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THE DIFFUSION LENGTH OF THERMAL NEUTRONS IN URANIUM

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May 27, 1944

Report Received: May 27, 1944 Figures Received: June 9, 1944
Issued: JUN 13 1944

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Abstract

Measurements made in a uranium cylinder result in a mean value of 1.55 cm. for the diffusion length L for distances of 1 to 4 cm. from the base of the cylinder. Calculations give a value which agrees with the experimental result and show further that L increases from 1.40 to 1.63 cm. as the neutrons diffuse a distance of 5 cm. into the uranium.

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THE DIFFUSION LENGTH OF THERMAL NEUTRONS IN URANIUM

The diffusion length of thermal neutrons in uranium has been determined from intensity measurements made in a cylindrical "sigma pile". The pile, 15.2 cm in diameter and 5.5 cm high, was composed of circular uranium disks of various thicknesses, covered on the sides and top by a layer of cadmium. A circular disk of paraffin, 1.5 cm thick, was placed beneath the metal pile, which itself was supported (in the original arrangement) about two feet above the upper face of the thermal column. The purpose of the paraffin layer was to secure a more nearly isotropic distribution of the thermal neutrons diffusing upward into the metal pile.

The diffusion equation for a cylinder, with a neutron source distributed in the plane of the base, has for a solution a sum of zero order Bessel harmonics, the relative amplitude of each depending on the source distribution. With increasing distance from the base, however, only the fundamental remains and its relaxation distance, b, is related directly to the diffusion length, L:

$$\frac{1}{L^2} = \frac{1}{b^2} - \frac{(\alpha)^2}{a^2} \qquad (1)$$

where $\alpha = 2.4048$ is the first root of the zero order Bessel function and a is the radius of the cylinder plus the augmentation distance of the neutron distribution.

Before b and a could be measured, it was necessary to determine if harmonics above the first were present in the metal pile. This was done by measuring the radial neutron density distribution in the $Z = 1$ plane (1 cm above the paraffin plate). Measurements were made by irradiating small equal indium foils placed between the uranium plates. Cd difference measurements showed an effect of less than 1 in 500 due to fast neutrons. The radial intensity function thus obtained was definitely flatter than $J_o(r)$, showing the presence of harmonics higher than the first at $Z = 1$.

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The set-up was then modified by placing a graphite cylinder, also of 15.2 cm diameter and 7.5 cm high, directly below the paraffin plate and extending the side cadmium cover down to cover this addition. Then in order to increase the neutron density the entire cylinder was placed directly on the graphite of the thermal column (this change caused no increase in the negligible neutron effect). All the following measurements were made with the modified arrangement of the metal pile.

Figure 1 shows the radial intensity distribution in the $Z = 1$ plane. The points are the experimental values and the solid curve is $J_o(2.4048\ r/a)$ with a taken as 8.66 cm, the point at which the experimental curve extrapolates to zero. The close fit of the points to J_o shows that $Z = 1$ the higher harmonics are now very small, and that L can be determined from the relaxation distance measured above $Z = 1$. (A traverse taken at $Z = 2.5$ also showed only the fundamental.) The experimentally determined value of a gives an augmentation distance of $8.66 - 7.62 = 1.04$ cm.

The intensity distribution along the Z axis was then measured, giving the points shown semi-logarithmically on Figure 2. Apparently the data fit a single exponential, shown by the straight line, of relaxation distance equal to 1.42 cm. An end correction was then applied to the points near the top of the pile. This was done by assuming the augmentation distance determined from the radial distribution, and the slope of the uncorrected vertical distribution, then calculating

$$I\ \text{corr} = \frac{I_{obs}}{1 - e^{-\frac{2(Z_o - Z)}{1.42}}}$$

where $Z_o = 5.5 + 1.04$ cm. The corrected experimental points are also shown in Figure 2. The correction is rather large at $Z = 5$ but is negligible below $Z = 4$. Because of some uncertainty in the correction it is difficult to attach significance to the curvature at the end of the line. It is still true that from $Z = 1$ to 4

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the experimental data show a single relaxation distance of 1.42 within experimental uncertainty. With $b = 1.42$ and $a = 8.66$, we then obtain

$$L = 1.55 \pm .01 \tag{2}$$

The limit of error given is larger than the statistical error (which is 1% for each point) and is estimated simply from the amount by which the points deviate from the straight line.

