CP-2541
G

Contract No. W-7401-eng-37

Section P-7

* * *

ON THE STATISTICS OF ROD WARPING

A. V. Martin and G. Young

December 28, 1944

* * *

ABSTRACT

The probability of exceeding assigned values of displacements or forces in simple rod models assembled from slugs picked at random is discussed, and some estimate for the statistical gain resulting from the use of shorter slugs is obtained. The probability that a rod with self-warping will touch the top of the tube appears great enough to justify a recommendation of the use of top ribs.

Report received: January 5, 1945
Issued: JAN 18 1945

3168-01

RESTRICTED

G-C

1/23/45

ERRATUM FOR CP-2541

On page 19 of this report the calculations given correspond to symmetric arches, instead of unsymmetric arches as stated. To obtain the values for the unsymmetric case, which corresponds more closely to the practical situation, make the following changes:

line 3, change 1.17 to 1.6 and 12×10^{-2} to 5×10^{-2}.

line 7, change 1/8 to 1/20.

line 10, change 12×10^{-2} to 5×10^{-2} and 28×10^{-2} to 2×10^{-1}.

line 12, change 1.31 to 1.8 and 1/10 to 1/30.

The rest of the text is unaffected.

8168-02

ON THE STATISTICS OF ROD WARPING

A. V. Martin and G. Young

Mechanics of a Simple Arch

Consider a horizontal rod of length 2ℓ, simply supported at the ends, and subject to a downward force w per unit length. As in

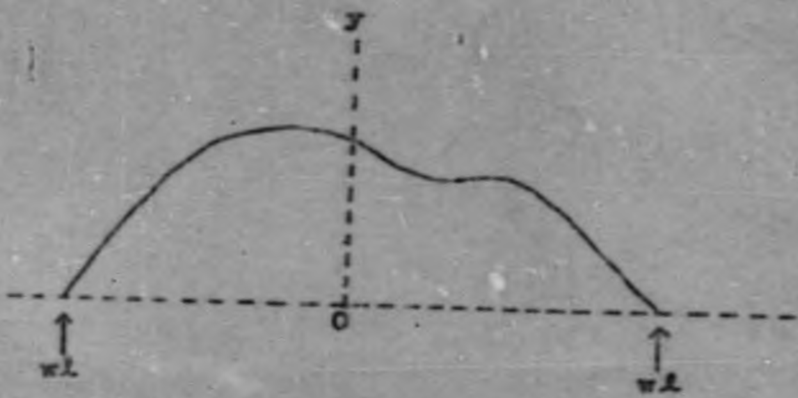

Fig. 1

CP-2274 we imagine the rod to have an intrinsic (i.e., in the absence of applied forces) downward curvature C(x). Then the equation governing the shape of the rod as a simple beam is

$$y''(x) = -C(x) + \frac{w}{2B}(\ell^2 - x^2) \quad , \qquad (1)$$

where B is the flexural rigidity of the rod. The general solution is

$$y(x) = a + bx - \int_{-\ell}^{x} (x - t)\, C(t)\, dt + \frac{wx^2}{24B}(6\ell^2 - x^2) \quad , \qquad (2)$$

where a and b are integration constants. Imposing the end conditions

8168-03

$$y(-\ell) = y(\ell) = 0 \qquad (3)$$

determines a and b and leads to the result

$$J(x) = y(x) + \varphi(x) = \frac{1}{2}\left(1 - \frac{x}{\ell}\right)\int_{-\ell}^{x} (\ell + t)\, C(t)\, dt + \frac{1}{2}\left(1 + \frac{x}{\ell}\right)\int_{x}^{\ell} (\ell - t)\, C(t)\, dt \qquad (4)$$

where

$$\varphi(x) = \frac{w}{24\,B}\,(5\ell^4 - 6\ell^2x^2 + x^4) \quad . \qquad (5)$$

Thus the final displacement of each point of the rod is expressed in terms of the intrinsic curvature at every point of the rod.

If the system is symmetrical about the midpoint, we obtain, upon setting $C(x) = C(-x)$,

$$J(x) = (\ell - x)\int_{0}^{x} C(t)\, dt + \int_{x}^{\ell} (\ell - t)\, C(t)\, dt \quad . \qquad (6)$$

Rod With Constant Intrinsic Curvature

For later reference we consider some features of the case in which the rod has a constant initial curvature downward, i.e., $C(t)$ equal to a positive constant k. From (1) it is seen that the final arch is everywhere concave downward, as in Fig. 2, provided that $w\ell^2 < 2Bk$. As

ℓ increases past this value the arch becomes concave upward in the center, as shown in Fig. 3 (cf. Fig. 5 through Fig. 8 of CP-2264).

