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PASSAGE OF ELECTRONS THROUGH MATTER

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A. Survey

1. Empirical

a. One of the oldest properties of electrons is their emission of x-rays (discrete and continuous spectra) when impinging on matter.

b. Cloud-chamber tracks show light ionization, and considerable multiple scattering, except for fast electrons. Tracks are long and range indefinite.

c. Measurements in aluminum show:

(α) For $\beta > 0.7$, $E > 1.4\ mc^2$, the range (i.e., "extrapolated range") of electrons of energy E, velocity βc, is linear in energy. (Feather Rule--see Rasetti; Cork).

(β) For $\beta < 0.7$ $(E < 1.4\ mc^2)$ range proportional to $(E - mc^2)^2$. Therefore, $\frac{-dE}{dx} \propto \frac{1}{E - mc^2}$.

(γ) For absorber not too thick, the absorption of an initial Fermi (β - decay) distribution is nearly exponential. (It is the extension of the fairly straight line on a logarithmic plot that gives the "extrapolated range" in this case.)

(δ) End of range is not marked by cessation of readings from detector, but by leveling off at a low value which decreases only slowly with further increase of absorber thickness.

(ε) Cosmic ray work shows apparent failure of theory--anyway a sharp break--at 200 Mev. This is due to the possibility of multiplicative processes, leading to formation of "showers".

2. Qualitative Theory

a. The observation (δ) is accounted for by Bremsstrahlung. This is obvious, as electrons are known to produce x-rays when they impinge on matter. Persistence at large thickness is then due to known penetrating power of x-rays and γ-rays.

b. We shall see that Bremsstrahlung may involve large energy losses, thus leading to considerable straggling. On the other hand, Bremsstrahlung is important only for fast electrons. But slow ones--as noted above--suffer large multiple scattering. The lack of a definite range (which sharply differentiates electrons from heavy particles) is a consequence of these effects.

c. The fact that the range varies as $(E - mc^2)^2$ for slow (not too slow) electrons is in accord with the theory of stopping power by ionization. Since the actual range is much longer than the thickness of absorber traversed (due to multiple scattering) it is not possible to make a more quantitative comparison.

3. Summary

The principal processes by which electrons are stopped are ionization (including excitation) and Bremsstrahlung. The former predominates at low energy, the latter at high. Due to their small mass, electrons undergo large multiple scattering at low energies. Bremsstrahlung entails large straggling and scattering. These ideas permit a qualitative account of the passage of electrons through matter, but a quantitative comparison is very difficult.

B. Ionization Loss

1. There are two differences from the corresponding process for heavy particles:

   a. In a 2-electron collision, the reduced mass is $\frac{1}{2}$ m, not m as in an electron-heavy particle collision. Thus, the deBroglie wave-length is doubled, and therefore b_{min} is doubled, which puts a factor $\frac{1}{2}$ under the logarithm.

   b. Wave functions have to be anti-symmetrized in the two electrons. We also for convenience, since the two electrons aren't distinguishable, take the one with higher final energy to be the incident one.

2. As a result of these two effects, the usual formula

$$-\frac{dE}{dx} = 4\pi N Z_o \frac{z^2 e^4}{mv^2} \ln \frac{2\gamma mv^2}{k Z_o R \hbar}$$

is replaced by

$$-\frac{dE}{dx} = 2\pi N Z_o \frac{e^4}{mv^2} \left[1 + \ln \frac{E^2}{2(k Z_o R \hbar)^2} \right].$$

If this change makes any difference, then the approximations are bad anyway, so it isn't too important. For the correction (in the non-relativistic energy range) replaces

$$\ln \frac{2 m v^2}{k Z_o R \hbar} \quad \text{by} \quad \ln \frac{2 m v^2}{k Z_o R \hbar} + \tfrac{1}{2} - \tfrac{1}{2} \ln 32 = \ln \frac{2 m v^2}{k Z_o R \hbar} - 1.2.$$

But the $\ln$ must be $\gg 1$, so the correction 1.2 is small.