It would seem that as the neutrons diffuse through the metal L should increase, as selective absorption raises the average temperature of the neutrons. This temperature change would cause a continuous increase of b with Z which, however, was not observed. Some calculations were then carried out to determine to what extent L should change with Z. This was done by assuming a Maxwellian velocity distribution present at $Z = 0$ and dividing it into 30 velocity groups. The value of L for each of these groups was then determined from the relation

$$\frac{2N}{N-1}\,\frac{\lambda}{L} \doteq \mathrm{Log}\,\frac{1-\frac{\lambda}{L}}{1-\frac{\lambda}{L}} \tag{3}$$

where N is the ratio of total to absorption cross-section, and λ the mean free path for all collisions. The necessary cross-sections were obtained from the Columbia data, taking

$$\sigma_s + \sigma_a = \left(8.86 - \frac{15080}{V(\text{m per sec})}\right) \times 10^{-24}\,\text{cm}^2 \tag{4}$$

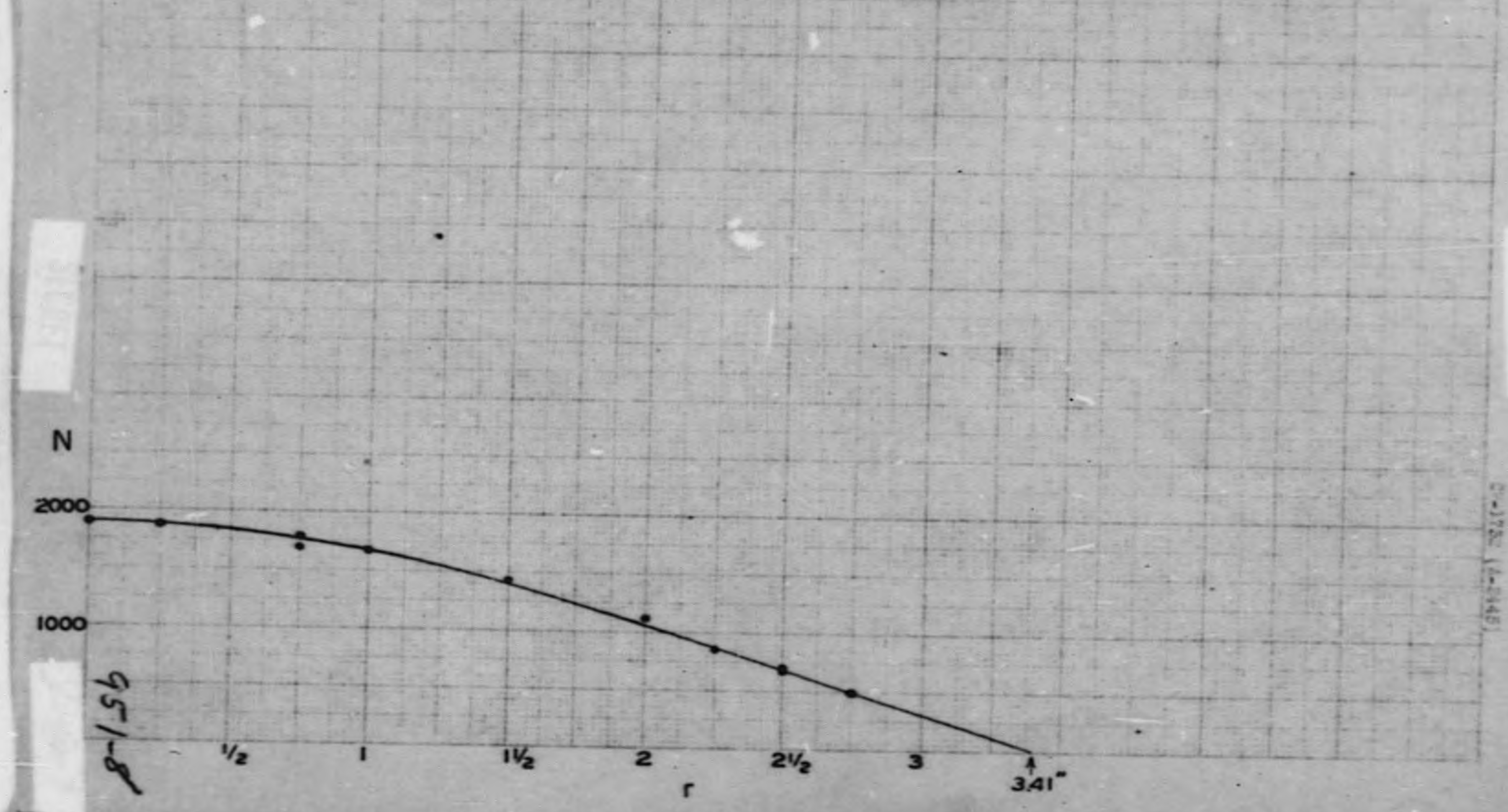
Figure 3 shows the number of neutrons present at each velocity and the corresponding value of L.

