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PILE NEUTRON PRODUCTION YIELD CURVES CALCULATED USING THE UCRL DIFFERENTIAL ANALYZER

R. J. Barrett, J. Killeen, J. O. Rasmussen, Jr., and S. G. Thompson

October 30, 1953

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ABSTRACT

A set of 69 pile yield curves for the production of plutonium, americium, and curium isotopes in a neutron flux of 5×10^{14} $cm^{-2}sec^{-1}$ is given. The differential analyzer of the University of California Radiation Laboratory was used in obtaining the curves. The curves are given also on log-log plots for comparison.

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I. INTRODUCTION

The irradiation of isotopes with neutrons at high fluxes results in changes in the isotopic composition of the irradiated material, and the magnitudes of the changes are dependent on the length of time involved. The problem of calculating an expected isotopic composition for a proposed irradiation even when the neutron flux and the neutron cross sections involved are known is often difficult and tedious. After an irradiation is completed and the composition has been determined, it may be in certain cases very time-consuming to compute the cross sections. Such problems as these play an important role in the work of preparing isotopes and measuring their properties. The success or failure of the experiments often depends on the preparation of certain isotopes in the optimum ratios or amounts for the measurements which follow. The difficulties are enhanced when many modes of decay or destruction occur simultaneously, as is the case in the heavy element region. In this region, in addition to the ordinary changes resulting from beta decay or electron capture, changes occur as a result of fissionability and alpha particle decay.

For many experiments it is desirable to use the highest neutron fluxes obtainable, and the transmutations occurring even in a short time may be of large magnitude. When a particular product produced is of high order in the reaction sequence, slight changes in the flux or nvt may produce large

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differences in its amount. Such problems as these are commonplace in utilizing the high neutron flux available in the Materials Testing Reactor (MTR) at the Reactor Testing Station in Idaho.

To facilitate certain experimental work in connection with the MTR, it has been useful to prepare sets of yield curves showing changes in composition of certain isotopes as the time of irradiation is varied. This report includes yield curves for some of the heavy isotopes. In this case the problems were first formulated as described in Appendix I using best available cross sections and other constants which could be obtained at that time. Then the curves were plotted by the UCRL differential analyzer as described in Appendix II. It was necessary to assume a reasonable value for the flux rate which would be obtained at MTR, since the calculations were made before the MTR was in actual operation; however, the assumed value was only slightly different from the flux which is actually being obtained now. The cross sections used have been in several cases superseded by more accurate values; however, the yield curves in general remain very useful in their present form and it does not seem worth-while as yet to revise them.

It should be mentioned that the cross sections used here are not for thermal neutrons or for a particular energy spectrum. These values are cross sections for pile neutrons as obtained in the Hanford and NRX Chalk River piles. For this reason, the apparent cross sections may be different in MTR irradiations, depending on the energy distribution of the neutrons. Differences in the cross sections apparently have been observed in irradiations involving different reactors and even in irradiations utilizing different parts of the same reactor. Nevertheless, from the practical standpoint the cross sections as used here give a good indication of the amounts of the products to be formed and are justified on this basis alone.

Since the results to be described here have been very useful at UCRL, it is believed they might also be useful to other laboratories engaged in similar work. This report has been prepared in the hope that the methods used and the results obtained will have some value in this connection.

II. YIELD CURVES

There follow exact reproductions of the curves plotted by the differential analyzer using the assumed "pile neutron cross sections" for radiative capture (σ_γ) and for total absorption (σ_t), the sums of the cross sections for radiative capture and for fission; the cross sections used are listed in Table 1. Each curve plots the yield of a particular isotope with one parent isotope present initially. The notation 40/49* indicates the yield curve for Pu^{240} with Pu^{239} initially present. In some cases, where production may proceed by alternate paths, additional information indicates the path plotted.

To convert the vertical scale to absolute ratio of the number of atoms of the isotope plotted to the initial number of atoms of the parent it is necessary only to multiply by the factor indicated at the upper right (i.e., x 0.035) of each curve. For isotopes produced by more than one path

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*The isotope abbreviation consists of two digits, the first of which is the last figure in the atomic number, and the latter of which is the last figure in the mass number.

it is necessary to add the contributions from the different paths. For a problem in which several isotopes are initially present the yield of any isotope may be obtained by summing the appropriate curves corresponding to the different parent isotopes, weighting each term according to the fraction of the particular parent in the original target mixture.

Some of the results from the analyzer curves are plotted together on log-log plots permitting direct comparison. These plots are labeled Fig. 1 through Fig. 6.

Table 1

Cross Sections Used for Differential Analytical Work[*]

Nuclide	σ_γ (barns)	σ_t (barns)	$T_{1/2}$ (α)	$T_{1/2}$ (β)
48	420	438	92 y	--
49	325	1043	2.4×10^4 y	--
40	410	410	6.6×10^3 y	--
41	307	1300	5×10^5 y	14 y
42	50	50	5×10^5 y	--
43	--	--	--	5 hr
51[†]	150 to 52 636 to 52^m	889	475 y	--
52	2000	8000	10^4 y	80 y
52^m	--	--	--	16 hr (β^- + EC)
53	150	150	10^4 y	--
54	--	--	--	25 m
62	23	23	162 d	--
63	assume 0[‡]	assume 0[‡]	[‡]	--
64	10	10	10y[‡]	--

[*] Pile neutron flux of 5×10^{14} sec^{-1}cm^{-2} was used in all calculations.

[†] Since 52^m decays β:EC = 4:1, effective σ for production of 62 from 51 is 588 b, and of 42, 148 b.

[‡] As stated in the text, the above values for cross sections and half-lives were used in the production of the curves published herein. More recent data, to be published, indicate the following possible assignments:

Nuclide	σ_γ (barns)	σ_0 (barns)	$T_{1/2}$ (α)
62	15	15	162 d
63	250	~500	35.1 y
64	10	10	19.1 y

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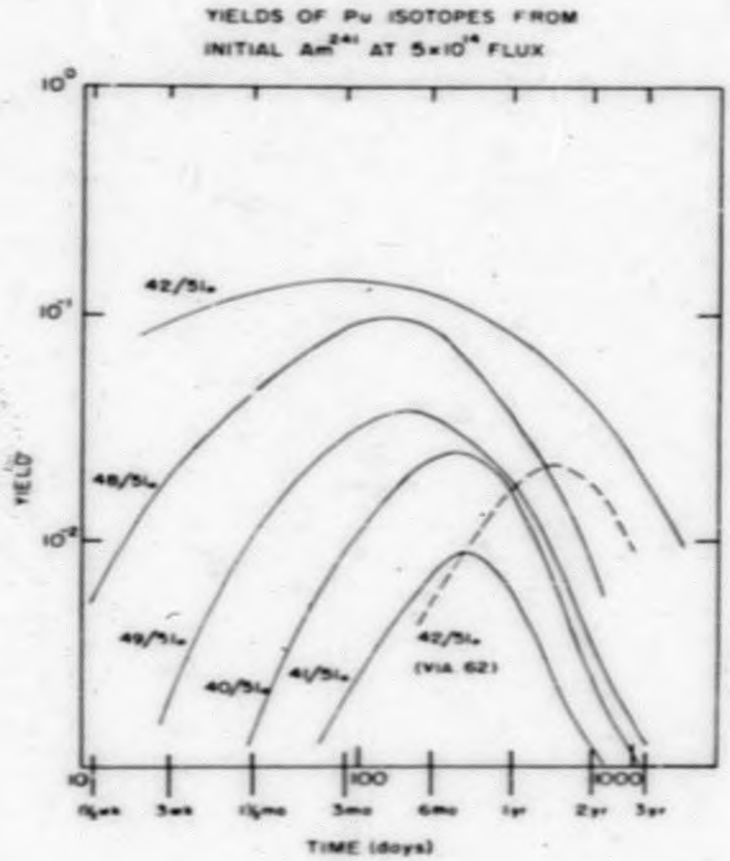

FIG 1

MU-4673

Figure 1

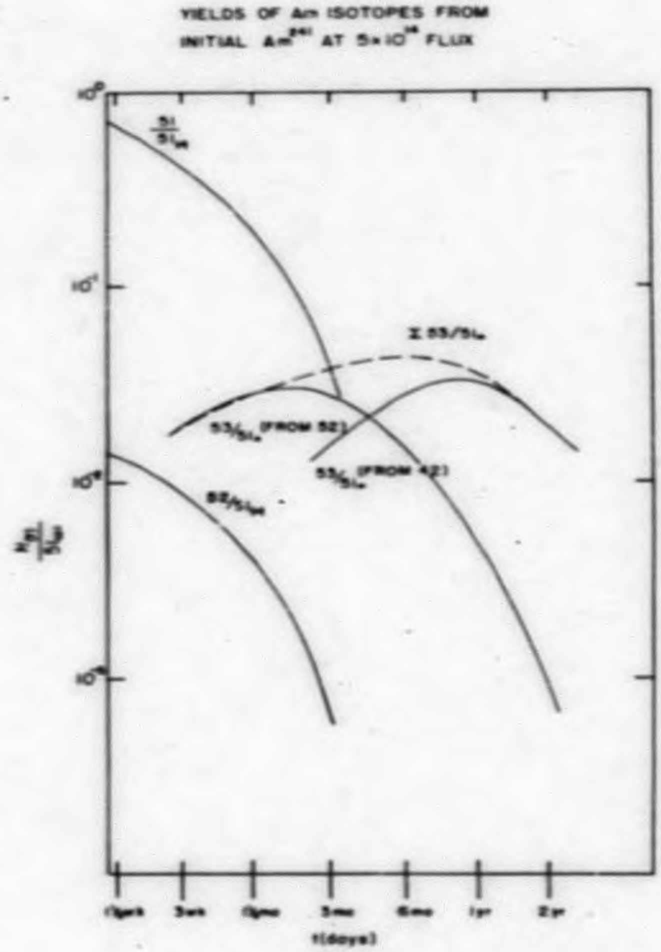

FIG. 2

MU-4674

Figure 2

YIELDS OF Cm ISOTOPES FROM INITIAL Am^{241} AT 5×10^{14} FLUX

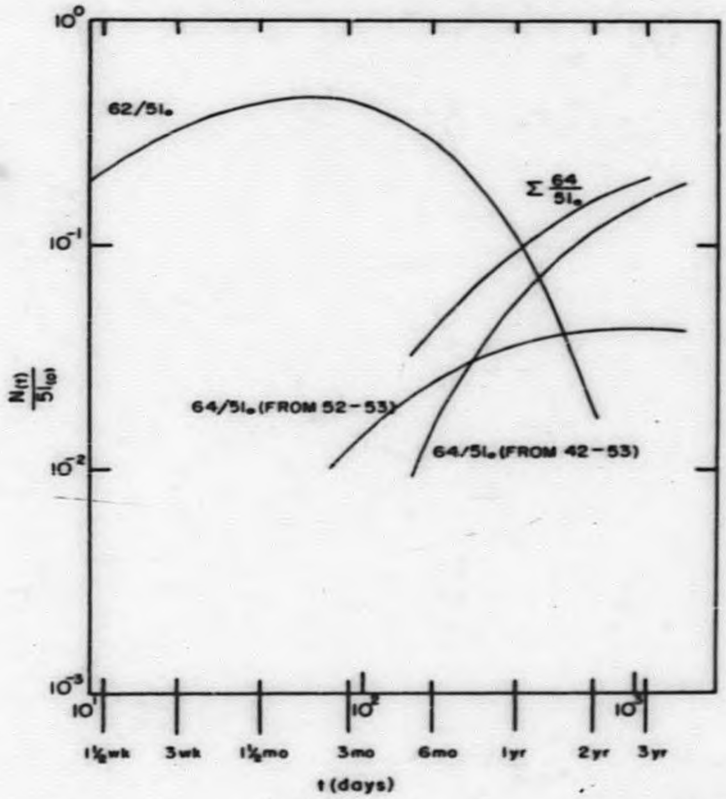

FIG. 3

MU-4675

Figure 3

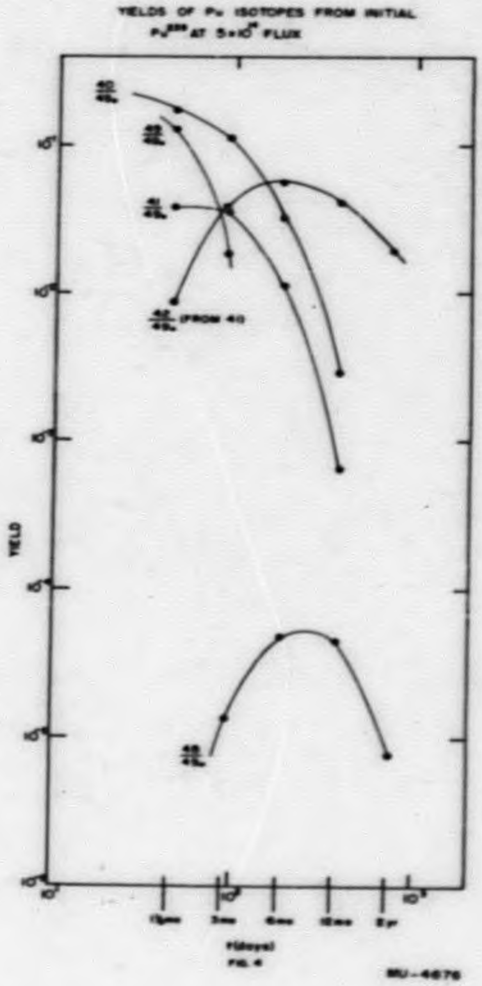
YIELDS OF Pu ISOTOPES FROM INITIAL
Pu239 AT 5×10 FLUX
YIELD
(FROM 41)
t(days)
FIG. 4
MU-4676

Figure 4

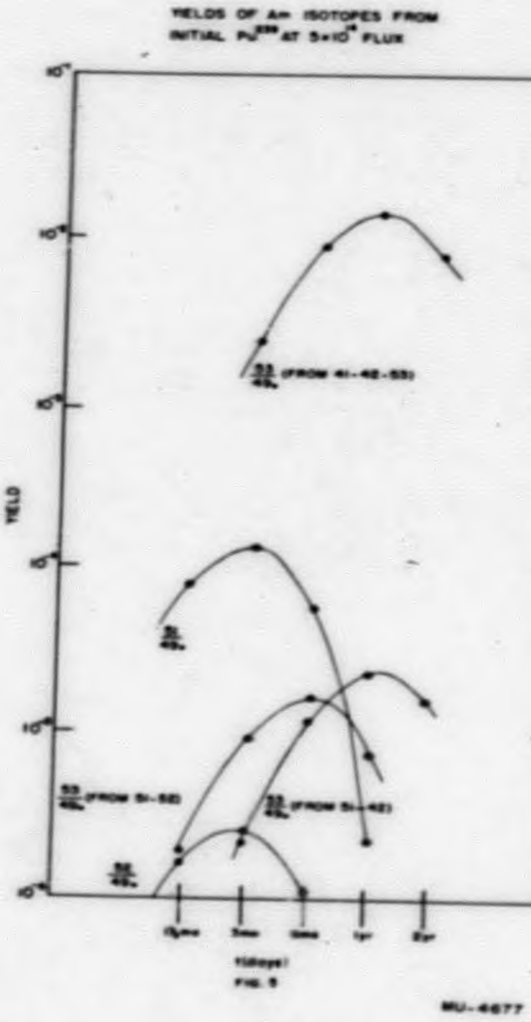
YIELDS OF Am ISOTOPES FROM
INITIAL Pu239 AT 5×10^14 FLUX
YIELD
(FROM 41-42-53)
(FROM 51-52)
(FROM 51-42)
1yr
2yr
FIG. 5
MU-4677

Figure 5

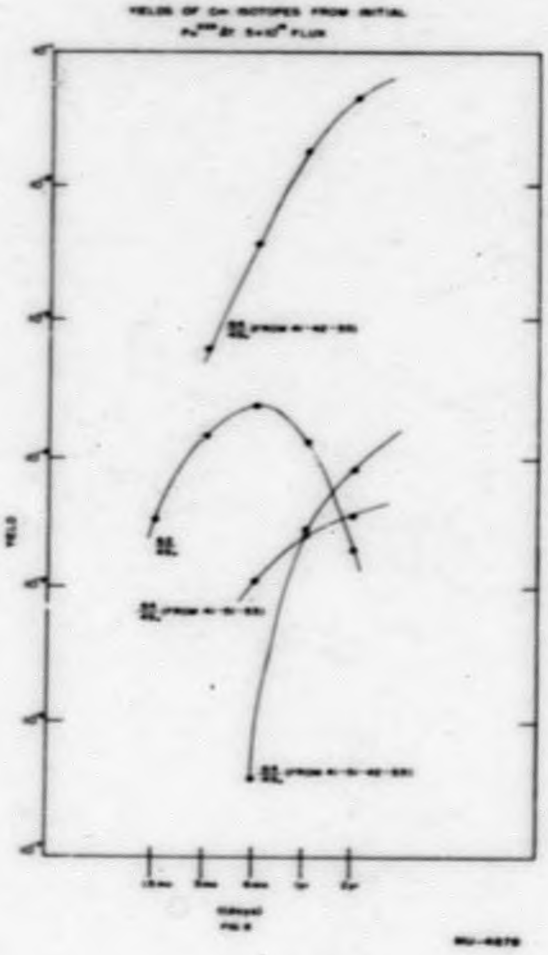

Figure 6

Index to curves.

The various chains are numbered 1 through 9 (5 and 8 omitted) and consist of the following sequences of nuclides:

Chain No. 1: $\underline{49}$-$\underline{40}$-$\underline{41}$-$\underline{42}$-53-64

2: $\underline{48}$-$\underline{49}$-$\underline{40}$-$\underline{41}$-51-62-48

3: $\underline{49}$-40-41-51-52-53-64

4: $\underline{49}$-40-41-51-42-53-64

6: $\underline{51}$-52-53-64

7: $\underline{51}$-42-53-64

9:* $\underline{51}$-$\underline{62}$-48-49-40-41-52

Curves are given starting with each underlined nuclide.

---

*Some of chain 9 is given a second time with expanded scale.

Chain No. 1: yield <u>vs</u> t at 5 x 10^{14}

$$\underbrace{49\text{-}40\text{-}41}_{\text{from chain 2}}\text{-}42\text{-}53\text{-}64$$

$b_1 = 16.48$ (3)

$b_2 = 6.466$ (7.7)

$b_3 = 20.55$ (2.4)

$b_4 = 0.7885$ (60)

$b_5 = 2.365$ (20)

$b_6 = 0.01577$

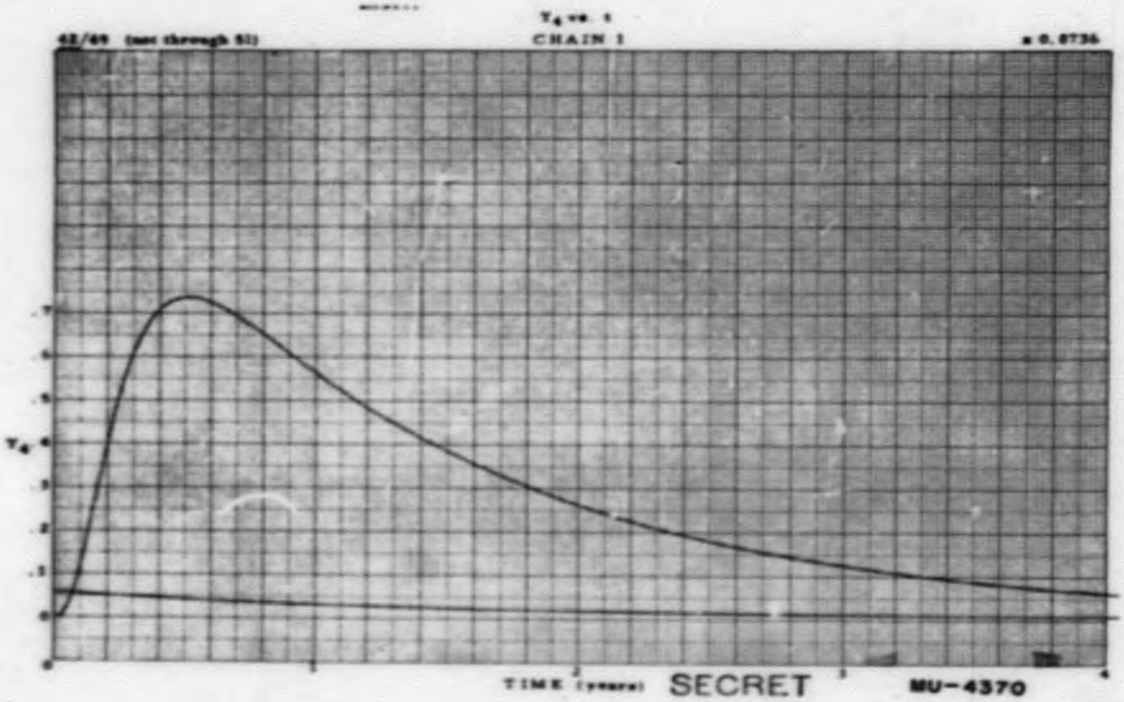

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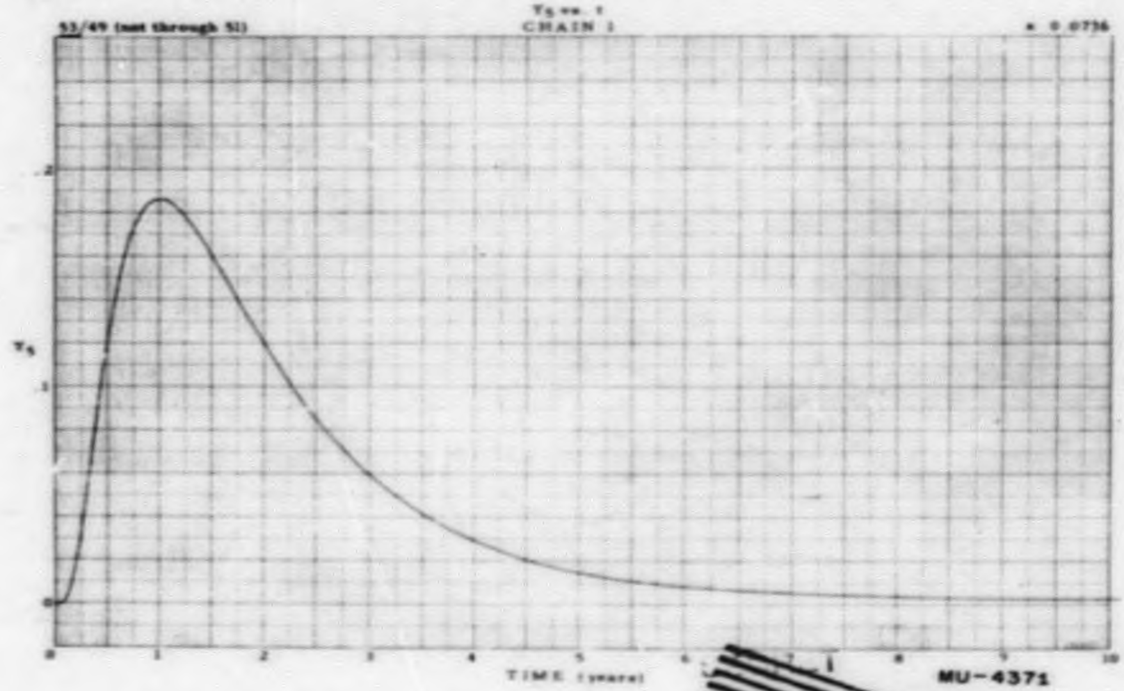
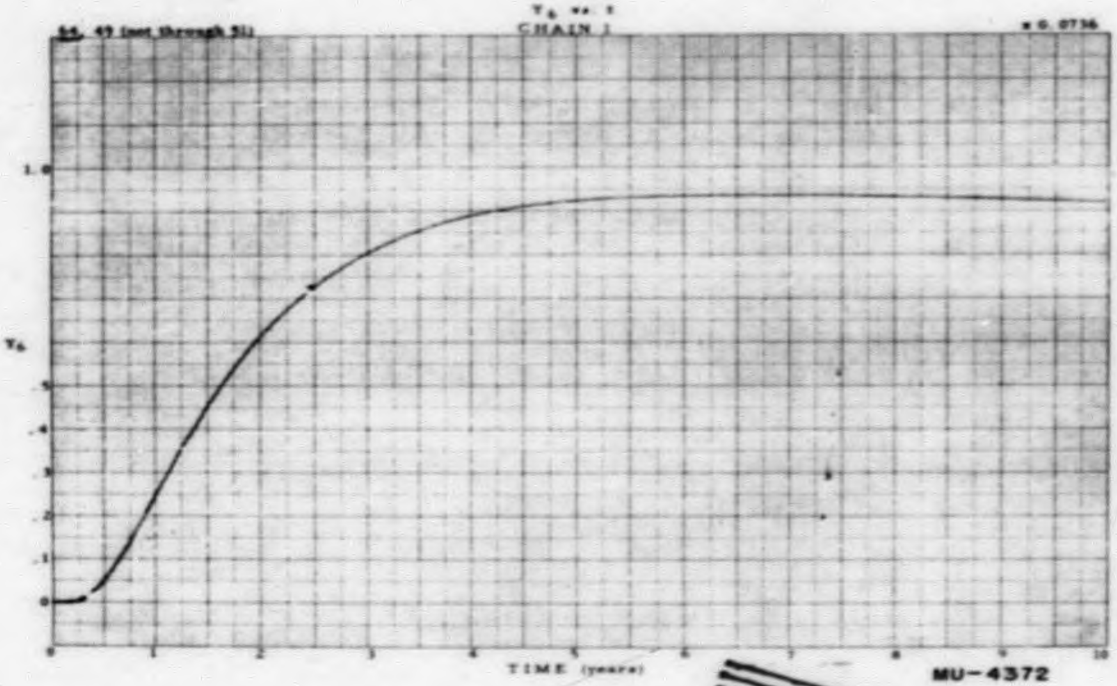
Figure 7



