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STOPPING POWER AND ION DENSITY

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A. Survey

1. A "particle" of charge Ze moves with velocity v, mass M, energy $E = Mv(1 - v^2/c^2)^{-\frac{1}{2}}$, through a "substance" of atomic number Z_o, density N atoms/cm^3. We have to consider the process of slowing down, and the nature of the trail of ions produced. Very slow particles are of little interest, as their range is very short. If Z is large, or if the particle is an electron, there are complications which we postpone. Thus, our considerations will apply particularly to protons, deuterons, α-particles, and mesons.

2. The important quantities are:

I → (a) Density of ions per cm of path ("Specific Ionization" I)

(Can be counted in cloud chamber and emulsion work, and is frequently employed in estimates of the mass of an ionizing particle.) Can also be determined by use of shallow ionization chamber, depth small enough so that ion density doesn't vary in a path of length = this depth. Plot as function of path length traversed is "Bragg Curve".

n → (b) Total Ionization Produced (n). This determines pulse size in ion chamber or proportional counter.

R → (c) Range (R). This is an important limiting factor in various experimental techniques. Being readily measured, it gives a useful measure of initial energy of particles. ("Range-Energy relation").

$F \rightarrow$ (d) Stopping Power, or rate of energy loss per cm (F). This is not observable, but provides the theoretical basis for treating the others.

$E_o \rightarrow$ (e) We may add the initial energy E_o, which determines the range.

3. Elementary observations--above all, tracks are straight and of uniform length. (Meson tracks are not straight, however.)

   (a) The energy required to produce an ion pair depends on the stopping substance (i.e., on N and Z_o) but not on the particle (Z, v, M). ~33 volts in air. Thus, $n \propto E_o$, and $I \propto F$.

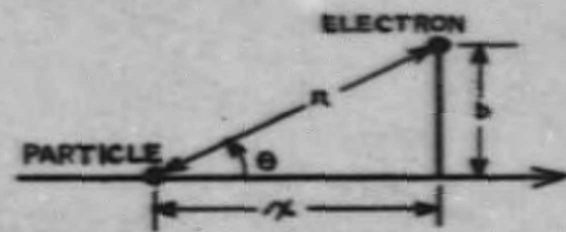
   (b) Geiger Rule. $R \propto E_o^{3/2}$ for moderate E_o. For example works for α-particles with ranges from 3 to 8 cm of air.

   (c) Bragg Rule. "Atomic stopping power" = $\frac{F}{N}$ is roughly proportional to $Z_o^{\frac{1}{2}}$ ($A^{\frac{1}{2}}$ in original statement; A = atomic weight) or "mass stopping power" = $\frac{F}{NA}$ is roughly proportional to $A^{-\frac{1}{2}}$.

4. Concept of "air equivalent", etc. Columns of two substances whose lengths are in inverse proportion to their respective stopping powers F will obviously produce the same energy loss in a particle, provided the columns are short compared to ranges. It is found that the final provision is unnecessary, so that equivalent thicknesses of two substances can be found for any range.

B. Basic Theory.

1. Momentum transfer in a single collision. Let particle velocity = v ; therefore put $x = vt$. Describe the collision by the "impact parameter" b. Suppose v so large and b so small that the electron behaves as if free, but that b is not so small that the electron's final velocity is comparable to v. According to the latter assumption, the electron will not move far during the time the particle is close enough to it to interact appreciably.



The momentum Δp acquired by the electron is given by $\int_{-\infty}^{\infty} F\,dt$, where F is the force acting on the electron. Since it is assumed free, the only force is the coulomb force due to the particle. As the electron doesn't move far during the collision, we compute F for a stationary electron. It is clear from symmetry that the component of Δp parallel to v is $\int_{-\infty}^{\infty} F_{\parallel}\,dt = 0$

$$F_{\perp} = \frac{Ze^2}{r^2}\,\frac{b}{r} \qquad v = \frac{dx}{dt}, \qquad dt = \frac{dx}{v}$$

$$r = b/\sin\theta \qquad x = -b/\tan\theta \qquad dx = b\,\frac{\sec^2\theta}{\tan^2\theta}\,d\theta = \frac{b\,d\theta}{\sin^2\theta}$$

$$\Delta p = \Delta p_{\perp} = \int_0^{\pi}\left(\frac{Ze^2 b}{b^3/\sin^3\theta}\right)\left(\frac{b\,d\theta}{v\sin^2\theta}\right) = \frac{Ze^2}{v b^2}\int_0^{\pi}\sin\theta\,d\theta = \frac{2Ze^2}{v b}.$$