3. Accordingly, the qualitative discussion is like that for heavy particles, namely, a rapid decrease with increasing energy to a broad minimum near $E = 3\ mc^2$, followed by a logarithmic increase. The rapid decrease at low energy is nearly proportional to $\frac{1}{E}$, so that the range is nearly proportional to E_o^2. An important result is the factor NZ_o, which tells that the mass stopping power of matter varies only slowly with Z_o.

4. Real comparison with experiment is difficult, due to:
   a. Scattering at low energy.
   b. The increase beyond the minimum is obscured by the rising effect of Bremsstrahlung.
   c. However, range proportional to E^2, and mass absorption coefficient nearly independent of Z_o, both seem roughly verified.

C. Bremsstrahlung

1. This process may be regarded in two ways: (a) as source of the continuous x-ray spectrum; (b) as contributing to the energy loss of electrons. From the former standpoint it is important over a wide energy range; from the latter, it is important only when not overshadowed by ionization loss--namely, at rather high energies. It is sufficient for the present purpose, and more simple, to center attention on the second aspect.

2. Bremsstrahlung Spectrum
   a. The cross-section for an electron of energy E to lose a fraction between f and $f + df$ of its energy by radiating a photon of energy $q = fE$ is, in absence of screening:

$$d\sigma = 4\alpha Z_o^2 (e^2/mc^2)^2 \left\{ f^2 + (4/3)(1 - f) \right\} \left\{ \ln \frac{2E(1-f)}{mc^2\, f} \right\} \frac{df}{f}$$

(where $\alpha = e^2/\hbar c$ as usual). (See Heitler, eq. 20, p. 168; Heisenberg, "Cosmic Rays", p. 80, 5th expression--which, however, has a puzzling difference.) Since this expression fails to include the screening of the nucleus of charge Z_o by its electrons, this may overestimate the cross-section. With <u>complete screening</u> we get:

$$d\sigma = 4\alpha Z_o^2 (e^2/mc^2)^2 \left\{ f^2 + (4/3)(1-f) \right\} \left\{ \ln \frac{137}{Z_o^{1/3}} \right\} \frac{df}{f}$$

(See Heitler, p. 170, eq. 26; Heisenberg, p. 12, eq. 1) The <u>smaller</u> of these expressions should be used.

b. It is possible for Bremsstrahlung to occur as the result of a close collision between the incident electron and one of the atomic electrons. The expressions above include only collisions with the nucleus. It appears from work by Lamb and Wheeler that the principal correction is the replacement of the factor Z_o^2 by $Z_o(Z_o + 1)$. This leads to the expressions:

$$d\sigma_1 = 4\alpha Z_o(Z_o + 1)(e^2/mc^2)^2 \left\{ f^2 + (4/3)(1 - f) \right\} \left\{ \ln \frac{2E(1-f)}{mc^2\, f} \right\} \frac{df}{f}$$

$$d\sigma_2 = 4\alpha Z_o(Z_o + 1)(e^2/mc^2)^2 \left\{ f^2 + (4/3)(1 - f) \right\} \left\{ \ln \frac{137}{Z_o^{1/3}} \right\} \frac{df}{f} .$$

c. The expressions can be made more perspicuous by further approximation, based on the fact that $\left\{ f^2 + (\frac{4}{3})(1 - f) \right\}$ is nearly 1 for $0 \leq f \leq 1$, while $\ln \left(\frac{1-f}{f}\right)$ varies slowly except when f is near 0 or 1 -- in fact the rapid variation near $f = 0$ is not physical, as the screening intervenes.

$$d\sigma_1 \approx 4\alpha\, Z_0(Z_0+1)(e^2/mc^2)^2 \frac{df}{f} \ln\frac{2E}{mc^2}$$

$$d\sigma_2 \approx 4\alpha\, Z_0(Z_0+1)(e^2/mc^2) \frac{df}{f} \ln\frac{137}{Z_0^{1/3}}$$

use smaller

A more exact calculation would give a continuous transition between these expressions. Curves given by Heitler (p. 170, fig. 14) show (for the more accurate expressions in (a)) that use of sharp transition--corresponding to a sharp change from the solid curve marked ∞ to the dotted curve, always using the lower--leads to small error.

