CP-1729

Metallurgical Project

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Metallurgical Laboratory

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NUCLEAR PHYSICS DIVISION

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* * *

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# REPORT FOR MONTH ENDING MAY 25, 1944

PART I *

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Report Received: June 2, 1944 - Figures Received: May 27, 1944
Issued: JUN 14 1944

898 001

* for PART II see Report CK-1761

# SUMMARY OF ACTIVITIES OF EXPERIMENTAL GROUPS OF NUCLEAR PHYSICS DIVISION

During the month of May the P-9 pile at the Argonne Laboratory has been partly filled with liquid to the point of actually reaching the critical condition. It turned out that the amount of water needed for this was about 20% less than had been estimated from the results of the exponential experiment. It is not yet quite clear what the reason is for the discrepancy. The construction of the pile is now practically complete, and the machine will be put in operation to power as soon as the balance of the P-9 arrives.

A number of experiments have been carried out in Mr. Anderson's section on nuclear properties of product, reported in CK-1761.

The long range alpha particles emitted by 25 under neutron irradiation have been studied by Mr. Hughes in the Wilson chamber. He reports a cross-section of about $2 \times 10^{-24}$ for the production of alpha particles of ranges between 5 and 10 cm., and a cross-section of $1.6 \times 10^{-24}$ for ranges between 10 and 15 cm. The total cross-section of 3.6 for emission of these alpha particles corresponds to about 1/200 of the absorption cross-section of 25.

Mr. Hughes carried out some measurements on the diffusion of thermal neutrons in uranium metal through a thickness of 5.5 cm. He finds that the average value of the diffusion length is 1.55 cm. with a slight indication of a gradual hardening of the neutrons with increasing depth.

Mr. Seren has carried out additional measurements on the activation cross-sections of various isotopes. His results are summarized in a table in the report.

898 003

A measurement was performed in order to determine the increase in neutron density at the end of a W-slug when aluminum spacers of 7/8 " thick are interposed between slugs. It was found that the density of neutrons increases by about 40%.

An experiment was performed in order to determine whether the decomposition products of water by fission fragments reach a saturation pressure. No indication of a saturation was observed up to 14 atmospheres pressure of the detonation mixture.

Mr. Anderson constructed a $BF_3$ chamber for the detection of very weak neutron sources. It is possible with this chamber to detect sources emitting about 1 neutron per second.

In Mr. Morrison's group work has been going on in order to determine the reproduction factor of various types of Hanford lattices. Some points on the technique of the measurement have to be cleared up before final results can be quoted.

## PHYSICS GROUP III

W. H. Zinn

The P-9 experimental machine was sufficiently completed on May 15 so that liquid could be pumped into it. There was available 8,200 lbs. of P-9, and it was decided to make an attempt to get a chain reaction with this amount, or more correctly, to find data which would give the critical size by extrapolation. The exponential experiment had indicated that the critical height in the reactor tank would be in the neighborhood of 146 cm. Actually, the machine became reacting at a height close to 122.5 cm. This is a critical volume of 3,214 liters and a critical mass of 3.44 metric tons. The discrepancy between the expected critical volume and the actual one is not understood, but must lie either in an underestimate of the effectiveness of the reflector or in an error in the exponential experiments. The critical size found calls for a laplacian of the order of 1000 x $10^{-6}$ cm.$^2$, whereas in the exponential experiment a value much closer to 900 x $10^{-6}$ cm.$^2$ was found. The question with which we were immediately faced was the one of making use of the excess K which is available in this machine. The table gives the height of P-9 and the exponential period which was measured for these various heights:

| Height of P-9 in Reactor | $\tau$ |
|---|---|
| 122.6 cm. | 533.0 sec. |
| 123.0 | 75.3 |
| 123.1 | 54.2 |
| 123.6 | 37.8 |
| 123.7 | 29.5 |
| 124.2 | 16.4 |
| 124.7 | 9.4 |

The relationship between ih and periods is given by

$$ih = \frac{54}{\tau} + \frac{33}{\tau+.7} + \frac{1139}{\tau+6.5} + \frac{1793}{\tau+34} + \frac{581}{\tau+83}$$

from which we find for the periods given in the table that

$$ih = 56 \times (h - 122.43)$$

where h is the height of P-9 in cm. and 122.43 cm. is the height of the critical level. Also

$$\Delta k = 2.32 \times 10^{-5}\ ih,$$

and therefore, 1 cm. represents about 0.13% in K. If the tank is filled, the level above the bottom of the tank will be 182 cm., and the reflector will have an effectiveness of about 18 cm. so that h = 200 cm. This would give an indicated excess K equal to 10%. This is much over estimated since the sensitivity was measured for a height in which the reacting volume was a cylinder of height less than its width, and therefore, was quite sensitive to change in this single dimension. It is estimated, however, that the excess K will be about 6%. This is more than is required for ordinary operation of the machine; in fact, about 3% is adequate for this purpose. A number of suggestions have been made for utilizing the spare K. These are (1) rejacket the rods with jackets as thick as 1/8"; (2) remove some of the rods in such a way as to peak the intensity at the central experiment thimble; (3) add several more control rods which should be adjustable; (4) remove some of the rods and replace them by rods made of chemicals in which useful isotopes may be synthesised; (5) add some poison to the P-9 which, at the same time, could be a corrosion inhibitor. Of these suggestions, No. 1 would be the most desirable, but at this stage it would introduce considerable delay in the operation of the machine.

It has been decided to add two more control rods whose position will be adjustable by hand and to remove a certain number of the heavy metal rods. How many rods must be removed will be determined by experiment.

Other than addition of these control rods, the construction of the machine is substantially completed, and power operation awaits the delivery of a sufficient amount of P-9. It is not thought desirable to operate the machine without some P-9 as reflector in the top part of the tank since excess radioactivity would be generated in the upper shield. Therefore, about two tons beyond what is now on hand must be received before power operation can begin.

## PHYSICS GROUP - IV

H. L. Anderson

**Activation Cross-Sections for n$\gamma$ Reactions** L. Seren, H. Friedlander and S. Turkel

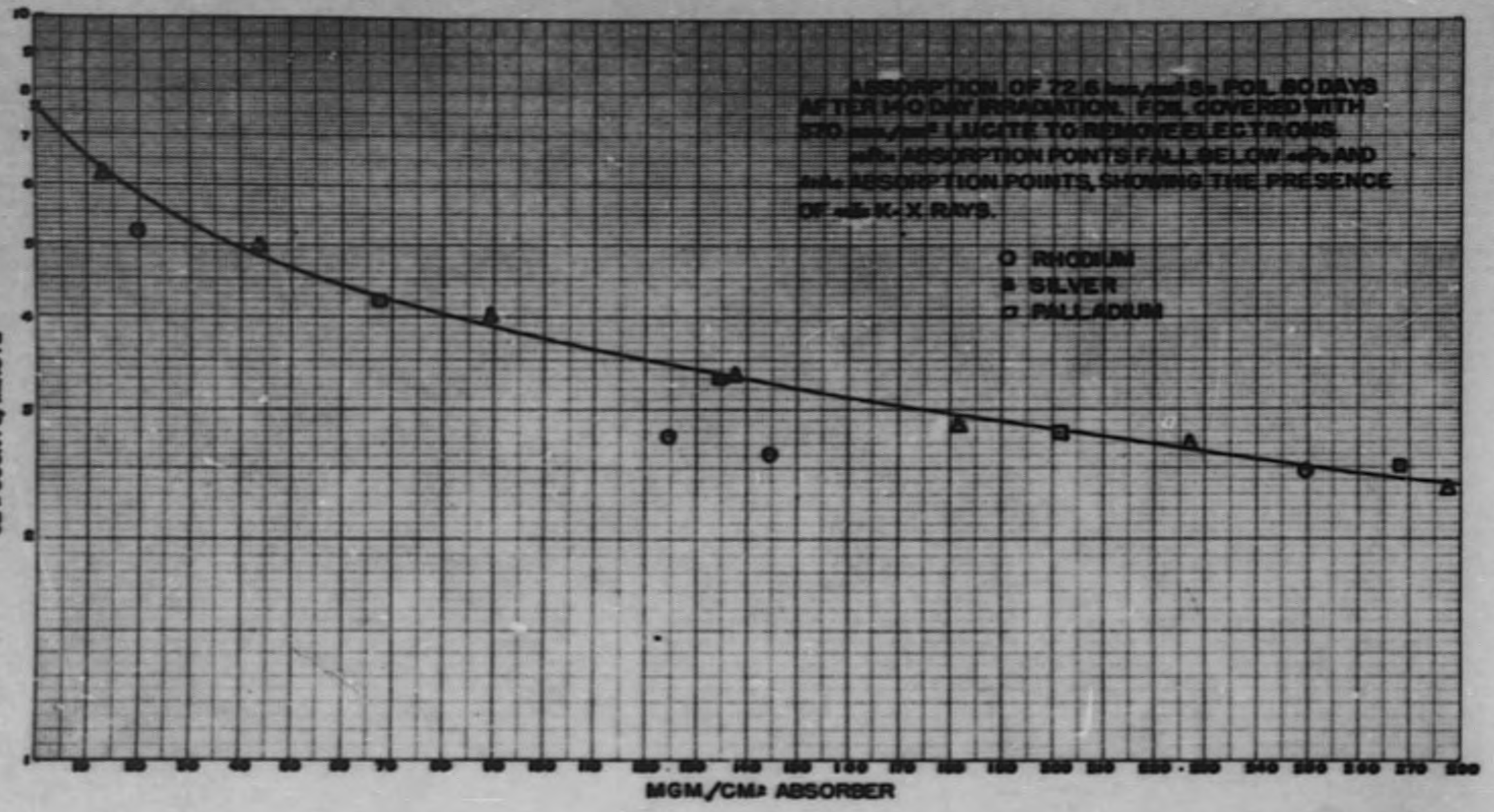
Additional activation measurements completed during the month are listed in the table below. Explanatory remarks concerning these tables are given in (CP-1175) and (CP-1592).

