20	10.34	8.37	6.17	4.85	3.97	3.30
Finished product (pounds per 100,000 ft.-lb.):								
—48-mesh.....	.99	2.03	2.96	3.98	5.83	7.09	8.42	9.76
—100-mesh.....	.99	2.03	2.96	3.95	5.40	5.91	6.43	6.29
—200-mesh.....	.99	2.02	2.86	3.51	4.08	4.19	4.35	4.08
Surface of —400-mesh, cm ² per gram.....	18.710	10.717	8.619	6.803	5.807	5.576	5.024	4.811
Total surface produced, cm ² per ft.-lb.....	83.00	92.76	95.63	87.36	85.10	84.32	78.76	72.85
Total efficiency based on 243 cm ² per ft.-lb.....	34.16	38.17	39.35	35.95	35.02	34.70	32.41	29.96
Useless work.....percent.....	94.05	89.22	85.43	80.11	74.39	71.92	67.37	63.67
Useful efficiency.....do.....	2.03	4.11	5.73	7.15	8.97	9.74	10.58	10.88

DISCUSSION OF RESULTS

The experiments under conditions of equal capacity do not indicate that the maximum input of work per revolution is necessarily accompanied by the best results but often do indicate the maximum usefless work and iron wear.

A consideration of the best results must take into account the useful efficiency, the amount of finished fine material produced per unit of work, the iron wear, the fineness of the minus 400-mesh material, and the return circulating load. The best results have been obtained under the following conditions of equal capacity for the variables investigated.

Varying mill speed.—The best work was accomplished at approximately 40 percent of the critical speed. Low speeds result in:

- Maximum useful efficiency.
- Maximum production of fine material.
- Least iron wear.
- Most-granular minus-400-mesh product.
- Largest return load (at 48-mesh), 44.7 percent.

With higher speeds the return load becomes very small owing to overgrinding.

Varying pulp density.—The best work was accomplished at a density of about 40 percent. This density results in:

- Maximum useful efficiency.
- Maximum production of fine material.
- Medium iron wear.
- Nearly the most-granular minus-400-mesh product.
- Nearly the smallest return load (at 48-mesh), 15.4 percent, smallest, 9.0 percent.

The fact that the best results were accompanied by a small return load suggests overgrinding. A greater capacity or shorter residence in the mill might have resulted in better work. A comparison of experiments 17 and 31 shows that the greater capacity in experiment 17 resulted in much better work.

Varying ball size.—The best work was accomplished with 1-inch balls. This size results in:

- Maximum useful efficiency.
- Maximum production of fine material.
- Medium iron wear.
- Most-granular minus-400-mesh product.
- Nearly the smallest return load (at 48-mesh), 7.1 percent, smallest, 3.1 percent.

The low return load suggests some overgrinding in this experiment. A larger capacity might have given better results.

Varying ball loads.—The best work was accomplished with the largest ball loads. These result in:

- Maximum useful efficiency.
- Maximum production of fine material.
- Least iron wear.
- Most-granular minus-400-mesh product.
- Next-to-largest return load (at 48-mesh), 56.9 percent.

Small return loads with medium ball charges indicate overgrinding.

Varying pulp charges.—The best work was accomplished with smaller charges, such as the 8¼-pound charge. This charge resulted in:

Maximum useful efficiency.

Maximum production of fine material.

Medium iron wear.

Nearly the most-granular minus-400-mesh product.

Nearly the smallest return load (at 48-mesh), 9.6 percent, smallest, 6.8 percent.

The best result with variable pulp charges, as with variable pulp density, is accompanied by a low return load and again suggests overgrinding or better results with a greater capacity. A comparison of experiments 64 and 72 shows that very marked improvement in all ways results from the greater capacity in experiment 64.

Unfortunately, it was impossible to continue these experiments. The effect of increased capacity on a small 40-percent pulp-density charge with a maximum ball size of 1 inch would doubtless have resulted in better work.