MU-4371

[redacted]

Figure 8



MU-4372

Figure 9

Chain 1A: 48-49-40-41-42-53-64

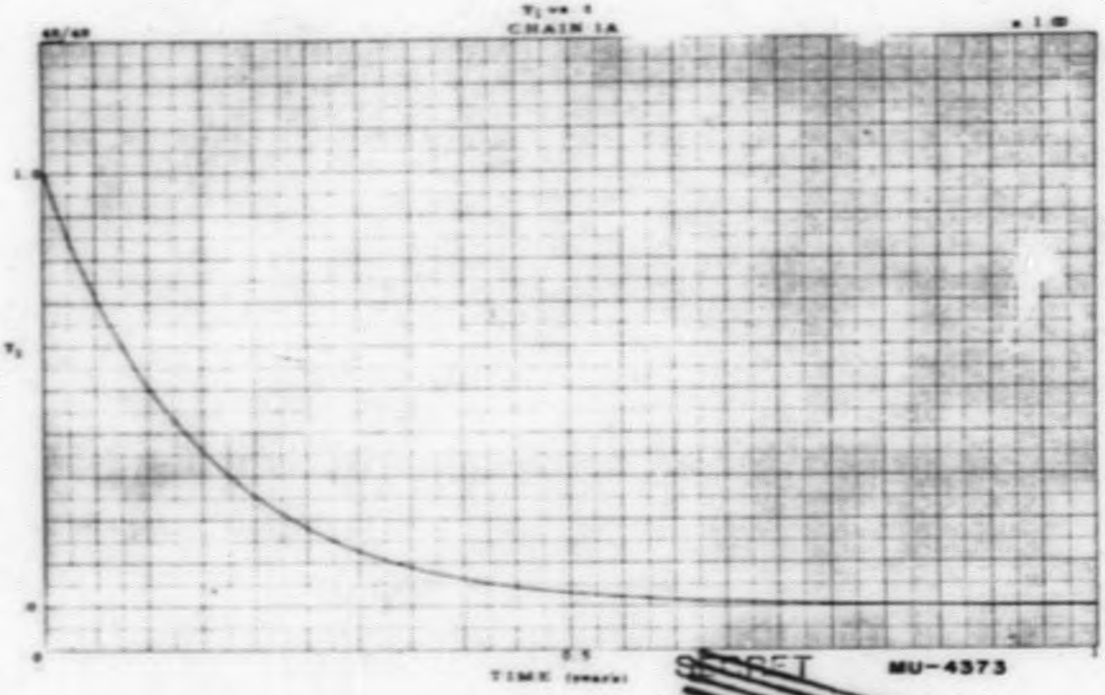


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Figure 10

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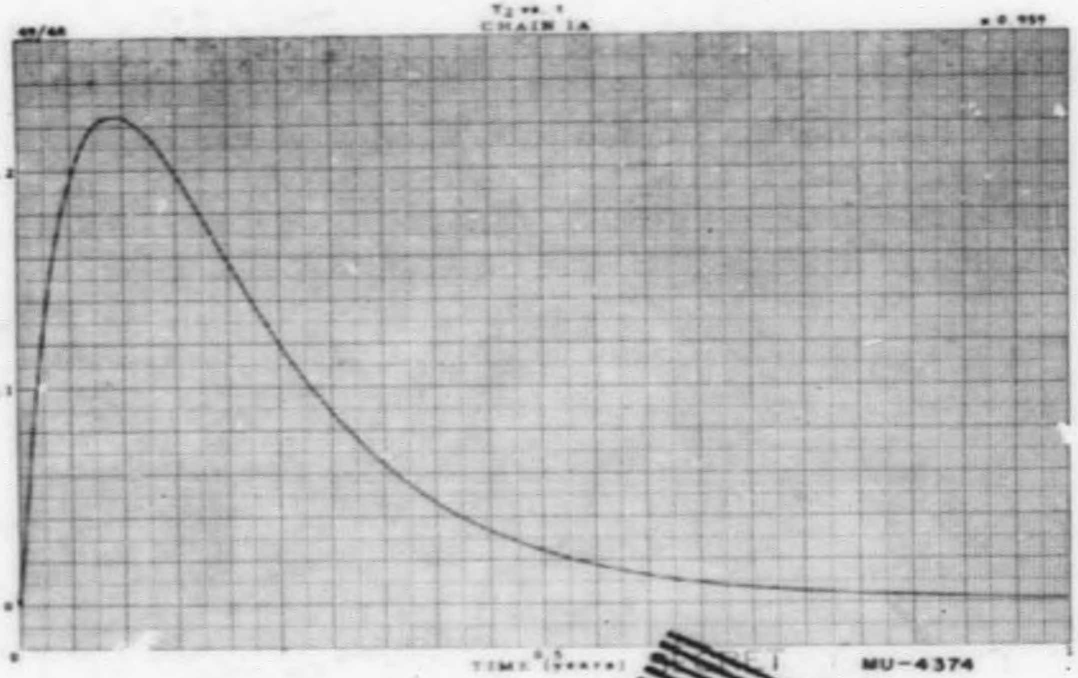

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Figure 11

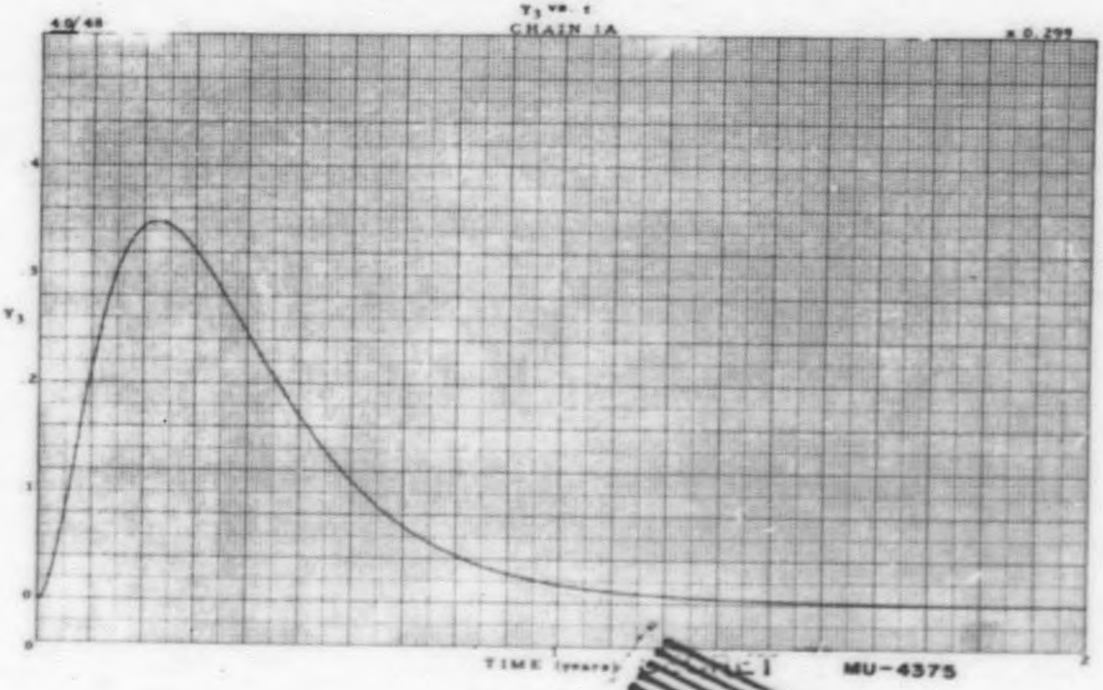
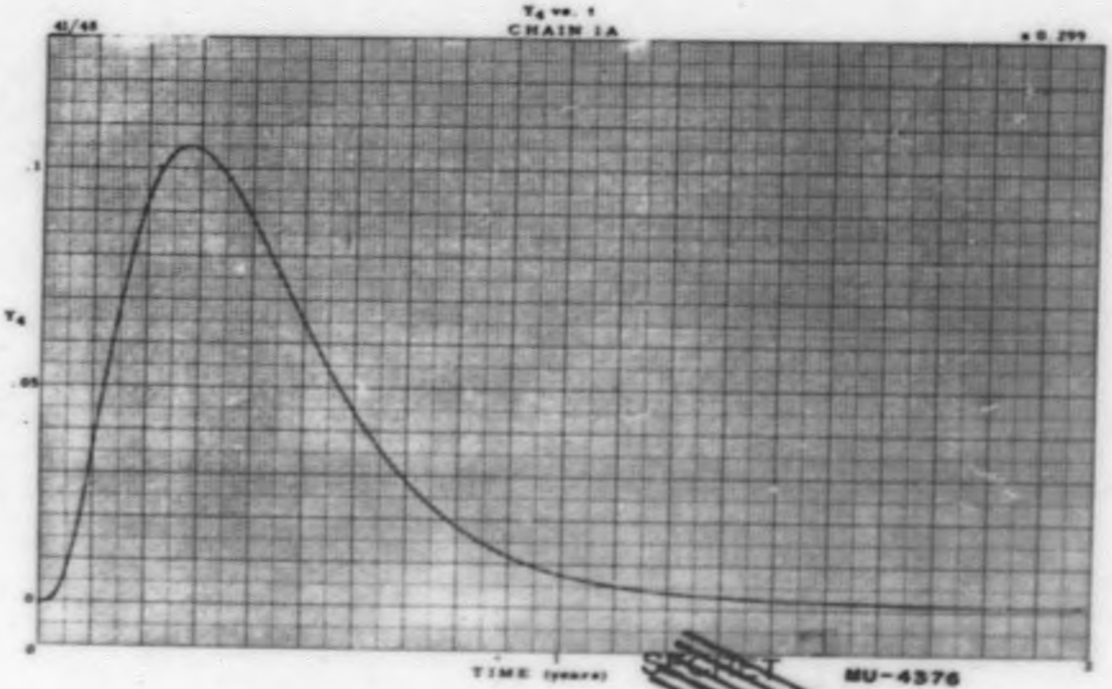


Figure 12



MU-4376

Figure 13

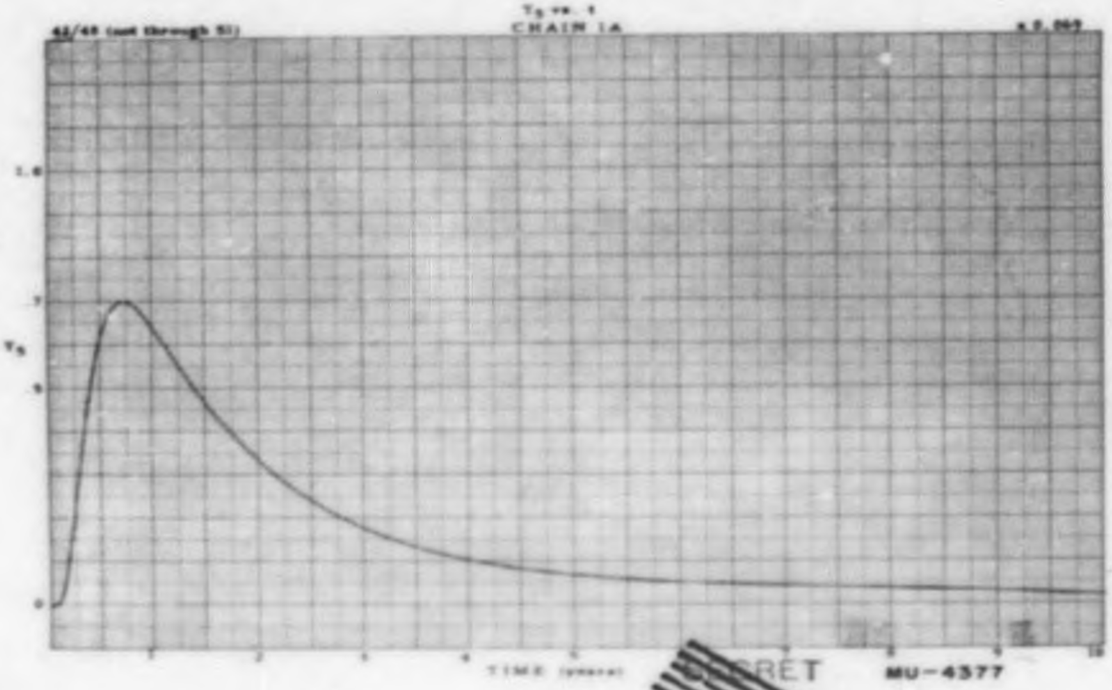


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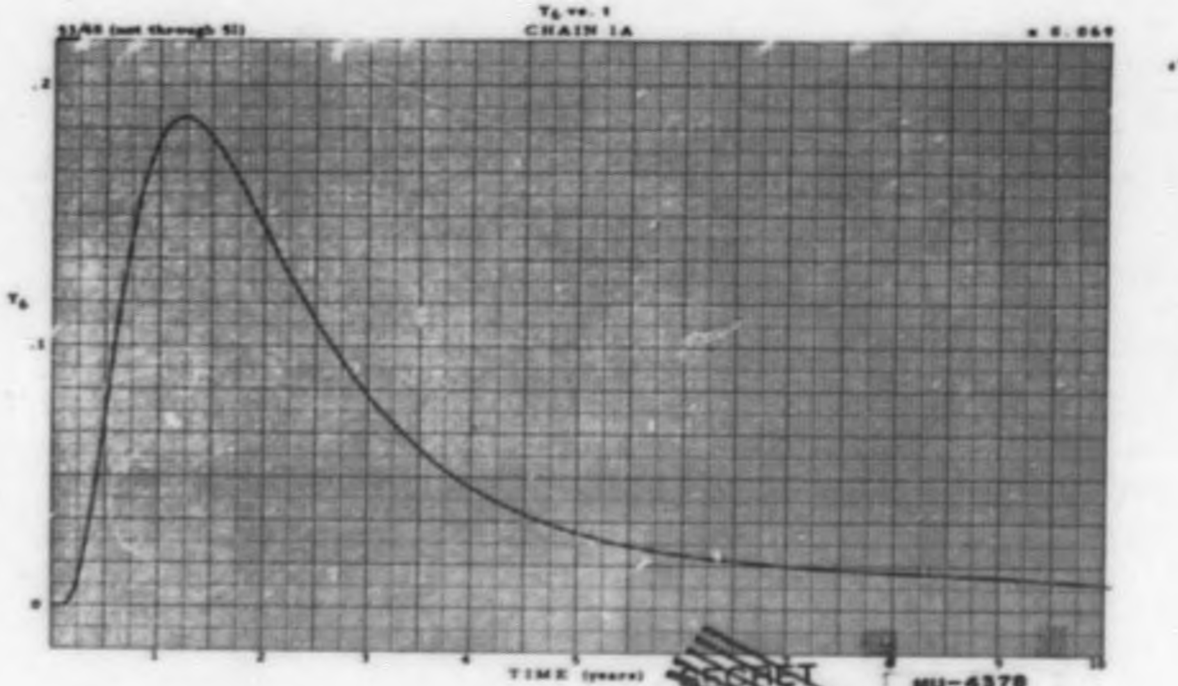
Figure 14



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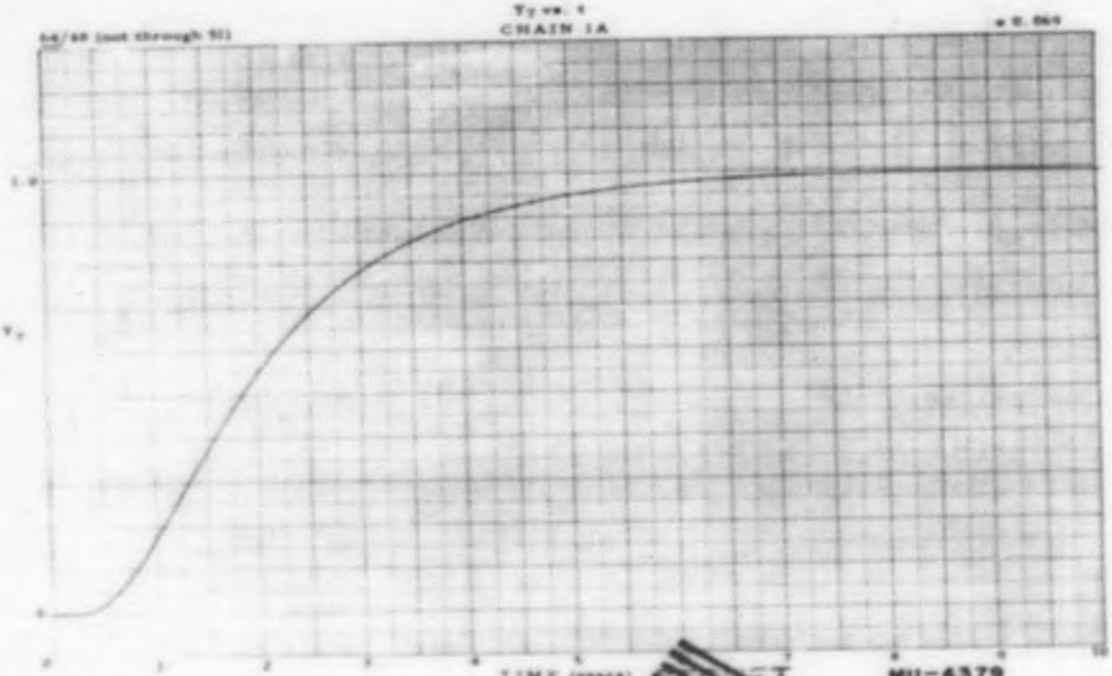
MU-4378

Figure 15



MU-4379

Figure 16

Chain 1b: 40-41-42-53-64

$b_1 = 6.466$ (7)

$b_2 = 20.55$ (2)

$b_3 = 0.7885$ (60)

$b_4 = 2.365$ (20)

$b_5 = 0.01577$ (3000)

$$Y_1^1 = -b_1Y$$

$$Y_2^1 = b_1Y_1 - b_2y_2$$

$$y_5^1 = b_4Y_4 - b_5Y_5$$

8

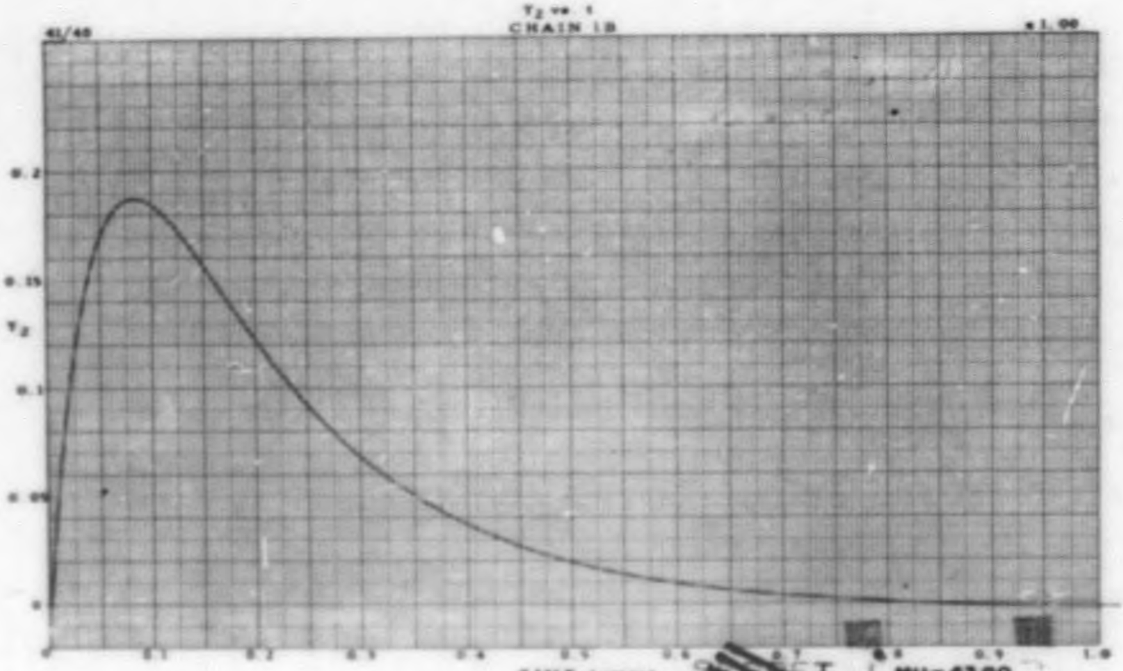


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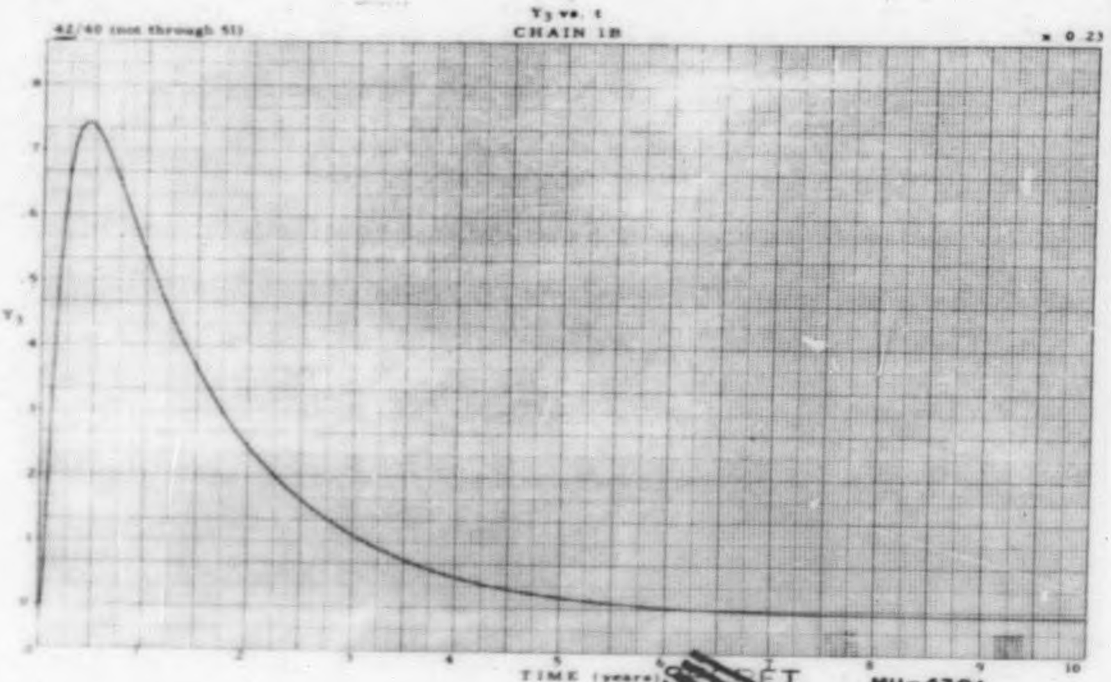
Figure 17



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MU-4381

Figure 18

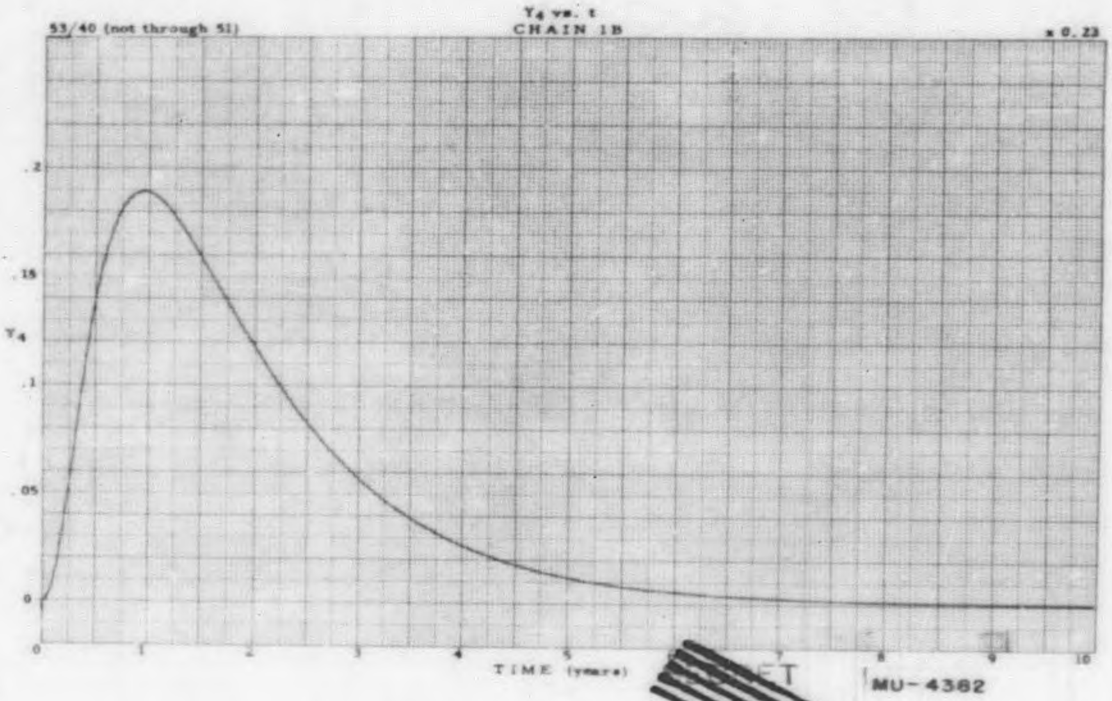

Y_4 vs. t
CHAIN 1B
53/40 (not through 51)
x 0.23
.2
.15
Y_4
.1
.05
0
0
1
2
3
4
5
6
7
8
9
10
TIME (years)
MU-4382