| Natural Isotope | Abundance % | Half Life of A + 1 | Isotopic Cross-Section $\times 10^{24}$cm$^2$ | Atomic Cross-Section $\times 10^{24}$cm$^2$ | $\frac{cm^2}{gm}$ Al Mass Absorption of $\beta$-Rays | Where Irradiated | Remarks |
|---|---|---|---|---|---|---|---|
| $_{20}Ca^{48}$ | 0.19 | 30 min. | 0.55 | .00105 | $\approx$5.85 | Pile | Gives rise to 57 min. $_{21}Sc^{49}$ daughter.<br>Nuclear Isomer |
| $_{20}Ca^{48}$ | 0.19 | 150 min. | 0.205 | .00039 | $\approx$5.85 | Pile | Gives rise to 57 min. $_{21}Sc^{49}$ daughter.<br>Nuclear Isomer |
| $_{20}Ca^{40}$ | 96.96 | 8.5 days | $<$0.00031 | $<$0.0003 | K-capture X-rays | Pile | Upper limit of cross-section calculated assuming 1.1 Mev $\gamma$-ray per disintegration. |
| $_{27}Co^{59}$ | 100. | 10.7 min. | 0.73 | 0.73 | 147 and 12.9 | Thermal Column | Two groups of particles observed. See description below. |
| $_{41}Cb^{93}$ | 100. | 6.6 min. | 0.0099 | 0.0099 | $\approx$11.6 | Thermal Column | |
| $_{50}Sn^{124}$ | 6.8 | 9 min. | 0.574 | 0.039 | 5.2 | Pile | |
| $_{50}Sn^{<125}$ | | 40 min. | | 0.0142 | 13.6 | Pile | |
| $_{50}Sn^{<125}$ | | 26 hr. | | 0.072 | 75. | Pile | |
| $_{50}Sn^{112}$ | 1.1 | 105 days | 1.1 | 0.012 | K-capture X-rays | Pile | Gives rise to 105 min. $_{49}In^{113*}$. The efficiency of the G-M counter for $_{49}$In X-rays was computed. |

| Natural Isotope | Abundance % | Half Life of A + 1 | Isotonic Cross-Section $x10^{24}cm^2$ | Atomic Cross-Section $x10^{24}cm^2$ | $\frac{cm^2}{gm}$ Al Mass Absorption of β-Rays | Where Irradiated | Remarks |
|---|---|---|---|---|---|---|---|
| $_{52}Te^{126}$ | 19.0 | 9.3 hr. | 0.78 | 0.15 | 27.2 | Pile | Nuclear Isomer |
| $_{52}Te^{126}$ | 19.0 | 90 days | 0.073 | 0.014 | ≈24. | Pile | Nuclear Isomer. Gives rise to 9.3 hr. $_{52}Te^{127}$. |
| $_{52}Te^{128}$ | 32.8 | 72 min. | 0.133 | 0.0436 | 12.6 | Pile | Nuclear Isomer |
| $_{52}Te^{128}$ | 32.8 | 32 days | 0.0154 | 0.00504 | 12.6 | Pile | Nuclear Isomer Gives rise to 72 min. $_{52}Te^{129}$ |
| $_{52}Te^{130}$ | 33.1 | 25 min. | 0.242 | 0.0801 | 7.1 | Thermal Column | Nuclear Isomer |
| $_{52}Te^{130}$ | 33.1 | 30 hr. | $<0.008$ | $<0.003$ | ≈7.1 | Pile | Nuclear Isomer Gives rise to 25 min. $_{52}Te^{131}$ |
| $_{52}Te^{130}$ | 33.1 | 8.0 days daughter | 0.242 | 0.0802 | 28 for $_{53}I^{131}$ | Pile | Cross-section calculated from 8 day $_{53}I^{131}$ daughter which grows from 25 min. and 30 hr. activity |
| $_{76}Os^{192}$ | 41.0 | 17 days | 5.34 | 2.19 | 187. | Pile | Range of β-rays ~20 mgm/cm² Al or 0.12 Mev |
| $_{80}Hg^{204}$ | 6.7 | 5.5 min. | 0.37 | 0.0248 | 9.4 | Thermal Column | |
| $_{81}Tl^{203}$ | 29.1 | 4.23 min. | 0.30 | 0.087 | 10.0 | Thermal Column | |

The decay of $_{20}Ca^{49}$ required a somewhat unusual treatment to obtain the cross-sections because its isomeric activities of 150 minutes and 30 minutes half life, respectively, both gave rise to the same 57 minute half life daughter, $_{21}Sc^{49}$. This in turn decayed to the stable $_{22}Ti^{49}$. Graphs were first drawn from values obtained from the theoretical equations of such relationship in order to study the decay. These were approximate because the equations assumed the initial number of daughter atoms to be zero, whereas they actually were grow-

CP-1729



ABSORPTION OF 72.6 mg/cm² Sn FOIL 80 DAYS AFTER 140 DAY IRRADIATION. FOIL COVERED WITH 370 mg/cm² LUCITE TO REMOVE ELECTRONS.

Rh ABSORPTION POINTS FALL BELOW Pd AND Ag ABSORPTION POINTS, SHOWING THE PRESENCE OF In K-X RAYS.

ing and decaying exponentially with the decay of the parent while being irradiated. A general equation was then derived in which the entire activity, without approximations, was extrapolated to include all the growth and decay, during irradiation and afterwards. The cross-sections submitted are a result of the application of this equation.

Concerning the 10.7 minute $Co^{60}$, Livingood and Seaborg[1] suggested that the radiation consisted largely of conversion electrons resulting from an isomeric transition to the longer lived (5.3 year) $Co^{60}$. Later Nelson, Pool and Kurbatov[2] claimed that the radiation was that of continuous beta rays of end point 1.35 ± 0.1 MEV and a $\gamma$-ray of 1.5 ± .02 MEV but no conversion electrons. Finally, Deutsch and Elliott[3] found that the direct transitions constitute only 10% or less of the disintegrations. The $\beta$-ray has a maximum energy of 1.50 ± 0.15 MEV and is followed by a $\gamma$-ray. At least 90% of the disintegrations proceed by an isomeric transition by a 0.056 ± 0.003 MEV $\gamma$-ray with corresponding conversion electrons. Our findings are, as a whole, in accord with that of Deutsch and Elliott. We find that 10.8% or less of the disintegrations produce a $\beta$-ray whose $\mu/\rho$ in Al is 12.9 $cm^2/gm$ while the rest of the disintegrations (89.2%) show up in the form of low energy electrons whose $\mu/\rho$ in Al is 147.5 $cm^2/gm$. These low energy electrons must have been produced by a $\gamma$-ray of greater than 0.07 MEV in order for our counter to observe them. The cross-section has been calculated on the basis of this $\beta$, $\gamma$ branching, assuming that each particle (hard or soft) represented a disintegration.