d. We are here most interested in the spectrum of emitted photons, which we see is just $\frac{1}{f}$. Since energy loss is fE, we see that all energy losses are equally probable--within the accuracy of our approximations. This uniformity of energy losses is the most characteristic feature of the spectrum of Bremsstrahlung at these energies--it is this that produces the large straggling, due to high probability of large energy loss.

e. The above expressions can be slightly improved (Heitler)

$$d\sigma_1 = 4\alpha Z_0(Z_0+1)(e^2/mc^2)^2 \left\{f^2+\left(\tfrac{4}{3}\right)(1-f)\right\} \frac{df}{f} \left[\ln\frac{2E(1-f)}{mc^2\, f} - \tfrac{1}{2}\right]$$

$$\approx 4\alpha Z_0^2(e^2/mc^2)^2 \frac{df}{f} \left[\ln\frac{2E}{mc^2} - \tfrac{1}{2}\right]$$

$$d\sigma_2 = 4\alpha Z_0(Z_0+1)(e^2/mc^2)^2 \left\{f^2+\left(\tfrac{4}{3}\right)(1-f)\right\} \frac{df}{f} \left[\ln\frac{183}{Z_0^{1/3}} + \tfrac{1}{9}\right]$$

$$\approx 4\alpha Z_0^2(e^2/mc^2)^2 \frac{df}{f} \left[\ln\frac{183}{Z_0^{1/3}} + \tfrac{1}{9}\right].$$

3. <u>"Energy-Loss Cross-Section"</u>

a. More interesting than the total cross-section for Bremsstrahlung ($\sigma = \int_0^1 d\sigma_f$) is the "energy-loss cross-section" $\phi = \int_0^1 f\, d\sigma_f$.

$$\left.\begin{aligned} \phi_1 &= 4\alpha Z_0(Z_0+1)(e^2/mc^2)^2\left[\ln \frac{2E}{mc^2} - \frac{1}{3}\right] \\ \phi_2 &= 4\alpha Z_0(Z_0+1)(e^2/mc^2)^2\left[\ln \frac{183}{Z_0^{1/3}} + \frac{1}{18}\right] \end{aligned}\right\} \text{ use \underline{smaller}}$$

To these expressions should be added the non-relativistic limiting form:

$$\phi_3 = \frac{16}{3}\alpha Z_0(Z_0+1)(e^2/mc^2)^2 .$$

Note these expressions are almost the ones gotten <u>retaining</u> the factor $\left\{f^2 + \left(\frac{4}{3}\right)(1-f)\right\}$ in $d\sigma$, but putting $\ln\left(\frac{1-f}{f}\right) \sim 0$, and integrating. But the precise expressions are gotten more elaborately by Heitler.

b. Energy loss.

$$\frac{dE}{dx} = -N\int_0^1 E f\, d\sigma \qquad \text{or} \qquad -\frac{dE}{dx} = N E \phi .$$

Thus, $\underline{E = E_0 e^{-N\phi x}}$, i.e. exponential absorption.

Here, we have assumed ϕ independent of E. This is certainly true of ϕ_2, and even ϕ_1 depends on E only logarithmically.
(In fact, for ϕ_1, $-\frac{dE}{E} = dx\, NA\left[B + \ln E\right] = -d(\ln E) = -d\left[\ln E + B\right]$.

$$-NAx = \ln\left(\frac{B+\ln E}{B+\ln E_0}\right).$$

$$B + \ln E = (B + \ln E_o)\, e^{-NAx}.$$

$$\frac{1.4\,E}{mc^2} = \left(\frac{1.4\,E_o}{mc^2}\right)^{e^{-NAx}}.$$

This expression differs from the one derived from ϕ_2, but is not generally important.) The exponential absorption only applies when the radiation loss >> ionization loss. This commonly occurs only for energies so large that ϕ_2 should be used, so that dependence of ϕ_1 on E doesn't spoil exponential absorption in region where exponential absorption is to be used.