Figure 19

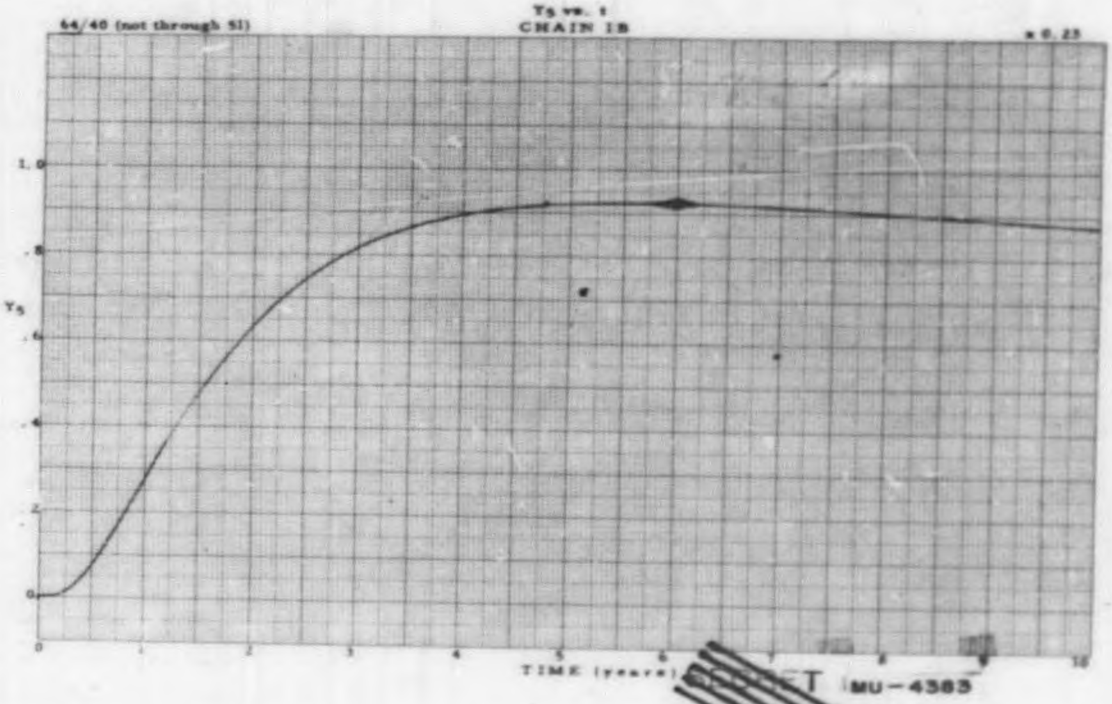

64/40 (not through 51)
Y5 vs. t
CHAIN IB
x 0.25
Y5
1.0
.8
.6
.4
.2
0
0
1
2
3
4
5
6
7
8
9
10
TIME (years)
~~SECRET~~
MU-4383

Figure 20

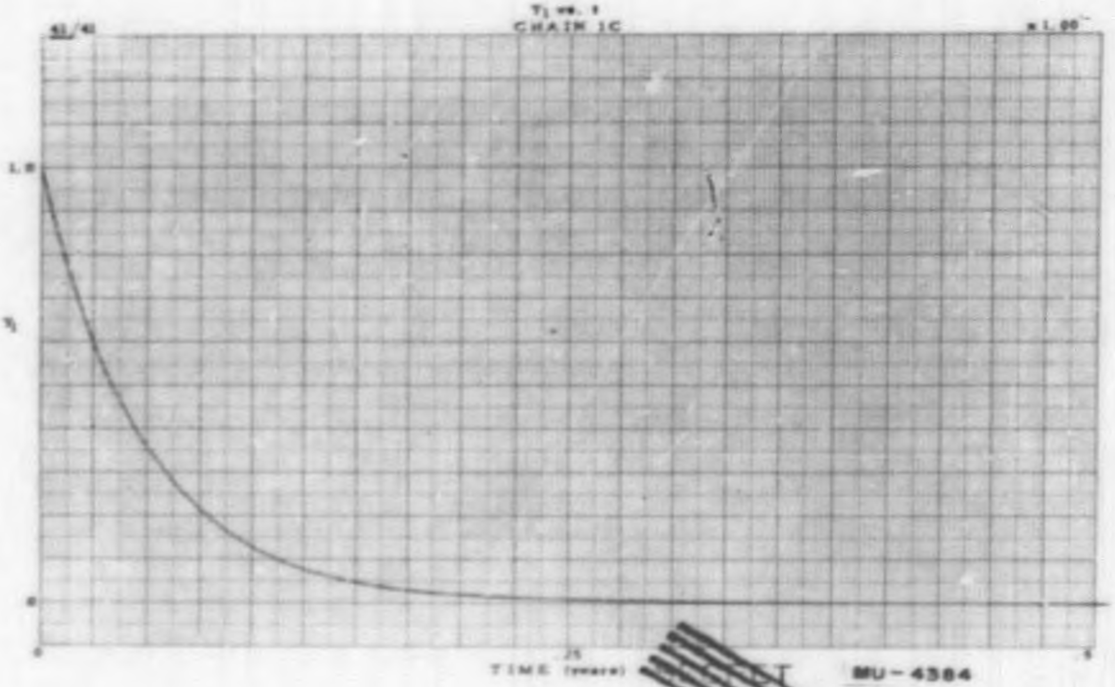
Chain 1c: 41-42-53-64 at 5 x 10^{14}

b_1 = 20.55 (2.4)

b_2 = 0.7885 (60)

b_3 = 2.365 (20)

b_4 = 0.01577 (3000)



MU-4384

Figure 21

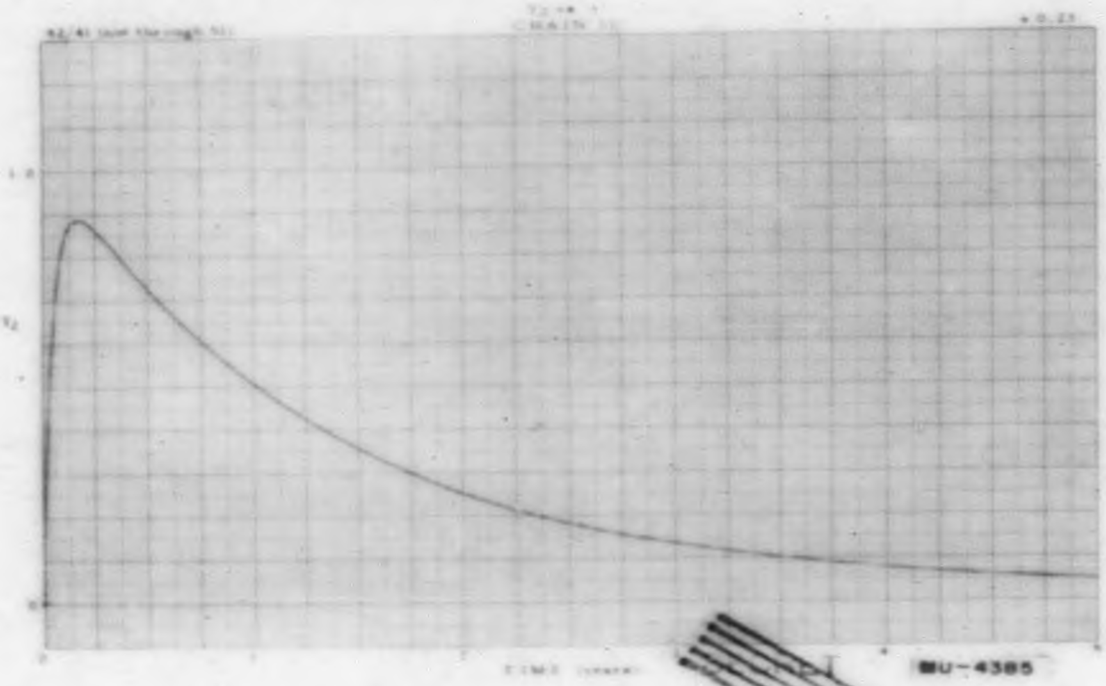


Figure 22

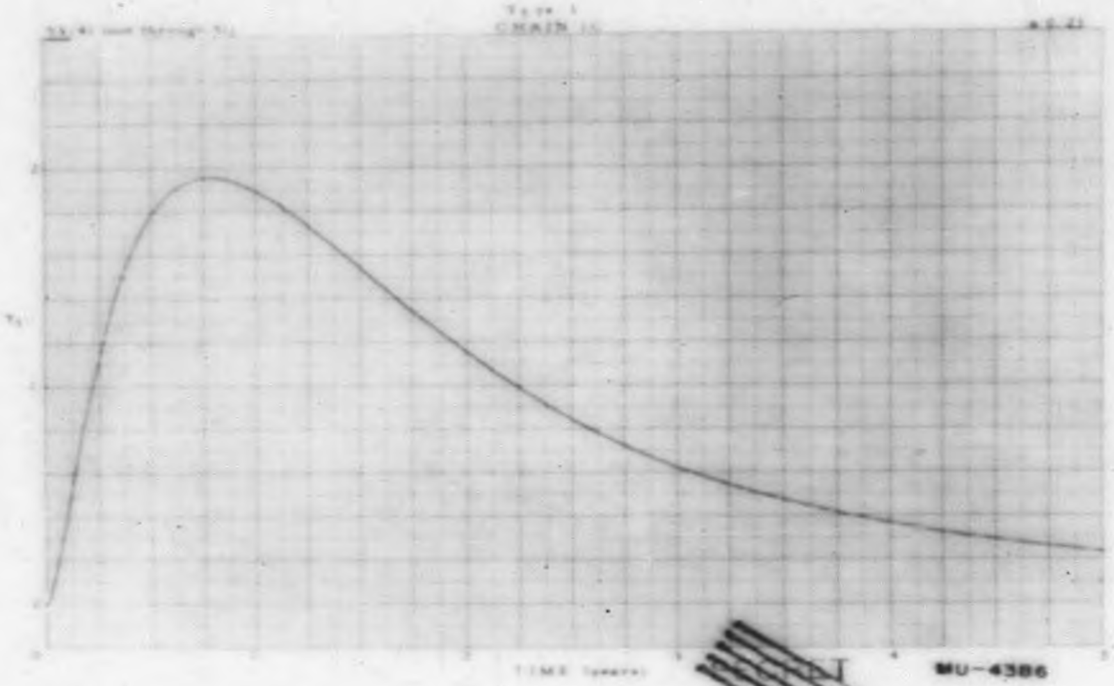


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Figure 23

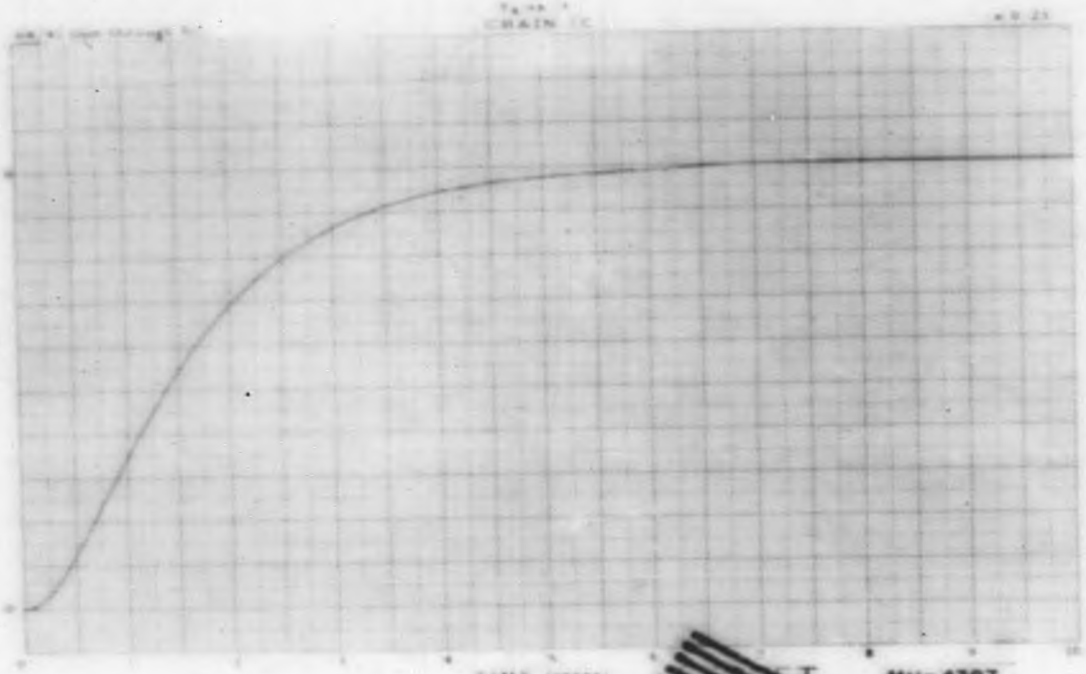

Figure 24

Chain 1D: 42-53-64 at 5×10^{14}

$b_1 = 0.7885$ (60)

$b_2 = 2.365$ (20)

$b_3 = 0.01577$ (3000)

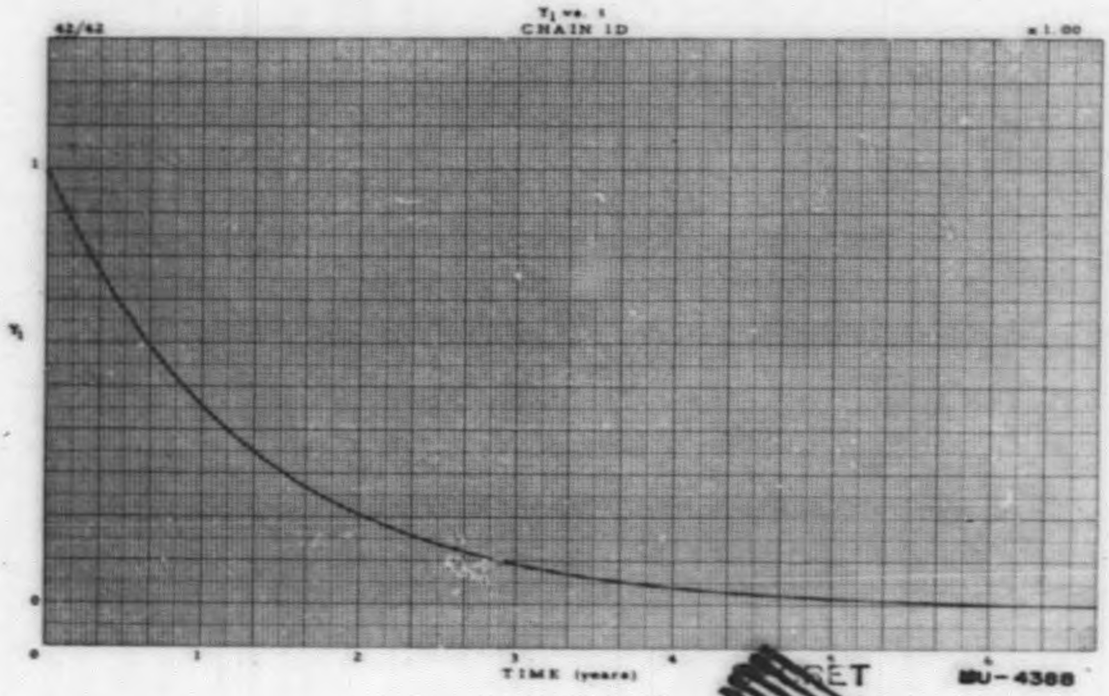

Figure 25

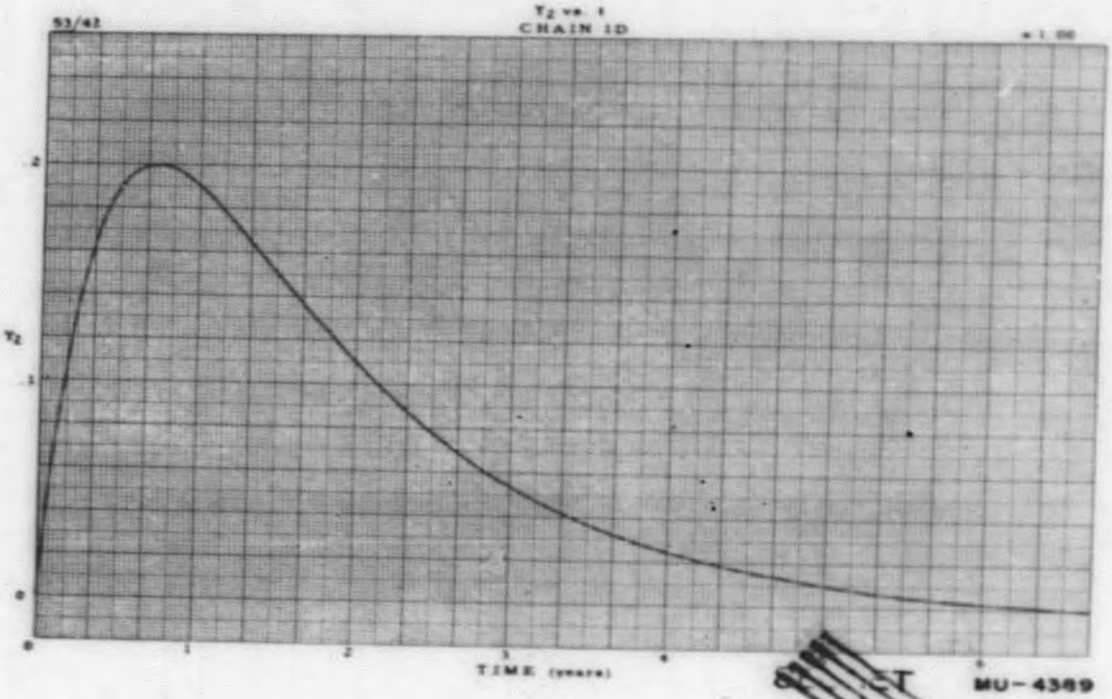

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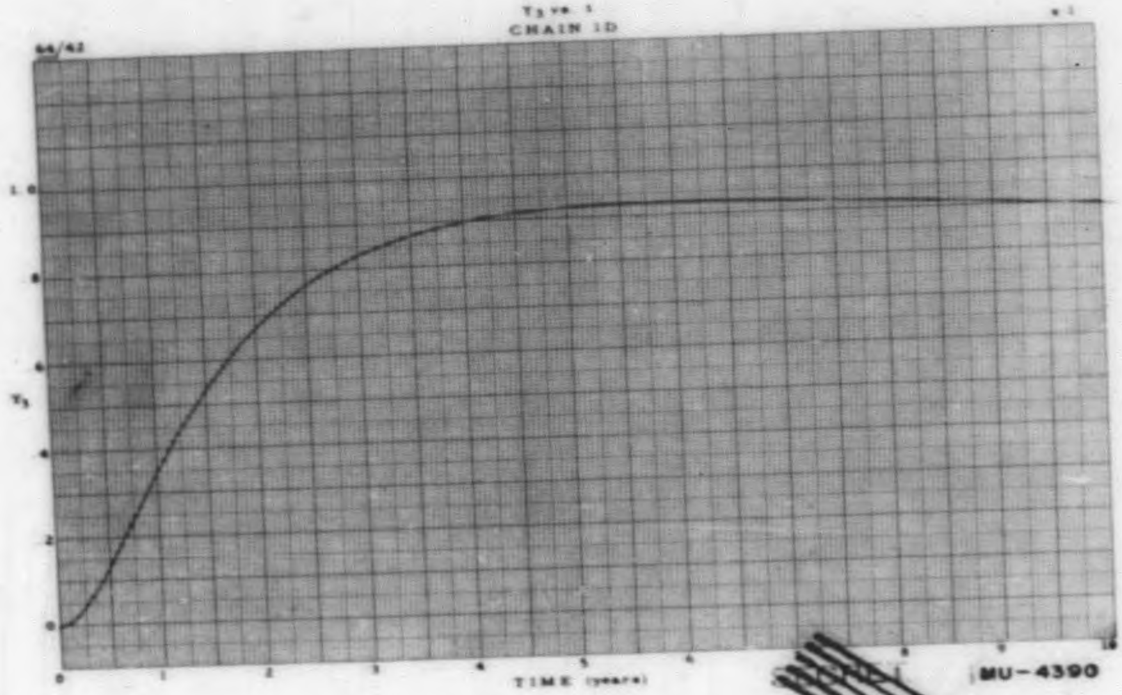
Figure 26



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MU-4390

Figure 27

Chain 2: 49-40-41-51-62-48

$b_1 =$ 16.48

$b_2 =$ $\underline{6.466}$* and 5.52**

$b_3 =$ 20.55

$b_4 =$ 14.02

$b_5 =$ 1.924

$b_6 =$ 6.907

---

*Corresponds to $\sigma_t^{40} = 410$

**Corresponds to $\sigma_t^{40} = 350$

NOTE: $\sigma_t^{62} = 23$ in curves; = 10 in factor. 41 factor = 0.311; checked.