The indium X-rays produced by the K-capture of $_{50}Sn^{113}$ were detected by taking absorption curves with $_{45}$ rhodium, $_{46}$ palladium and $_{47}$ silver absorbers. From Compton and Allison's book on X-rays:

---

[1] Livingood and Seaborg, "Phy. Rev." 60, 913 (1941)

[2] Nelson, Pool and Kurbatov, "Phy. Rev.", 62, 1(1942)

[3] Deutsch and Elliott, "Phy. Rev.", 62, 559(1942)

for 49 Indium

$\lambda$ $K_{\alpha_2}$ = .5155 A°

$\lambda$ $K_{\alpha_1}$ = .5111 A°

$\lambda$ $K_{\beta_3}$ = .4542 A°

$\lambda$ $K_{\beta_1}$ = .4536 A°

$\lambda$ $K_{\beta_2}$ = .4441 A°

K-critical absorption $\lambda$'s

for $_{45}$Rh, $\lambda$ = .5330 A°

$_{46}$Pd, $\lambda$ = .5080 A°

$_{47}$Ag, $\lambda$ = .4845 A°

Note that both the $K_{\alpha_1}$ and $K_{\alpha_2}$ X-ray lines of $_{49}$In would be strongly absorbed in $_{45}$Rh but not in $_{46}$ Pd or $_{47}$Ag. The $K_{\alpha_1}$ and $K_{\alpha_2}$ comprise 84.3% of the X-ray transitions of $_{49}$In and the $K_{\beta}$ lines the remainder. On the accompanying absorption curve, the absorption points taken with $_{45}$Rh absorbers fall lower than the $_{46}$Pd or $_{47}$Ag absorber points due to the presence of In $K_{\alpha}$ X-rays. The Sn foil was covered first with lucite to remove all the electrons. S. W. Barnes has produced the $Sn^{113}$ isotope by proton bombardment of $_{49}$In and finds 2 $\gamma$-rays, .085 MeV and 0.39 MeV in equilibrium with the 105 day X-rays. The production and detection of $Sn^{113}$ by a slow neutron $(n,\gamma)$ reaction has not been reported previous to this.

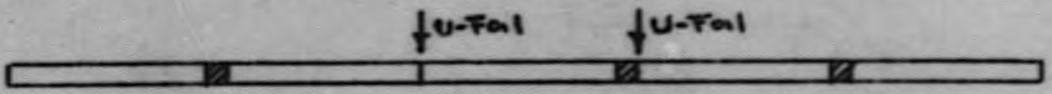
Both the 25 min. and 30 hr. periods induced in $_{52}Te^{130}$ by neutron capture decay into 8.0 day $_{53}I^{131}$. The cross-section computed on the basis of the latter activity is almost exactly equal to the cross-section computed on the basis of the two former activities, but this is more of a coincidence rather than a demonstration on the accuracy of our measurements. Note that the cross-section of all three isotopes $_{52}Te^{126,128 \text{ and } 130}$ is only .293 x $10^{-24}$ cm$^2$ per natural atom.

898 012

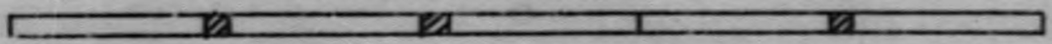
**Effect of Aluminum End Caps on # Slugs** - J. Tabin and J. Bistline

Aluminum end caps have been proposed to cool the ends of Hanford slugs. These end caps, however, separate the slugs so much that the neutron density at the ends is increased. For slugs separated by 7/8" aluminum spacers the heat production at the ends is 40% higher than if the ends are in contact.

The heat production in uranium is due almost entirely to fission. Since the $\gamma$ activity of irradiated uranium is also almost entirely due to fission, the heat production is conveniently measured by observing the $\gamma$ activity. Uranium metal disks of diameter equal to the slug diameter and .002" thick were placed at the ends of the slugs. In a given irradiation one of these disks was sandwiched between the ends of two bare uranium slugs and another between the end of a slug and the aluminum separator. The distance between the two disks was very nearly equal to one period for the Argonne lattice (8¼") so that the neutron environment of the two disks was quite similiar. To avoid any asymmetry in the lattice the positions of the uranium-uranium interface and uranium-aluminum interface in successive experiments were interchanged. The sketch shows the disposition of the slugs, the wafers and the disks in the two cases.



Position I



Position II

All irradiations were made identically and all $\gamma$ activities were measured through an aluminum absorber with an ionization chamber connected to an FP54 amplifier. The $\gamma$ activities observed were all reduced to a fixed time after the end of irradiation by reference to a typical decay curve for the particular irradiation used.

The experiment was done first without water in the annulus and later with water in the annulus. The results are tabulated below.

| | No Water | | With Water | |
|---|---|---|---|---|
| Interface | Position | Activity | Position | Activity |
| U-Al | I | 1055, 1098 | I | 956, 945 |
| U-Al | II | 1077, 1045 | II | 1009, 998 |
| U-U | I | 772, 783 | I | 698, 690 |
| U-U | II | 798, 775 | II | 711, 700 |
| $\frac{\text{U-Al}}{\text{U-U}}$ | 1.37 | | 1.40 | |

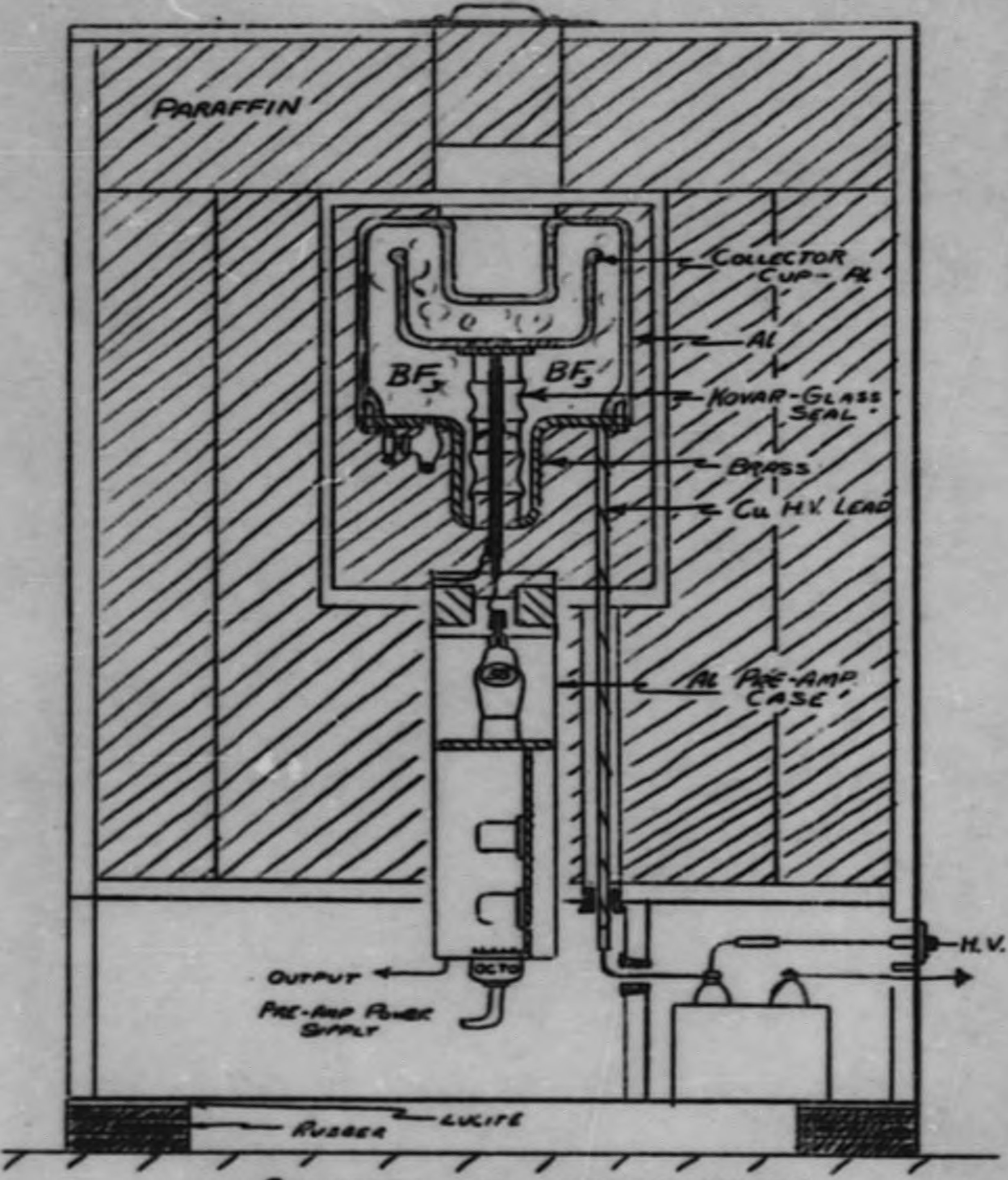
The advantage of the thick aluminum end caps is diminished by these results.

High Sensitivity Neutron Detector - H. L. Anderson, W. J. Sturm and J. Dabbs

A large $BF_3$ particle ionization chamber has been constructed which is intended to measure the number of neutrons emitted from weak neutron emitters. The chamber contains 0.8 moles of $BF_3$ and is embedded in a large paraffin block. The neutron source is placed in a cavity which extends into the center of the chamber. Fast neutrons emitted by the source are slowed down in the surrounding paraffin and detected as thermal neutrons in the chamber.

The background is largely due to $\alpha$ contamination of the chamber walls. About 1/3 of the background pulses are somewhat larger than those of the boron disintegrations. Using a discriminator circuit designed by Mr. Brill the larger pulses are not counted, and it is possible to operate with a background count of 10 per minute.