The curves in figures 62 to 66 show that when mill speed and pulp charge vary and capacity is the same the effect on the mill operation is small. The variables that seriously affect operation are pulp density, ball load, and size of balls.

The experiments that fulfilled the conditions of best work for the different variables are summarized in table 89.

From a comparison of the results of a 16-pound charge at a pulp density of 50 percent by volume, using 142 pounds of mixed balls at 60 revolutions per minute, with the experiments giving the best results it appears that a combination of the best conditions for each variable might result in a useful efficiency of 15.2 percent with a considerable increase in finished fine material, less iron wear, a large circulating load, and a more-granular minus-400-mesh product. Total efficiency would not increase much, but the useless work would be materially decreased.

TABLE 89.—*Experiments fulfilling conditions of best work; equal capacity*

Experiment	Variables	Useful efficiency	Finished fines (pounds per 100,000 ft.-lb.)			Iron wear (pounds per ton of quartz)	Return load (plus 48-mesh)
			Minus 48-mesh	Minus 100-mesh	Minus 200-mesh		
1.....	Mill speed, 33 r. p. m.....	10. 41	8. 47	6. 31	4. 28	3. 02	44. 7
17.....	Pulp density, 40 percent.....	10. 18	7. 20	6. 26	4. 50	5. 71	15. 4
43.....	Ball size, 1-inch.....	10. 24	6. 49	6. 18	4. 37	4. 21	7. 1
53.....	Ball load, 249 pounds.....	11. 95	10. 53	6. 84	4. 47	2. 54	56. 9
64.....	Pulp charge, 8¼ pounds.....	11. 11	8. 14	6. 68	4. 75	5. 89	9. 6

Effect of capacity.—A comparison of the results under conditions of equal input of work and equal capacity shows the effect of variations in capacity under otherwise similar conditions. A few comparisons for different capacities, given in table 90, show that although the total efficiency is often less the increased capacity is almost invariably accompanied by the following:

Greater useful efficiency.

Greater production of minus-48-mesh material per unit of work input.

More granular minus-400-mesh material.

Less iron wear.

Greater percentage of plus-48-mesh material as return load.

The lower return load and poorer efficiency at lower capacities show the effect of overgrinding. The advantages of an increased circulating load have been shown by Fahrenwald (83).

Probably the 15.2-percent useful efficiency calculated as possible with the most favorable variables might be raised considerably by increasing the capacity.