Note: (a) Except for the factor 2, this result can be gotten by a dimensional argument.

(b) The factor $\frac{1}{v}$ appears, because Δp is proportional to time of collision, and time of collision is inversely proportional to v. It might be thought that a faster particle could hit the electron harder and transfer more momentum. This is indeed true, but as we shall see only affects close impacts, which aren't considered here.

2. Energy Loss. The energy transferred to the electron--whose initial momentum is supposed $\ll \Delta p/m$, and therefore taken as zero--is $\frac{\Delta p^2}{2m} = (\Delta T)_b$. Then $-\frac{dE}{dx} = F = \int (\Delta T)_b \cdot$ (no. of impacts per cm with impact parameter b) $= \int (2\pi b\, db) N Z_o (\Delta T)_b$. Therefore,

$$F = \int_{b_{min}}^{b_{max}} \frac{(\Delta p)^2}{2m} N Z_o 2\pi b\, db = \int_{b_{min}}^{b_{max}} \frac{4\pi z^2 e^4}{v^2 b^2 m} N Z_o b\, db$$

$$= \frac{4\pi z^2 e^4 N Z_o}{v^2 m} \ln \frac{b_{max}}{b_{min}} .$$

b_{min} and b_{max} are to be determined as limits on range of validity of our approximation. Due to logarithm, the determination need not be accurate.

3. Discussion of Approximation.

(a) First approximation is use of classical (not quantum) mechanics.

This is O.K. so long as well-defined classical orbits exist--i.e., so long as the de Broglie wavelength is much smaller than the distance of closest approach b. We know $\lambda\!\!\!^{-}_{part} = \frac{\hbar}{Mv}$. But we get a larger number by considering a coordinate system with particle at rest, and electron coming past with velocity v. Then $\lambda_{elec} = \frac{\hbar}{mv}$, and this is the quantity b must be greater than if we want to use classical mechanics. In fact, since the electron (as seen from the particle) may be anywhere within a region of this size, we see that Δp can no longer increase as 1/b when b decreases below λ_{elec}. We then have the condition

$$b \gg \frac{\hbar}{m v} .$$

(b) Second approximation is that electron stands still while struck.

This is O.K. if electron gets a velocity small compared to v. Therefore, $\frac{\Delta p}{m} \ll v$. A different approach is to note that, even in a head-on collision, the electron only gets a velocity 2 v. For the particle may be supposed to have ∞ mass, and in a coordinate system moving with it, the electron approaching with velocity v can at most be reflected back with velocity $+$v. Thus, our expression for Δp cannot possibly be right if it leads to $\Delta p > 2\,m\,v$, and is presumably right only if $\Delta p \ll 2\,m\,v$. This is the same condition as before.

Since $\Delta p = \frac{2 Z e^2}{b v}$, we have

$$b \gg \frac{Z e^2}{\frac{1}{2} m v^2} .$$

Note this is the distance at which the potential energy $\frac{Z e^2}{b}$ = the total energy $\frac{1}{2} m v^2$; i.e., it is the classical turning point. We then see that the electron won't stand still if the distance of closest approach to its initial position is less than the classical turning point--this is obvious, and gives another way to derive the condition.

(c) Third approximation is that the electron is free.

When we look more carefully, we see that really the <u>only</u> approximation about the electron is that it stands still while struck. Being free just means that the binding forces don't cause it to move during the collision. This will be the case if the collision time t is small compared to times which characterize the motion of the bound electron.