The spontaneous neutron emission from a 1.91 kilograms uranium metal lump gives 30 counts per minute above background. Since the neutron emission from such a lump is known to be 28 neutrons per second, the efficiency of the chamber is estimated to be 1.8%. This efficiency depends somewhat on the energy distribution of the source but for most neutron emitters the dependence is not very marked.



GEOMETRY OF $BF_3$ CHAMBER

J Daues

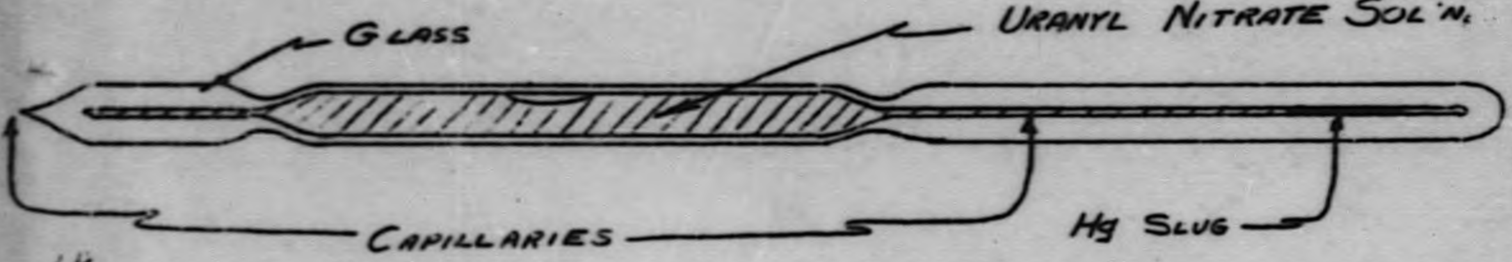
The present apparatus can measure the emission of 1 neutron per second in 6 hours of observation with a statistical accuracy of ± .25 neutrons per second.

### Dissociation Pressure of Water Due to Fission - H. L. Anderson and E. Fermi

The gas pressure developed in a closed vessel containing a water solution of uranyl nitrate under neutron bombardment was observed up to 14 atmospheres. The pressure increased linearly with uniform irradiation. There was no indication of a saturation pressure at which the rate of combination equals the rate of dissociation. The low rate of recombination may be due to removal of oxygen by some alternative process, or to some inhibiting action of the walls - we have not been able to check these points as yet.

A sketch of the arrangement is given which shows a solution of 2.4 x $10^{-4}$ moles of 25 in about 1.2 cc of solution as UNH. The solution is enclosed in a Pyrex glass tube provided with capillary tubing at each end. The pressure was observed by observing the displacement of a mercury pellet toward the closed end of one of the capillary tubes.

The amount of gas produced was approximately what might be expected from the number of fissions after 622 KWH (3 x $10^{13}$) and a yield of 1 molecule of water dissociated per 100 ev of fission energy.


GLASS
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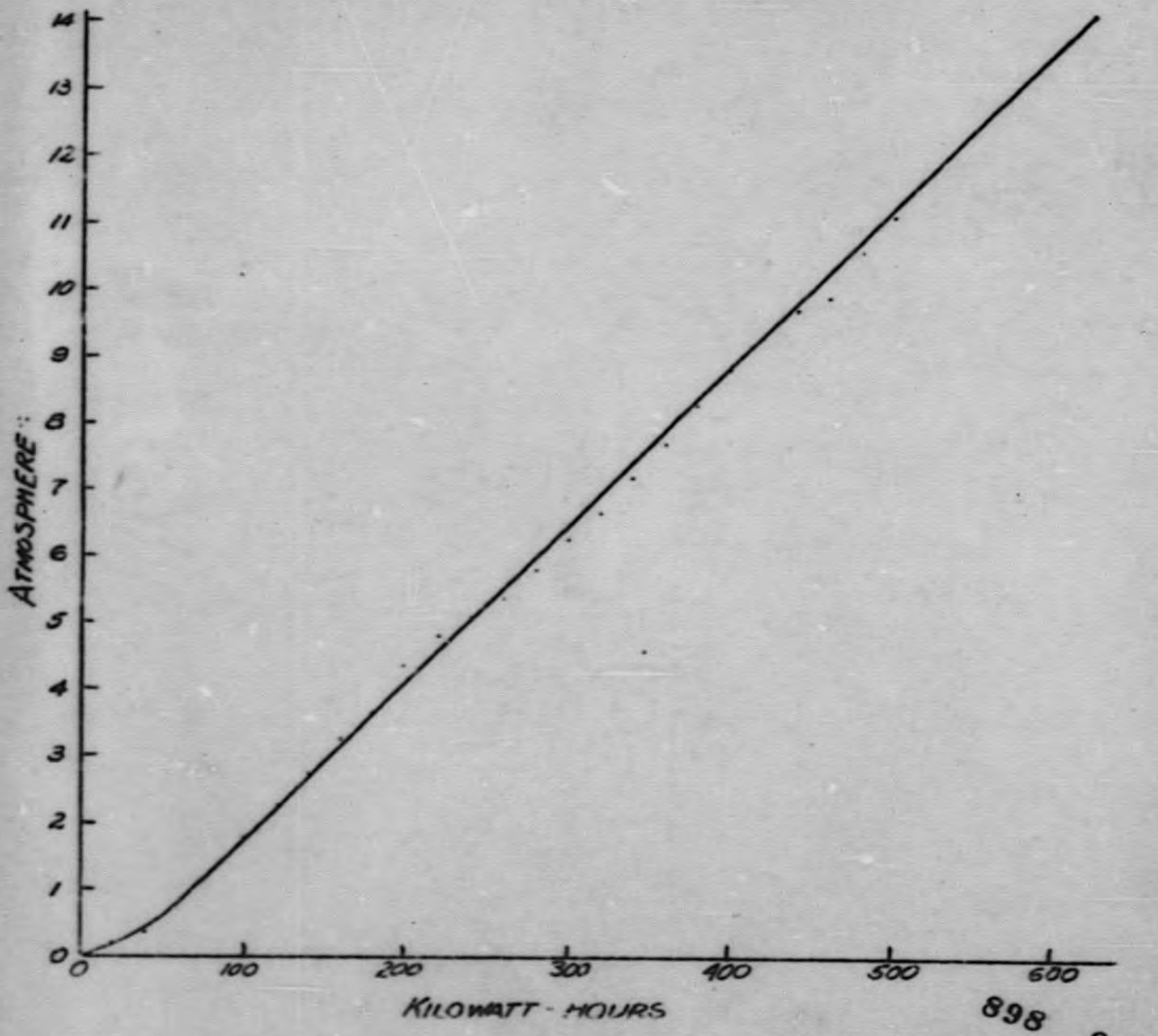
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## DIFFUSION EXPERIMENTS IN THE WEST STANDS

D. Hall

The work of this group on exponential piles in the West Stands is nearly terminated. Measurements on Hanford lattices, besides those already reported, have been made to show the effect on the Laplacian of thick Al end-caps, of safety rods, of poison slugs, and of the replacement of air by tank He.

These results will be reported about June 1, when a measurement of the effect of a suspected wall reflection, which influences all of them, will have been completed.

## Physics Group VI
(Written by D. J. Hughes)

**Diffusion Length of Thermal Neutrons in Uranium** D. Hughes and E. Bragdon

This experiment has been finished and a report is now being published (CP-1732 ). The measurements, which were made in a uranium cylinder, resulted in a mean value of 1.55 cm for the diffusion length L for distances of 1 to 4 cm from the base. Calculations give a value which agrees with the experimental result and show further that L increases from 1.40 to 1.63 cm as the neutrons diffuse a distance of 5 cm into the uranium.

**Long $\alpha$'s from Uranium** D. Hughes, C. Eggler, and H. Kanner

Since the last monthly report we have done some work to check the occurrence of $\alpha$'s only slightly longer (say 4 - 8 cm) than the natural $\alpha$'s from uranium. This was done by photographing the rays from an uncovered enriched uranium oxide foil (40 mg oxide on 30 $cm^2$) in the cloud chamber. There were about 150 natural $\alpha$'s from the foil on each expansion (max. range 3.2 cm air) and one long $\alpha$ (range $>$ 4) per 50 expansions. Had the reported "one $\alpha$ per two fissions" of Segre's been true we would have observed two long $\alpha$'s per expansion. The number actually obtained shows that there are about as many $\alpha$'s in the 5 - 10 cm as in the 10 - 15 cm range.