TABLE 90.—Comparison of experiments with variable capacity

Variables	Ex-periment no.	Capac-ity (pounds per minute)	Total effi-ciency (per-cent)	Useful effi-ciency (per-cent)	Finished fine prod-uct per unit of work			Surface minus 400-mesh (cm ² per gram)	Iron wear (pounds per ton of quartz)	Return load (plus 48-mesh)
					-48	-100	-200			
Mill speed (r. p. m.):										
33.....	7	0.400	31.56	9.32	6.20	5.68	4.27	5,050	4.63	26.4
33.....	1	.762	31.60	10.41	8.47	6.31	4.28	4,937	3.02	44.7
80.....	6	.762	32.70	8.93	6.38	5.39	3.86	5,658	5.21	.7
80.....	12	.889	32.07	9.61	7.46	5.74	4.05	5,500	5.12	2.7
Pulp density (percent):										
10.....	28	.122	17.77	2.78	1.46	1.44	1.38	8,313	24.44	1.8
10.....	14	.762	24.79	7.88	6.25	4.80	3.26	5,194	6.99	32.2
40.....	31	.552	31.65	8.58	5.49	5.17	4.08	5,950	7.52	7.3
40.....	17	.762	33.57	10.18	7.20	6.26	4.50	5,209	5.71	15.4
80.....	37	.748	31.61	6.60	4.80	3.63	2.79	7,313	2.36	64.1
80.....	23	.762	32.21	6.71	4.94	3.68	2.81	7,378	2.08	63.6
Dry charge.....	26	.762	25.74	8.12	6.96	4.33	2.93	5,644	2.37	40.9
Do.....	40	.994	26.06	8.87	7.74	4.64	3.06	5,392	1.74	51.0
Ball load (pounds):										
71.....	55	.444	27.99	9.14	7.54	5.45	3.67	5,025	4.90	10.1
71.....	47	.762	26.97	9.60	8.59	5.51	3.58	4,859	3.05	37.6
178.....	59	.640	36.71	10.07	7.67	5.96	4.22	5,785	4.60	2.7
178.....	51	.762	37.69	10.85	8.66	6.37	4.41	5,678	4.51	7.3
249.....	61	.308	35.10	10.58	7.92	6.38	4.47	5,210	4.74	8.1
249.....	53	.762	32.26	11.95	10.53	6.84	4.47	4,593	2.54	56.9
Pulp charge (pounds):										
2 ³ / ₄	70	.080	34.16	2.03	.99	.99	.99	18,710	24.19	.4
2 ³ / ₄	62	.762	39.81	10.50	7.21	5.92	4.56	5,748	8.59	24.9
8 ¹ / ₄	72	.250	39.35	5.73	2.96	2.96	2.86	8,619	10.34	0
8 ¹ / ₄	64	.762	39.77	11.11	8.14	6.68	4.75	5,653	5.89	9.6
27 ¹ / ₂	68	.762	31.40	10.12	7.61	6.21	4.26	4,930	4.59	11.8
27 ¹ / ₂	76	.870	32.41	10.58	8.42	6.43	4.35	5,024	3.97	15.0
38 ¹ / ₂	69	.762	30.31	10.45	8.61	6.23	4.17	4,784	3.81	11.3
38 ¹ / ₂	77	1.120	29.96	10.88	9.76	6.29	4.08	4,811	3.30	31.8

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where d_1 = maximum diameter and d_2 = minimum diameter.

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Gives experiments made in rolls which show that most of the energy required in rock crushing appears as heat, but surface formation itself does not generate heat. Doubtless the energy absorbed in the formation of surface is in accord with the Rittinger law, and the heat must be due to friction. Energy curves, plotted according to Rittinger and Kick, do not check with the actual energy, and a new formula is proposed. Results are based on sieve sizing, assuming a figure for -200-mesh of approximately 35 microns average size. "The energy required for the actual process of crushing (disregarding losses in the machine itself) is absorbed mainly in three ways: (1) Actual formation of new surface or final rupture; this is a small part of the total energy. (2) Internal friction accompanying

the distortion prior to rupture. (3) Surface friction of particles on particles and of particles on crushing surface."

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73. TAGGART, ARTHUR F. *Handbook of Ore Dressing*. New York, 1927. Mechanics of the ball mill, pp. 381-384. Crushing efficiency, pp. 488-497.
Gives cost comparisons and Kick, Rittinger, Del Mar, and Gates methods of calculating efficiencies. Discusses Kick against Rittinger controversy, as well as settling rates and particle size, pp. 550-556.
74. GROSS, JOHN, AND ZIMMERLEY, S. R. *Crushing and Grinding: I. Surface Measurement of Quartz Particles*. *Tech. Pub.* 46, *Am. Inst. Min. and Met. Eng.*, 1928. *Milling Methods* vol., 1930, p. 7.

Describes method for surface determination of quartz by its dissolution rate in hydrofluoric acid. The mathematical extrapolation of a rate curve to zero time gives the rate before any change in surface or concentration of solution has taken place; this initial rate (I. R.) is a true relative measure of surface, and the calibration of the I. R. value for the conversion to definite surface units was done by silver coating of quartz. The accuracy of the method is discussed.

75. LIPSETT, S. G., JOHNSON, F. M. G., AND MAASS, O. The Surface Energy of Solid Sodium Chloride: III. The Heat of Solution of Finely Ground Sodium Chloride. *Jour. Am. Chem. Soc.*, vol. 50, 1928, p. 2701. For former work, see *idem*, vol. 49, 1927, pp. 925, 1940.