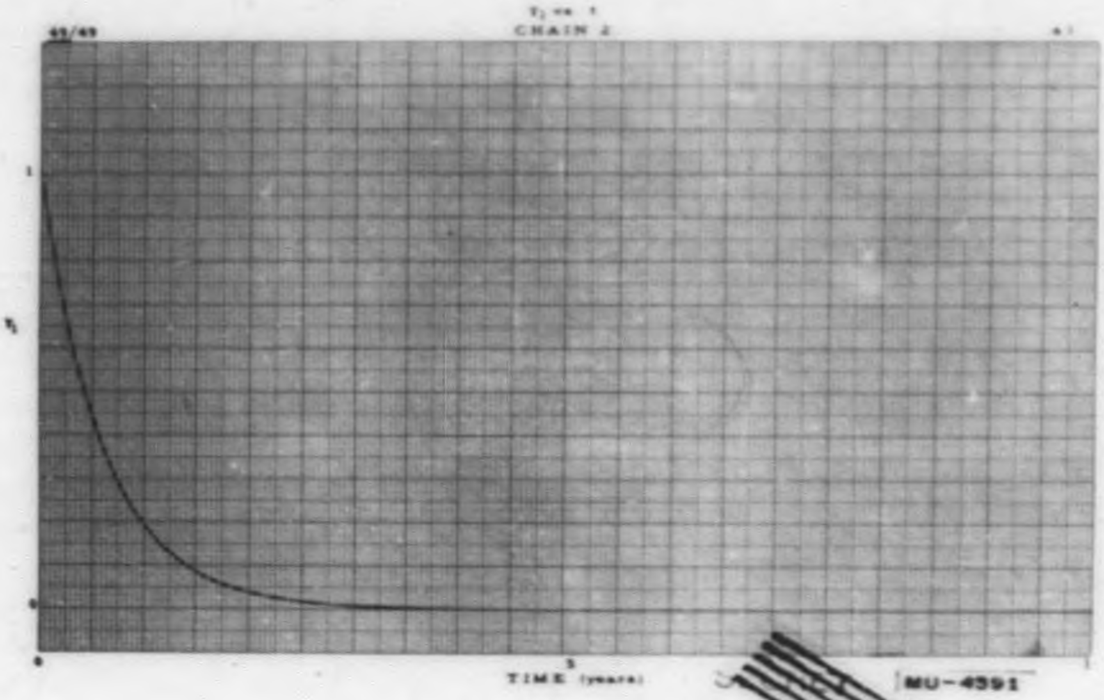

Figure 28

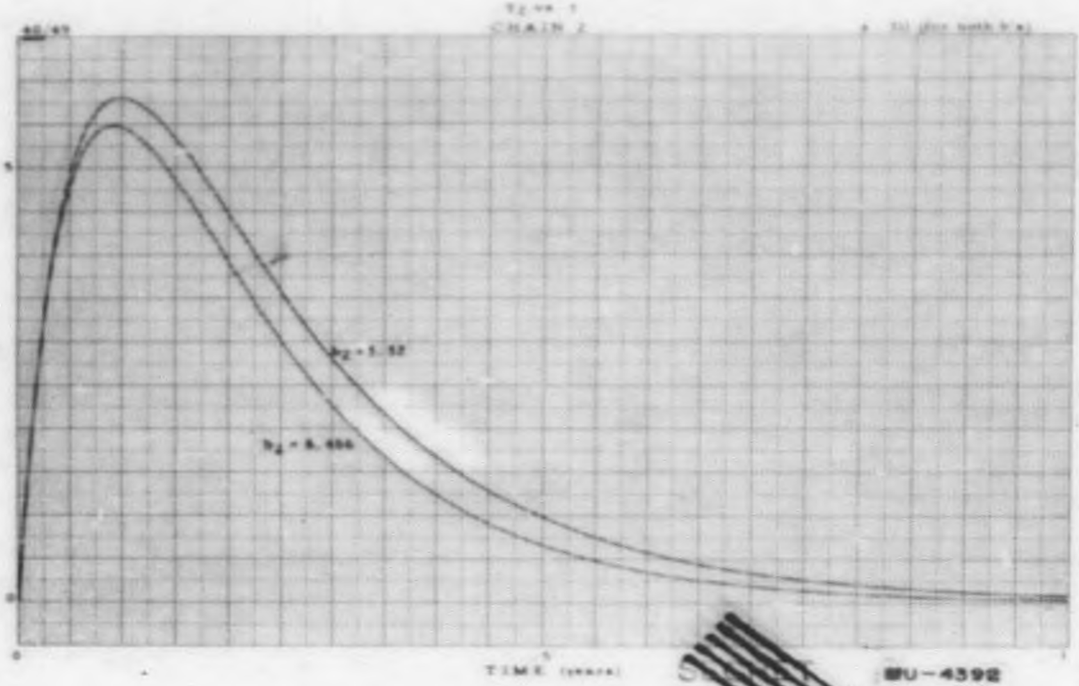

Figure 29

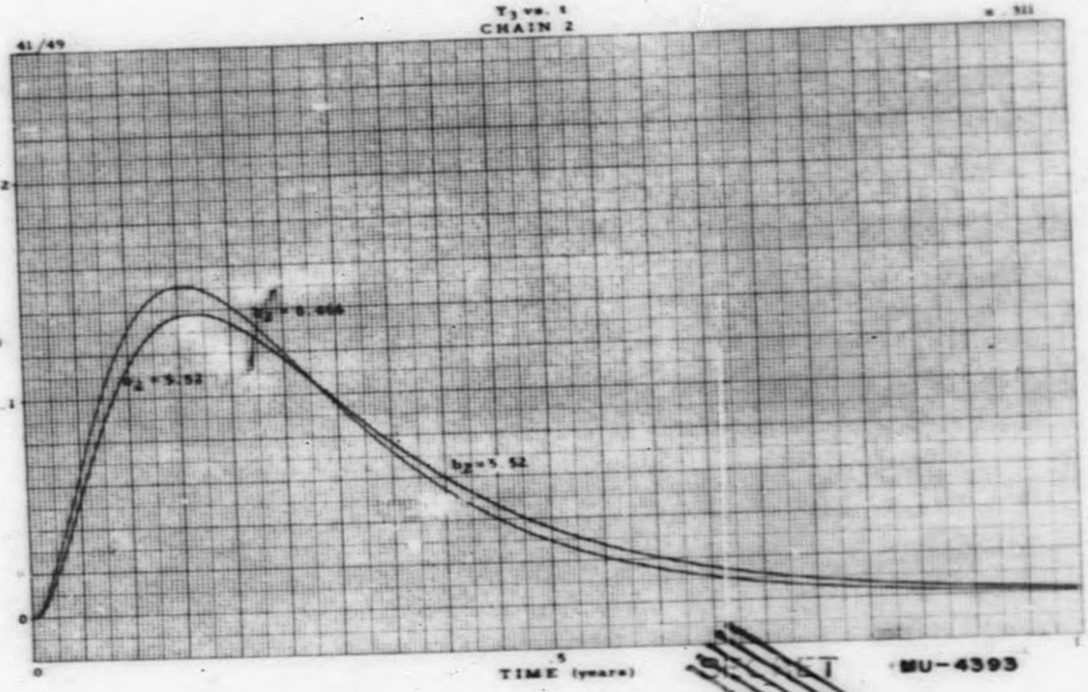

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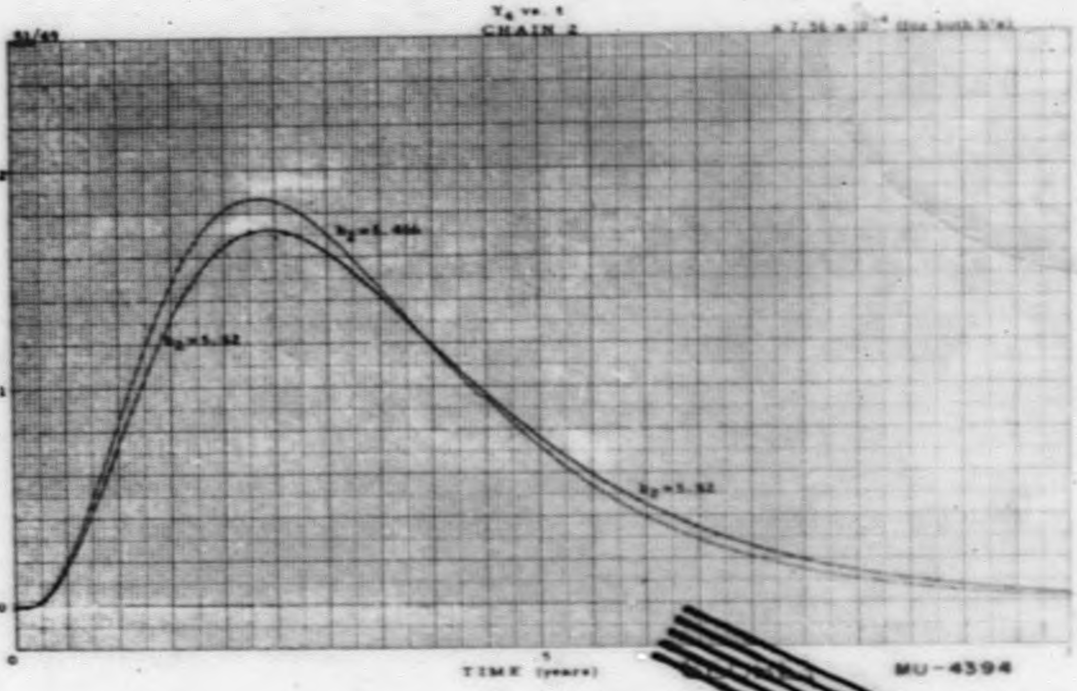
Figure 30



[redacted]

Figure 31

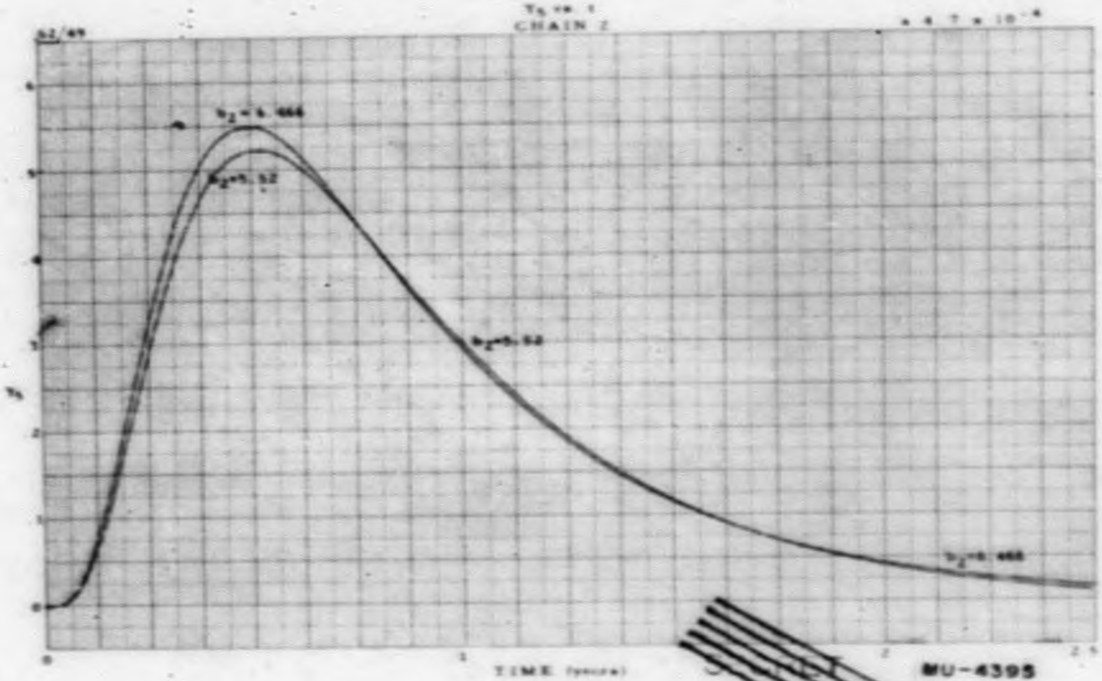


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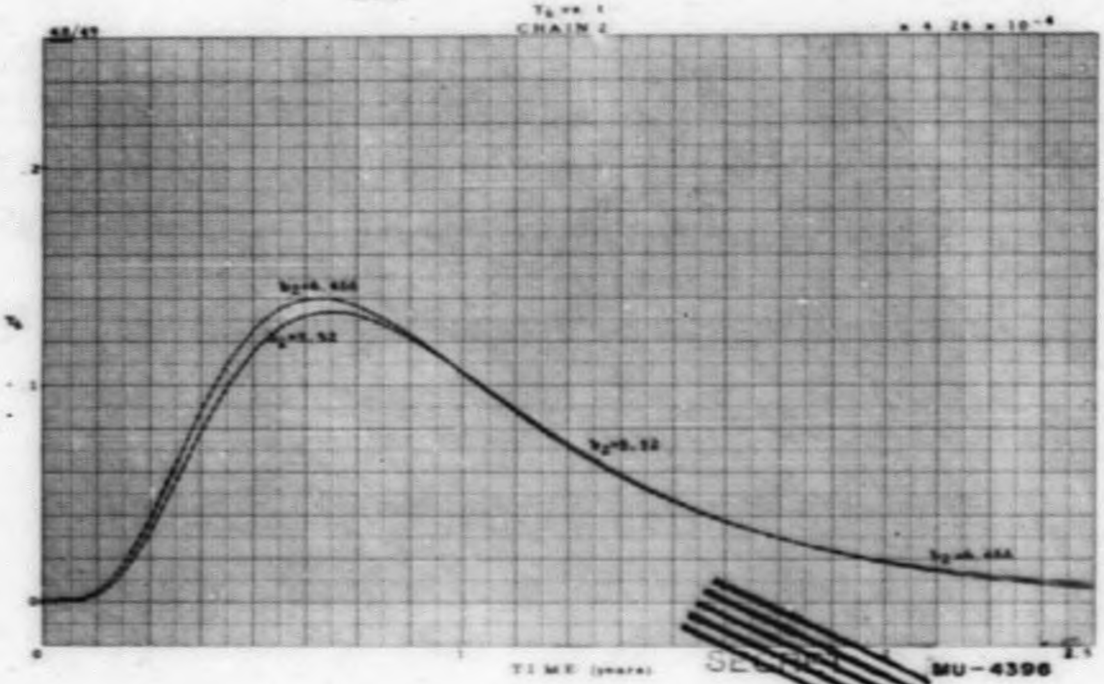
Figure 32



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MU-4396

Figure 33

Chain 2A: $\underbrace{48\text{-}49\text{-}40\text{-}41}_{\substack{\text{from chain 1a} \\ (1000\ Y_4\ \text{input})}}\text{-}51\text{-}62\text{-}48$

$${}_{41}b_4 = 20.55$$

$${}_{51}b_5 = 14.019$$

$${}_{62}b_6 = 1.924$$

$${}_{48}b_7 = 6.9073$$

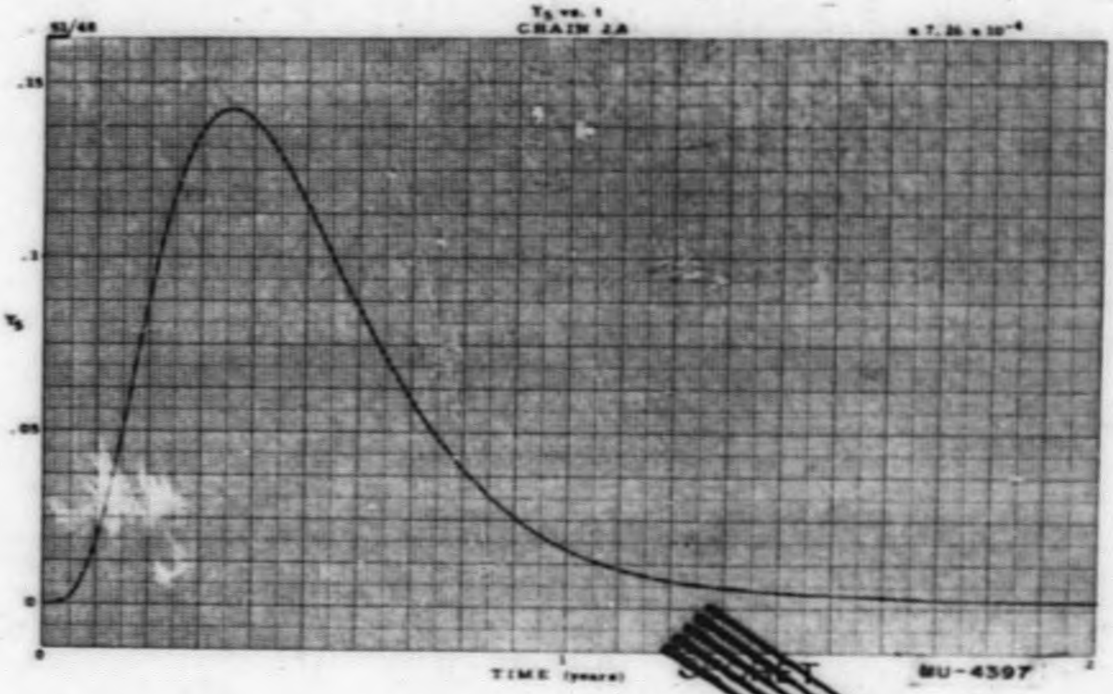
51/48
T5 vs. t
CHAIN 2A
a 7.26 x 10^-4
.15
.1
T5
.05
0
0
1
2
TIME (years)
MU-4397

Figure 34

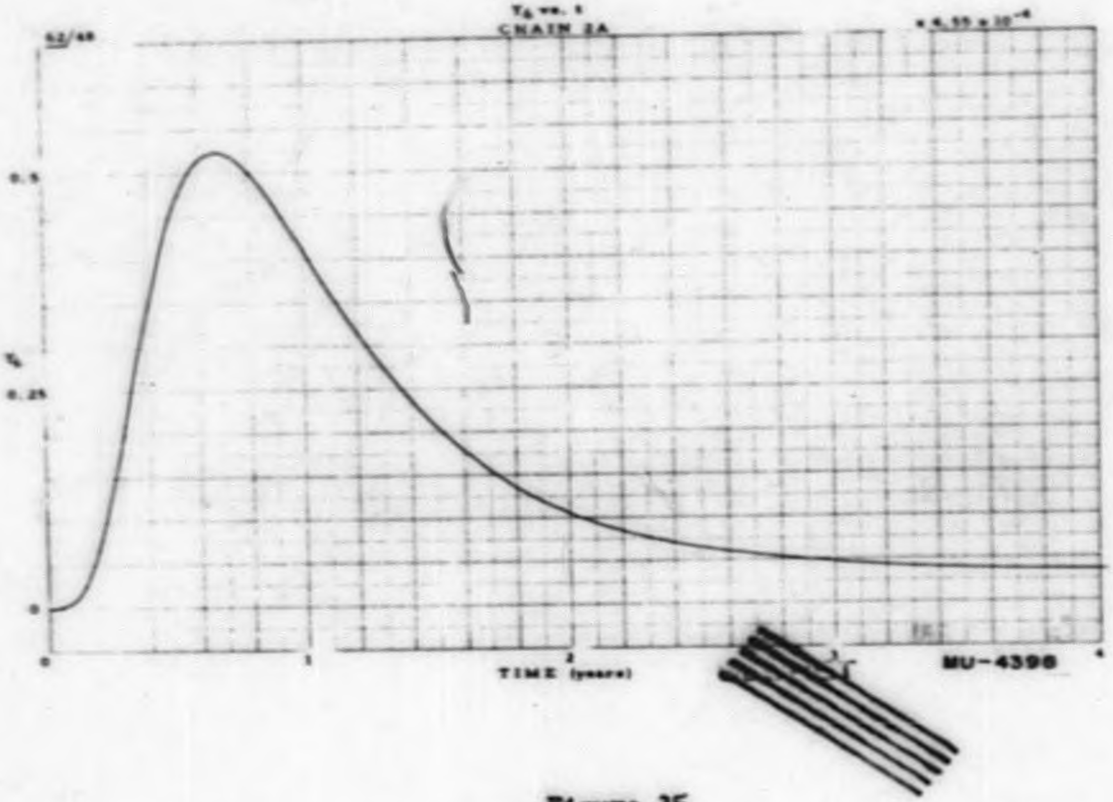
Y_4 vs. t
CHAIN 2A
× 4.55 × 10^{-4}
52/48
0.5
Y_4
0.25
0
0
1
2
3
4
TIME (years)
MU-4398

Figure 35

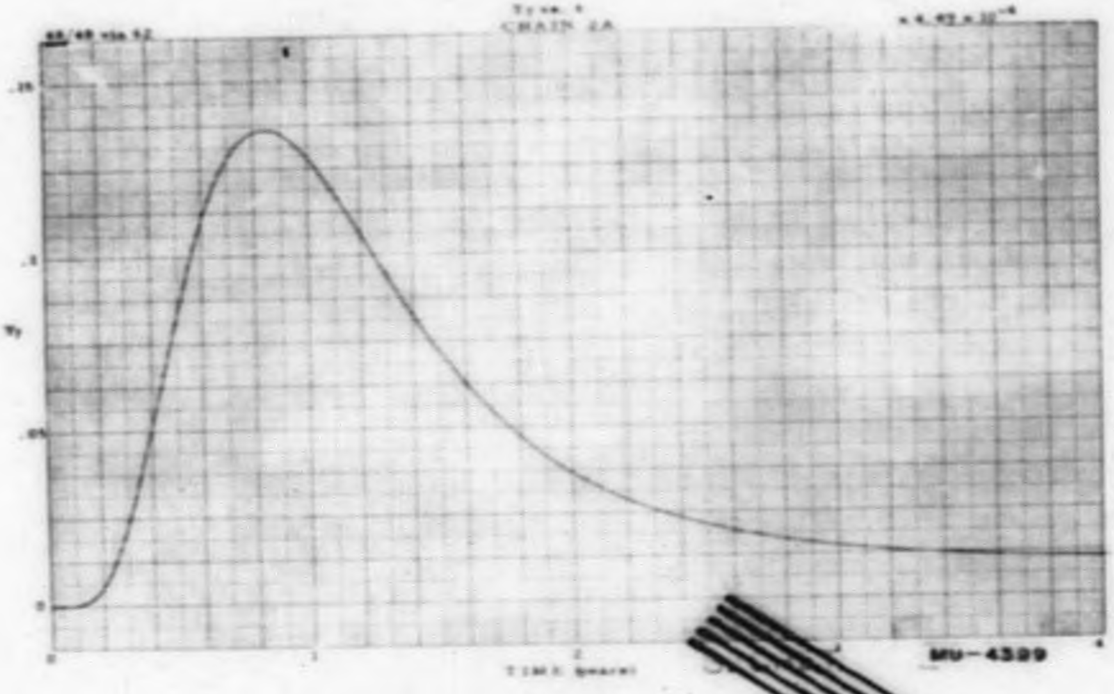

Figure 36

Chain 2b: $\underbrace{40-41}_{\text{from chain 1b (input 500 } Y_2\text{)}}-51-62-48$

$_{41}b_2 = 20.55$

$_{51}b_3 = 14.019$

$_{62}b_4 = 1.924$

$_{48}b_5 = 6.9073$

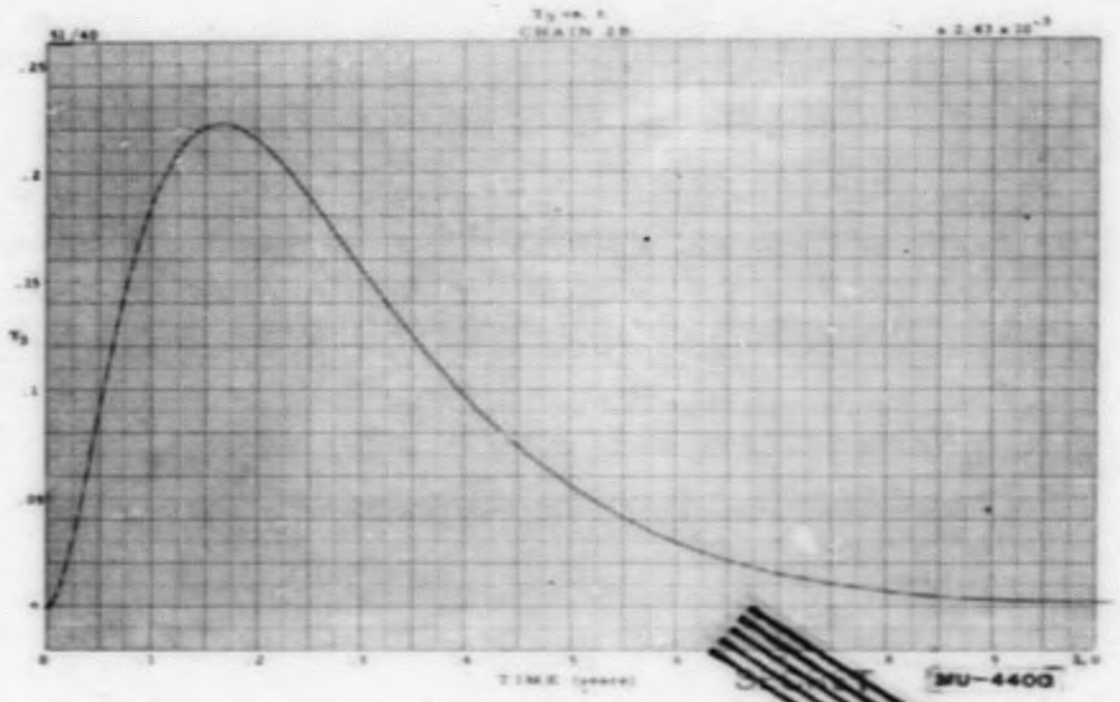
Y3 vs. t
CHAIN 2B
51/40
a 2.43 x 10^-3
.25
.2
.15
Y3
.1
.05
0
0
1
2
3
4
5
6
7
8
9
10
TIME (years)
MU-4400

Figure 37

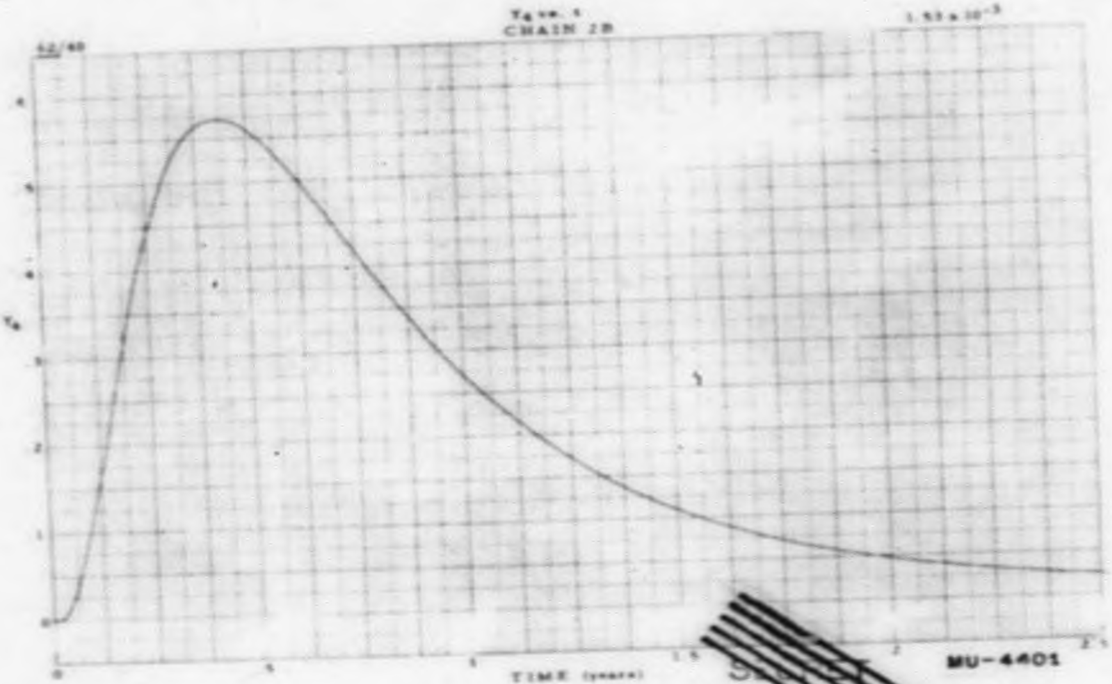
T_4 vs. t
CHAIN 2B
1.53×10^{-3}
12/40
T_4
6
5
4
3
2
1
0
0
.5
1
1.5
2
2.5
TIME (years)
MU-4401

Figure 38

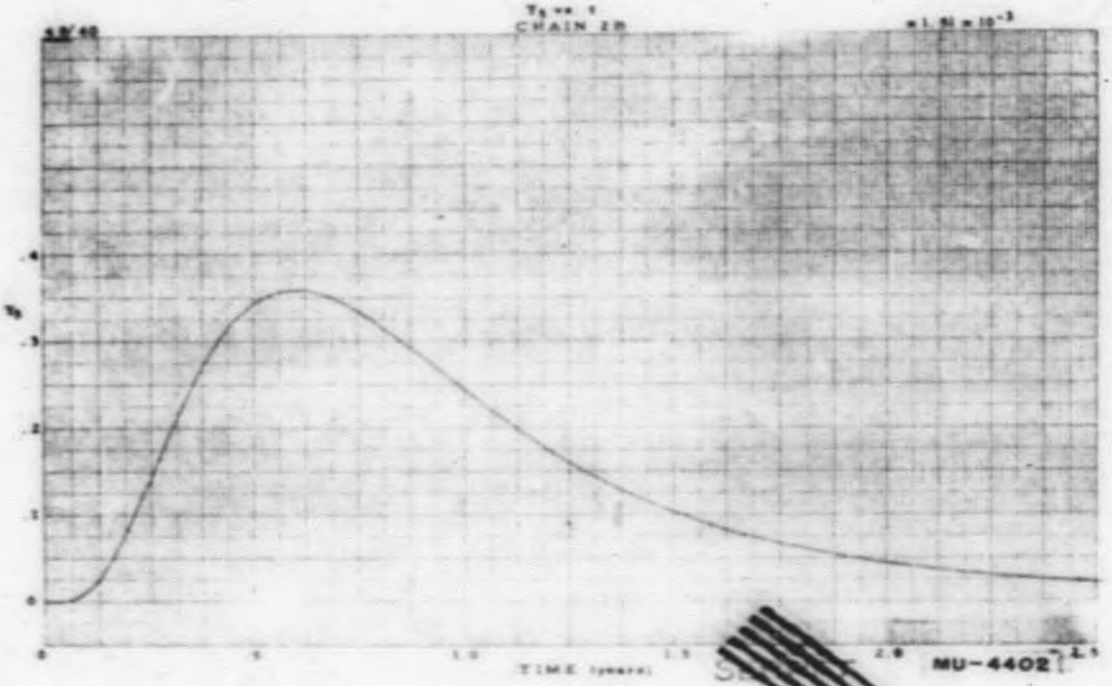
Y5 vs t
CHAIN 2B
α 1.51 x 10^-3
Y5
.4
.3
.2
.1
0
0
.5
1.0
1.5
2.0
2.5
TIME (years)
MU-4402