As no additional work on the number vs. range relationship is planned for the near future, we give in the accompanying figure the results to date. Part A shows 40 tracks obtained from thick uranium metal covered with Al. The curves are drawn as integral curves (number of tracks of range greater than R versus R) because of the thickness of the source. For the lower curve the stopping power of the chamber plus Al was 12 cm air equivalent (4 tracks with range $>$12) for the upper 15 cm (3 tracks with range $>$ 15). If the $\alpha$'s had a single range these curves would extrapolate to that range--about 15 cm--but the extrapolation is not justified in the light of the work with thin foils discussed below. It seems likely though from curves A that a majority of the $\alpha$'s have a maximum range of about 15 cm. Part B gives the differential range spectrum of 112 long $\alpha$'s obtained with thin (3cm air equivalent) enriched oxide foils. The curve is a composite of runs made with Al absorbers varying from 0 to 20 cm air equivalent. Due to geometry, the chamber is less sensitive to long tracks, and the data which make up the composite curve were corrected approximately for this factor. The number of tracks for each range is roughly constant up to 15 cm, with only a few somewhat longer.

Knowing the neutron flux, amount of enriched material present and the sensitive time of the chamber, it is possible to calculate a rough value for the cross-section for long $\alpha$ production. The result is

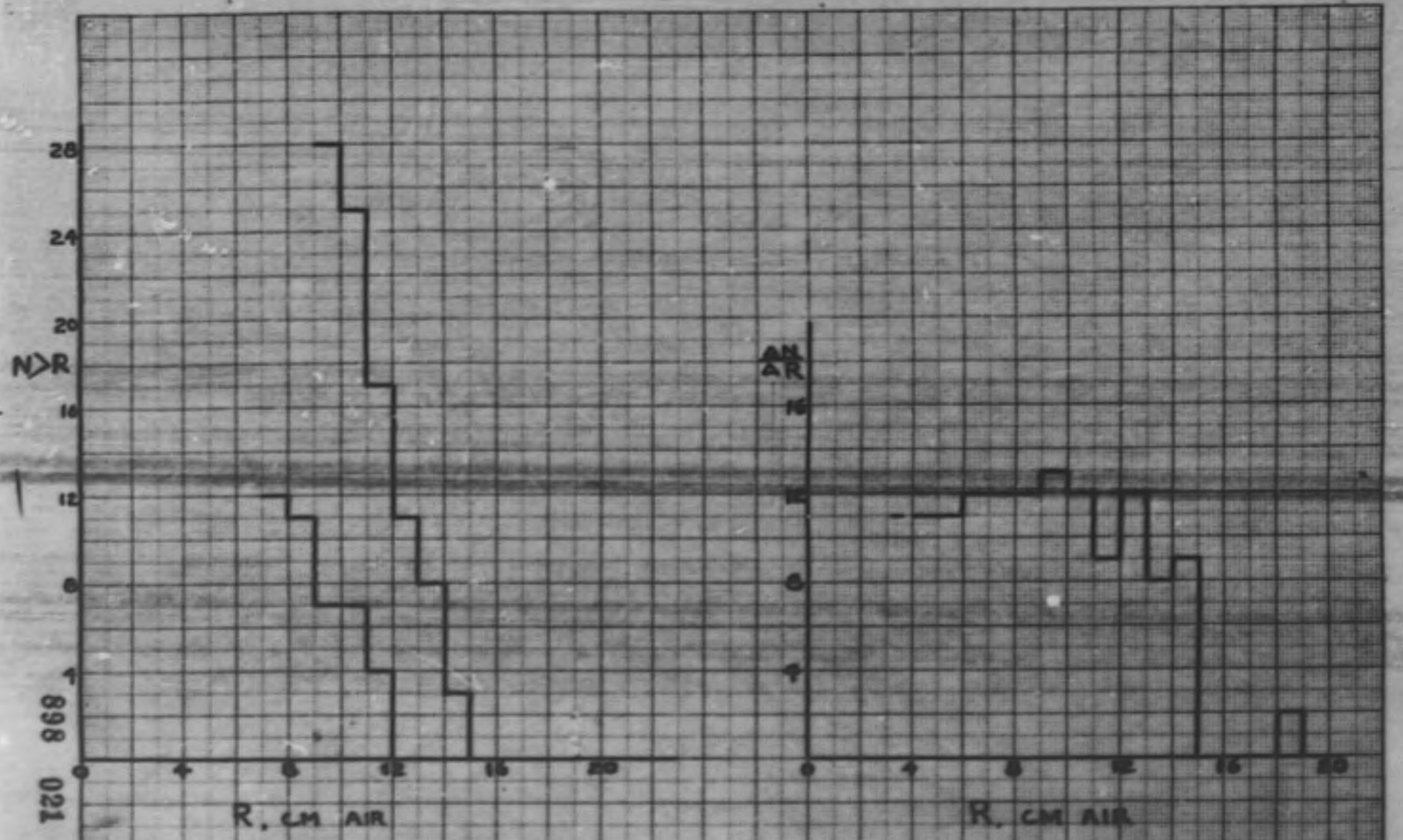
1.3 barns for the 10 - 15 cm group
2.0 barns for the 5 - 10 cm group

or a total of 3.3 barns. The calculations were also carried out very approximately for the tracks from the thick uranium metal also giving about 2 barns for the 10 - 15 cm group. This is interesting mainly in that it indicates that the $\alpha$'s are from 25 because the 12 fold enrichment factor entered the calculation in the former case and not in the latter.

Work is now being started on the coincidence of long $\alpha$'s with fissions and the angular relationships involved. For this, thin oxide foils on Zapon films were prepared so that both fission tracks are observed in the chamber. In addition a 5 mg Pu foil is to be used in a search for the long $\alpha$'s from Pu.

Decay of $Se^{75}$ C. Eggler and H. Kanner

At the request of Mr. Seren we have investigated the mode of disintegration of $Se^{75}$. This isotope could decay either by positron emission or K electron capture and emission of X-rays. The sample was placed in the cloud chamber and



A-LONG α'S FROM THICK U METAL

B- FROM ENRICHED U OXIDE FOILS

CP-1729 (A-2665)

photographed both with He and A gas in the chamber. In A many short photoelectrons originating in the gas were observed, of range about one mm, but no longer electrons (above the background) originating at the foil. In He only a few photoelectrons were seen in the gas and an upper limit on the number of electrons starting from the foil could be fixed. These results agree with what would be expected for AsK X-rays, as will be reported by L. Seren, and show that less than 1% of the disintegrations are by positron emission.

**Thermal Neutron Scattering Cross-Sections** D. Hughes and E. Bragdon

Preliminary experiments are underway to establish a method for determining elastic scattering cross-sections at thermal energies. The method is essentially to measure the backscattering for a fixed solid angle relative to that from a standard, such as graphite, using samples thin enough so that absorption effects will be negligible.

## SUMMARY OF THE REPORTS OF THE THEORETICAL PHYSICS GROUP

E. P. Wigner

The blueprint work and other direct work for W has sharply decreased during the month under review. Most of the work in this connection was concerned with shielding problems, relating in particular to optical instruments. It was carried out chiefly by Miss Way. Its main result is that the shielding of optical instruments is a very serious problem even if plastic rather than glass lenses are used. Thus, for instance, it has been found that even a borescope with plastic lenses will darken after a few hours use at W unless not only the tube under consideration but also some of the neighboring tubes are emptied of the heavy metal. Similar results were obtained for other arrangements.

The Lattice Design Group under Mr. Weinberg continued to cooperate with Mr. Friedman's group on the elaboration of the two- and three-group theories. In particular the experimentally obtained distributions of the production of thermal neutrons by a point source of fission neutrons in different moderators was analyzed and the constants for the two- and three-group theories obtained. The work on the improvement of the diffusion theory was continued with particular reference to the velocity distribution of thermal neutrons. Finally, this group devoted considerable attention to the water-uranium lattices and obtained the temperature coefficient of such lattices. These calculations show, as was to be expected, that the leveling effect plays a very important role in water moderated lattices and gives in some cases an increase of the multiplication constant of the order of one to five times $10^{-4}$ per degree.

The Pile Design Group under Mr. Young continued the work on the thick end caps. It was found in this connection that the energy production at the end of the slugs is increased by as much as 30% as compared with the energy production in a cross section towards the middle of the slug. This effect is caused by the neutrons having rather free access to the ends of the slugs; this increases the heat production at the end of the slug and makes a somewhat increased thickness of the conductor type end cap necessary. A good deal of work was devoted to an analysis of the frost test and of the possible causes of warping. The theoretical work on cartridge type loading (common jacket for all the slugs in a tube) was continued with special emphasis to cartridges with high helium pressure but without circulation of helium. These cartridges have a pressure gauge on one end which permits detection of a leak by the ensuing drop of the pressure but are otherwise completely sealed off. Mr. Ohlinger collaborated closely in this work. The main problem remains the handling of the cartridge during and after discharge which is quite difficult on account of the high radioactivity.