Describes results when NaCl was ground in an agate mortar and air elutriated. (Contained no SiO₂ by analysis.) Particle size was measured by means of photomicrograph. "The heat of solution was determined at 25° at a concentration of 4.62 percent with three different samples of salt."

Heat of solution of finely ground NaCl

Sample	Heat of solution (calories per mole)	Difference in heat of solution (calories per mole)	Diameter of average particle (μ)
1.....	-903.7	24.9	1.3
2.....	-903.6	25.0	1.2
3.....	-887.9	40.7	1.4

Column 3 gives "the difference between the heat of solution of coarsely ground sodium chloride (-9,286 calories per mole) and the finely ground salt measured." From previous work sublimed salt 1.3 μ diameter "would have approximately a heat of solution only 11 calories less than that of coarse salt." The large difference is attributed to the uneven surface of ground NaCl, microscopic examination showing sublimed salt to be regular in shape and to have smooth surfaces.

76. MARTIN, GEOFFREY, AND BOWES, EDGAR. The Theory of Fine Grinding: IX. Connection Between the Statistical Diameter and the Statistical Volume of Irregularly Shaped Particles of Crushed Sand. *Trans. Ceram. Soc.*, vol. 27, 1928, pp. 247-258.

Determines volume constant and the shape by count on air-elutriated products as being the same for different-size products.

77. ——— The Theory of Fine Grinding: X. The Connection Between the Statistical Diameter of Crushed-sand Particles and Their Statistical Surface. *Trans. Ceram. Soc.*, vol. 27, 1928, pp. 259-284.

Makes microscopic measurement on air-elutriated products to determine size, which ranged from 0.0333 to 0.8325 mm. Relation between surface and diameter was found to range from 2.00 to 2.49; the largest value was for the coarse material.

78. ——— The Theory of Fine Grinding: XI. Calculations Relating to Diameters, Surfaces, and Weights of Homogeneous Grades of Crushed Quartz Sand. *Trans. Ceram. Soc.*, vol. 27, 1928, pp. 285-289.

Gives data on 20 grades of quartz, with 31,000,000 to 1,240 particles per gram. Discusses methods of calculation for volumes, surface, and weights.

79. GROSS, JOHN, AND ZIMMERLEY, S. R. Crushing and Grinding Studies of Quartz. *Rept. of Investigations 2880, Bureau of Mines, 1928, 10 pp.*

Describes dissolution method for the surface measurement of quartz. Surface values for sieve-sized quartz show increasingly large interior surfaces (or cracks) for coarser particles. Experiments on crushing show that the surface produced is in direct proportion to the work expended, which proves that the Rittinger law holds in a crushing operation.

80. ——— Crushing and Grinding: II. The Relation of Measured Surface of Crushed Quartz to Sieve Sizes. *Am. Inst. Min. and Met. Eng., Tech. Pub. 126, 1928. Milling Methods vol., 1930, p. 27.*

Shows that the ratio of measured surface of crushed quartz to theoretical surface (on perfect cubes or spheres) increases greatly with coarser sizes and is due to cracks or interior surface and that the relation of measured surface to average size in a logarithmic plot can be extended to the unit crystal size, this curve being useful in obtaining average size of particle when the surface has been determined. Minus 200-mesh material is discussed. From a knowledge of the -200-mesh material the amount of useless work done in crushing can be determined.

