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WARPING INSTABILITY IN LONG RODS

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Abstract

If a pile rod gets bowed within its cooling tube it becomes warmer on the side which approaches the tube wall, and thermal expansion tends to warp it in the same direction as the original displacement. This was discussed roughly in N-601, and it was there concluded that the mechanism was not important for an isolated short slug. In connection with current development of continuous jacket (cartridge) assemblies, it seems desirable to look at this question again. In this case it is possible that the effect may be of some concern.

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514-1

WARPING INSTABILITY IN LONG RODS

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Variable Transfer Coefficient

One source of heating on the side where the rod approaches the outer wall is the decrease of the transfer coefficient to the water on that side. The rough considerations of N-601 suggested that this coefficient h might be expected to vary somewhat as the cube root of the thickness of the water layer. Kratz and Schlagel have obtained in one instance an experimental indication that the variation is more nearly as the first power of the water thickness. This latter formulation may be approximately expressed as

$$h = h_o \left(1 - \frac{y}{t} \cos \theta\right) , \qquad (1)$$

where h_o is the normal value of h when the rod is centered, y is the displacement of the rod from its central position, t is the normal thickness of the water layer, and the angle θ is measured around the perimeter of the rod from the direction of its displacement. Thus θ = 0 is the place where the water layer is thinnest.

Small temperature variations at the rod surface do not greatly affect the flow of heat G from unit area of the rod. Thus for small displacements it will be correct, on the assumption (1), to write the film drop $T = \frac{G}{h}$ as

514 - 2

$$T = \frac{G_o}{h} = \frac{G_o}{h_o}\left(1 + \frac{y}{t}\cos\theta\right) = T_o\left(1 + \frac{y}{t}\cos\theta\right), \quad (2)$$

where G_o and T_o are the normal values when the rod is centered.

If then we suppose the bulk water temperature to be constant around the rod, and if the slug-to-jacket contact is also constant around the rod, the temperature field in the rod will have a constant gradient $g = \frac{T_o y}{Rt}$, where R is the radius of the rod. This gives rise (CP-1464; CP-1698) to a bending moment of magnitude αBg, α being the linear coefficient of expansion of the rod and B = EI being its flexural rigidity. The usual beam equation is By" = bending moment, and upon choosing signs properly we obtain for a weightless rod

$$y'' + K^2 y = A \qquad K^2 = \frac{\alpha T_o}{Rt}, \quad (3)$$

where A depends upon how the ends of the rod (where y = 0) are secured and upon the magnitude of the rod displacement.

Suppose the power production along the rod to be uniform, so that T_o and K are constant, and that the ends of the rod are free to turn, so that A = 0. Then (3) has a simple solution in the form of cos Kx which vanishes at the ends of the rod if the rod half-length L satisfies $KL = \frac{\pi}{2}$. This corresponds to equation (7) of N-601. Using the same values as in that memorandum, namely $T_o = 20^o$, $\alpha = 1.5 \times 10^{-5}$, t = .22 cm, and R = 1.75 cm, gives L = 56 cm instead of the value 85 cm obtained there.

If the ends of the rod are clamped to make y' vanish there in addition to y, the solution becomes $y = \frac{A}{K^2}(1 + \cos Kx)$ with $KL = \pi$ and with A an arbitrary constant. Thus clamping the ends of the rod doubles its critical length.

The values of L obtained in this way are those at which the assembly becomes unstable; for smaller values of L a disturbance in the rod shape would be unable to maintain itself, and for larger values of L a disturbance of the right shape would grow larger.

The above discussion was limited, in (2), to very small displacements, and this led to a temperature gradient g given by $\frac{Rg}{T_o} = \frac{y}{t}$. For large displacements the temperature field in a bonded U slug has been studied by Murray under the assumptions of constant water bulk temperature and transfer coefficient as in (1). From his calculations Mr. Murray has kindly furnished us values for the effective warping gradient g, which varies as sketched in Fig. 1. Our calculations