Miss Way's group, in addition to the work mentioned in the first paragraph, continued the review of the exponential experiments carried out hitherto. In particular a comparison of the experiments on the Hanford type lattices gives rather consistent results showing, however, that the effect of the water on the multiplication constant may have been slightly—perhaps by as much as ½%—underestimated. The cause of this discrepancy is not known at present.

The Radiation Group under Mr. Seitz continued the theoretical investigation of the effect of radiation on materials. It also co-operated with the Technical Division to prepare for the experimental detection of possible hazards at Hanford.

As mentioned before, the Shielding Group under Mr. Friedman continued to develop the two- and three-group theories in collaboration with Mr. Weinberg's group. These theories endeavor to give a description of the pile and are thus substitutes for Fermi's pile equations. Their main advantage is that they are much more easily amenable to an accurate solution than Fermi's equations are. The accuracy of the equations is somewhat lower than that of Fermi's equations in case graphite is used as a moderator. However, the accuracy is actually higher if water is used instead of graphite. The idea is to distinguish only between two or three kinds of neutrons. One kind are always the thermal neutrons while the non-thermal neutrons are the other kind in case of the two-group theory. The three-group theory distinguishes further between fast neutrons which are above resonance and fast neutrons within the resonance region. It is evident that the restriction to only two or three kinds of neutrons as compared with a continuous variety introduces a considerable simplification. Last month's work was mainly concerned with the effect of the resonance absorption and with applications of the general theory to water reflectors.

In addition, the effect of resonance fission (as found lately in 49) on the critical size has been investigated.

## LATTICE DESIGN GROUP

A. M. Weinberg

Synthetic slowing down kernels in water

Mr. Cahn has fitted various analytic functions to Anderson's experimental distribution (at Indium resonance) of fission neutrons in water. Each type of function was adjusted to give the same age (32.33 $cm^2$) as the experimental distribution. The functions used (for a plane source) were

1. Single exponential; this gives rise to the simple two group pile theory.

2a. Convolution of two exponentials with equal relaxation lengths.

2b. Convolution of two exponentials with unequal relaxation lengths. Functions of type 2 give rise to the three group pile theory.

3. Convolution of exponential and Gaussian ("Christy" kernel).

4. Single Gaussian function.

Of these five functions, the Christy kernel gives the best representation of the experimental distribution while the Gaussian is the worst. The single exponential is found to be a much better fit in the case of water than in the case of graphite.

While such empirical curve fitting is of no particular theoretical significance, it is useful for purposes of calculating critical masses of small $H_2O$ moderated systems. In such calculations the slowing down kernel is required; if this kernel can be represented adequately by a simple analytic function (such as an exponential) which is a source function for a simple differential equation, the solution of the pile equation, especially with a reflector, is enormously simplified.

Temperature coefficient in $H_2O$ lattice

The neutron temperature coefficient of k in a water moderated system is very probably positive near the optimum; i.e., as the neutron temperature rises, such a structure becomes more chain reacting. In this respect a water lattice differs from a graphite one, and the reason for this difference is two-fold:

(a) The thermal part of the migration area in water is practically negligible compared with the fast part, and so the leakage of thermal neutrons is correspondingly unimportant. For this reason there is very little increase in the total leakage of neutrons as the neutron temperature rises. This statement is not true, of course, if the water temperature rises sufficiently to lower the water density significantly.

(b) The leveling effect in a water lattice near the optimum is almost twice as large as in a graphite lattice near the optimum. The reason for this is that the scattering cross section in water decreases strongly as the temperature increases (roughly as $T^{-1/3}$); consequently the diffusion of hot neutrons in the water is considerably easier than the diffusion of cold neutrons. "Leveling" in a water lattice therefore results both from the $\frac{1}{v}$ dependence of the metal and $H_2O$ absorption cross sections and from the $\frac{1}{v^{2/3}}$ dependence of the $H_2O$ scattering cross section.

These considerations may prove to be of some importance in determining whether an $H_2O$-metal lattice will chain react. The old calculations of the probable values of k were based on a water diffusion length of 2.85 cm; i.e., the value for room temperature neutrons. These calculations gave a best k of about 0.97%. However in a mixture which contains the very large amounts of metal found in a water lattice, the average neutron temperature is probably several hundred degrees higher. Consequently the water diffusion length is much longer in a lattice, and the leveling effect is appreciable even if the $H_2O$ is at room temperature.

Very rough estimates of the <u>neutron</u> temperature coefficients in several water lattices follow:

| $\frac{\text{Water volume}}{\text{Metal volume}}$ | $\frac{\Delta k}{k}$ per degree neutron temperature (at 293°C) |
|---|---|
| .44 | $-4.4 \times 10^{-5}$ |
| 1.25 | $2.7 \times 10^{-5}$ |
| 2.24 | $14.7 \times 10^{-5}$ |
| 3.42 | $30 \times 10^{-5}$ |
| 4.76 | $47 \times 10^{-5}$ |
| 9.9 | $98 \times 10^{-5}$ |

The $\eta$ effect has been assumed the same for each lattice ($= -6 \times 10^{-5}/°C$) in these estimates.

While the temperature coefficients probably decrease with higher temperatures, they are still sufficiently big to give important increases in k as the neutron temperature rises.

## Erratum: Transport kernel in cylindrical coordinates

The transport kernel in cylindrical coordinates quoted in last month's report was incorrect. The correct expression (S. Dancoff) for the probability that a neutron starting anywhere on an infinitely long cylindrical surface of radius r' will make a collision on its first flight in a unit volume element anywhere on a concentric cylindrical surface of radius r is

$$p(r,r') = \int_1^\infty K_0(\alpha r)\, I_0(\alpha r')\, d\alpha \qquad r' < r$$

$$= \int_1^\infty K_0(\alpha r')\, I_0(\alpha r)\, d\alpha \qquad r' > r$$

where the unit of length is the mean free path. This kernel leads, by a simple integration, to the values for the fast effect in cylinders quoted in CP-644.

<u>Table of binding energies</u>

Miss Castle has prepared a table which gives the binding energies for all the known isotopes of the 15 lightest elements. The values are based on the nuclear masses given in the project handbook. This table, which will be issued as a report, is convenient for calculating the reaction energies of reactions involving light nuclei.

<u>Effect of Maxwellian energy distribution on neutron diffusion in a lump</u>

Mr. Plass has continued his investigation of the variation of $\kappa$, the reciprocal diffusion length, with the radius of the metal lump, caused by the distribution in velocity of thermal neutrons in a pile. Last month's report contained the variation with radius of a $\kappa$ defined as that single diffusion length which gives a fit to the neutron density curve at the center and the surface of the lump. Mr. Plass has now investigated the variation of another $\varkappa$, defined so that the actual flux into the metal lump given by this theory equals the flux given by the simple $\frac{\mathrm{sh}\,\varkappa r}{\varkappa r}$ curve. The dependence of this $\varkappa$ on the radius is very similar to that reported last month. It is interesting that with either definition of $\varkappa$, in the limit of small radii, the same $\varkappa$ is obtained whose square is equal to $\int_0^\infty \varkappa^2(v)\, M(v)\, dv$, where $M(v)$ is the normalized Maxwell distribution function. A complete report on the subject has been written and will be distributed soon.

## PILE DESIGN GROUP

Gale Young

### Thick End Caps

Mr. Wilkins has carried out a rather difficult calculation and obtained the neutron density distribution in a slug with thick end caps. The density is higher and more uniform over the end of the slug than it is over a cross-section some distance from the end. The density decreases as one moves into the interior from the slug end, the decrease being more rapid along the axis than along the surface. For caps of 1 cm thickness he computes the average density over the end of the slug to be 1.34 times the average over a cross-section far from the end. This increase is thought to be roughly proportional to the thickness of the cap.

Mr. Weinberg arranged with Mr. Anderson for an experiment to be made on this point, and the value for the above ratio obtained by Mr. Anderson is 1.37.

This effect will increase somewhat the cap temperatures reported in CP-1580. Calculations have not yet been made to see how much this increase may be.

## Warping and Thermal Stresses

A number of results on these topics have been assembled in CP-1698. From these results one can compute, for example, the stresses produced by a long defective strip in the bond, which problem was proposed by Quinn in MUC-TDGE-173. The greatest tensile stress in this case occurs near the edge of the defect and is in the longitudinal direction. For a defect .6 cm wide this maximum stress is about 6% of the stress existing normally at the surface in the absence of a defect. This calculation is given in CP-1707.

Mr. Murray has made calculations to determine the shape assumed by the cross-section of a slug which is heated unsymmetrically.

## Frost Test

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Mr. Murray's calculations of the power distribution in a long core within a long induction coil are given in CP-1692.

Mr. Karush and Mrs. Monk discuss in CP-1671 the temperature rise of a coating over metal and aluminum cores, under certain simplifying assumptions. Mr. Karush is now extending the theory to allow the power production to be any function of time, and to include a finite bond resistance to heat flow.