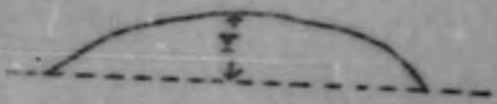

Fig. 2

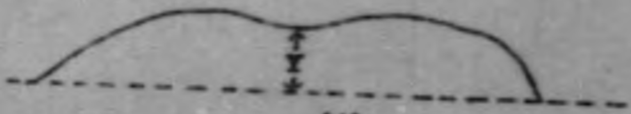

Fig. 3

Upon evaluating (6) with $C(t) = k$ we find for the central displacement $Y = y(0)$

$$96\,Y = \ell^2\left(48\,k - \frac{20\,w\ell^2}{B}\right) , \tag{7}$$

from which it follows that $24\,BY - w\ell^4$ has the same sign as $2Bk - w\ell^2$. Thus we have Fig. 2 if $24\,BY > w\ell^4$, and Fig. 3 if $w\ell^4 > 24\,BY$; a statement in which the value k of the constant curvature does not appear.

Upon differentiating (7) with respect to ℓ it is found that the maximum value of Y occurs in Fig. 2 and is given by

$$\ell^2 = \frac{6\,Bk}{5\,w}$$

$$Y_{max} = \frac{3\,Bk^2}{10\,w} \tag{8}$$

[redacted]

In fact (cf. CP-2274) this is the maximum possible displacement for any point of the rod, central or not, whatever be the length of the rod. From (8) it follows that at this maximum point

$$24\,EY = 5\,w\ell^4 \,, \qquad (9)$$

which is again a statement in which k does not appear.

Statistical Distribution of the Value of an Integral

With a view to application to the continuous jacket or long cartridge assemblies, we now consider that the rod is comprised of a number of slugs of length Δ picked at random, and thus that in the integrals of (4) and (6), $C(t)$ has within each interval of length Δ a constant value drawn at random from a statistical population. If $N = \frac{\ell}{\Delta}$ is not too small, an integral

$$I = \int_0^{\ell} F(t)\, C(t)\, dt \qquad (10)$$

may be approximated by the sum

$$I \simeq \sum_{i=1}^{N} F(t_i)\, C(t_i)\,\Delta \,, \qquad (11)$$

and thus its value appears as a linear combination of the random quantities $C_i = C(t_i)$. If the C_i are normally distributed about zero with a standard deviation σ, then (11) will be normally distributed about zero with a standard deviation σ_I, given by

$$\sigma_I^2 = \sigma^2 \sum_{i=1}^{N} F^2(t_i)\,\Delta^2 \simeq \sigma^2 \Delta \int_0^{\ell} F^2(t)\, dt \,. \qquad (12)$$

[redacted]

8168-06

With a normal population having zero mean and standard deviation σ, the probability of drawing a value greater than $f\sigma$ is given by

$$H(f) = \frac{1}{\sqrt{2\pi}} \int_f^\infty e^{-\frac{t^2}{2}}\, dt \;, \qquad (13)$$

which is listed in tables. For f large we may use

$$H(f) = \frac{1}{\sqrt{2\pi}} \frac{e^{-\frac{f^2}{2}}}{f} \left(1 - \frac{1}{f^2} + \ldots.\right) \;. \qquad (14)$$

For ease of analysis, it will be assumed throughout that the statistical distributions are normal.

<u>Statistical Distribution of Heights of Points on a Simple Arch</u>

Applying the considerations of (10) through (12) to the result given in (4) shows that the quantity $J(x)$ is normally distributed about zero with standard deviation σ_1 as expressed by

$$\frac{\sigma_1^2}{\Delta\sigma^2} = \frac{1}{4}\left(1 - \frac{x}{l}\right)^2 \int_{-l}^{x} (l + t)^2\, dt + \frac{1}{4}\left(1 + \frac{x}{l}\right)^2 \int_{x}^{l} (l - t)^2\, dt = \frac{1}{6l}(l + x)^2 (l - x)^2. \qquad (15)$$

To get a value $y(x) > Y$ we must draw a value of $J(x) > Y + \beta(x)$, and the probability of doing this is $H(f)$ where

$$f(x) = \frac{Y + \phi(x)}{\sigma_1(x)} = \sqrt{\frac{6\ell}{\Delta\sigma^2}} \left[\frac{Y}{\ell^2 - x^2} + \frac{w}{24B}(5\ell^2 - x^2) \right] . \quad (16)$$

Upon studying this as a function of x it is found that we have Fig. 4 if $24BY > w\ell^4$, and Fig. 5 if $24BY < w\ell^4$.