Figure 39

Chain 2c: 41-51-62-48 at 5 x 10^{14}

b_1 = 20.55 (2.4)

b_2 = 14.019 (3.5)

b_3 = 1.924 (25)

b_4 = 6.9073 (7)

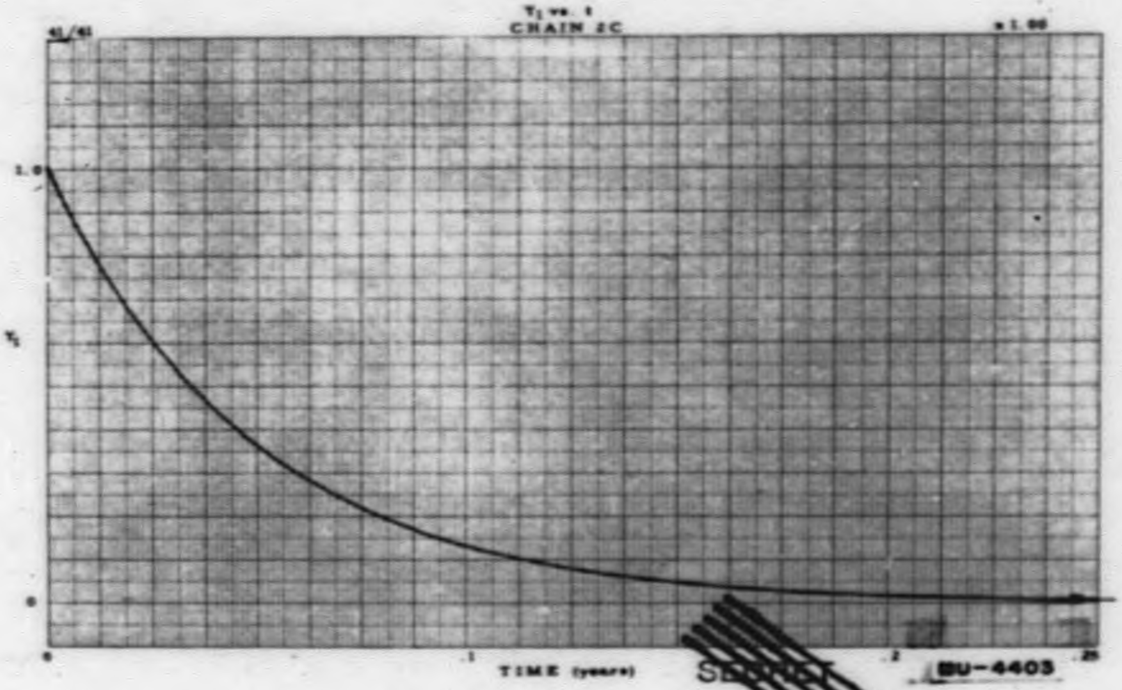

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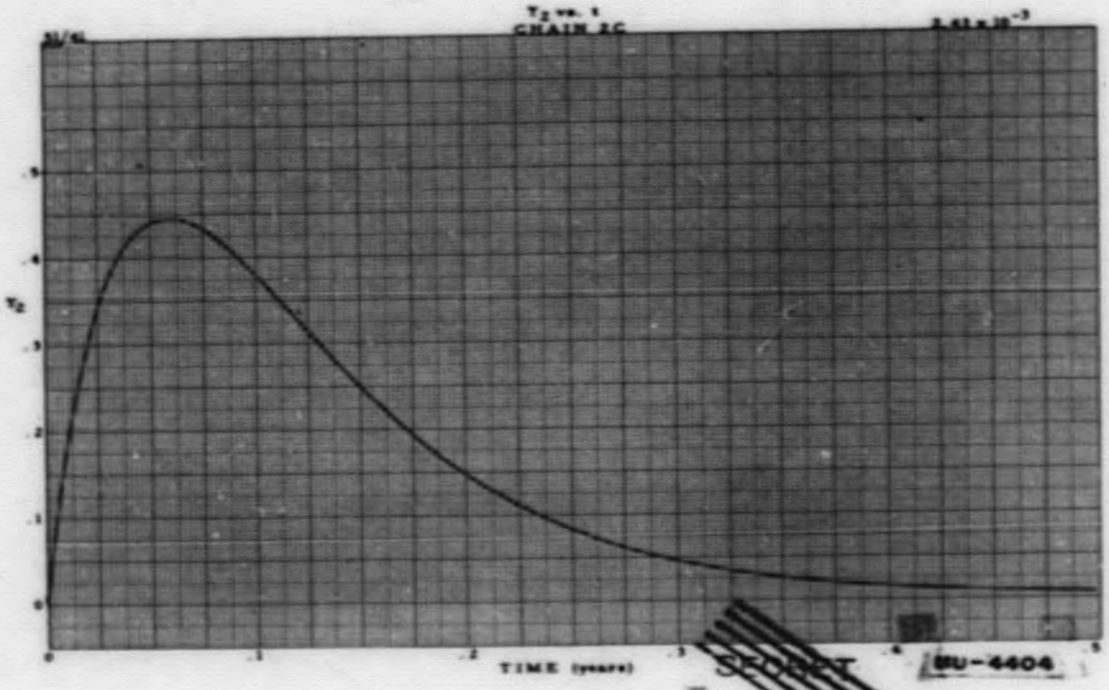
Figure 40



~~SECRET~~

Figure 41

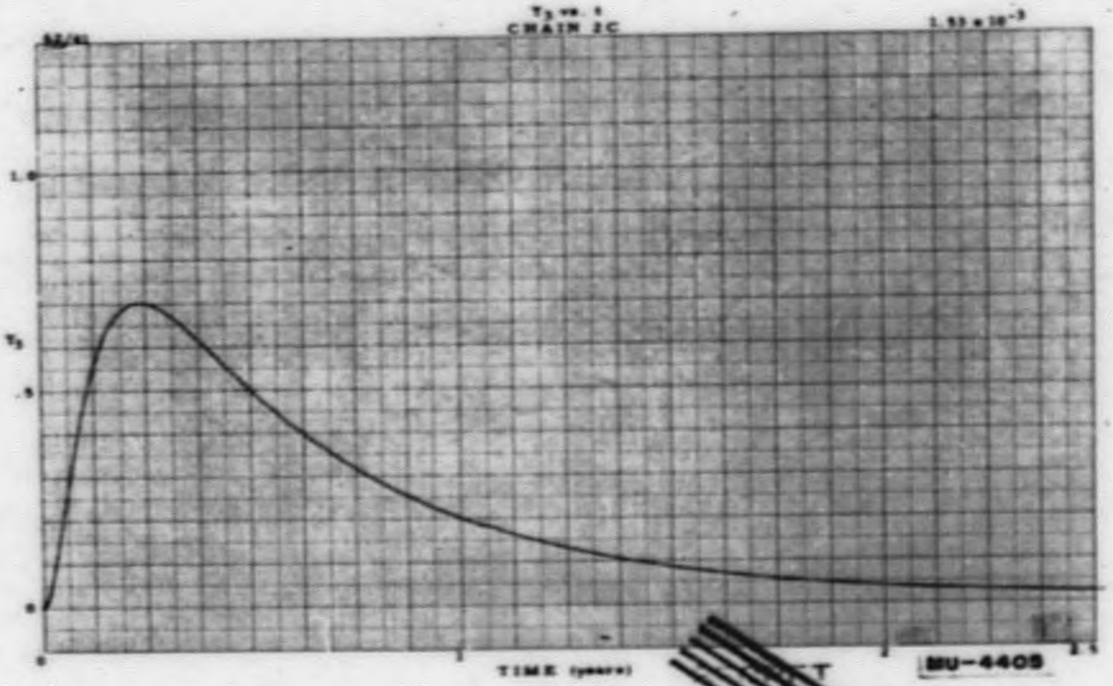


Figure 42

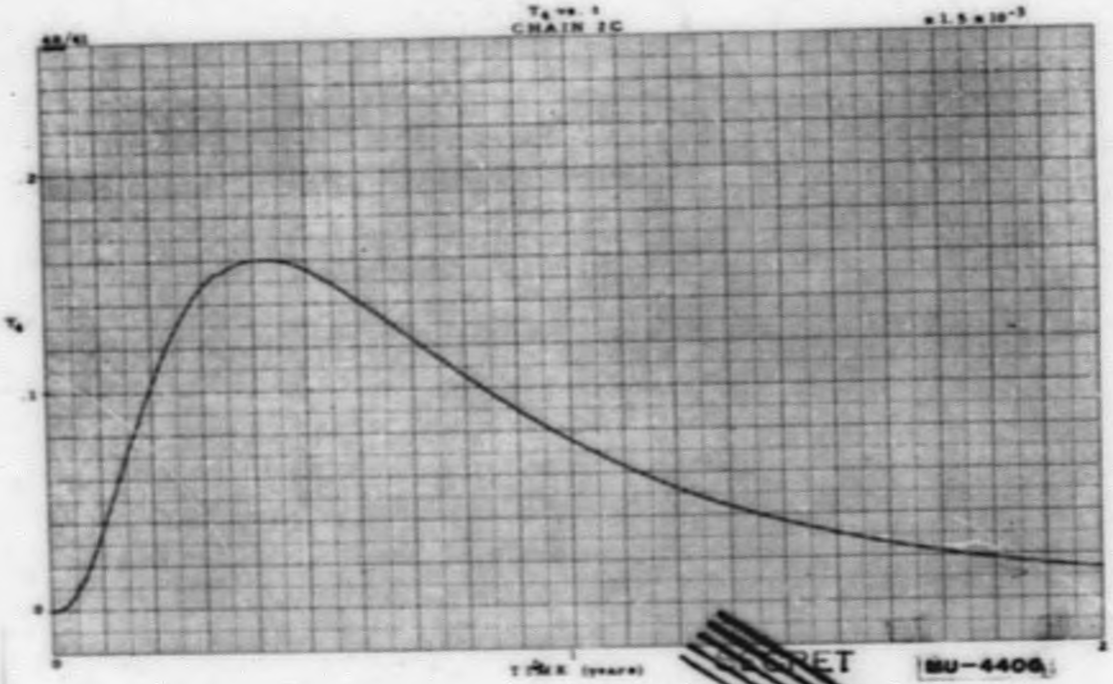


Figure 43

Chain 3: $\underbrace{49-40-41-51}_{\text{from chain 2}}-52-53-64$

$b_4 = 14.019$

$b_5 = 126.2$

$b_6 = 2.365$

$b_7 = 0.01577$

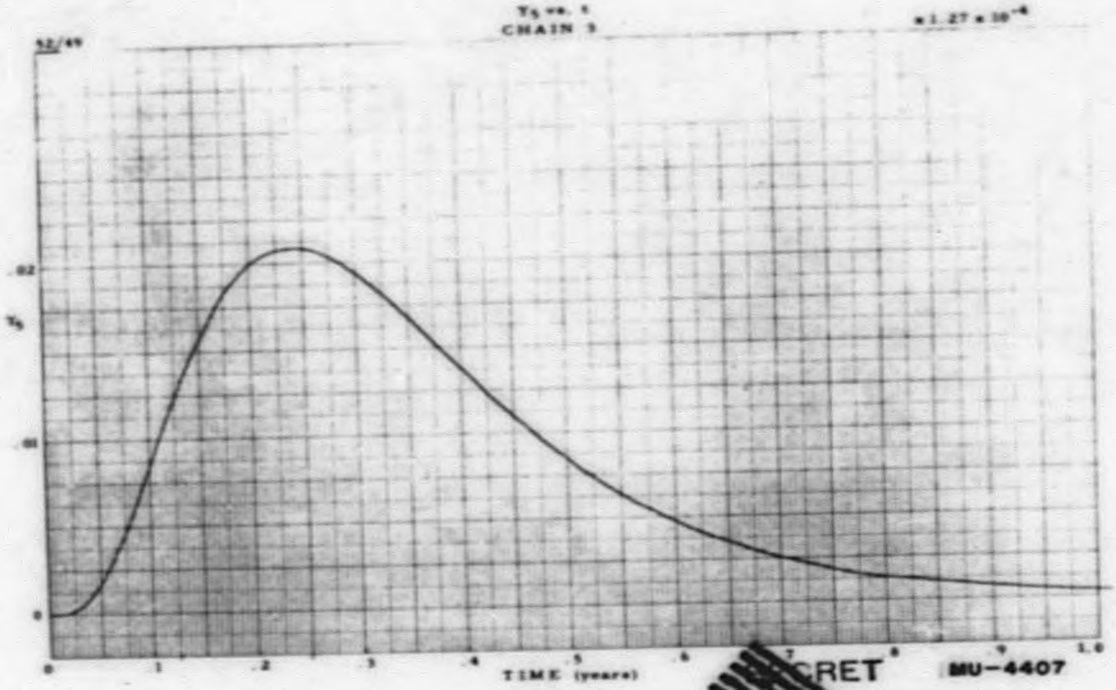
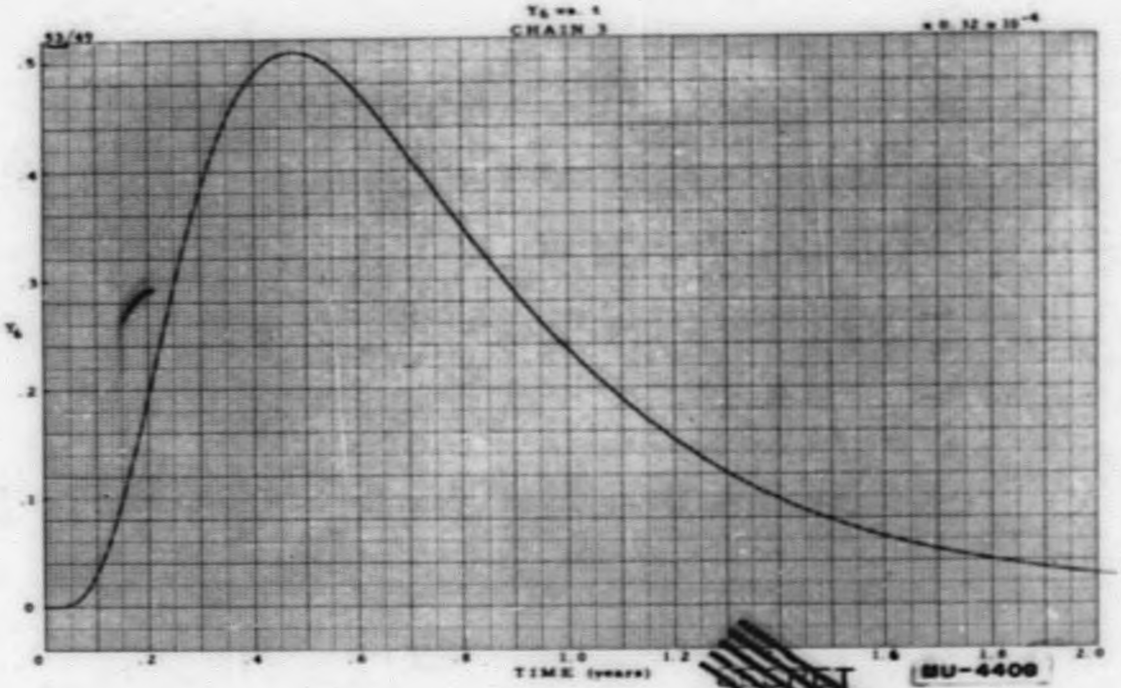

Y_5 vs. t
CHAIN 3
× 1.27 × 10^-6
52/49
Y_5
.02
.01
0
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
TIME (years)
~~SECRET~~
MU-4407

Figure 44



~~SECRET~~

MU-4406

Figure 45

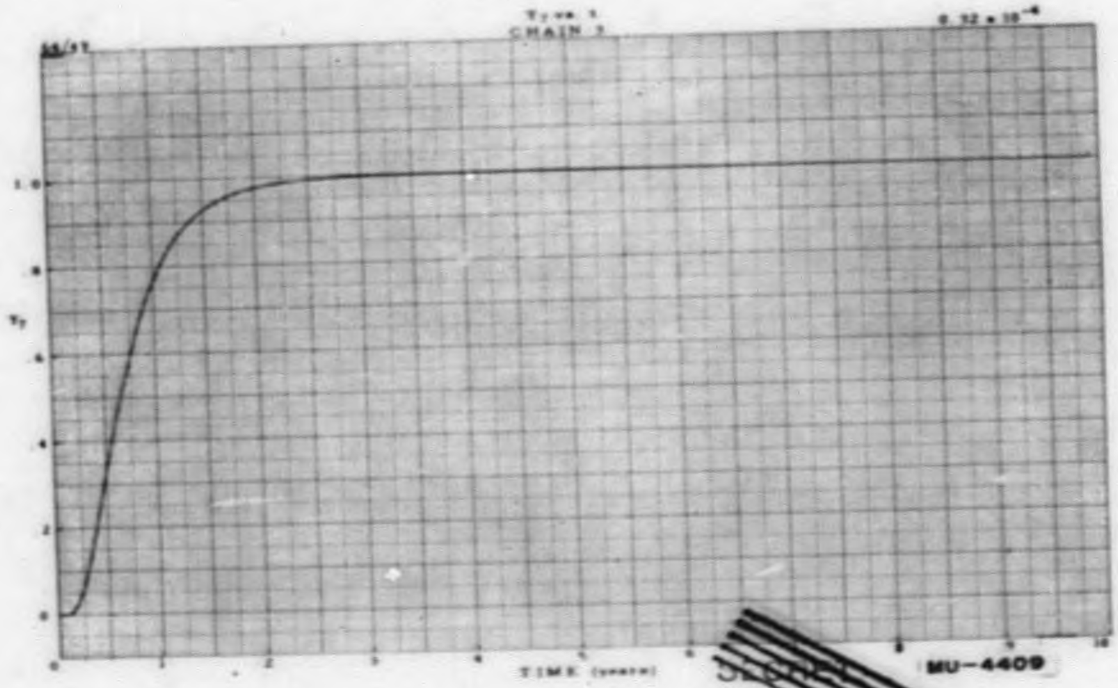
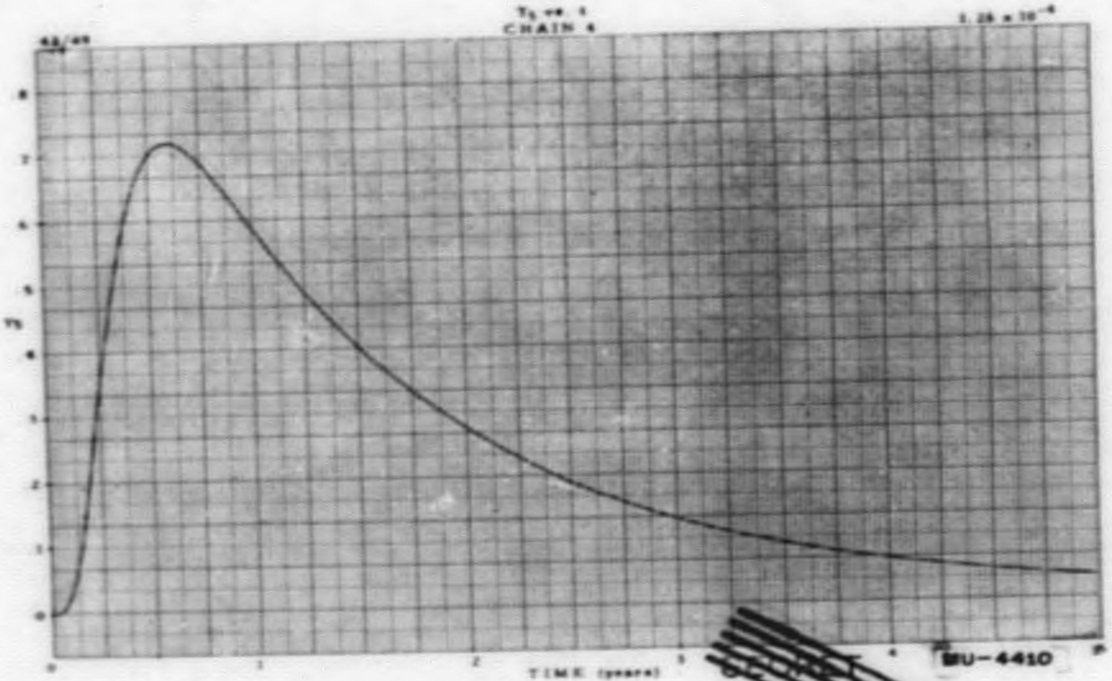


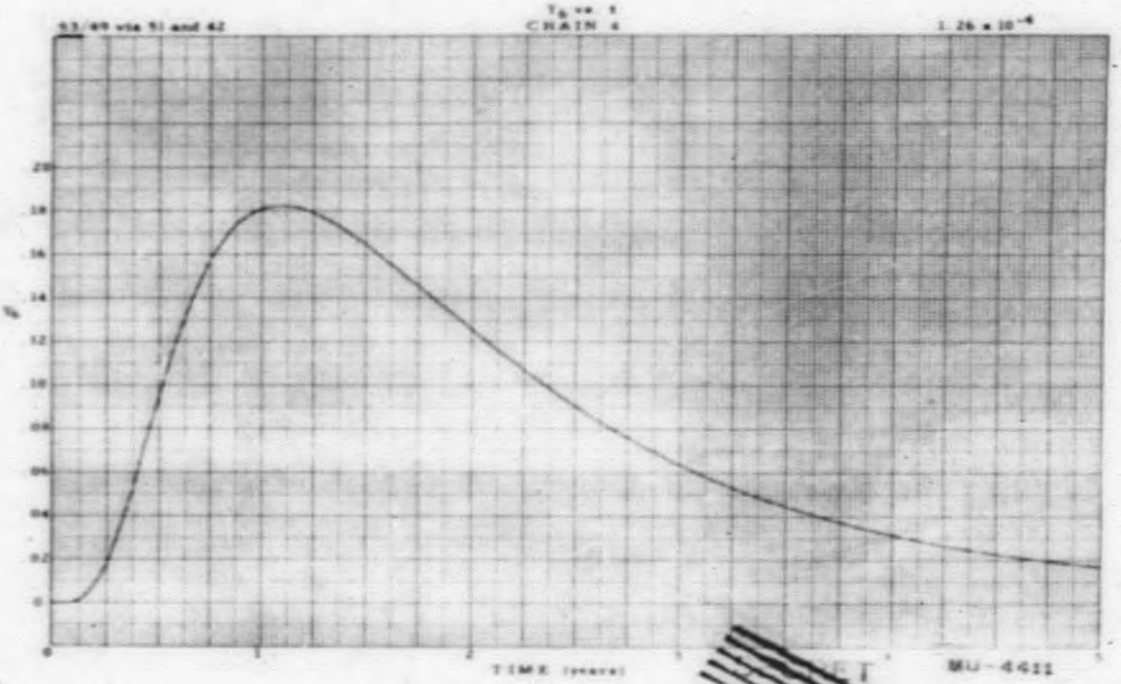
Figure 46

Chain 4: $\underbrace{49\text{-}40\text{-}41\text{-}51}_{\text{from chain 2}}\text{-}42\text{-}53\text{-}64$



~~SECRET~~

Figure 47



~~SECRET~~

Figure 48

DECLASSIFIED

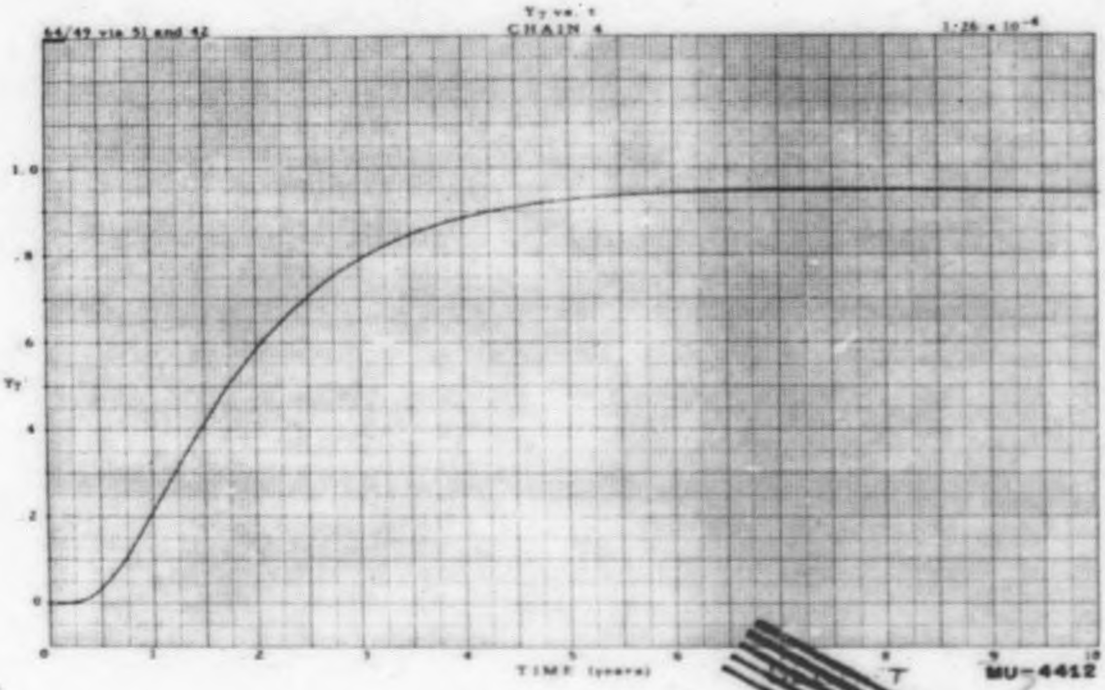

64/47 via 51 and 42
Y7 vs. t
CHAIN 4
1.26 x 10^-4
1.0
.8
.6
.4
.2
0
Y7
0
1
2
3
4
5
6
7
8
9
10
TIME (years)
MU-4412