It appears that the standard frost test procedure may not be suitable for testing unbonded slugs, since the heating of the jacket may expand it away from the slug and materially reduce the conductance between them. Studies of the transfer in unbonded slugs are being made by Mr. Kratz by other experimental methods.

this Information Not classified
decided by Stehn and Kahn
12-15-55

## Insulating End Caps

The current design (detail 64299) of insulating end cap for unbonded slugs involves considerable waste of k because of the large amount of aluminum used. A design which is superior in this respect is discussed in MUC-GY-9.

For certain types of sensitive leak tests (see CT-1599) a large air volume within the can is a disadvantage. The thickness of the air in a conventional air gap insulator is much greater than is needed thermally, but the gap would be difficult to control mechanically if made much thinner. However, the resistance to heat flowing perpendicularly through a pile of thin plates may be considerable without a great deal of air being present in the interfaces between plates, and to some extent this makes use of very thin air films as insulation. This idea of an insulator made up of a stack of plates was suggested by Mr. Lyon, and has been recently revived by Mr. Creutz and Mr. Szilard as a possible way of reducing air volume within the can. Mr. Kratz is making measurements on the thermal resistance of plates stacked up in this manner. Since aluminum reacts with the slug metal at high temperatures, Mr. Szilard suggests that at least some of the plates next to the slug would presumably be of magnesium. This type of insulator raises questions about the thermal contact and flow of heat at the lateral edges of the discs.

In the absence of difficulties connected with bonding, a solid insulator such as MgO may again be considered. However, there remain fundamental uncertainties about the behaviour of such substances under operating conditions.

## Detection of Swellings in Pile

Mr. Lyon has set up equipment and gained some preliminary experience with the behaviour of wires, cords, and collars in an annular stream. Actually, however, the situation is still much as outlined in the monthly report CP-1369, except that it is now known that swelling may go so far as to break the graphite. Mr. Morrison thinks that #3 of the suggestions listed in the above report may actually be used, namely to push the slugs back and forth occasionally. In MUC-GPC-202, Mr. Cooper points out that the hotter metal of the slug end might play the role of the water sensitive substance of suggestion #5, and by its reaction encourage the release of activity into the water stream. Mr. Ohlinger adds that it might also function as the swelling substance of #6, and give a signal by expanding the row of slugs lengthwise, though it may be difficult to disentangle this small motion from the larger thermal expansion. Since the effects of water reaching the slug end have to be reckoned with in any case, it would seem desirable to undertake heated experiments to see to what extent these effects may be thus turned to useful purpose.

## Cartridge Loading

The system which puts a number of slugs in one long jacket appears to be gaining favor, and may receive greater emphasis.

Present preference is for an arrangement which puts high pressure helium into the cartridge, but is free of connections to any outside helium supply system. The activity of products recoiling into the helium is serious. However, even with the slugs uncoated, it is hoped that by suitable arrangement of free volume within the cartridge and care in the design of the cartridge end it may be possible to install a pressure gauge at the end of each cartridge and thus have a signal if a leak occurs.

## EXPONENTIAL EXPERIMENTAL GROUP

Katharine Way

Calculations for k, the multiplication factor, and for $\eta$, the number of neutrons produced per thermal neutron absorbed in uranium, have been made for seven recent exponential piles by Mr. Cashwell. The lattices involved are the "old" Hanford lattice, the Hanford 305 lattice, and the current Hanford lattice.

The bearing of the experiments on the k situation at Hanford has already been discussed by Mr. Weinberg so special attention is directed here only to the calculations for the quantities entering into k and the significance of the computed values of $\eta$. A table describing the piles and showing the results is attached.

The structure of the lattice does not affect $\eta$ in any way. Therefore when nearly equal values of this quantity are found by calculations based on different exponential experiments the results are taken to indicate reliable methods of calculation and a good choice of the constants involved. The figures found for the seven experiments in question give a value of $\eta$ equal to 1.315 ± .004. The deviation seems gratifyingly small but the piles in question were all very similar and so the results for $\eta$ do not really furnish a good test of many of the points involved in lattice theory.

In all the piles in question the metal was in the form of cylinders and the lattice spacing was either 8" or 8-3/8". Ames metal was used throughout and the graphite-metal ratio remained very nearly the same. The chief difference between piles was the introduction of aluminum or aluminum and water, two of the series having been planned to measure just the effect of the addition of these materials. The constancy of $\eta$ in this group of experiments shows therefore only that the methods of calculation take the aluminum tubes and the cooling water into account pretty well. There does seem to be a slight consistent drop in $\eta$ between #39 and #40 and between #45 and #46. Each of the latter of these pairs differ from the former only by the addition of water.* However there is practically no difference between #45 and #47. These last two piles differ in water and also in graphite, and give very nearly the same $\eta$, but this might mean that the diffusion length of Kendall used in the calculations is too small.

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*This suggests that the hydrogen to uranium absorption cross-section ratio of .0415 (based on $\sigma_H = 0.295$ and $\sigma_U = 7.1$) is too low.

## EXPONENTIAL EXPERIMENTS ON HANFORD TYPE LATTICES

| Number of Experiment | #42 | #39 | #40 | #43 | #45 | #46 | #47 |
|---|---|---|---|---|---|---|---|
| Special Feature | No Al<br>No $H_2O$ | Al Tubes | Al Tubes plus $H_2O$ | Hanford 305 lattice | Hanford A with Al but no $H_2O$ | Hanford A with Al and $H_2O$ | Hanford P with Al and $H_2O$ |
| Radius of Ames Metal Cylinders | 1.62 | 1.62 | 1.62 | 1.829 | 1.726 | 1.726 | 1.726 |
| Graphite | --Agot | L = 50.16, p = 1.62 --- | | Gulf Cleves | L = 51.7, | p = 1.62 | Kendall<br>L = 50.0<br>p = 1.686 |
| Lattice | -8" square | horizontal | rods --- | --- 8-3/8" | square | vertical | rods ---- |
| Graphite Metal Ratio by Weight | 4.19 | 4.19 | 4.19 | 3.59 | 4.20 | 4.20 | 4.37 |
| $-\Delta \times 10^6$ | 111 ± 3 | 93 ± 2 | 56 ± 3 | 120 ± 3 | 91.2 ± 3 | 61.8 ± 4 | 64.3 ± 3 |
| $K^2$ | 611 | 604 | 596 | 603 | 604 | 605 | 568 |
| $k_{reported}$ | 1.068 | 1.056 | 1.033 | 1.072 | 1.055 | 1.037 | 1.036 |
| $p_1$ | .862 | .862 | .876 | .848 | .864 | .877 | .877 |
| $p_2 \cdot f$ | .911 | .897 | .872 | .921 | .897 | .872 | .86 |
| $\varepsilon$ | 1.034 | 1.034 | 1.034 | 1.037 | 1.034 | 1.034 | 1.034 |
| $\eta_{reported}$ | 1.315 | 1.322 | 1.308 | 1.323 | 1.317 | 1.313 | 1.31 |

## RADIATION GROUP

Frederick Seitz

Work has progressed on plans for the testing program at Hanford. At the present time the Technical Division is continuing its preparation of an assembly for one of the test holes at Hanford. This will be inserted in a simulated section of the Hanford pile which is being constructed at the Armory under the supervision of Dr. Ohlinger.

Suggestions were submitted to the Operating Division for a device which will determine the temperature of the graphite by measurement of the expansion of a cylinder containing helium and a device for measuring the electrical conductivity of a specimen of graphite continuously. The second arrangement incorporates a feature which makes it possible to keep the specimen cool.

Mr. Schweinler has been reviewing the calculations which were recently completed (see report CP-1661) on the ranges of knocked-on atoms in pile materials and has started a more refined series of calculations on the same subject.

Dr. Goldberger is undertaking calculations on the influence of crystal structure on the scattering of slow neutrons in graphite and uranium. These calculations were stimulated by the experimental work of Rainwater and Havens at Columbia University, described in the previous monthly report.

## SHIELDING GROUP

F. L. Friedman

**The Relation of Multigroup Pile Theories to Fermi Theory (Friedman).**--The difference between the multigroup theories and Fermi's pile theory is in the description of the slowing down process. The Fermi theory employs a Gaussian to describe the spatial distribution of neutrons slowed to a given energy from a high energy plane source. In the n group theory, the convolution of n - 1 exponentials is used to describe the slowing from a plane source. By the Fourier transform convolution theory this convolution may be written

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(s)F_2(s)\ldots\ldots F_{n-1}(s)\, e^{-isx}ds$$

where $F_k(s)$ is the Fourier transform of the exponential $e^{-\alpha_k x}$

$$F(s) = \frac{\alpha_k^2}{\alpha_k^2 + s^2}$$

The distribution of neutrons from a plane source may be found by performing the integral which may always be done by contour integration. Of particular interest in showing the relation to Fermi's Gaussian is the case of all $\alpha$'s equal and $n - 1 \rightarrow \infty$. The integral when the $\alpha$'s are equal is

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{e^{-isx}}{\left[1 + (\frac{s}{\alpha})^2\right]^{n-1}}\, ds$$

Since the age $\tau$ equals $\frac{n-1}{\alpha^2}$, the denominator of the integrand becomes

$$\left[1 + \frac{\tau}{n-1} s^2\right]^{\frac{n-1}{\tau s^2} \tau s^2} \longrightarrow e^{\tau s^2} \quad \text{as } n \longrightarrow \infty ,$$

and

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-\tau s^2} \, e^{-isx} \, ds = \frac{e^{-x^2/4\tau}}{2\sqrt{\pi\tau}}$$

More details of the transition from n group to Fermi theory have been developed and the characteristics of n group theories in general should be reported soon.