To estimate the latter, we recall that a bound particle has a multiply periodic motion, with a set of frequencies w_1, w_2, etc., and thus a set of times $\frac{1}{w_1}$, $\frac{1}{w_2}$, ... We now note that the electron is found in an atom, and that the frequencies associated with the set of Z_o electrons in the atom are simply those of the lines of its absorption spectrum. This includes the continuous spectrum, as well as the discrete spectrum. We may in fact, in the dipole approximation, treat an atom as a set of oscillators of these frequencies, the oscillator of frequency w_i being treated, not as a single degree of freedom, but as f_i degrees of freedom. f_i is the "oscillator strength", and is clearly usually fractional, since there is a not unnatural sum rule $\sum_i f_i = Z_o$ and there is an ∞ of frequencies w_i to run over.

Expression $Z_o \ln \frac{b_{max}}{b_{min}}$ should be replaced by $\sum_i f_i \ln\left(\frac{b_{max}}{b_{min}}\right)$.

We now have:

(i) $\tau \ll \frac{1}{w}$. To estimate the collision time τ, note that the main contribution to $\int_0^{\pi} \sin\theta \, d\theta$ is from

$$-\frac{\pi}{4} < \theta < \frac{\pi}{4}, \left[\text{i.e., } \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin\theta \, d\theta \doteq \sqrt{2}, \text{ (the largest part of 2)}\right].$$

Therefore τ is time for particles to go from $x = -b$ to $x = b$, or $\tau = \frac{2b}{v}$. Then,

$$b \ll \frac{v}{2 w_i} .$$

If this condition fails, we may consider the other extreme case, namely, $\tau \gg \frac{1}{w_i}$. In this case, we can use the adiabatic approximation; the electron will adjust its orbit to the slowly changing condition, and will be left in the same state it started in, i.e., there is an elastic collision. We see that for $b \gg \frac{v}{2 w_i}$, there is no contribution to the ionization.

(2) Dipole approximation must hold. This means $b \gg a_o$, where a_o is the radius of the atom. But this condition is only a condition on the validity of the approximation used to establish the relation $b \ll \frac{v}{2 w_i}$, that is, what it really means is only

$$\frac{v}{2 w_i} \gg a_o .$$

We note that the frequencies w_i which differ in order of magnitude are associated with different electrons, e.g., K x-rays with electrons in K-shell, etc. So we should put for each w_i some a_i which measures the radius, not of the whole atom, but just of the proper shell. Then

$$\frac{v}{2 w_i} \gg a_i .$$

(d) The fourth approximation is neglect of relativity.

The effect of relativity is to flatten the field (reducing its extent parallel to x-axis) and to increase its transverse component $f_\perp$. Due to the first effect, conditions at distance x should be replaced by those at $\frac{x}{\gamma} = \sqrt{1-\beta^2}\, x$. This means $t \rightarrow \sqrt{1-\beta^2}\, t$, and in particular the collision time t must be reduced by a factor γ. Due to the second effect, $f_\perp$ is increased by a factor γ, and we see that $f_\perp\, dt$ is unchanged. Thus, the previous calculation of the momentum transfer leads to the correct result. However, the condition $\mathcal{T} \ll \frac{1}{w_i}$ must be replaced by $\sqrt{1-\beta^2}\, \mathcal{T} \ll \frac{1}{w_i}$, and we get

$$b \ll \frac{v\gamma}{2 w_i}.$$

4. Determination of b_{min} and b_{max}. Results and Range of Validity.

(a) The result depends on b_{min} and b_{max} only through $\ln \frac{b_{max}}{b_{min}}$. Thus, we need not give an accurate estimate, and conditions of the form $b \ll \alpha$ or $b \gg \beta$ can be replaced by $b < \alpha$ and $b > \beta$, so that $\alpha = b_{max}$, $\beta = b_{min}$.