Figure 49

Chain 6: 51-52-53-64

$b_1 = 14.019$

$b_2 = 126.2$

$b_3 = 2.365$

$b_4 = 0.01577$

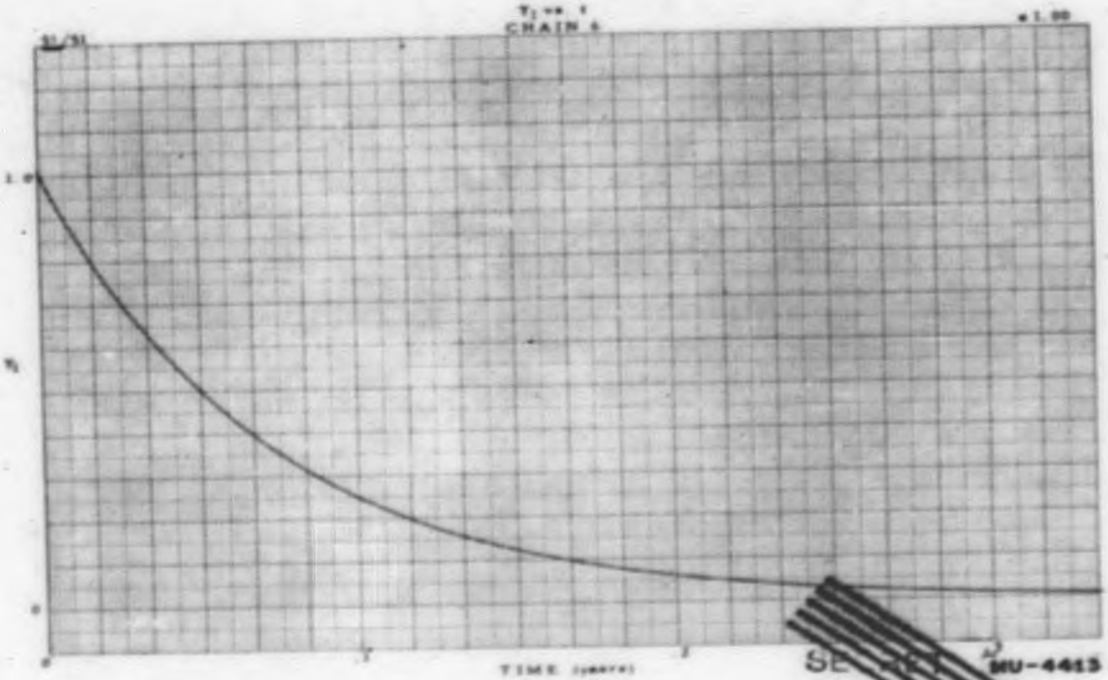
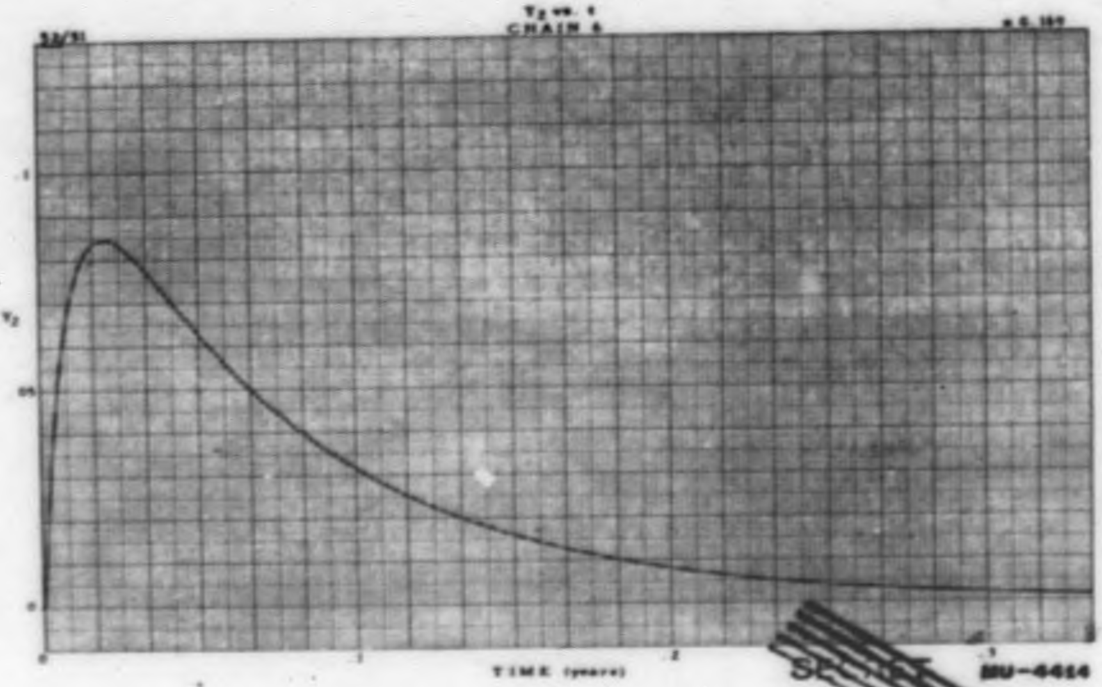


Figure 50



~~SECRET~~

Figure 51

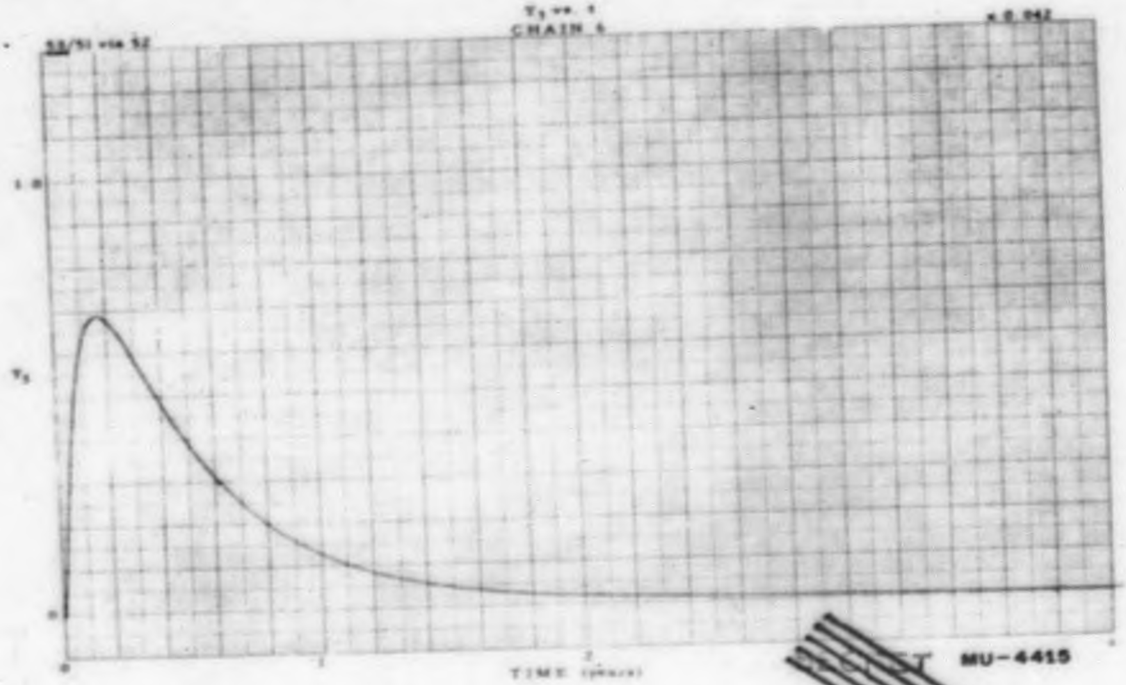
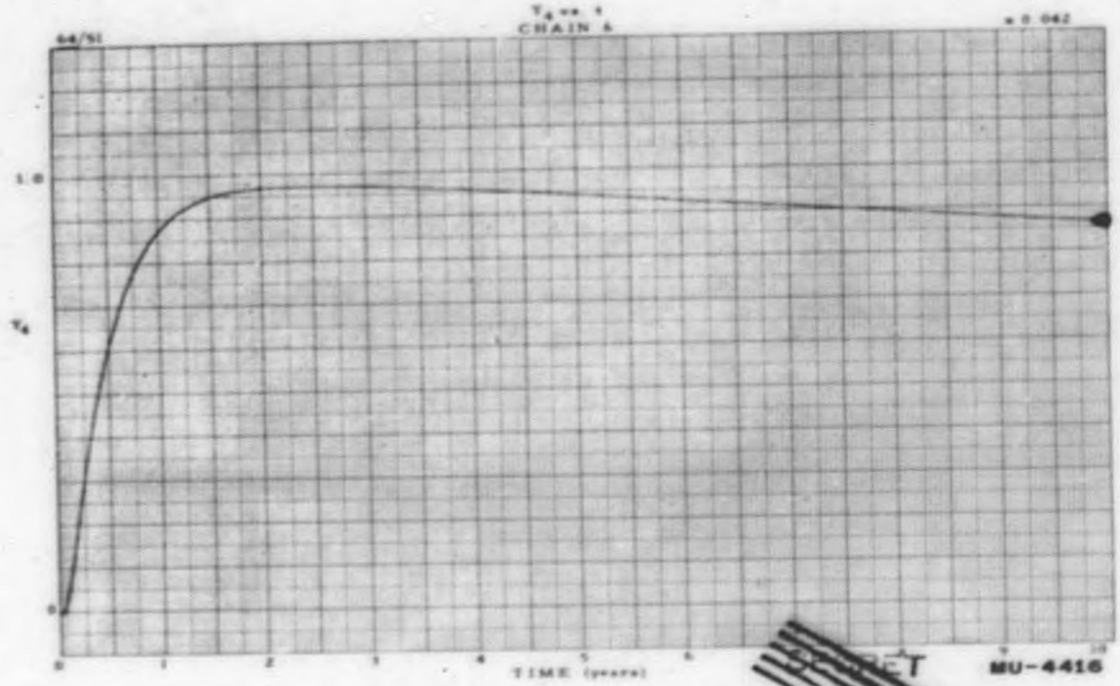

53/51 via 52
Y5 vs. t
CHAIN 6
x 0.042
1.0
Y5
0
0
1
2
4
TIME (years)
~~SECRET~~ MU-4415

Figure 52



~~SECRET~~

MU-4416

Figure 53

Chain 7: 51-42-53-64 at 5×10^{14}.

$b_1 = 14.019$

$b_2 = 0.7885$

$b_3 = 2.365$

$b_4 = 0.01577$

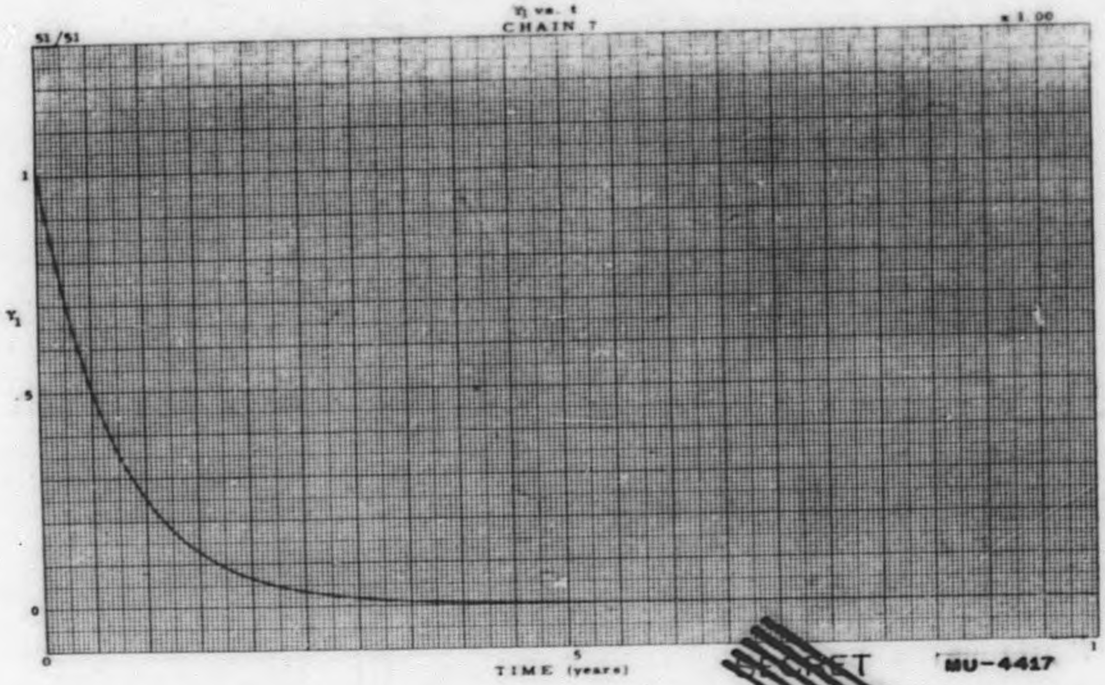
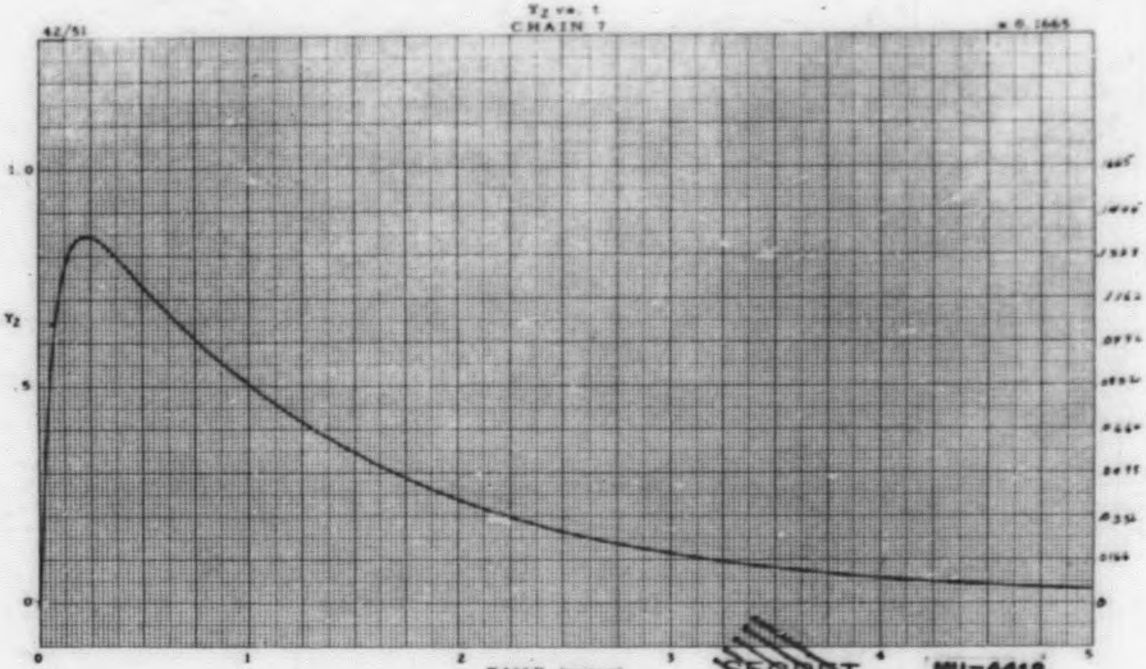


Figure 54



~~SECRET~~ MU-4418

Figure 55

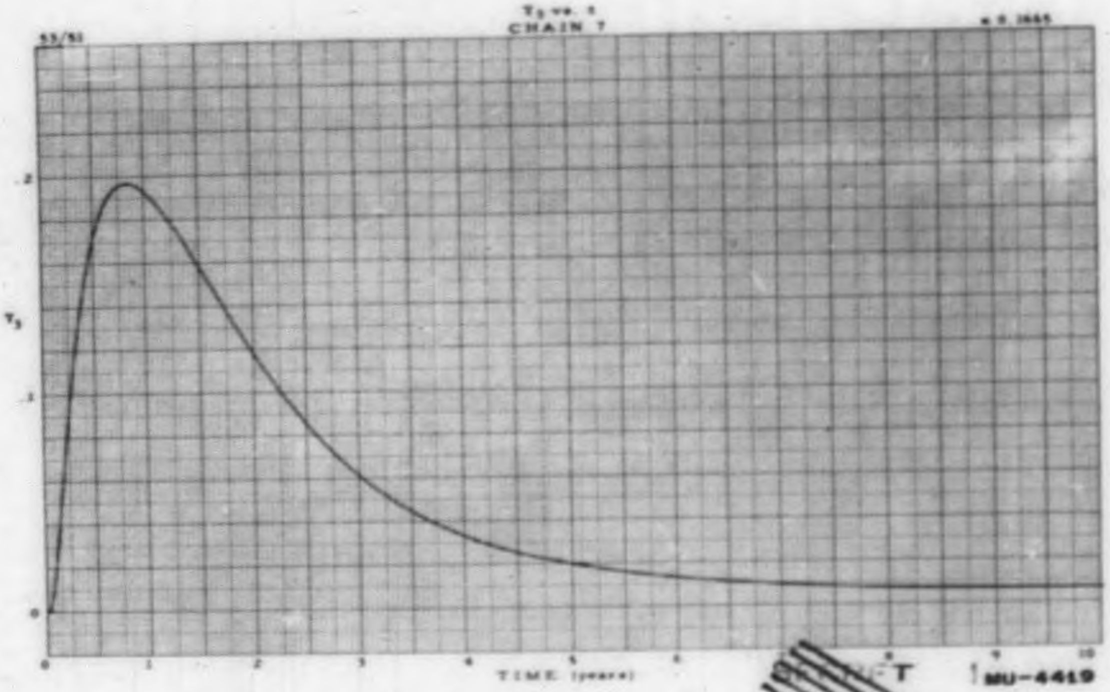


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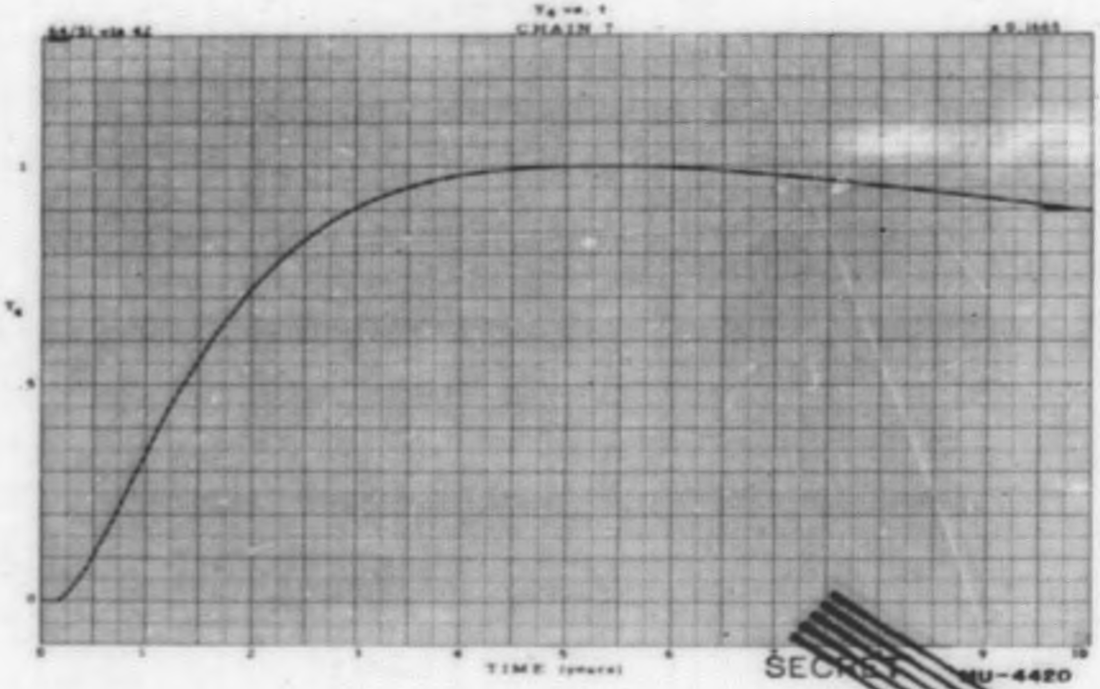
Figure 56



~~SECRET~~ MU-4420

Figure 57

Chain 9: $51-\underset{(23)}{62}-\underset{(438)}{48}-49-40-41-42$ at 5×10^{14}

$b_1 = 14.019$

$b_2 = 1.924$

$b_3 = 6.9073$

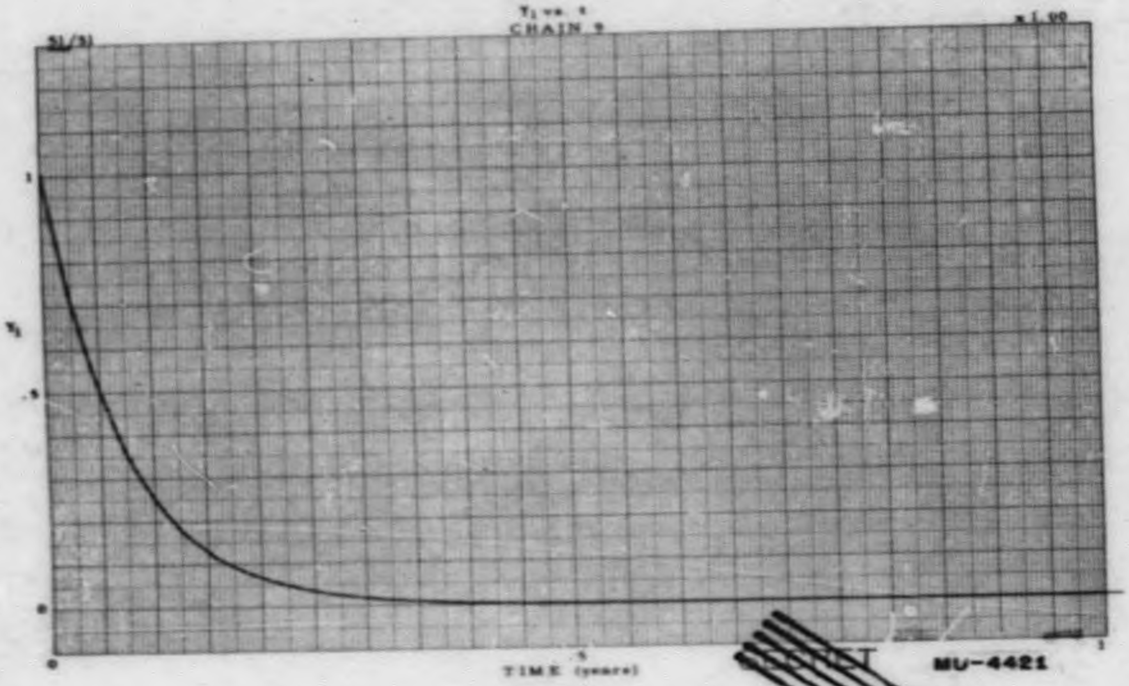
$b_4 = 16.480$

$b_5 = 6.466$

$b_6 = 20.55$

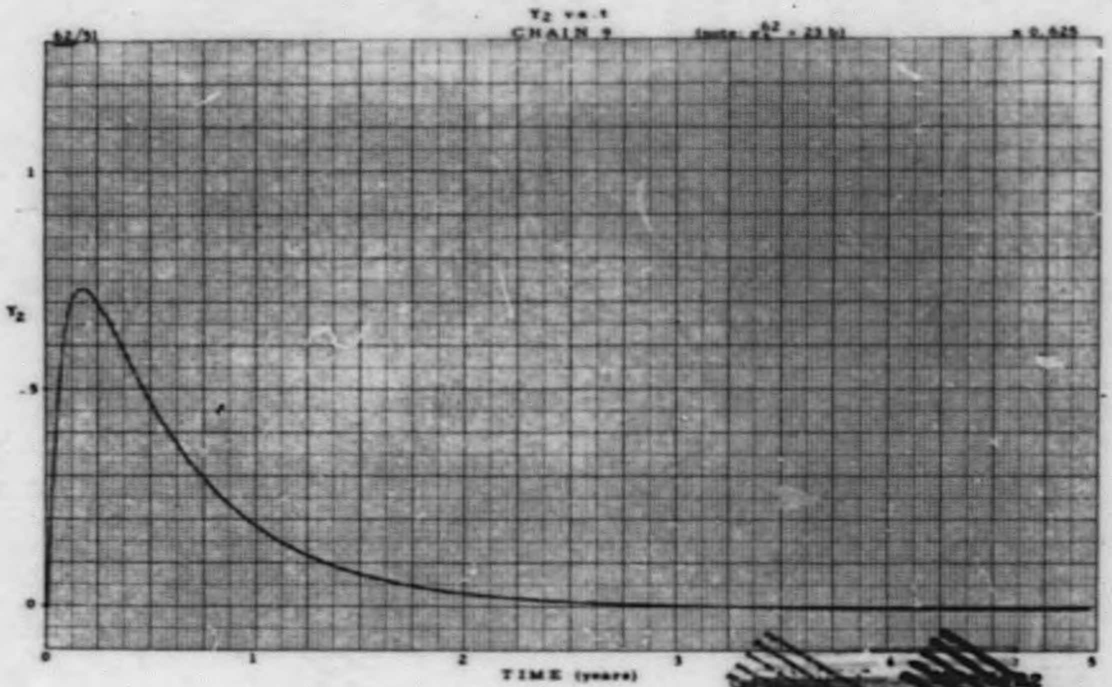
$b_7 = 0.7885$

NOTE: $\sigma_t^{62} = 23$; factors use 10)