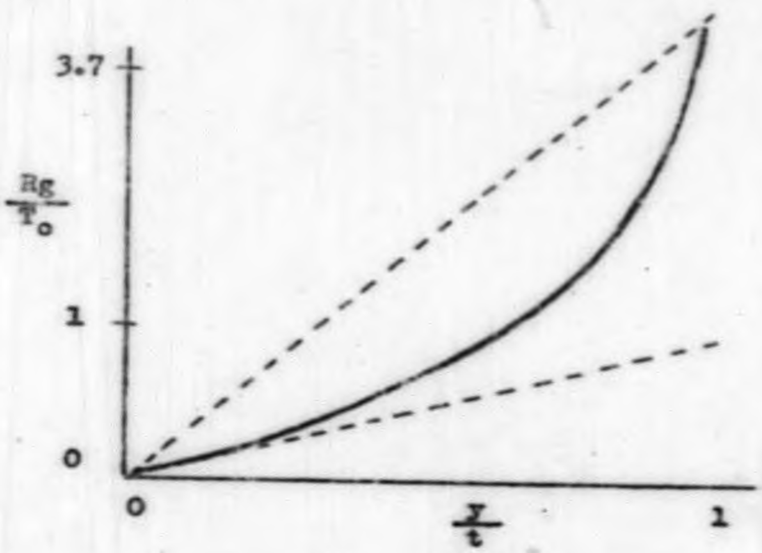

Fig. I

514 4

above were based on the lower dotted line which underestimates the warping moments set up. Use of the upper dotted line overestimates the moments, and has the effect of increasing our previous value of K^2 by a factor of 3.7. This in turn would divide the critical lengths by $\sqrt{3.7} = 1.9$.

Thus under small displacements the (weightless) rod will become unstable at the lengths first calculated. Under large displacements it becomes unstable at lengths which are somewhat shorter but greater than $\frac{1}{2}$ the first values. Murray's values for the bending moment as a function of y could be used to replace the second term in (3), and the rod length needed to maintain a given central displacement could then be found by numerical quadratures. This, however, has not been carried through. It is not known how good assumption (1) is, and it surely breaks down to considerable extent when boiling sets in.

Variable Bulk Temperature

Another source of heating on the side of the rod where the water is thin is the increase in water bulk temperature there because of poor mixing around the annulus (see eq. CEK-MUC-114). We give only a rough calculation on this point.

Let T_1 be the normal rise in bulk temperature of the water in moving unit distance along the centered rod. The usual hydraulic formulae indicate that under a given pressure drop the total rate of flow in a layer varies as the 5/3 power of the layer thickness. For small displacements the outward heat flow is practically uniform around the rod, as mentioned earlier, and thus the temperature rise per unit length is

$$\frac{T_1}{(1 - \frac{y}{t}\cos\theta)^{5/3}} = T_1(1 + \frac{5}{3}\frac{y}{t}\cos\theta) \ , \qquad (4)$$

which in form is analagous to (2).

Thus, assuming no mixing, we have after the water has travelled a distance 2L along a rod with constant displacement y, a gradient $g = \frac{2L\, T_1\, 5y}{R\, 3t}$; and, averaged over the length of the rod,

$$\bar{g} = \frac{5\, T_1\, L}{3 R t}\, \bar{y} \ . \qquad (5)$$

If we suppose, crudely, that this on-the-average relation holds between g and y at each point along the rod, then the result is to increase the value of K^2 in (3) to

$$K^2 = \frac{\alpha}{Rt}(T_0 + \frac{5}{3} T_1 L) \ . \qquad (6)$$

Considering that the water temperature rises about 70° along 700 cm of rod, and that its rate of rise at the center is $\frac{\pi}{2}$ times the average, gives $T_1 = \frac{\pi}{20}$ degrees per cm. Then the parenthesis in (6) becomes $20 + \frac{L}{3.8} = 20\,(1 + \frac{L}{76})$. Solving $L\sqrt{1 + \frac{L}{76}} = 56$ gives L = 45 cm, and $L\sqrt{1 + \frac{L}{76}} = 112$ gives L = 79 cm. These are the values to which the former critical lengths for small displacements are reduced when variation in bulk temperature around the annulus is taken into account in the present rough fashion.

The above considerations for weightless rods apply in the case of vertical rods, as contemplated in some of the P-9 designs, and to horizontal warping of sections of W rods which may have been lifted by vertical warping to clear the ribs below. They apply best to solid rods, and less well to separate slugs in a continuous jacket where B varies along the rod (cf. CP-1940).