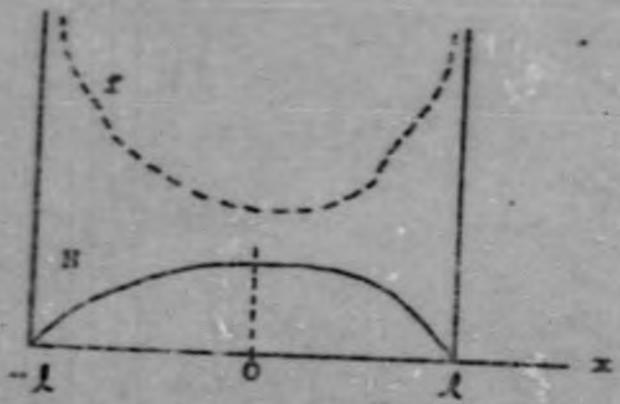

Fig. 4

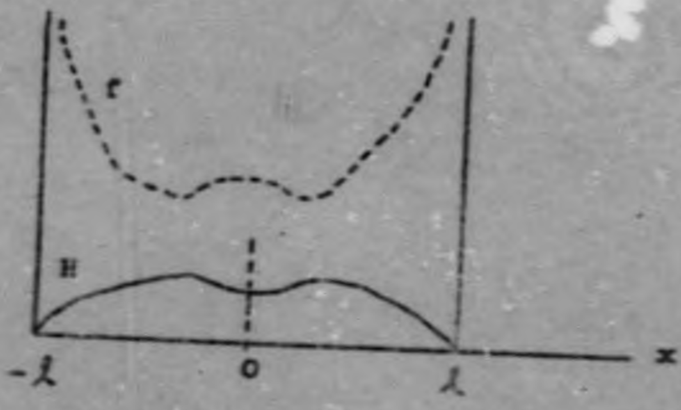

Fig. 5

8168-08

The agreement with Fig. 2 and Fig. 3 and the discussion following Eq. (7) is clear. Thus there are some resemblances between a rod with the same initial curvature at every point and a rod with the same probability of curvature at every point.

At the center of the rod we have

$$f(0) = \sqrt{\frac{6}{\Delta\sigma^2}}\left(Y\ell^{-3/2} + \frac{5w\ell^{5/2}}{24B}\right) , \qquad (17)$$

and upon adjusting ℓ to minimize f, and so maximize H, we obtain

$$\ell^4 = \frac{72BY}{25w}$$

$$f_{min} = \sqrt{\frac{6}{\Delta\sigma^2}}\,\frac{1.075\,w^{3/8}\,Y^{5/8}}{B^{3/8}} . \qquad (18)$$

This occurs in Fig. 4, and can be shown to be the smallest value of f for any point, central or not, and for any length of rod. Upon comparing with (9) it is seen that the value of ℓ^4 given by the first equation of (18) is 3/5 of that given by (9).

For the symmetrical case, we find that, if we let σ_2 denote the standard deviation of the quantity $J(x)$ as given by (6), then we have

$$\frac{\sigma_2^{\,2}}{\Delta\sigma^2} = (\ell - x)^2 \int_0^x dt + \int_x^\ell (\ell - t)^2\, dt = \frac{(2x + \ell)(\ell - x)^2}{3} . \qquad (19)$$

8168-09

We note that

$$\sigma_1^2(0) = 2\sigma_2^2(0) \, , \qquad (20)$$

and hence the assumption of symmetry multiplies $f(0)$ by $\frac{1}{\sqrt{2}}$. This relation (20) holds also in more general cases to be considered later.