Note we don't have an exponential loss of <u>electrons</u>, but of <u>energy</u>.

c. Discussion.

The energy-loss cross-section, given by ϕ_3 at low energies, increases linearly with $\ln E$ until it is equal to ϕ_2, after which it levels off and retains this value for all higher energies. Note $\phi \propto Z_o^2$ pretty nearly. Also note (Heitler, p. 173, fig. 15) that transition from ϕ_3 (non-relativistic) to ϕ_1 occurs about at $E \simeq 2\,mc^2$ or $E \simeq 3\,mc^2$. Also note that ϕ_2 is only three or four times ϕ_3, so ϕ never changes very much with energy.

<u>Note:</u> For particles of mass M, get $e^2/mc^2 \longrightarrow e^2/Mc^2$; therefore, $\phi_M = (m/M)^2\, \phi_m$. Thus, <u>Bremsstrahlung is small for heavy particles</u>.

D. Overall Results on Energy Loss

1. Unit of length for Bremsstrahlung--mean free path--"Radiation Length".

$E = E_o\, e^{-N\phi x} = E_o\, e^{-x/\lambda}$.

$\lambda = \frac{1}{N\phi}$.

λ = "mean free path for radiation of energy" = distance in which energy decreases by factor e^{-1}.

= Radiation Length.

When Bremsstrahlung is a dominant process, it is convenient to use λ as a unit of length. Usually put $t = x/\lambda$.

Then rate of production of quanta of energy fE by Bremsstrahlung is

$$\frac{dn}{dt} \approx \frac{df}{f} .$$

2. Critical Energy.

As Energy E increases, Bremsstrahlung increases in proportion--sometimes faster. But ionization decreases rapidly to a broad minimum, then increases only logarithmically. Thus, ionization dominates at low energy, Bremsstrahlung at high energy. There is accordingly a "critical range" where the two effects are comparable. For use in shower theory, one defines a critical energy β at which

$$\left(\frac{dE}{dx}\right)_{coll} = \left(\frac{dE}{dx}\right)_{rad} = -NE\phi = -\frac{\beta}{\lambda} .$$

Therefore,

$$\beta = -\lambda \left(\frac{dE}{dx}\right)_{coll} \quad \text{or} \quad \beta = \lambda(\beta)\, F(\beta) .$$

In fact, λ and F are nearly independent of β in the critical region, since this turns out to be one in which F varies at most logarithmically, and so does λ . Roughly,

$$\lambda \propto Z_o^{-1}(Z_o+1)^{-1}, \quad F \propto Z_o \text{ ; therefore}$$

$$\beta \propto \frac{1}{Z_o+1}.$$

3. Tabulated Data.

a. The following data are from Rossi, High Energy Particles, p. 295.

Substance	C	Air	H_2O	Al	A	Fe	Cu	Pb
λ in gm cm^{-2}	44.6	37.7	37.1	24.5	19.8	14.1	13.1	6.5
β in Mev	102	84.2	83.8	48.8	35.2	24.3	21.8	7.8