Warping of Weighted Rod

Let the rod have weight w per unit length, let it be of length 2L between points of support, and measure x from the center. Then (3) becomes

$$y'' + K^2 y = a(L^2 - x^2) + A \qquad a = \frac{w}{2B}, \qquad (7)$$

with the general solution

$$y = \frac{A}{K^2} + \frac{2a}{K^4} + \frac{a}{K^2}(L^2 - x^2) + F \sin Kx + G \cos Kx \qquad (8)$$

where A, F, G are arbitrary constants. Determining these to make $y'(0) = y'(L) = y(L) = 0$ gives

$$y = \frac{a}{K^2}(L^2 - x^2) + \frac{2a}{K^4}\frac{KL}{\sin KL}(\cos KL - \cos Kx)$$

$$y'' = -\frac{2a}{K^2}\left(1 - \frac{KL}{\sin KL}\cos Kx\right)$$

$$y(0) = \frac{2a}{K^4}\left(\frac{KL}{\tan KL} + \frac{K^2L^2}{2} - \frac{KL}{\sin KL}\right) \qquad (9)$$

$$y''(L) = A = \frac{2a}{K^2}\left(\frac{KL}{\tan KL} - 1\right) .$$

For $KL < \pi$ the quantity $y(0)$ is negative and hence a physically meaningful solution does not exist, this corresponding to the result obtained above for a weightless rod clamped at the ends. For $\pi < KL \leq 4.49$ (which is a root of $x = \tan x$) there exists a solution which is everywhere non-negative, and thus physically meaningful. For $KL > 4.49$ the solution again becomes negative at some points. Upon differentiating the parenthesis on the right side of the expression for $y(0)$ it is found to have a positive minimum for $KL = \tan KL = 4.49$, and this minimum value gives

$$y(0) = 33.4\,\frac{a}{K^4} \qquad (10)$$

This value of KL also makes $A = 0$, so that there is no bending moment transmitted across the ends to the portions of the rod adjoining the warped arch.

If we set $A = 0$ and remove the requirement that $y'(L) = 0$, which amounts to cutting the rod at its ends so these are free to turn, we get instead of (9)

$$y = \frac{a}{K^2}(L^2 - x^2) + \frac{2a}{K^4}\left(1 - \frac{\cos Kx}{\cos KL}\right)$$

$$y'' = \frac{2a}{K^2}\left(\frac{\cos Kx}{\cos KL} - 1\right) \qquad (11)$$

$$y(0) = \frac{2a}{K^4}\left(1 + \frac{K^2L^2}{2} - \frac{1}{\cos KL}\right) .$$

It is now found that y(0) is negative for $KL < \frac{\pi}{2}$, in agreement with the previous result for an unclamped weightless rod; for $\frac{\pi}{2} < KL \leq \pi$ there is a solution which is everywhere non-negative; and on this range the parenthesis in y(0) has its minimum value for $KL = \frac{\tan KL}{\cos KL} = 2.21$, this minimum giving

$$y(0) = 10.2 \frac{a}{K^4} . \qquad (12)$$

With $B = 6 \times 10^6$ kg cm^2, w = .18 kg per cm, we find $a = 1.5 \times 10^{-8}$ cm^{-3}. For the lower dotted line in Fig. 1 we have $K = 2.8 \times 10^{-2}$ cm^{-1}. With these values (10) gives y(0) = .82 cm and (12) gives y(0) = .25 cm.

Such y(0) values represent the displacements at which the rod becomes unstable; smaller displacements would not be maintained. The above values are greater than the thickness of the W annulus, so that such forces as used in the computation would not be able to maintain an upward warp against gravity. However, we have here neglected the warping due to variable bulk temperature, and have used the lower dotted line of Fig. 1 which underestimates the warping forces produced by large displacements. If we used instead the upper dotted line, the above $y(0)$ values would be divided by $3.7^2 = 13.7$ and would thus become considerably less than the thickness of the annulus, and so these larger forces are quite capable of holding the rod up against the top of the tube. Thus it is possible that this may happen in the pile.

END