The b for each velocity group was calculated from (1), using the measured augmentation distance, then the intensity due to each velocity group as a function of Z. Adding the separate velocity group intensities gives the "theoretical" intensity curve shown in Figure 4 (intensity set to 1900 at $Z = 1$). The experimental and "corrected experimental" points are also shown on the same curve. It is seen that the calculated curve does show a slight curvature but of magnitude too small to be detected experimentally. The average relaxation distance of the calculated curve for various ranges of Z and the corresponding L's are as follow:

Z	b	L
0 - 1 cm.	1.308 cm.	1.404 cm.
1 - 2	1.373	1.485
2 - 3	1.420	1.546
3 - 4	1.458	1.594
4 - 5	1.489	1.635
1 - 4	1.416	1.541

It is seen that the calculated L for Z = 1 to 4 agrees almost exactly with the measured value. It was intended that the calculated curve would merely show how the measured L should change with distance, but the close agreement means that the actual values of the calculated L's above are probably nearly correct. We can then say that in going a distance equal to 3.38 relaxation distances in the uranium the diffusion length increases from 1.40 to 1.63 cm.

Fig. 1 Radial Intensity Distribution



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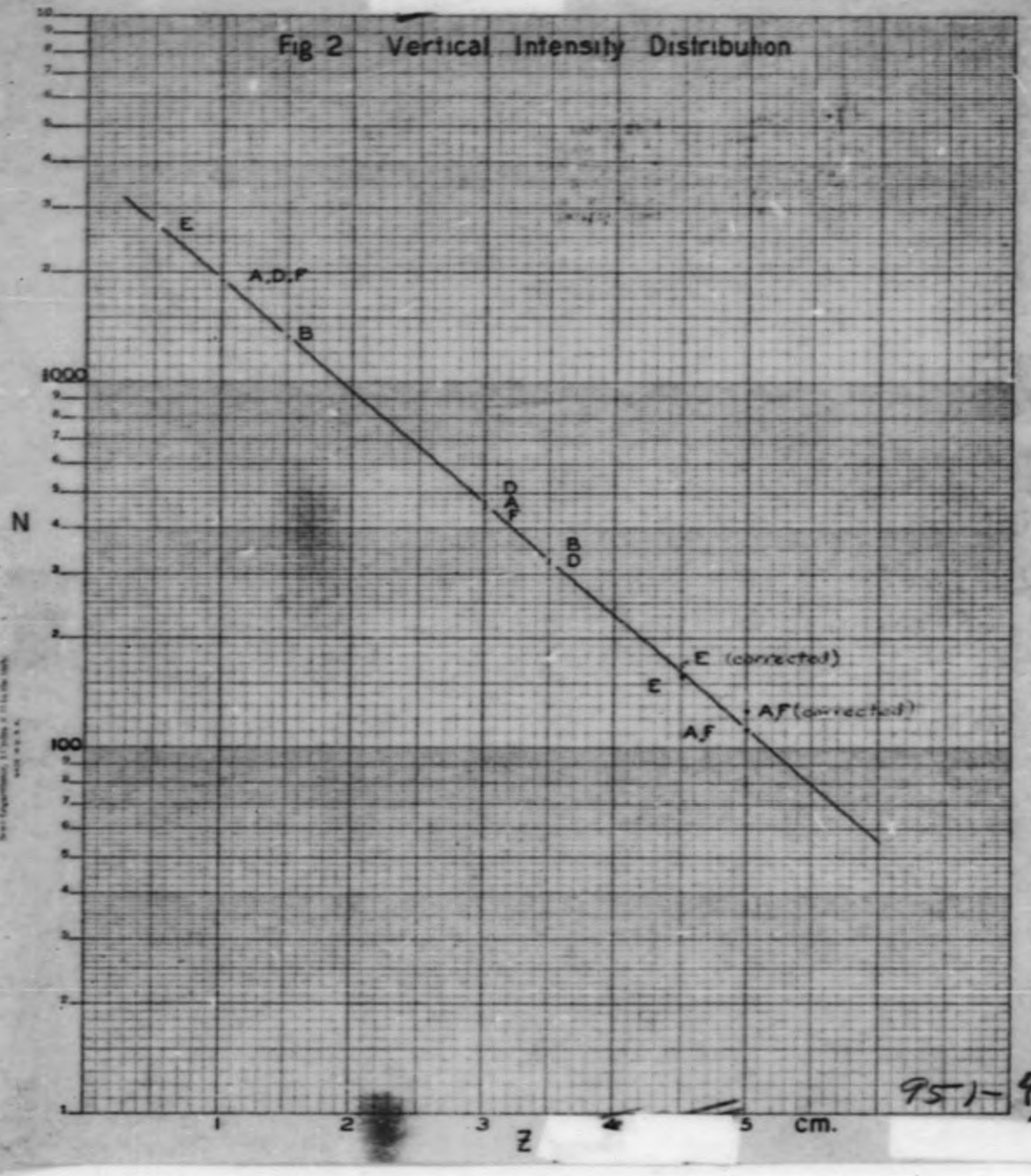

