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BROOKHAVEN NATIONAL LABORATORY
Associated Universities, Inc.
Upton, L.I., N.Y.

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Informal Report

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Internal Report

LONG COIL MEASUREMENTS SATISFY TWO-DIMENSIONAL
FIELD EQUATIONS

R. A. Beth

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The amount by which the field of a magnet bends the path of a charged particle is proportional to the integral of $B_z ds$ along the trajectory. Instead of making tedious point by point measurements of B in magnets and performing the integrations numerically, it has been found useful to measure

$$I_y = I_y(x, y) = \int_{z_1}^{z_2} B_y dz \tag{1}$$

directly, by using a search coil whose winding consists of long and narrow turns extending through the magnet gap from z_1 to z_2 in the direction of the trajectory. It should be noted that the integral I_y is taken along a straight $x = \text{constant}$, $y = \text{constant}$ line and not along the actual curved trajectory path; for small curvature the difference is small.

It may be shown as follows that I_y and I_x from long coil measurements satisfy two-dimensional field equations:

Let the field in the region under discussion have a potential $\Omega = \Omega(x, y, z)$ and assume constant permeability, μ , throughout the region. Then we integrate

$$\frac{\partial^2 \Omega}{\partial x^2} + \frac{\partial^2 \Omega}{\partial y^2} + \frac{\partial^2 \Omega}{\partial z^2} = 0 \tag{2}$$

over z from z_1 to z_2 at constant x and y . The third term yields (at fixed x and y)

$$\int_{z_1}^{z_2} \frac{\partial^2 \Omega}{\partial z^2} dz = \int_1^2 d \left(\frac{\partial \Omega}{\partial z} \right) = \frac{\partial \Omega}{\partial z} \Big|_2 - \frac{\partial \Omega}{\partial z} \Big|_1 = (H_z)_1 - (H_z)_2 \quad (3)$$

Therefore, by keeping the ends of the coil in $z = \text{constant}$ planes, z_1 and z_2 , so that

$$(H_z)_1 = (H_z)_2 \quad (4)$$

for all x, y positions used, the result of integrating (2) is

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0, \text{ where} \quad (5)(a)$$

$$V = V(x, y) = \int_{z_1}^{z_2} \Omega(x, y, z) dz \quad (b)$$

The condition (4) may be satisfied in various ways: by extending the long coil completely through the magnet so that both ends lie in regions of negligible field or by having one (or both) ends in a $z = \text{constant}$ plane where $H_z = 0$, e.g. in the middle symmetry plane of a magnet. Obviously, any superimposed $H_z = \text{constant}$ field will still satisfy (4).

Since $B_y = \mu H_y = -\mu \frac{\partial \Omega}{\partial y}$ we have

$$I_y = -\mu \frac{\partial V}{\partial y} = \int_{z_1}^{z_2} B_y dz \quad (6)(a)$$

$$I_x = -\mu \frac{\partial V}{\partial x} = \int_{z_1}^{z_2} B_x dx \quad (b)$$

Hence, from (5) and (6)

$$\frac{\partial I_x}{\partial x} + \frac{\partial I_y}{\partial y} = 0 \quad (7)(a)$$

$$\frac{\partial I_x}{\partial y} = \frac{\partial I_y}{\partial x} \quad (b)$$

$$\frac{\partial^2 I_x}{\partial x^2} + \frac{\partial^2 I_x}{\partial y^2} = 0 \quad (c)$$

$$\frac{\partial^2 I_y}{\partial x^2} + \frac{\partial^2 I_y}{\partial y^2} = 0 \quad (d)$$

In other words, the integrals I_x and I_y behave just like the components of a two-dimensional field vector as long as (4) is satisfied with constant z_1 and z_2 .

One can also prove (7)(a) by applying Gauss' theorem to a rectangular parallelepiped with cross section Δx by Δy extending from z_1 to z_2 . The condition (4) says that no flux goes through the ends of the parallelepiped. (7)(b) can be obtained by integrating the z component of $\mu \text{curl } \mathbf{H} = 0$ from z_1 to z_2 . (7)(c) and (d) follow from (a) and (b) by suitable differentiation and combination to eliminate I_y or I_x .

This long coil theorem provides an internal check on measurements of any component of I as a function of x and y ; alternatively, if (as in our beam separator magnetic measurements) I_y is measured as a function of x only on the median plane ($y = 0$) the theorem enables us to predict values above and below the median plane with confidence. The theorem helps in considering the effect of fringing fields on particle trajectories; thus it would apply from the middle of one magnet to the middle of another so long as (4) is satisfied.

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