<u>Arch Subject to a Central Restraining Force</u>

If we assume the existence of a point force $2F$ (which may be either upward or downward) applied to the center of the rod as shown in

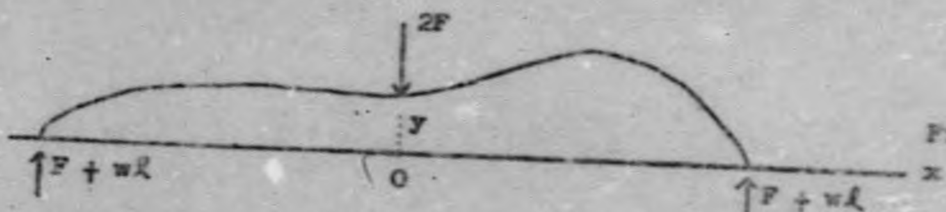

Fig. 6

Fig. 6, then the governing equation (1) is changed simply by the addition of a term $\frac{F}{B}(\ell - |x|)$, giving

$$y''(x) = -C(x) + \frac{w}{2B}(\ell^2 - x^2) + \frac{F}{B}(\ell - |x|) \, . \qquad (22)$$

Clearly then, the solution of (22) subject to (3) differs from that of (1) subject to (3) simply by the addition of the solution of the equation $y''(x) = \frac{F}{B}(\ell - |x|)$ subject to (3). This leaves unchanged the integral expression $J(x)$ given by (4), but replaces (5) by

$$\phi(x) = \frac{w}{24B}(5\ell^4 - 6\ell^2x^2 + x^4) + \frac{F}{6B}(2\ell^3 - 3\ell x^2 + |x^3|) \, . \qquad (23)$$

If we limit our attention to the point $x = 0$, and denote $y(0)$ by Y, we have

$$Y + \frac{5w\ell^4}{24B} + \frac{F\ell^3}{3B} = \frac{1}{2}\int_{-\ell}^{0} (\ell + t)\, C(t)dt + \frac{1}{2}\int_{0}^{\ell} (\ell - t)\, C(t)dt = J(0). \quad (24)$$

Any two of the three quantities Y, $\frac{w}{B}$, and $\frac{F}{B}$ may be assigned fixed values and the statistical distribution of the third studied in terms of those values and the parameter ℓ. We choose to regard Y, w, and B as fixed and study the distribution of F. Proceeding as in (15) through (20) we see that (15), (19) and (20) obviously continue to hold, since $J(x)$ has not been changed. It follows that the assumption of symmetry again multiplies f, as given below in (25) for the non-symmetric case, by $\frac{1}{\sqrt{2}}$. To get a value of $F > F_o$ we must draw a value of $J(0) > Y + \frac{5w\ell^4}{24B} + \frac{F_o\ell^3}{3B}$, and the probability of doing this is $H(f)$, where

$$f = \frac{Y + \frac{5w\ell^4}{24B} + \frac{F_o\ell^3}{3B}}{\sigma_1(0)} = \sqrt{\frac{6}{\Delta\sigma^2}}\left(Y\ell^{-\frac{3}{2}} + \frac{5w\ell^{5/2}}{24B} + \frac{F_o\ell^{3/2}}{3B}\right), \quad (25)$$

which reduces to (17) in case $F_o = 0$.

If we seek to minimize f by adjusting ℓ, which is the only parameter in (25) which is not yet fixed, we find the condition

$$25w\ell^4 - 24F_o\ell^3 = 72BY, \quad (26)$$

which is easily solved when numerical values have been assigned. In case w = 0, this gives

$$f_{min} = 2\sqrt{\frac{2F_0Y}{\Delta\sigma^2B}} \quad . \tag{27}$$

Arch with Clamped Ends

In order to analyze the behavior of a non-symmetric rod whose ends are clamped, it is necessary to assume unequal forces and moments at the two ends and to assign different values to the integration constants for positive and negative ranges of x. We shall not carry through this somewhat longer analysis, but state without proof that here again the quantity f(0) in the non-symmetric case is equal to $\sqrt{2}$ times that quantity for the symmetric case, which we now proceed to find.