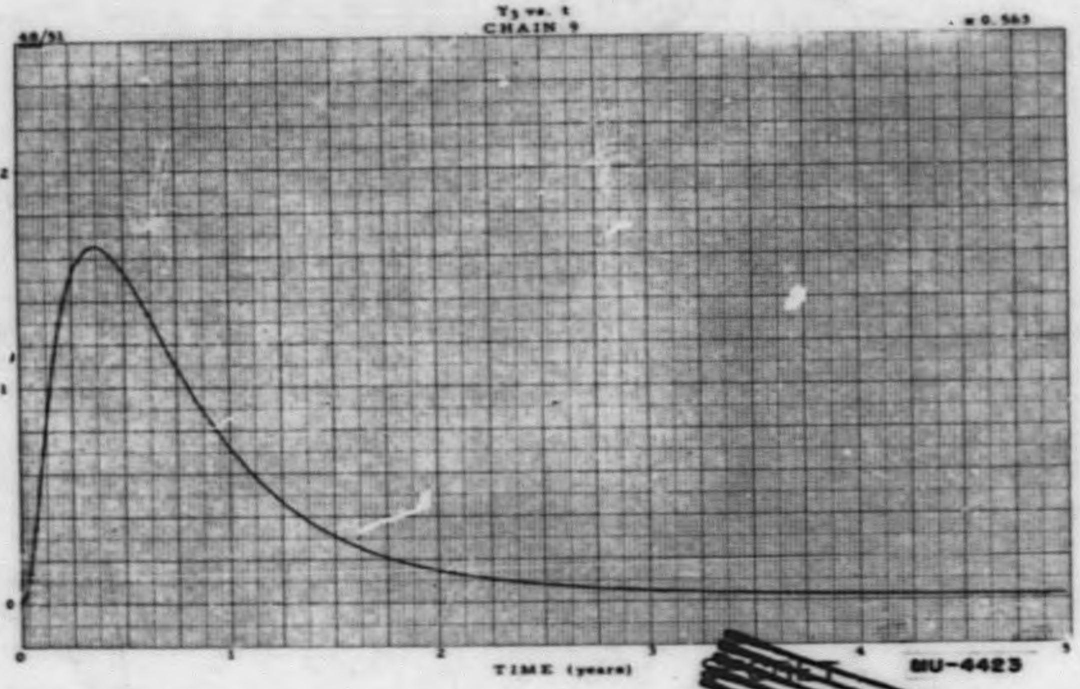
~~SECRET~~ MU-4421

Figure 58



[redacted]

Figure 59



~~Secret~~

MU-4423

Figure 60

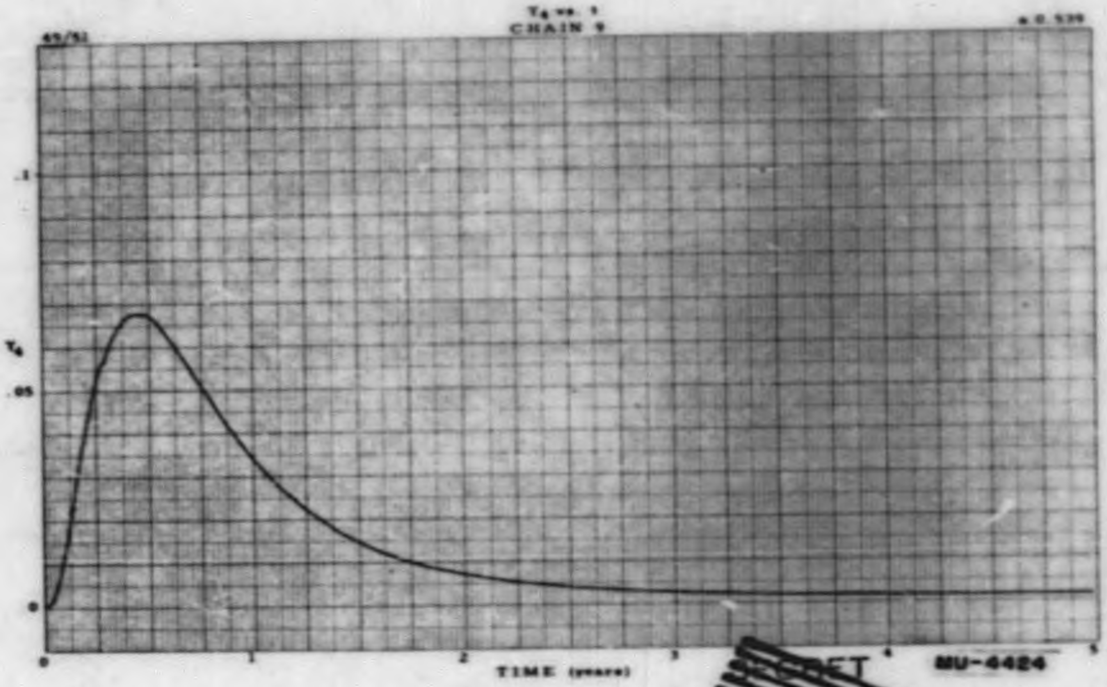


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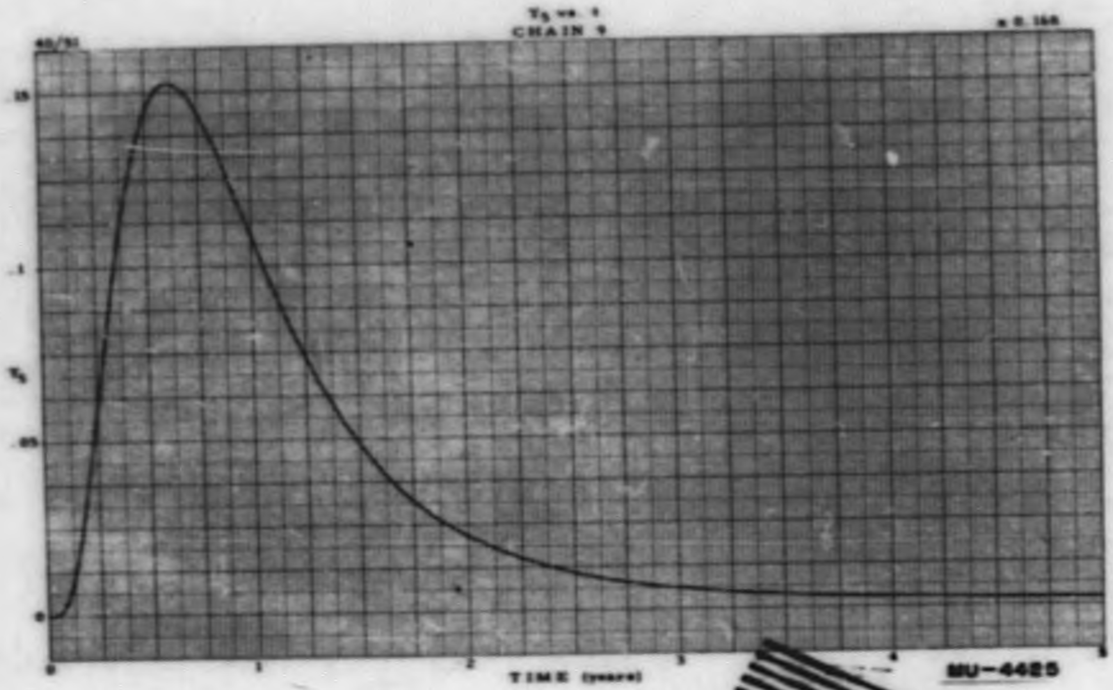
Figure 61

DECLASSIFIED



[redacted]

MU-4425

Figure 62

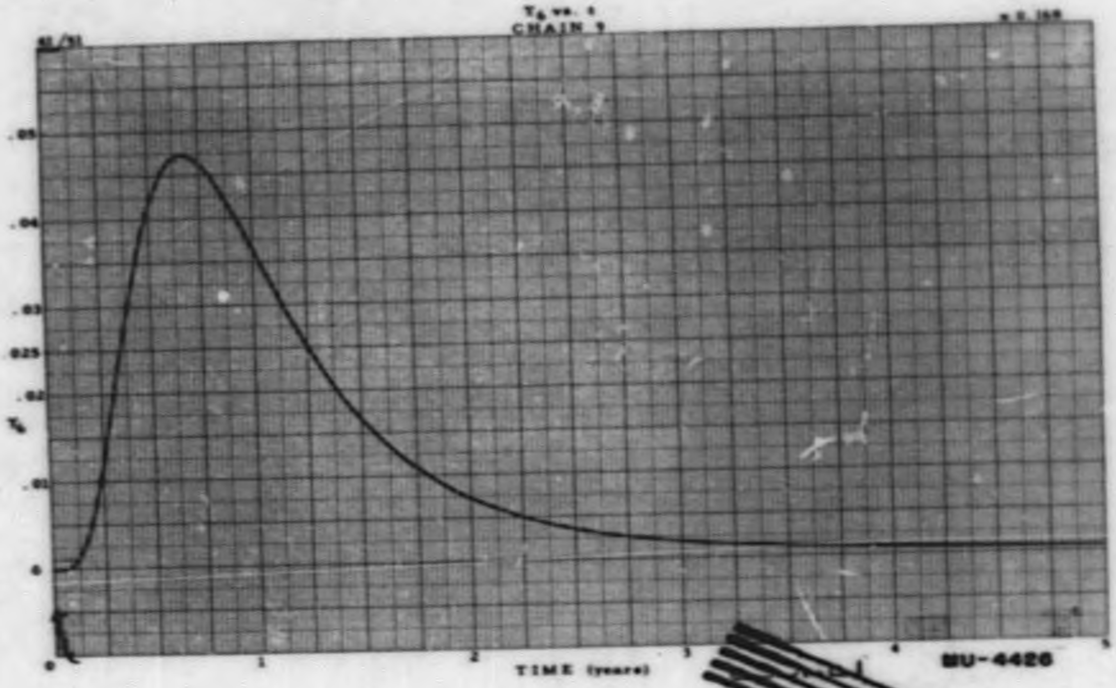


Figure 63

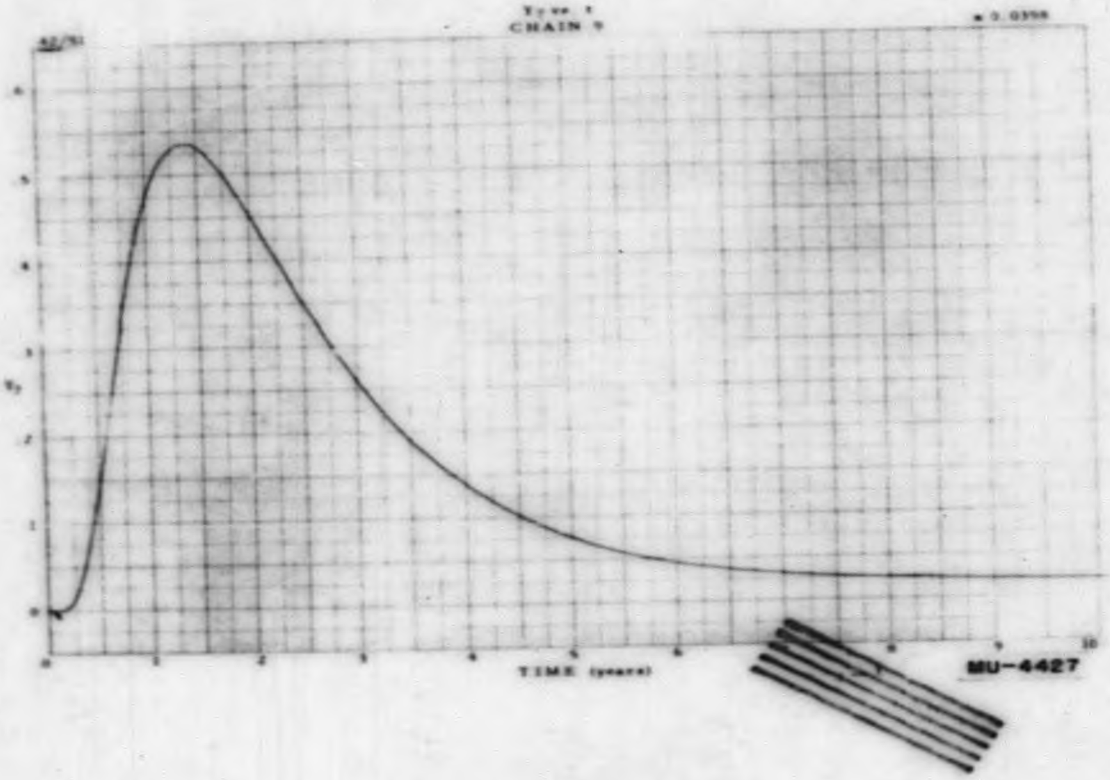


Figure 64

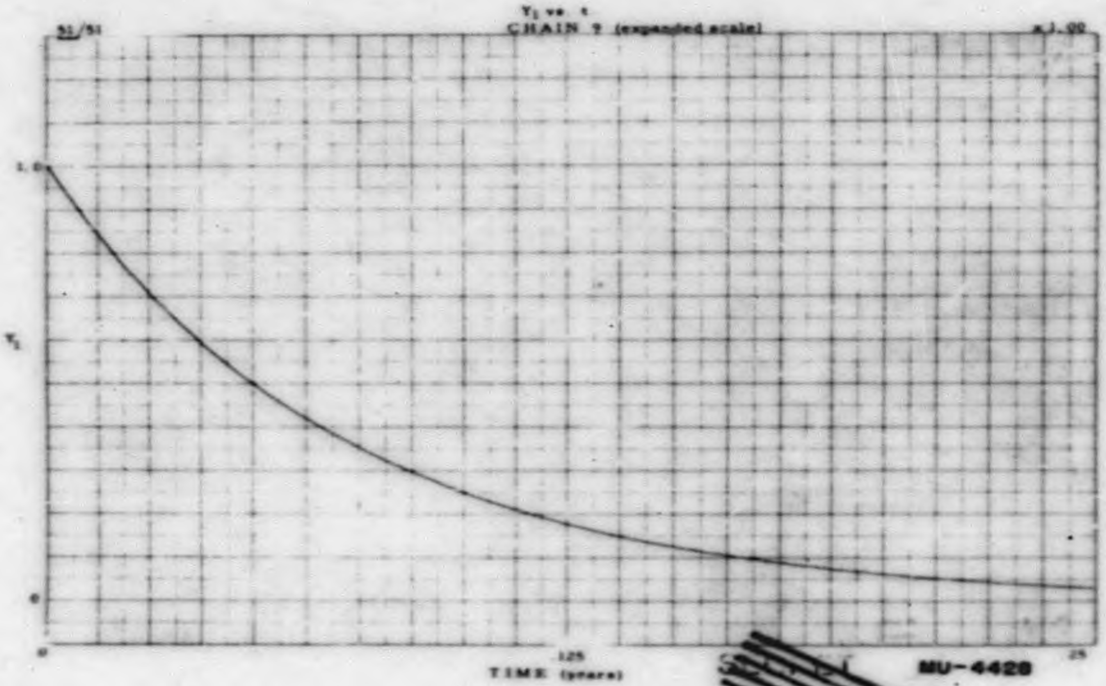
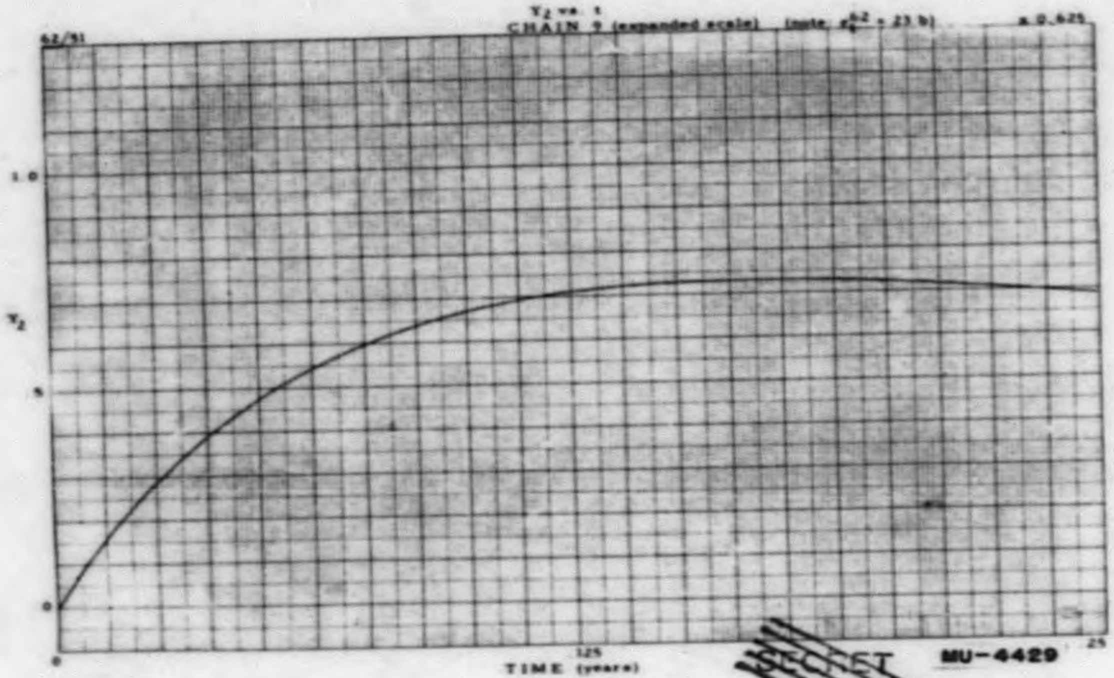
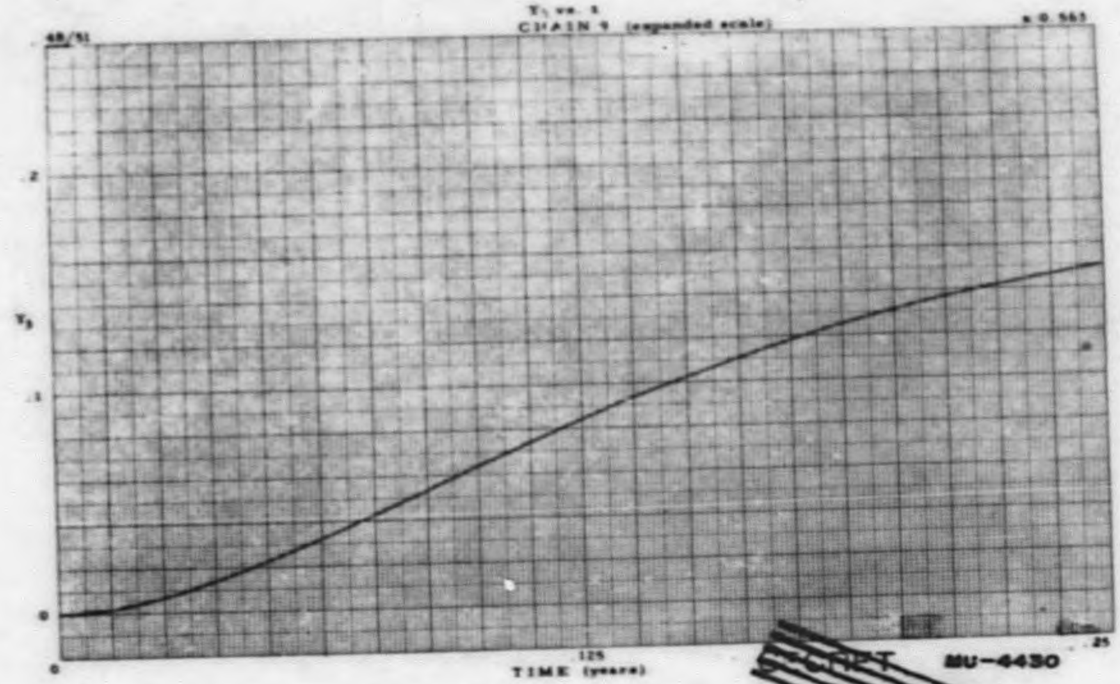

Y_1 vs. t
CHAIN 7 (expanded scale)
51/51
a 1.00
1.0
Y_1
0
0
125
25
TIME (years)
MU-4428

Figure 65



~~SECRET~~ MU-4429

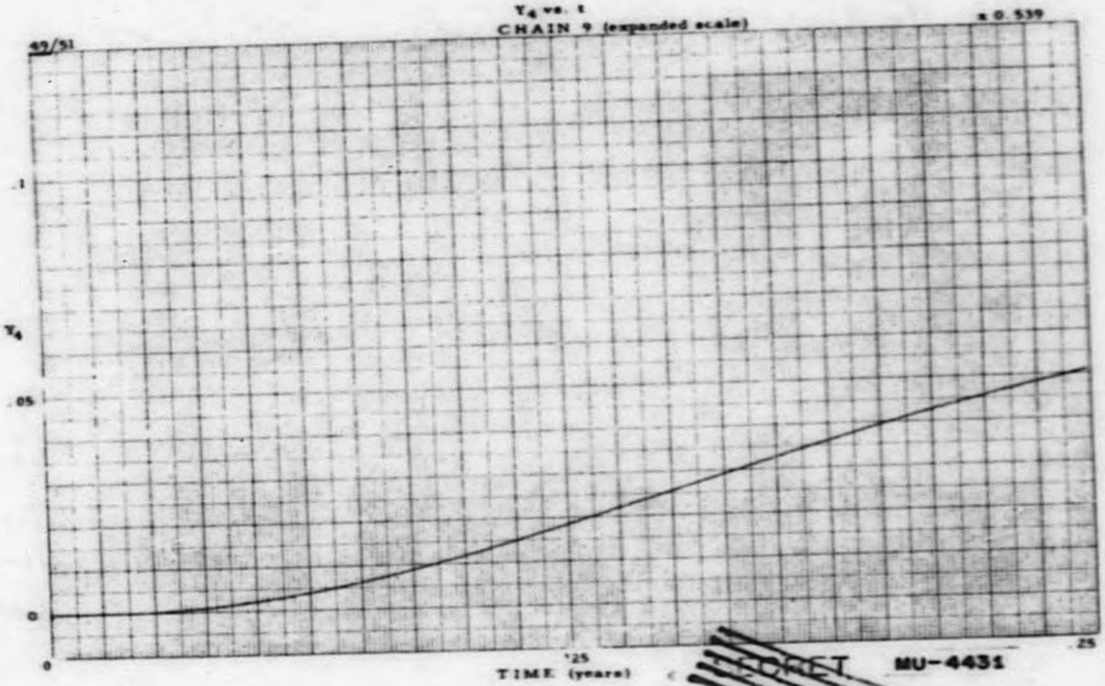
Figure 66



~~SECRET~~ MU-4430

Figure 67

CLASSIFIED



~~SECRET~~ MU-4431

Figure 68

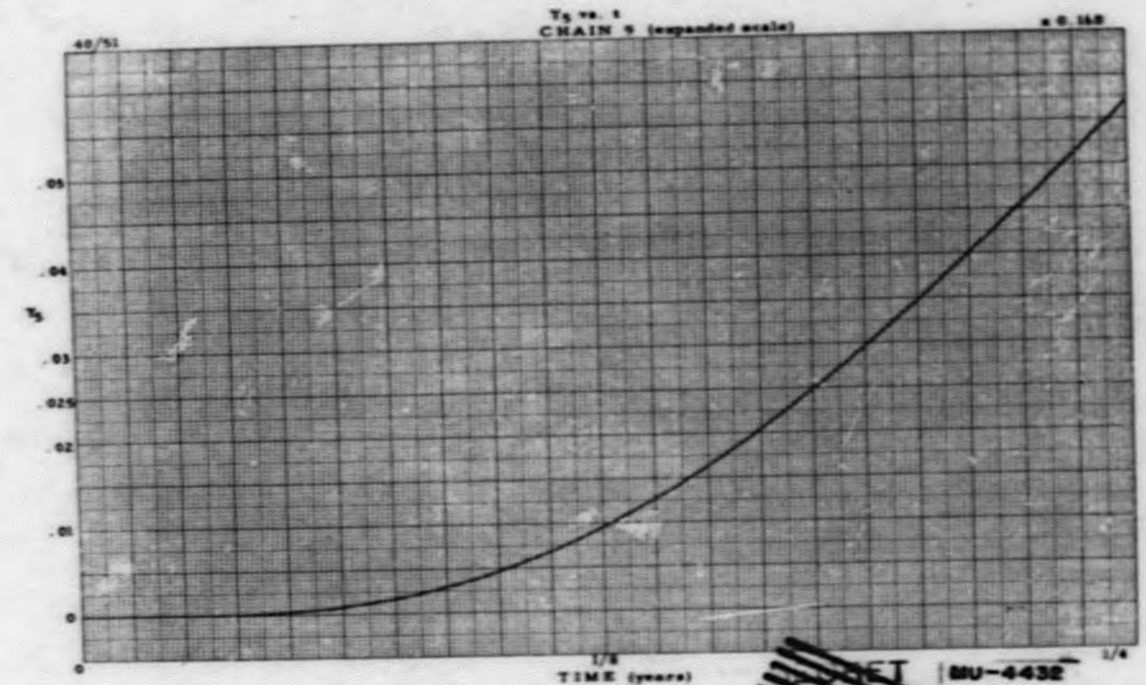
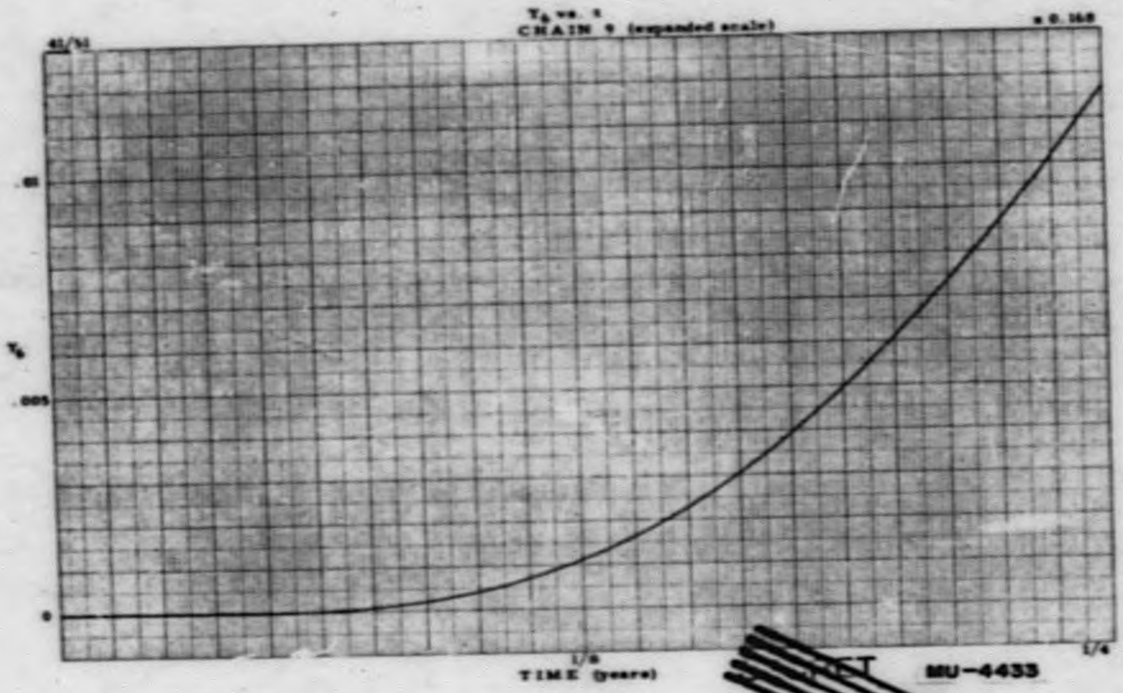