81. GROSS, JOHN, AND ZIMMERLEY, S. R. *Crushing and Grinding: III. Relation of Work Input to Surface Produced in Crushing Quartz.* Am. Inst. Min. and Met. Eng., Tech. Pub. 127, 1928. *Milling Methods* vol., 1930, p. 35.
Describes device for accurately determining the work input to crushing, utilizing work performed by a falling ball. Thirteen experiments, in which work input ranged from 3 to 46 units of work, show that surface produced is directly proportional to input of work. From these experiments it is concluded that crushing follows Rittinger law. Efficiency of crushing is compared to theoretical efficiency based on surface energy of quartz. Distribution of the work in various sizes shows importance of -200-mesh material, which represents major part of work.
82. COGHILL, W. H. *Evaluating Grinding Efficiency by Graphical Methods.* Eng. and Min. Jour., vol. 126, 1928, pp. 934-938.
Uses "force diagram" to obtain "mean mesh" as a measure of advance in a crushing operation. By sieve sizing and surface figures based on size of particles the power efficiency is measured. Minus 200-mesh material is considered as unnecessary work, and the machine should not be credited therefor. Counts on sieve sizes of chert and dolomite from 2-mesh to 60- or 65-mesh check with theoretical number of particles; therefore, similar shapes exist in all sizes, and a surface figure in proportion to the theoretical for all sizes is allowable.
83. FAHRENWALD, A. W. *Flotation Practice in the Coeur d'Alene District, Idaho.* Am. Inst. Min. and Met. Eng., volume on "Flotation Practice", 1928, pp. 107-132.
Gives the advantages of large circulating load. Laboratory experiments indicate a greater efficiency with higher pulp densities. Poor classification results in lower mill capacity and overgrinding.
84. ANDREASEN, A. H. M. *Zur Kenntnis des Mahlgutes.* Kolloidchem. Beihefte, vol. 27, 1928, p. 349.
Shows that increase in surface area per unit weight is generally not sufficient indication of the grinding while the distribution of the particles is; the frequency curve follows Martin's compound-interest law; no general law exists governing the distribution of grain sizes, as this distribution varies with the manner of grinding and with the material; and the compound-interest law does not hold for ball-mill products.
85. WORK, LINCOLN T. *The Graphical Analysis of Fineness Distribution Curves for Pulverized Materials.* Proc. Am. Soc. Test. Mat., vol. 28, 1928, pt. II.
Reviews various methods of measuring particle size of fine material. Favors "direct" method of microscopic measurement involving statistical analysis.
86. ANABLE, ANTHONY. *Closed-circuit Fine Grinding and What it Should Accomplish in the Cement Industry.* Rock Products, vol. 32, 1929, pp. 66-72.
Discusses classification. Compares mill efficiency and work done by tons of size produced by the Rittinger law and by the Kick law. That the classifier determines the capacity, not the mill, is based on examples given.
87. GOOSKOV, W. A. *Application of Cinematograph to the Study of the Fall of Coal Particles in Still Water.* Am. Inst. Min. and Met. Eng., Tech. Pub. 18, 1929.
Discusses use of motion pictures to analyze fall of particles in sedimentation.
88. HATCH, THEODORE, AND CHOATE, SARAH P. *Statistical Description of the Size Properties of Nonuniform Particulate Substances.* Jour. Franklin Inst., vol. 207, no. 3, 1929.
Describes microscopic count-statistical method for determining average size from frequency curves. Tyndall meter is used to measure surfaces of suspensions. Includes good bibliography.