We assume then that we have a symmetric rod subject to gravity and a central force 2F as in Fig. 6, but with each of its ends clamped parallel to the x axis by a moment M. This rod has been analyzed in CP-2274, equations (1) through (4), and Fig. 4. From the first of quations (4) of that report we have

$$Y + \frac{w\ell^4}{24B} + \frac{F\ell^3}{12B} = \int_0^\ell \left(\frac{\ell}{2} - t\right) C(t)\,dt = J(0). \tag{28}$$

Hence for the standard deviation $\sigma_2(0)$ of the quantity J(0) we find

$$\frac{\sigma_2^{\,2}(0)}{\Delta\sigma^2} = \int_0^\ell \left(\frac{\ell}{2} - t\right)^2 dt = \frac{\ell^3}{12} \quad . \tag{29}$$

8168-12

We have for this case, corresponding to (25),

$$f = \frac{Y + \frac{w\ell^4}{24B} + \frac{F\ell^3}{12B}}{\sigma_2^2(0)} = \sqrt{\frac{3}{\Delta\sigma^2}}\left(2Y\ell^{-3/2} + \frac{w\ell^{5/2}}{12B} + \frac{F_o\ell^{3/2}}{6B}\right) . \qquad (30)$$

Adjusting ℓ to minimize f leads to the condition

$$5w\ell^4 + 6F_o\ell^3 = 72BY , \qquad (31)$$

corresponding to (26). If we set $w = 0$ and drop the assumption of symmetry (i.e., multiply the right hand number of (30) by 2), we find

$$f_{min} = 2\sqrt{\frac{2F_oY}{\Delta\sigma^2B}} , \qquad (32)$$

which is identical with (27).

<u>Numerical</u>

Taking $B = 2 \times 10^6$ kg cm^2, $w = 12 \times 10^{-2}$ kg per cm, $Y = 10^{-1}$ cm = 40 mils, and $\Delta = 5$ cm gives with (18)

$$\ell = 47 \text{ cm}$$

$$f_{min} = \frac{5.5}{\sigma} 10^{-4} \qquad (33)$$

To get a value for σ we observe that σ^2 is the average of the square of the curvature of a large number of slugs; and that, for normal distributions, σ is $\sqrt{\frac{\pi}{2}} = \frac{5}{4}$ times the average absolute value of the curvature of a large number of slugs. By "curvature of a slug" in the above treatment is meant the component of curvature in the xy plane for

8168-13

which the analysis has been carried out. Actually, if a slug is warped in a plane with curvature C_o, the curvature of its projection on another plane is $C_o \cos\varphi$ where φ is the angle between the planes. Since the average value of $|\cos\varphi|$ is $\frac{2}{\pi}$ it is seen that for slugs randomly oriented with respect to the xy plane we shall have σ equal to $\sqrt{\frac{2}{\pi}} = .8$ times the average of the maximum curvatures C_o of a large number of slugs. If (cf. CP-2274) the average warp, in its own plane, of an 8" slug is 2.5 mils = 6.3×10^{-3} cm, we have $\bar{C}_o = 1.25 \times 10^{-4}$ and $\sigma = 10^{-4}$ per cm.

Another consideration which would arise in a more detailed study is the compounding of displacements in two perpendicular planes, as in the last section of CP-2274. This will not be gone into here, since the arches studied above are quite far from representing a model of a pile rod anyway.

Using $\sigma = 10^{-4}$ in (33) gives $f = 5.5$ and then from (34) we have an H of about 2×10^{-8}. Changing Δ to 20 cm increases H to 3×10^{-3}, which gives some idea of the statistical improvement resulting from the use of short slugs. This statistical improvement with short slugs is in addition to the improvement (CP-2143) which they may make in reducing the rigidity B of the rod. The quantity f varies with Δ in the same fashion in all the cases discussed above.

Taking in (27) the values $2F = 20$ kg, $Y = 10^{-1}$ cm, $\Delta = 20$ cm, $B = 10^7$ kg cm^2, and $\sigma = 10^{-4}$ per cm gives $f_{min} = 2$ and a probability H of 2.3×10^{-2}.

8168-14

[redacted]

RODS WITH SELF-WARPING

Distribution of Fourier Coefficient

A rod placed horizontally and simply supported at the ends will hang in some shape depending on gravity, the intrinsic curvature $C(x)$ of the rod, and the rigidity B of the rod. Let 2ℓ denote the length of the rod and w the force of gravity per unit length. Take the origin for x to be at the center of the rod. Then (see Eq. (37) of CP-2370) the actual curvature of the rod in the above situation is

$$C_1(x) = C(x) - \frac{w}{2B}(\ell^2 - x^2) . \qquad (34)$$

Let now self-warping, as measured by a certain constant K (discussed in CP-2065 and CP-2370), come into operation and increase until $K\ell$ approaches $\frac{\pi}{2}$. Then, as shown in CP-2370, the arch in the rod will bend further and go to $+\infty$ or $-\infty$ according as the coefficient of the term $\cos \frac{\pi x}{2\ell}$ in the Fourier expansion of $C_1(x)$ is positive or negative. In equation (39) of that report was calculated the smallest constant value for $C(x)$ which makes this coefficient positive, and so enables the rod to run away upward as $K\ell \rightarrow \frac{\pi}{2}$. In the present approach we wish to calculate the probability of this happening, supposing the curvatures of the separate slug lengths of the rod to have been drawn from a given statistical population as discussed above.