Y_5 vs. t
CHAIN 5 (expanded scale)
40/51
a 0.168
Y_5
.05
.04
.03
.025
.02
.01
0
0
1/8
1/4
TIME (years)
[redacted]
MU-4432

Figure 69



[redacted] MU-4433

Figure 70

Chain 9a: 62-48-49-40-41-42

b_1 = 1.924

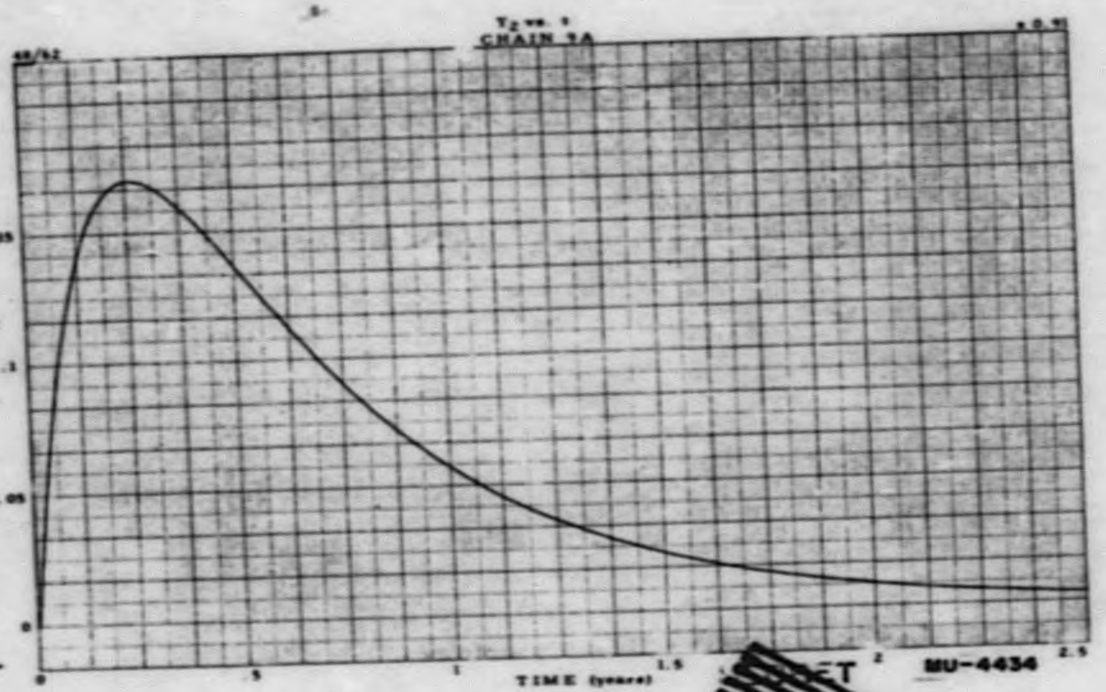
b_2 = 6.9073

b_3 = 16.48

b_4 = 6.466

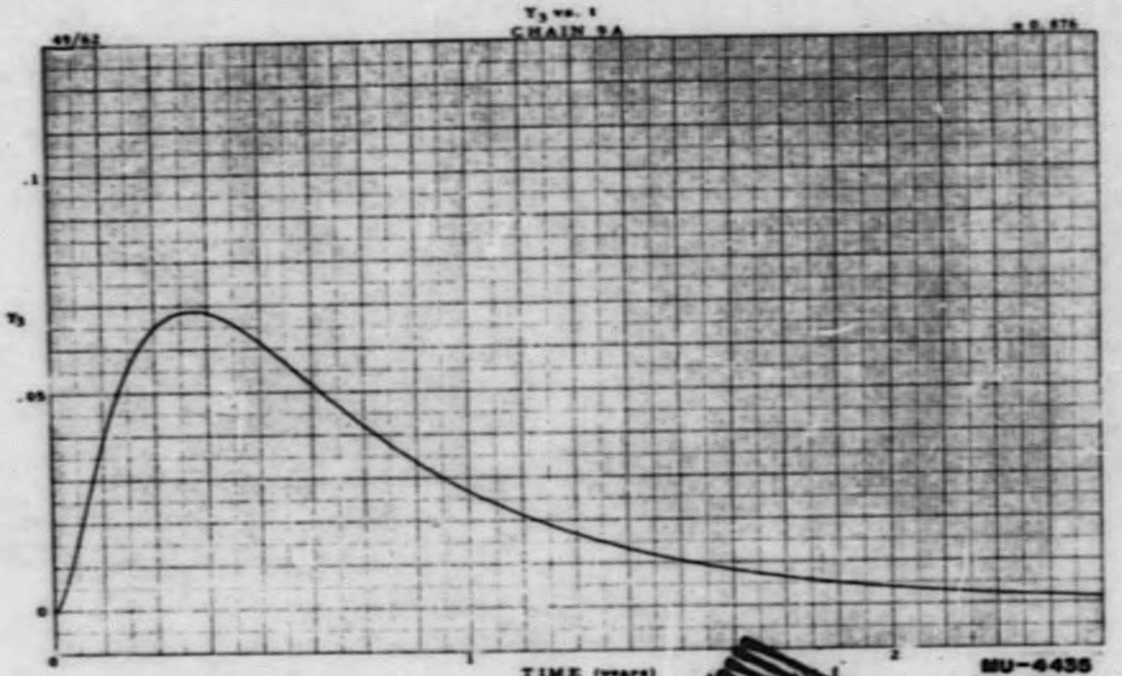
b_5 = 20.55

b_6 = 0.7885



~~SECRET~~

Figure 71



[redacted]

MU-4435

Figure 72

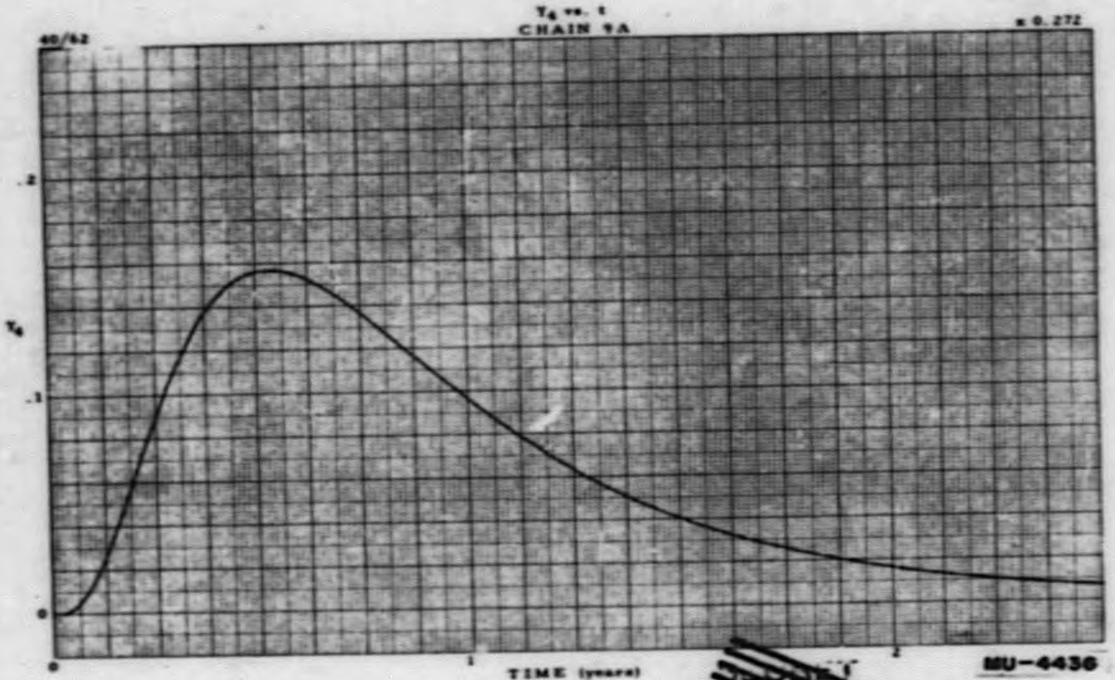


Figure 73

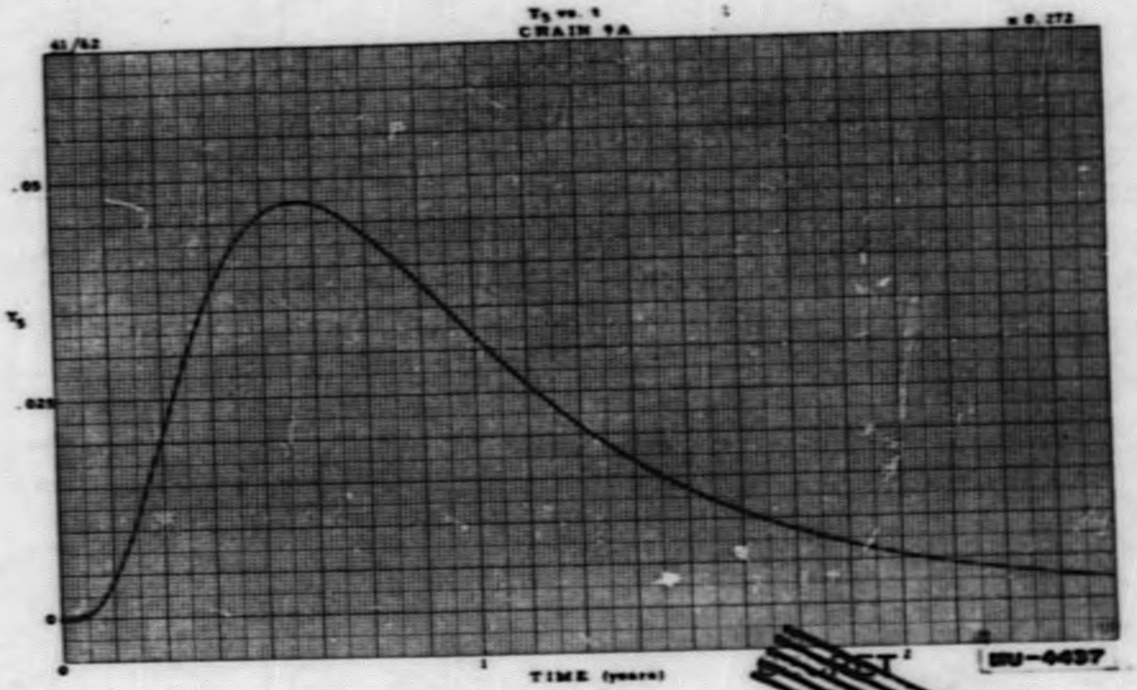


Figure 74

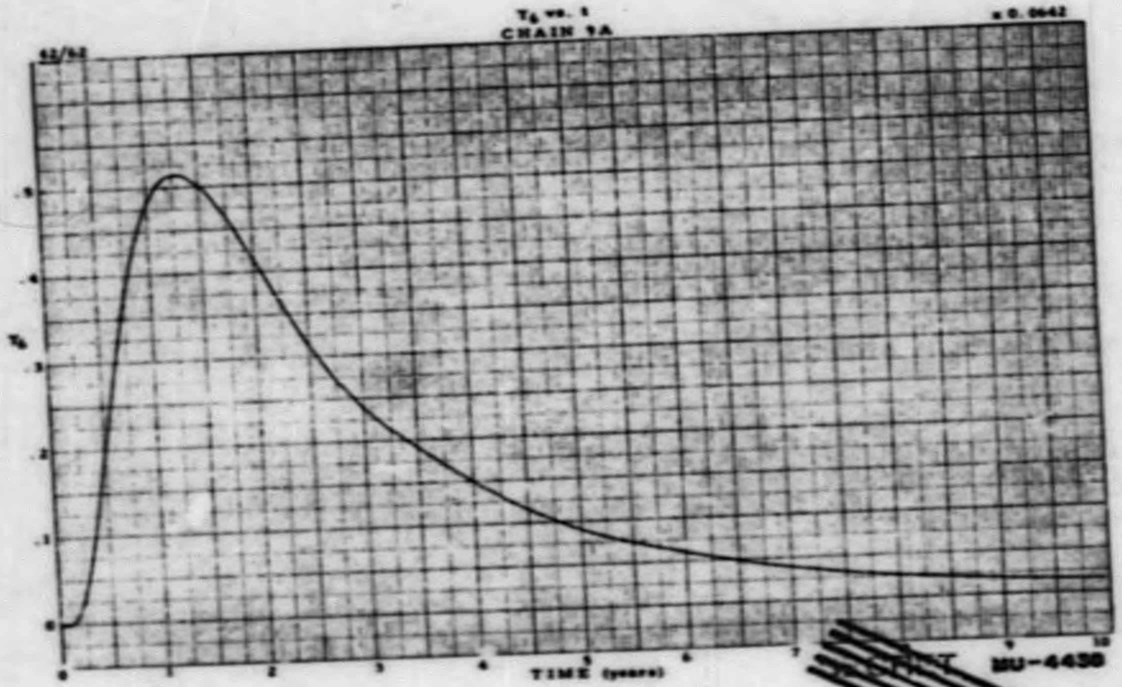

T_4 vs. t
CHAIN 9A
a 0.0642
12/52
T_4
.5
.4
.3
.2
.1
0
0
1
2
3
4
5
6
7
8
9
10
TIME (years)
MU-4438

Figure 75

APPENDIX I

The problem of radioactive growth and decay of members of a radioactive chain is represented mathematically by a system of simultaneous linear differential equations of first order. The inclusion of neutron-induced production and destruction of various nuclei can be introduced into the same differential equation formulation quite simply where the neutron flux is constant in time.

In any network of growth and decay not involving closed cycles, the network can be divided into its component unbranched chains, since the differential equations are linear.

Consider a chain of first order transformations (as a series of n,γ reactions at constant neutron flux* or a radioactive decay chain):

$$\begin{tikzcd}
A_1 \arrow[r, "a_2"] \arrow[d, "b_1-a_2"] & A_2 \arrow[r, "a_3"] \arrow[d, "b_2-a_3"] & A_3 \arrow[r, "a_4"] \arrow[d, "b_3-a_4"] & \cdots \arrow[r, "a_n"] & A_n \arrow[dr, "b_n"] & {} \\
{} & {} & {} & {} & {} & {}
\end{tikzcd}$$

---

*If no radioactive decay processes are significant in the chain, the condition of constant neutron flux need not be imposed. The total integrated flux (nvt) is all that need be known. But if radioactive decay processes are significant, the flux variations give to the set of differential equations constants variable with time. If the variations can be treated as stepwise constant, the problem may be broken up and treated in separate segments. A suitable polynomial in time t might be fitted to the flux variation and the resulting differential equations in turn treated by the Laplace transformation method. We shall not, however, be concerned with any but the constant flux case here.

where a_i is the constant coefficient governing the rate of production of isotope A_i from its parent A_{i-1}. If this production is by radioactive decay,

$$a_i = \lambda_i = 0.693/\text{half-life}$$

of A_{i-1} in seconds; and if the production is a bombardment particle-induced reaction, then $a_i = I\sigma_i$, where I is the flux of bombarding particles in particles per square centimeter per second, and σ_i is the capture cross section of A_{i-1} in square centimeters.

b_i is the constant coefficient governing the rate of destruction of isotope A_i by all processes. It is then the sum of all ${}_i\lambda_j$ for radioactive decay of A_i by all modes j plus the sum of all $I({}_i\sigma_k)$ for all particle-induced reactions k destroying A_i where σ_i is the cross section of A_i in square centimeters for process k. That is,

$$b_i = \sum_j {}_i\lambda_j + I\sum_k {}_i\sigma_k$$

The dimensions of the a_i and b_i are reciprocal time.

The differential equations relating the amounts of y_i of species A_i present* are:

---

*The quantity y_i may be number of atoms, number of moles, or other convenient measure of number of atoms.

$$\begin{aligned} dy_1/dt &= -b_1 y_1 \\ dy_2/dt &= a_2 y_1 - b_2 y_2 \\ &\text{-----------------} \\ dy_n/dt &= a_n y_{n-1} - b_n y_n \;. \end{aligned} \tag{1}$$

It can be shown that if the above set of equations is replaced by the following

$$\begin{aligned} dz_1/dt &= -b_1 z_1 \\ dz_2/dt &= b_1 z_1 - b_2 z_2 \\ &\text{-----------------} \\ dz_n/dt &= b_{n-1} z_{n-1} - b_n z_n \;, \end{aligned} \tag{2}$$

where each a_n is replaced by b_{n-1}, z_n is related to y_n in a simple manner, namely,

$$y_n = \frac{\prod_{2\leq i\leq n} a_i}{\prod_{1\leq j\leq n-1} b_j} \; z_n \tag{3}$$

For the differential analyzer calculations the set of equations (2) was used instead of (1) in order to simplify the operation of the analyzer. Thus, for each curve from the analyzer (i.e., z_n) one must apply as vertical scale factor the coefficient of z_n in (3); this scale factor is given numerically in the upper right hand side of each analyzer curve.

APPENDIX II

APPLICATION OF THE UCRL DIFFERENTIAL ANALYZER (1,2)

The differential analyzer is an instrument for evaluating the solutions of ordinary differential equations. It solves linear and nonlinear equations with a single independent variable.

The idea of the differential analyzer is well known.(4,3) It consists of a number of integrators which can be connected together so as to solve an ordinary differential equation directly for given boundary conditions. The output of one integrator may be used continuously as the integrand or variable of integration of one or more other integrators and this system of interconnected integrators generates the solution of the equation.

As an example of this method consider the differential equation:

$$\frac{d^2y}{dx^2} + y = 0.$$

Expressing the highest order derivative explicitly the equation becomes

$$y'' = -y .$$

Now integrating with respect to x

$$y' = - \int ydx \qquad \text{Integrator 1}$$

and integrating once more

$$y = \int y'dx \qquad \text{Integrator 2.}$$

The solution of this equation is then obtained using two integrators, connected so that the output of integrator 2 is the integrand of integrator 1 and the output of integrator 1 is the integrand of integrator 2 with a sign change. The variable of integration of both integrators is the independent variable of the machine. The solution is then obtained from integrator 2 and corresponds to the given initial conditions of $y(x_0)$ and $y'(x_0)$.

The UCRL machine is a mechanical differential analyzer--that is, the system of differential equations is replaced by an analogous mechanical system of gears and connecting linkages. An integrator is a continuously variable gear of the ball and disk type. The connection system is an electromechanical one. In order to transmit mechanical rotations, self-synchronous electric motors (selsyns) are used.

The variables of the differential equations are represented by rotations of selsyn shafts. The numerical value of the function is directly proportional to the number of rotations of the shaft which represents it. The proportionality factor is a pre-calculated scale factor for that function.

Power is supplied to the independent variable selsyn which drives all the other selsyns by means of the connections made between units. The actual connections between selsyns are made by patch cords on a plug board where there is a socket for each selsyn properly labeled by its mathematical use.

The solutions of the system of equations are plotted on input-output tables. When using the tables as input tables a known functional relationship is previously plotted on the regular graph paper and a person following the curve with a hand crank feeds in rotations proportional to the function plotted. When used as output tables a graphical solution is obtained directly. Any two functions generated in the machine may be plotted against each other.

The UCRL differential analyzer has fourteen integrators and six plotting tables. There are also fourteen adders and twelve scaling multipliers. All of these components are described in detail in references (1) and (2).

The differential analyzer is well suited for solving the systems of equations described in this report. Consider the system

$$\begin{aligned} y_1 &= -b_1y_1 \\ y_2 &= b_1y_1 - b_2y_2 \\ y_3 &= b_2y_2 - b_3y_3 \\ &\text{--------------} \\ y_n &= b_{n-1}y_{n-1} - b_ny_n \end{aligned} \qquad (1)$$

where the b_1's are known constants. The initial conditions given are

$y_1 = 1,\ y_2 = y_3 = y_4 = \ldots = y_n = 0$ at $t = 0$.

The system (1) can be written as

$$\begin{aligned} y_1 &= -\int b_1y_1dt \\ y_2 &= \int(b_1y_1 - b_2y_2)dt \\ &\text{------------------} \\ y_n &= \int(b_{n-1}y_{n-1} - b_ny_n)dt \end{aligned} \qquad (2)$$

The equations, written in this form, indicate the method of setting up the problem for analyzer solution. A system of n equations requires n integrations to be performed simultaneously. In addition, the dependent variables, y_i, must be multiplied by the constants b_i and the indicated subtractions performed. The multiplication by constants is generally performed on an integrator. In this case the integrand displacement is fixed proportional to the constant. The coefficients can then be set to three significant figures. These products can be written in the following form

$$b_i y_i = \int b_i dy_i \; .$$

The variable of integration in this case is a dependent variable generated simultaneously on another integrator. These products which are all integrator outputs are combined according to the equations on the adders of the analyzer. Six of the fourteen adders on the UCRL machine are also integrands of integrators. The selsyns used for this purpose are called differential selsyns and have two input terminals. The rotation of the shaft of such a selsyn is then the algebraic sum of the input rotations. These integrators then generate integrals of the form

$$\int (u + v) dx$$

where u and v are any two functions being generated in the machine. They are ideal for this problem since the integrals

$$\int (b_{i-1} y_{i-1} - b_i y_i) dt$$

are to be performed.

The given initial conditions of the problem determine the initial displacements of the integrands. Except where y_1 appears these initial displacements are zero. Also in this problem the $y_i \longrightarrow 0$ as $t \longrightarrow \infty$. So after the various yields have grown and decayed the integrand displacements approach zero. This is an undesirable situation as it leads to inaccuracies for long running times (relative to the machine). To avoid this the integrand is initially displaced by an amount ±h, and the integrator output is then added on a regular adder to the function ±ht. So a yield y_i is generated as

$$y_i = \int(b_{i-1}y_{i-1} - b_i y_i - h_i)dt + h_i t \ .$$

The ht is some convenient multiple of the independent variable.

Obviously the number of equations that can be solved simultaneously on such a machine is limited by the number of integrators available; however, in this problem indefinitely long series can be solved by cutting the sequence off where necessary and then using a yield solution obtained as an input function to solve the succeeding equations. For example, y_1, y_2, y_3, y_4 may be solved and plotted simultaneously. The graph of $y_4(t)$ can then be used on an input table and with this information the equations for y_5, y_6, y_7, y_8 can be solved on a later run of the analyzer.

We can now proceed to write down the analyzer setup for a sample system of equations.

$$A_1y_1 = -\frac{1}{50}\int B_1(b_1y_1 - h_1)d(C_1t) - \frac{B_1C_1}{50}h_1t$$

$$B_1b_1y_1 = \frac{1}{50}\int \frac{50B1}{A_1}b_1d(A_1y_1)$$

$$A_2y_2 = \frac{1}{50}\int B_2(b_1y_1 - b_2y_2 - h_2)d(C_2t) + \frac{B_2C_2}{50}h_2t$$

$B_2b_1y_1$ comes from $B_1b_1y_1$ through a scaling multiplier.

$$B_2b_2y_2 = \frac{1}{50}\int \frac{50B2}{A_2}\ b_2d(A_2y_2) \qquad \text{etc.}$$

The 1/50 comes from the fact that on the UCRL machine the radius of integral wheel is equivalent to fifty threads of the integrand lead screw. This is called the integrator constant and figures in all scale factor calculations. The A_i, B_i, C_i are the scale factors associated with each function in the problem. They are determined by the range of the variables and the maximum allowable displacement of an integrator along with the size of the plotting tables. The scale factor is then the proportionality factor between the value of the function in its natural units and the number of rotations representing that value on some analyzer component. The scale factors are then expressible in

$$\frac{\text{selsyn rotations}}{\text{given units of variables}} .$$

The main task in analyzer setup is to determine the best scale factors. It is generally a trial process as the range of the variables is not known; however, in this problem the maximum possible value of any of the yields is known and this helps considerably.

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