The Christy-Kernel (Friedman).--In order to approximate the slowing down distribution, for example in water, the convolution of an exponential and a Gaussian, a monochromatic diffusion with a Fermi type slowing down, has been used by Christy and others (cf. Weinberg's monthly report). In order to use the description in the theory of a going pile we add a description of the thermal diffusion and look for the characteristic equation connecting the diffusion lengths, the age, and the reproduction constant with the "Laplacian" of the pile. This equation is

$$e^{-\tau\rho^2}(1 - L_f^2\rho^2)(1 - L_s^2\rho^2) = k$$

$L_f$ is the fast diffusion length, $\tau$ the age of the Gaussian, $L_s$ the thermal diffusion length, k the reproduction factor and $\rho^2$ is the "Laplacian".

There is only one negative real root for $\rho^2$. As expected, its value for $k - 1 << 1$ is $(1 - \kappa)/M^2$ where

$$M^2 = L_s^2 + L_f^2 + \tau \ .$$

Pile Theory with Resonance Fission (Weinberg and Friedman).-- The discovery of a strong resonance in 49 at 0.3 volts makes it worthwhile to investigate the "Laplacian" of a region in which resonance fission is supposed to occur just above thermal energy. This has been done both on the two-group theory (exponential slowing down) and on the Fermi theory (Gaussian slowing). The negative "Laplacians" are

$$\frac{\left\{\kappa_t^2 + \kappa_f^2\left[1 - (1 - p)\nu_R\right]\right\}}{2}\left\{\sqrt{1 + \frac{4(k - 1)\kappa_f^2\,\kappa_t^2}{\left\{\kappa_t^2 + \kappa_f^2\left[1 - (1-p)\nu_R\right]\right\}^2}} - 1\right\}$$

on the two group theory and $\beta^2$, the positive real root of

$$\left[1 - (1-p)\nu_R e^{-\beta^2/\kappa_f^2}\right](\beta^2 + \kappa_t^2) = \left[k - (1-p)\nu_R\right]\kappa_t^2\, e^{-\beta^2/\kappa_f^2} \quad ,$$

on the basis of Fermi's theory. In the above p is the resonance escape probability, $\nu_R$ the number of neutrons produced per resonance capture, $\kappa_t$ the reciprocal of the thermal diffusion length, $\kappa_f$ the reciprocal of the fast diffusion length or of the square root of the age, k the reproduction constant.

When $k - 1 << 1$ we may write

$$\beta^2 = \frac{k - 1}{M^2}$$

with

$$M^2 = \frac{1}{\kappa_f^2} + \frac{1}{\kappa_t^2}\left[1 - (1 - p)\nu_R\right]$$

for the two group and this applies also in Fermi's theory.

It must be remembered that k is changed by the resonance fission effect when $k_o$ is the k assuming $p = 1$

$$k = pk_o + (1 - p)\tau_R$$

Effects of Resonance Capture in Piles (Williamson and Friedman).-- A change in the amount of resonance capture in a reproducing region changes the "Laplacian" of the region not only through its effect on k, the reproduction factor, but also by changing the migration area $M^2$. The migration area is the sum of $L^2_f$, $L^2_m$ and $L^2_s$, the diffusion area in the slowing down process above the resonance region, in the resonance region, and during the thermal diffusion in the thermal region. Of these three contributions to $M^2$ only $L^2_m$ is changed by a change in p, the resonance escape probability. (Of course other constants may be changed simultaneously as in the addition of more metal.) When p is near 1, $L^2_m$ varies directly as p.

The division into above, in, and below resonance suggests that a three group theory is the natural theory to apply to calculations of resonance effects. For this reason, we have computed the "Laplacians" for various k's and p's using the three group theory.

The "Laplacian" determines the size of a pile without a reflector. When reflectors are used the other two roots of the characteristic equation of the region (see report CP-1574) are needed to describe the effect of the reflectors. These roots have also been studied and will be used in computations to determine the resonance escape bonus, the additional saving effected in the pile size by the use of reflectors arising from the possibility that neutrons may escape resonance capture by entering the reflector above the resonance region and returning to the pile beneath it.

For the special case of $k - 1 << 1$, the formula for the "Laplacian" is always

$$\frac{1 - pf\varepsilon\eta}{L^2_f + p(L^2_m)_{p=1} + L^2_s}$$

where $f$ = thermal utilization and $\varepsilon\eta$ = the number of fast neutrons per thermal neutron captured in uranium.

The detailed results of the three group theory will be reported soon.

**Extrapolation Distance in a Water Lattice (Ginsburg, Cahn and Friedman).**--When $k - 1 << 1$ the two group and three group theories of piles with reflectors give simple forms for the effective size of piles and for the neutron distributions near the pile boundaries (see CP-1554 and CP-1574).

The two group theory has been applied to these problems for the water lattice experiments at X. The effect of different diffusion constants inside the pile and in the reflector is not as yet considered. The three group theory is also under way. The preliminary results of the two group theory for the increase in the size of the pile size are:

| $\frac{\text{Cell radius}}{\text{Rod radius}}$ | Addition to the length of each side |
|---|---|
| 1.46 | 17.9 cm |
| 1.64 | 17.1 cm |

Mr. Weinberg has kindly supplied us the thermal utilization and resonance escape constants used.

On the Neutron Distribution in Media with Anisotropic Scattering (Friedman).—We investigated the neutron distribution in a system in which a scattering but non-absorbing medium fills the space from $x = -\infty$ to 0 and the space from $x = 0$ to $\infty$ is empty. The neutrons are considered monoenergetic. This problem has been discussed by Placzek and his collaborators by the Hopf-Wiener method. The problem becomes more complex as the scattering of a neutron in a single collision departs from isotropy. For slight anisotropy, however, that is when the probability of scattering through a given angle is linear in the cosine of that angle, the solution is simply related to the solution in the case of isotropic scattering. The angular distribution of neutrons emerging from the plane surface at $x = 0$ is the same, and the neutron density is related to the density in the isotropic case by the formula

$$\rho = \rho_{\text{isotropic}} + 3j\,\overline{\cos\theta}\,\frac{x}{\lambda}$$

$\rho$ = the density, $j$ = the current, $\overline{\cos\theta}$ = the average cosine of the angle of scattering in one collision (positive for forward anisotropy), and $\lambda$ = the free path.

We are indebted to Professor Placzek for pointing out this possibility; the result was also found by Mark using the Hopf-Wiener method.

**Angular Distribution of Neutrons at a Plane Boundary (Lloyd).**-- For the problem described in the preceding section no solution in simple form is known. Good approximations to the angular distribution of neutrons emerging from the plane surface may, however, be given in terms of well known elementary functions. The exact solution given numerically by Placzek (MT-6) serves for comparison. Three particularly simple approximations are

$$\varphi(\mu) = \frac{1 + \sqrt{3}\,\mu}{1 + \frac{1}{2}\sqrt{3}} \qquad (1)$$

$$\phi(\mu) = \frac{1}{2}\left[1 + \sqrt{3}\,\mu + .345793\,\mu\,\ln\left(1 + \frac{2}{3\mu}\right)\right] \qquad (2)$$

and

$$\varphi(\mu) = \frac{1}{2}\left[1 + 1.31926\,\mu + .412788\,\mu\,(\mu + 1.05847)\,\ln\left(1 + \frac{1}{\mu}\right)\right] \qquad (3)$$

where $\mu$ is the cosine of the angle between the outward normal of the surface and the direction of the emergent neutron and $\varphi(\mu)d\mu$ is the number of neutrons at the surface with direction $\mu$ in $d\mu$. The density at the surface, $\int_0^1 \varphi(\mu)\,d\mu$, is normalized to 1.

The current at the surface is the rigorously $\frac{1}{\sqrt{3}}$ (see Placzek and Seidel, MT-5). (1) and (3) give this answer exactly, while (2) fails by only 1 in 58000.

The first of those approximations, Fermi's, has a maximum error of 7%, the second 0.2%, and the last .06%.

These approximations are useful, for example, in Albedo theory. Details of the work will be reported later.