The coefficient in question will be positive if $\int_{-\ell}^{\ell} C_1(x) \cos \frac{\pi x}{2\ell}\, dx$ is positive, and with (34) this is found to be the case if

8168-15

$$J > \frac{16 w \ell^3}{\pi^3 B} = \frac{2w}{BK^3} \tag{35}$$

where

$$J = \int_{-\ell}^{\ell} C(x) \cos\frac{\pi x}{2\ell}\, dx \,. \tag{36}$$

For a symmetric system with $C(x) = C(-x)$ this reduces to

$$J = 2\int_{0}^{\ell} C(x) \cos \frac{\pi x}{2\ell}\, dx \,. \tag{37}$$

If the ends of the rod are clamped, we make use of the analysis of Fig. 2 and equations (9) through (13) of CP-2370. Since the origin was there taken at one end of the rod instead of at the center, we must translate the origin used in these equations to agree with that of $C_1(x)$ as given by (34), which latter expression we then substitute for $C(x)$ in the equations transformed by the translation of origin. When this is done, we find that the condition that the rod bend to $+\infty$ as $K\ell \longrightarrow \pi$ is that

$$\int_{-\ell}^{\ell} C_1(t) \cos \frac{\pi t}{\ell}\, dt > 0.$$

Replacing $C_1(t)$ in this integral by its value as given by (34) leads to the condition

$$J > \frac{2\ell^3 w}{\pi^2 B} = \frac{2\pi w}{BK^3}\,, \tag{38}$$

where

$$J = \int_{-\ell}^{\ell} C(t) \cos \frac{\pi t}{\ell}\, dt. \tag{39}$$

If the rod is symmetric so that $C(x) = C(-x)$, then (39) becomes

$$J = 2\int_0^{\ell} C(t) \cos \frac{\pi t}{\ell}\, dt \ . \qquad (40)$$

Let us apply the methods of equations (10) through (12) to the inequalities (35) and (38), and let us denote by $H(f)$ the probability that the rod will go to $+\infty$ as $K\ell$ approaches the critical value. Then we find for the case of free ends that

$$f = \frac{16w\ell^{5/2}}{\pi^3 B\sqrt{\Delta\sigma^2}} = \frac{4}{\sqrt{2\pi}}\,\frac{w}{BK^{5/2}\sqrt{\Delta\sigma^2}} \ , \qquad (41)$$

and for clamped ends,

$$f = \frac{2w\ell^{5/2}}{\pi^2 B\sqrt{\Delta\sigma^2}} = 2\sqrt{\pi}\,\frac{w}{BK^{5/2}\sqrt{\Delta\sigma^2}} \ . \qquad (42)$$

Moreover, comparison of (36) with (37) and of (39) with (40) shows (in view of (12)), that in each case the assumption of symmetry multiplies the standard deviation of the integral by $\sqrt{2}$ and hence f by $\frac{1}{\sqrt{2}}$.

More General Treatment

The above discussion concerns only certain critical values of $K\ell$. We now remove this restriction, and also include a restraining force $2F$ operating at the center of the rod.

We first consider the symmetric case, and observe that if we assume a downward force $2F$ applied at the center of the rod in Fig. 7

of CP-2370, and an upward force F at each end, then the first of equations (40) of that report becomes

$$y''(x) + K^2 y(x) = -C(x) + \frac{w}{2B}(\ell^2 - x^2) + \frac{F}{B}(\ell - |x|) + \frac{M}{B} \qquad (40')$$