(b) We accordingly take

$$b_{max} = \frac{\gamma v}{2 w_i}$$

and

$$b_{min} = \left\{ \begin{matrix} \hbar/mv \\ \\ \frac{2 Z e^2}{m v^2} \end{matrix} \right\} \text{ whichever is greater.}$$

Therefore,

$$F = 4\pi N \frac{Z^2 e^4}{m v^2} \sum_i f_i \ln \begin{Bmatrix} \dfrac{\gamma m v^2}{2\hbar w_i} \\ \\ \dfrac{\gamma m v^3}{4 Z e^2 w_i} \end{Bmatrix}$$ whichever is <u>smaller</u>.

(c) We recall that if $b \gg b_{max}$, the collision is adiabatic, and therefore elastic. Also, if <u>$b_{max} \gg b_{min}$</u>, then most of the scattering involves $b > b_{min}$, so it is safe to neglect collisions for which $b < b_{min}$, these occurring rarely, and not involving larger momentum transfers than are involved in our approximation by those having $b = b_{min}$. Thus, by including all impact parameters $b_{min} < b < b_{max}$, we get all which contribute appreciably to the energy loss. But this involves the condition $b_{min} < b_{max}$, and in fact (as we see from our result containing $\ln \frac{b_{max}}{b_{min}}$) $b_{min} \ll b_{max}$ is necessary in order for these collisions to produce a large energy loss, and thus mask whatever may have been neglected by the roughness of our approximations.

We conclude

$$b_{max} \gg b_{min}$$

is a necessary conditon for the validity of the calculations. Here, we cannot replace $\gg$ by $>$.

(d) (α) Using both expressions for b_{min}, we find we must have both

$$\frac{v}{2 w_i} \gg \frac{\hbar}{m v} \quad \text{and} \quad \frac{v}{2 w_i} \gg \frac{2 Z e^2}{m v^2}.$$ (We shall see that these non-relativistic expression are good enough.) We recall also the condition for validity of dipole approximation in estimating b_{max} was $\frac{v}{2 w_i} \gg a_i$.

(β) We know that

$$\hbar w_i \approx \frac{Z_i e^2}{a_i} \approx \frac{m}{2} v_i^2 \quad \text{(virial theorem)}$$

where v_i = velocity of electron in shell associated with frequency w_i, and Z_i is the effective charge acting on the said shell. $1 \leq Z_i \leq Z_o$. We also have the uncertainty relation $m v_i a_i \sim \hbar$.

Therefore

$$\hbar w_i a_i \approx \frac{m}{2} v_i^2 a_i \sim \tfrac{1}{2} \hbar v_i \; ; \quad w_i a_i \sim \tfrac{1}{2} v_i .$$

(γ) Then the third condition becomes $v \gg v_i$. The first condition is $\frac{1}{2} m v^2 \gg \hbar w_i \approx \frac{1}{2} m v_i^2$, or again $v \gg v_i$. We see that the dipole approximation is always good, if the calculation can be done at all, since the condition for its validity is the same as the condition $b_{max} \gg b_{min\ Q.M.}$.

(δ) Second condition is:

$$v \gg (4 Z e^2 w_i / m)^{1/3} = (2 \frac{Z}{Z_i} v_i^2 a_i w_i)^{1/3} \approx v_i \left(\frac{Z}{Z_i}\right)^{1/3} .$$

From $Z_i \geq 1$ follows $\frac{Z}{Z_i} < Z$. Therefore if $v \gg v_i Z^{1/3}$, it will follow a fortiori that condition 2 is satisfied. This will follow from $v \gg v_i$ for protons and deuterons $(Z = 1)$ and α-particles $(Z = 2)$. It will fail for fission fragments and the Minnesota heavy particles.

(ϵ) The ratio of

$$\frac{b_{min\ class}}{b_{min\ Q.M.}} = \frac{2 Z e^2}{\hbar v} = \frac{2}{v} \frac{Z}{Z_i} w_i a_i = \frac{Z}{Z_i} \frac{v_i}{v} < Z \frac{v_i}{v} .$$

Therefore condition for use of Q.M., not classical b_{min} is $v \gg Z v_i$.

This again follows from $v \gg v_i$ for p, d, α, but may fail at low v for fission fragments and the Minnesota particles.

(e) Results.