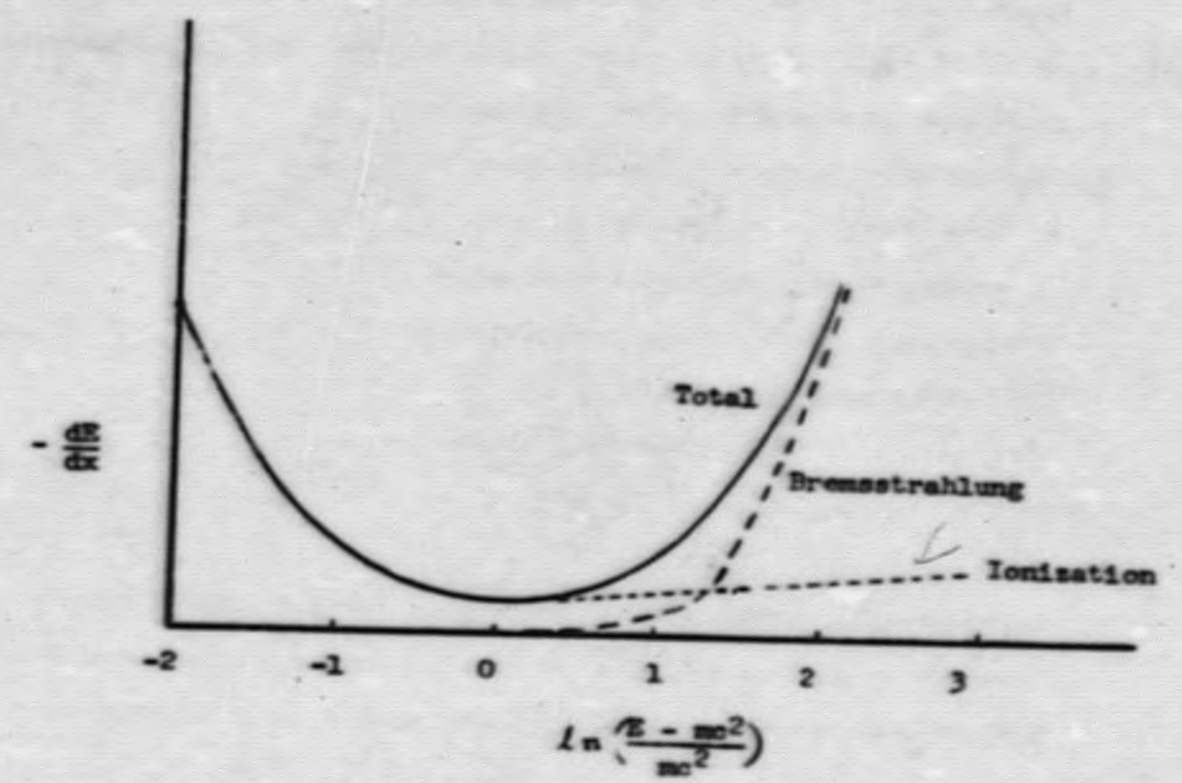
b. These numbers were calculated using $\lambda = \frac{1}{N\phi_2}$. That is, they are based on the formula with complete screening. They contain an additional refinement, namely, correction for inaccuracy of the Born approximation used in deriving the formulas. The correction is semi-empirical; theory shows that the cross-section should be increased (and λ and β decreased) by a factor $(1 + \alpha Z_o^2)$, but the constant α is evaluated empirically. Rossi gives $\alpha = \frac{0.12}{(82)^2}$, so that the correction is 12% for Pb, but less than 2% for Cu.

c. The values of β are in need of further correction. However, their only use is in shower theory, where the crudeness of the theory makes it unprofitable to carry the accuracy of the numbers very far.

4. Further Comments.

a. The slowing down of a fast electron in matter N, Z_o, can be described as follows: it loses energy by radiation at a rate proportional to its energy--so that the energy decreases exponentially with path traversed--until its energy is of the order of β. Here it makes a transition to the collision loss mechanism, but--since $\beta >$ ionization minimum--continues for a short time to lose energy at a decreasing rate. It quickly passes through the minimum, however, and then suffers ionization loss at an <u>increasing</u> rate until stopped. This behavior is displayed in the figure, which is for Pb.

b. Heitler's discussion of experiments of Blackett and Wilson shows, however, that this description is true only for electrons having $E < \sim 200$ Mev--the cascade process coming in at higher energies.



$-\frac{dE}{dx}$
Total
Bremsstrahlung
Ionization
-2
-1
0
1
2
3
$ln\left(\frac{E - mc^2}{mc^2}\right)$

END