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RAB-8

BNL 6808
Informal Report

ACCELERATOR DEPARTMENT
(AGS)
Internal Report

LONG COIL MEASUREMENTS SATISFY TWO-DIMENSIONAL FIELD EQUATIONS

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March 1, 1963

The amount by which the field of a magnet bends the path of a charged particle is proportional to the integral of $B_t ds$ along the trajectory. Instead of making tedious point by point measurements of B in magnets and performing the integrations numerically, it has been found useful to measure

$$I_y = I_y(x,y) = \int_{z_1}^{z_2} B_y dz \qquad (1)$$

directly, by using a search coil whose winding consists of long and narrow turns extending through the magnet gap from z_1 to z_2 in the direction of the trajectory. It should be noted that the integral I_y is taken along a straight x = constant, y = constant line and not along the actual curved trajectory path; for small curvature the difference is small.

It may be shown as follows that I_y and I_x from long coil measurements satisfy two-dimensional field equations:

Let the field in the region under discussion have a potential $\Omega = \Omega(x,y,z)$ and assume constant permeability, μ, throughout the region. Then we integrate

$$\frac{\partial^2\Omega}{\partial x^2} + \frac{\partial^2\Omega}{\partial y^2} + \frac{\partial^2\Omega}{\partial z^2} = 0 \qquad (2)$$

over z from z_1 to z_2 at constant x and y. The third term yields (at fixed x and y)

$$\int_{z_1}^{z_2} \frac{\partial^2\Omega}{\partial z^2}\, dz = \int_1^2 d\left(\frac{\partial\Omega}{dz}\right) = \frac{\partial\Omega}{\partial z}\Big|_2 - \frac{\partial\Omega}{dz}\Big|_1 = (H_z)_1 - (H_z)_2 \ . \qquad (3)$$

Therefore, by keeping the ends of the coil in z = constant planes, z_1 and z_2, so that

$$(H_z)_1 = (H_z)_2 \qquad (4)$$

for all x,y positions used, the result of integrating (2) is

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0, \text{ where} \qquad (5)(a)$$

$$V = V(x,y) = \int_{z_1}^{z_2} \Omega(x,y,x)dz \ . \qquad (b)$$

The condition (4) may be satisfied in various ways: by extending the long coil completely through the magnet so that both ends lie in regions of negligible field or by having one (or both) ends in a z = constant plane where $H_z = 0$, e.g. in the middle symmetry plane of a magnet. Obviously, any superimposed H_z = constant field will still satisfy (4).

Since $B_y = \mu H_y = -\mu \frac{\partial\Omega}{\partial y}$ we have

$$I_y = -\mu \frac{\partial V}{\partial y} = \int_{z_1}^{z_2} B_y dz \qquad (6)(a)$$

$$I_x = -\mu \frac{\partial V}{\partial x} = \int_{z_1}^{z_2} B_x dx \ . \qquad (b)$$

Hence, from (5) and (6)

$$\frac{\partial I_x}{\partial x} + \frac{\partial I_y}{\partial y} = 0 \qquad (7)(a)$$

$$\frac{\partial I_x}{\partial y} = \frac{\partial I_y}{\partial x} \qquad (b)$$

$$\frac{\partial^2 I_x}{\partial x^2} + \frac{\partial^2 I_x}{\partial y^2} = 0 \qquad (c)$$

$$\frac{\partial^2 I_y}{\partial x^2} + \frac{\partial I_y}{\partial y^2} = 0 . \qquad (d)$$

In other words, the integrals I_x and I_y behave just like the components of a two-dimensional field vector as long as (4) is satisfied with constant z_1 and z_2.

One can also prove (7)(a) by applying Gauss' theorem to a rectangular parallelepiped with cross section Δx by Δy extending from z_1 to z_2. The condition (4) says that no flux goes through the ends of the parallelepiped. (7)(b) can be obtained by integrating the z component of μcurl H = 0 from z_1 to z_2. (7)(c) and (d) follow from (a) and (b) by suitable differentiation and combination to eliminate I_y or I_x.

This long coil theorem provides an internal check on measurements of any component of I as a function of x and y; alternatively, if (as in our beam separator magnetic measurements) I_y is measured as a function of x only on the median plane (y = 0) the theorem enables us to predict values above and below the median plane with confidence. The theorem helps in considering the effect of fringing fields on particle trajectories; thus it would apply from the middle of one magnet to the middle of another so long as (4) is satisfied.

RAB/yew
3/4/63
Distr.: AD B1, B2, B3