(α) If $\frac{1}{2} m v^2 \gg Z \hbar w_i$ (for all i); then

$$F = 4\pi N \frac{Z^2 e^4}{m v^2} \sum_i f_i \ln \frac{\gamma m v^2}{2\hbar w_i} .$$

(β) If $\frac{1}{2} m v^2 \gg \hbar w_i$ but not $\gg Z \hbar w_i$; then

$$F = 4\pi N \frac{Z^2 e^4}{m v^2} \sum_i f_i \ln \frac{\gamma m v^3}{4 Z e^2 w_i} .$$

(γ) If $\frac{1}{2} m v^2$ is not $\gg \hbar w_i$ for any particular w_i; then that w_i will not be included in the sum. This is because the failure of our approximations is indicated by the contribution to energy loss being small. Only if there is no w_i for which $\frac{1}{2} m v^2 \gg \hbar w_i$ holds, will the calculation fail completely. But when this happens, we may say the particle is practically stopped, so the above formulae for F can be used down to $v = 0$ in estimating the range.

Note use of non-relativistic approximation throughout the section (4d). It is clear that condition $b_{max} \gg b_{min}$ will hold even better if b_{max} is increased by a factor γ.

(δ)
$$\sum_i f_i \ln \frac{\gamma m v^2}{2\hbar w_i} = \ln \frac{\gamma m v^2}{2} \sum_i f_i - \sum_i f_i \ln \hbar w_i$$

$$= Z_0 \ln \frac{\gamma m v^2}{2} - Z_0 \ln E_I .$$

E_I = average ionization energy is defined by this equation

$$\ln w_o = \ln \frac{E_I}{\hbar} = \frac{1}{Z_o} \sum_i f_i \ln w_i .$$

We get for p, d, α

$$F = 4\pi N Z_o \frac{z^2 e^4}{mv^2} \ln \frac{\gamma mv^2}{2 E_I} .$$

provided

$$\tfrac{1}{2} mv^2 >> \hbar w_i \qquad \text{or} \qquad E_{part} >> \frac{M}{m} \hbar w_i$$

for all atomic absorption frequencies w_i.

(E) Bloch's Estimate of E_I. Use Fermi-Thomas atom model. Electrons have momentum p, and are confined to volume of radius a. Between uncertainty relation and exclusion principle, get $p^3 a^3 \sim Z\hbar^3$ or $p\,a \sim Z^{1/3}$. By the virial theorem, the potential and kinetic energies are of the same order of magnitude, so that

$$\frac{Z_o e^2}{a} \sim \frac{p^2}{2m} \qquad \text{or} \qquad p^2 a \sim Z_o .$$

Therefore,

$$p \sim Z_o^{2/3} , \qquad a \sim Z_o^{-1/3} .$$

Instead of now estimating the energy

$$w \sim \frac{Z^2 e^2}{a} \;\; N\, Z_o^{7/3} ,$$

Bloch argues that (in the spirit of the harmonic oscillator approximation) it is now necessary to compute the frequency of vibrations in the Fermi gas. He says this is given by $w \sim v/a$, where v = sound velocity in the gas. But v should be the same order of magnitude as electron velocity;

therefore $\sim p$, so that

$$w \sim p/a \sim Z_0 \, .$$

Thus,

$$E_1 = k Z_0 \, .$$

Therefore

$$F = 4\pi N Z_0 \frac{z^2 e^4}{mv^2} \ln \frac{2\gamma mv^2}{k Z_0 R\hbar} \, .$$

k is a numerical constant (not given by Bloch's estimate).

Experiments on gold (quoted by Heitler) give $k = 2$[illegible].

Note

$$kR\hbar = Rh = \underline{13.5 \text{ volts}} .$$ (For air, $Z_0 \sim 7\frac{1}{2}$. This gives $k Z_0 R\hbar \sim 98$ volts. Experiment gives 80, which is a good enough check, considering crudeness of the Bloch argument.) "Serber Says" quotes a value $kR\hbar = 11.5$ volts obtained by Wheeler from experiments by Wilson on 2 - 4 Mev α's in aluminum.