The addition of this term $\frac{F}{B}(\ell - |x|)$ introduces corresponding changes in equations (41) and (45) of CP-2370 which now become

$$Y = \frac{\int_0^\ell \sin K(\ell - t)\, C(t)dt}{K \cos K\ell} - \frac{w}{BK^4}\left(\frac{1}{\cos K\ell} - 1 - \frac{K^2\ell^2}{2}\right)$$

$$- \frac{F}{BK^3}(\tan K\ell - K\ell) - \frac{M}{BK^2}\left(\frac{1}{\cos K\ell} - 1\right), \qquad (41')$$

and

$$Y = \frac{1}{K \cos \frac{K\ell}{2}} \int_0^\ell \sin K\left(\frac{\ell}{2} - t\right) C(t)dt - \frac{2F + w\ell}{BK^3}\left(\frac{1}{\sin K\ell} - \cot K\ell - \frac{K\ell}{2}\right), \qquad (45')$$

which last equation results from the assumption of clamped ends.

Now if we assume that the ends are free to turn, then we have equation (41') with $M = 0$, from which we obtain

$$\frac{F(\sin K\ell - K\ell \cos K\ell)}{BK^2} + YK \cos K\ell + \frac{w}{BK^3}\left(1 - \cos K\ell - \frac{K^2\ell^2}{2}\cos K\ell\right) =$$

$$\int_0^\ell \sin K(\ell - t)\, C(t)dt\,. \qquad (43)$$

If we assume that the ends are clamped, we find from (45')

[redacted]

$$J \equiv KY \sin K\ell + \frac{F + \frac{w\ell}{2}}{BK^2}(2 - 2\cos K\ell - K\ell \sin K\ell) = \int_0^\ell \left[\cos Kt - \cos K(\ell - t)\right] C(t)dt. \tag{44}$$

In each of the last two relations we regard B, Y, F and w as fixed and let H(f) denote the probability that the integral in that relation will exceed the value of the resulting expression in B, Y, F and w. Then for free ends we find from (43) that

$$f = \frac{2\left[F(\sin K\ell - K\ell\cos K\ell) + \frac{w}{K}\left(1 - \cos K\ell - \frac{K^2\ell^2}{2}\cos K\ell\right) + BYK^3\cos K\ell\right]}{\sqrt{\Delta\sigma^2}\; K^{3/2}\, B\sqrt{2K\ell - \sin 2K\ell}}, \tag{45}$$

and for clamped ends we have from (44)

$$f = \frac{\left(F + \frac{w\ell}{2}\right)(2 - 2\cos K\ell - K\ell\sin K\ell) + BYK^3\sin K\ell}{\sqrt{\Delta\sigma^2}\; K^{3/2}\, B\sqrt{1 - \cos K\ell}\,\sqrt{K\ell - \sin K\ell}}. \tag{46}$$

We note that if we set $F = 0$ and replace $K\ell$ by $\frac{\pi}{2}$ in (45) and by π in (46), then we obtain (41) and (42) respectively, just as we should.

Calculations similar to those given above, but somewhat more lengthy, show that if we drop the assumption that the rod is symmetric then the expressions on the right hand sides of (45) and (46) become multiplied by $\sqrt{2}$, but are otherwise unchanged.

SECRET

8168-19

Numerical

We take $B = 4 \times 10^6$ kg cm^2, $\Delta = 5$ cm, $\sigma = 10^{-4}$ per cm, $w = 12 \times 10^{-2}$ kg per cm, and K = 2.[illegible] $\times 10^{-2}$ per cm. Then (41) gives $f = 1.17$ which corresponds to $H = 12 \times 10^{-2}$. This is the probability that a length $2\ell = 110$ cm of rod, if cut out and simply supported at its ends in the cooling stream at the center of the pile, would warp upward to touch the tube*. The pile contains about 10^4 times this length of rod, and thus if each such length near the center has a probability 1/8 of warping to reach the tube wall it would be practically certain that some of the lengths would do so. Increasing the slug length Δ to 20 cm changes the probability H from 12×10^{-2} to 28×10^{-2}.

If the rod length is 220 cm and the ends are clamped, we find from (42), for $\Delta = 20$ cm, that $f = 1.31$ and thus H = about 1/10.

While the rods analyzed represent at best a pretty crude model of the rods in the pile, the probabilities obtained would tend to discourage operation without top ribs. It is seen in (41) and (42) that the results are quite sensitive to the value of K.

*Actually, with the value used for w, the probability computed is that for the rod to move upward to reach the wall in a plane 45° from the horizontal (cf. last section of CP-2274). The probability of it touching the wall anywhere in the top quadrant is somewhat greater, but the more detailed calculation for this has not been made.