Fig 2 Vertical Intensity Distribution

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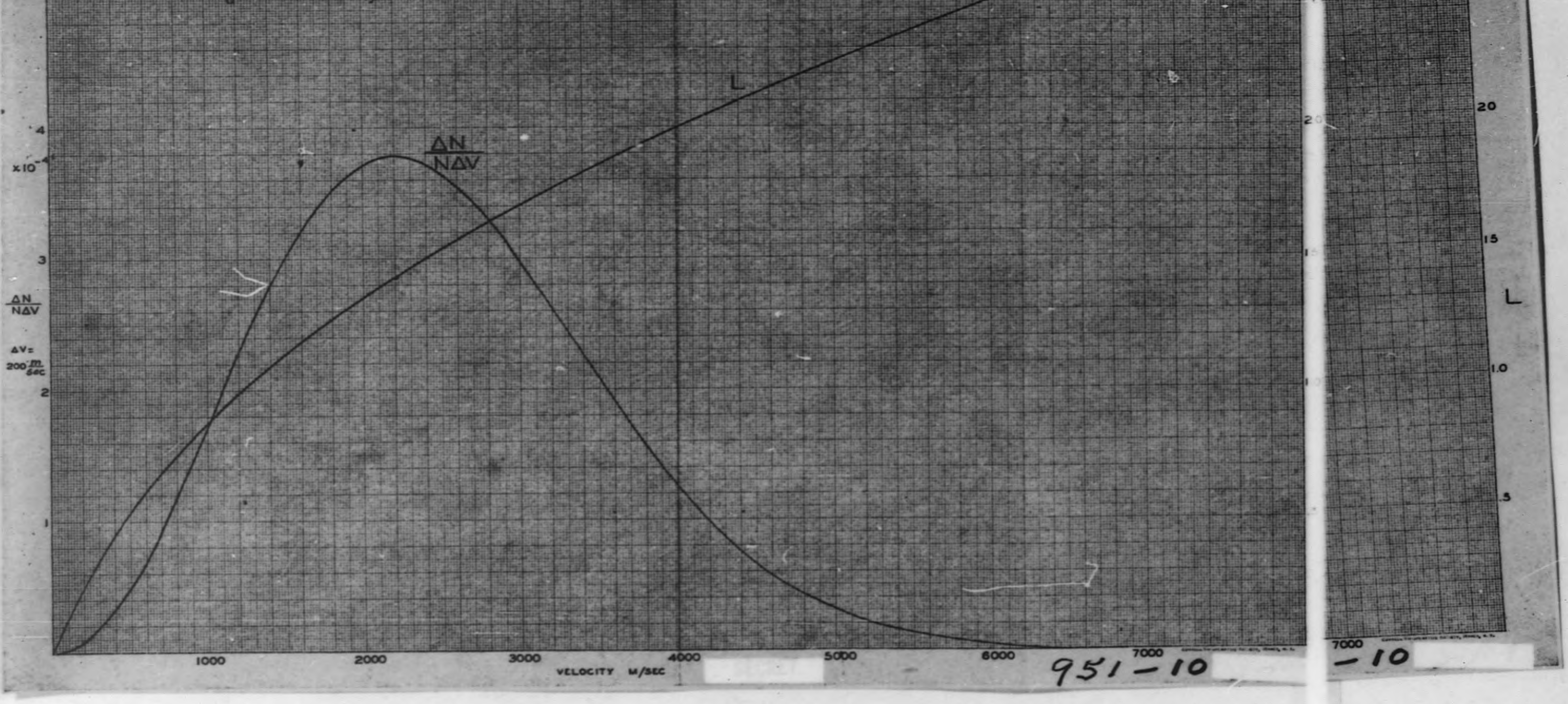
Fig 3 Velocity Distribution and L Values