89. COGHILL, W. H. Fine Grinding in the Tri-State. Reports and addresses, 11th Ann. Meeting Am. Zinc Inst., St. Louis, Apr. 15, 1929, pp. 100-117. Gives historical data on ball milling. Discusses rod mill, the multiple-compartment mill, and short-length laboratory mills, principally in regard to the mechanics of ball milling. A new formula is given for ball paths.
90. ANDREASEN, A. H. M. Über die Gültigkeit des Stokes'schen Gesetzes für nicht Kugelförmige Teilchen. *Kolloid Ztschr.*, vol. 48, no. 2, 1929, pp. 175-179. Proves Stokes' law theoretically and practically for spherical particles of less than 85 microns diameter, using spherical quartz particles in water. Shows that counting and weighing method to obtain average diameter of particles greater than 5 microns is correct to 1 to 2 percent. Separates particles by sedimentation with repeated decantings and determines a constant use of which Stokes' law applies to ground particles.
91. LUNNON, R. G. The Laws of Motion of Particles in a Fluid. *Min. and Met.*, vol. 10, 1929, p. 333. Abstract of paper read before Inst. Min. Eng. (England). Gives the three laws (Stokes, Allen, and Newton) on falling particles in a liquid.
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93. ZIMMERLEY, S. R., AND GROSS, JOHN. Crushing Resistance of Minerals. Rept. of Investigations 2948, Bureau of Mines, 1929, 4 pp. Gives crushing resistance of galena, sphalerite, pyrite, and calcite in relation to quartz based on surface produced under accurately measured work input.
94. GROSS, JOHN, ZIMMERLEY, S. R., AND PROBERT, ALAN. A Method for the Sizing of Ore by Elutriation. Rept. of Investigations 2951, Bureau of Mines, 1929, 8 pp. Determines sizes of mineral particles obtained in elutriation products from -200-mesh ore by comparison with quartz particles obtained under identical conditions of elutriation. Surface figures for galena, sphalerite, and pyrite are given for sieve sizes and elutriation products.
95. GROSS, JOHN, AND ZIMMERLEY, S. R. Efficiency of Grinding Mills. Rept. of Investigations 2952, Bureau of Mines, 1929, 23 pp. Gives method for determining surface on ore by comparing sieve sizes and elutriation products with quartz, surface of which has been determined. Results given show over-all and useful efficiency of ball mills. Gives classifier efficiencies and discusses minus 200-mesh material.
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97. BUDNIKOFF, PETER P., AND NERKRIKTSCH, M. I. Zur Zermahlungstheorie. *Zement*, vol. 18, 1929, pp. 194-198, 230-233. Discusses conditions that affect crushing efficiency. Gives tensile strength of some minerals. Discusses particle-size determination by sieves and by settling rate, effect of irregular particles on surface measurements, energy necessary for grinding a unit weight of solid to a unit weight of gas which might be assumed as the grinding energy theoretically required, effect of particle size on chemical action and on fusion with the relation of surface thereto, Martin's dissolution method, and Koehler's method of ThO adsorption for surface measurements. As the theoretical efficiency of grinding is so low it becomes advisable to look for other methods of grinding; electric fields of high frequency and waves of high frequency are mentioned as possible means.

98. GAUDIN, A. M., GROSS, JOHN, AND ZIMMERLEY, S. R. The So-Called Kick Law Applied to Fine Grinding. *Min. and Met.*, vol. 10, 1929, pp. 447-448.
Shows theoretically that in grinding fine sizes the efficiency, according to the Kick law, increases enormously when compared to surface energy. Efficiency passes 100 percent at about 0.1 micron size and at 0.001 micron size reaches several thousand percent.
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Discusses advantage of closed circuits and particular advantage of bowl classifiers in removing a finished product after primary mills and return of heavy minerals for additional grinding. Flow sheets of copper plants show changes and results due to these changes.
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Advances new theory of ball action. Maximum grinding is at 65 percent of critical and greatest efficiency at 50 percent of critical speed. Gives efficiency of mills of various diameters.
101. DEAN, R. S., AND RYJORD, J. E. Alloys for Cable Sheathing. *Metals and Alloys*, vol. 1, 1930, pp. 410-414.
States that on lead-calcium alloys load varies inversely as log of time of rupture.
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Determines heat of wetting of energy of immersion. Describes calorimeter used and method of operation.
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106. ROLLER, PAUL S. The Bulking Properties of Microscopic Particles. *Ind. and Eng. Chem.*, vol. 22, 1930, p. 1206.
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109. GROSS, JOHN, AND ZIMMERLEY, S. R. A Device for Determining Work Input to a Laboratory Ball Mill. *Rept. of Investigations 3056*, Bureau of Mines, 1931, 3 pp.
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Describes various methods and air analyzer, with results obtained. Smallest separation is 0 to 5 microns. Method is checked by microscope. Gives frequency distribution of particle sizes.
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Describes method approved by Am. Stand. Assoc., July 6, 1932.
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