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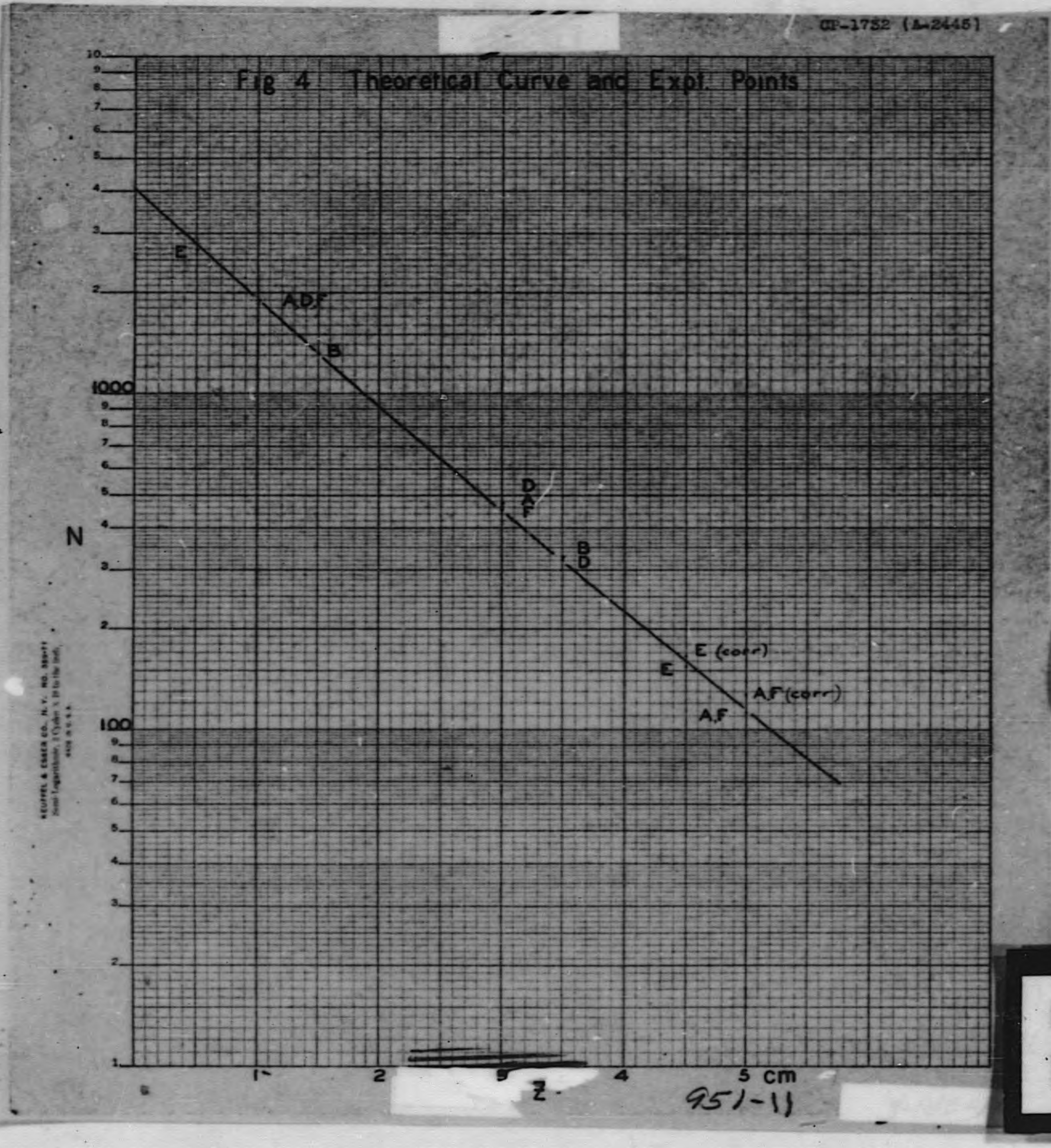

Fig 4 Theoretical Curve and Expt. Points

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