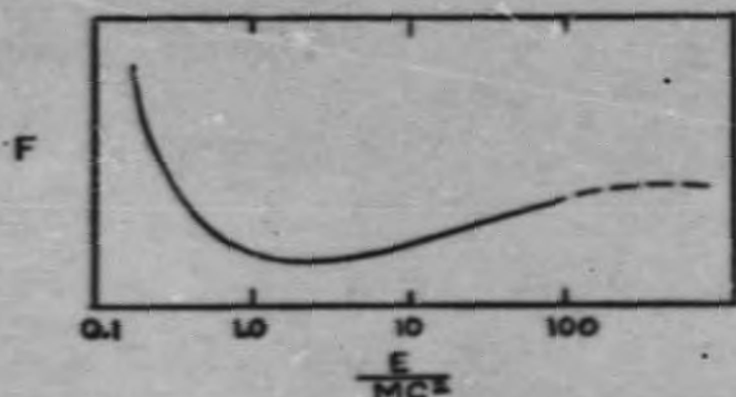
C. Comparison of Theory and Experiment--Definitive Results--Application.

1. Stopping Power

(a)

$$F = 2\pi N Z_0 \frac{M}{m} \frac{z^2 e^4}{\frac{1}{2} M v^2} \ln \frac{4 \frac{m}{M} E}{k Z_0 RH} .$$

This holds provided $\frac{m}{M} \frac{M v^2}{2}$ >> energy of K-electrons. The formula shows that F decreases rapidly as E increases, until E becomes relativistic. Then $\frac{1}{2} M v^2$ ceases to increase, while E continues to increase, giving a slow (logarithmic) increase of F. There is thus a minimum of F in the transition region from non-relativistic to extreme relativistic E. This is at perhaps $3 M C^2$.



(b) The minimum is hard to detect--or rather the rise at higher energies. This rise ceases at quite high energies due to the Fermi effect. This effect comes in strongly when v is greater than the velocity of light in the stopping substance. The effect is a strong polarization of the medium by the field of the particle, which shields the more distant parts of the medium, and thus cuts off the increase of b_{max} by the factor γ.

(c) Bragg rule. $\frac{F}{N} \sim Z_o^{\frac{1}{2}}$. This appears to hold simply because $Z_o^{\frac{1}{2}}$ is a fair approximation to $Z_o \ln \frac{\alpha}{Z_o}$ when $\frac{\alpha}{Z_o} >> 1$.

(d) We will see that the Geiger rule $(R \propto E_o^{3/2})$ means $F \propto v^{-1}$, or $F \propto E^{-\frac{1}{2}}$, but in fact, $F \propto E^{-1} \ln \frac{E}{\beta}$, which ought to be closely $F \propto E^{-1}$, since the $\ln$ varies slowly. Here are two suggestions.

(i) $E^{-\frac{1}{2}}$ is a good approximation to $E^{-1} \ln \frac{E}{\beta}$ in the range where the Geiger rule holds.

(ii) Teller observes that slow particles can not excite inner shell electrons; therefore as particles slow down below $\frac{M}{m}$. K-energy, they are able to excite fewer and fewer electrons; thus, the stopping power doesn't decrease as fast as expected. In fact, the number of electrons remaining is about proportional to v (Teller says), giving $E^{-\frac{1}{2}}$ outside the logarithm.

2. Range.

$$R = \int dx = \int \frac{dx}{dE} dE = \int_0^{E_o} \frac{dE}{F(E)}$$

Also,

$$\frac{dR}{dE_o} = \frac{1}{F(E_o)} .$$

(a) Geiger rule:

$$R \propto E_o^{3/2} , \quad \frac{dR}{dE_o} \propto E_o^{\frac{1}{2}} ; \text{ therefore, } F(E_o) \propto E_o^{-\frac{1}{2}} .$$

(b) Theory:

$$F(E) \propto E^{-1} \ln \frac{E}{\beta} \sim E^{-1} .$$

$$R \propto \int_0^{E_o} \frac{dE\, E}{\ln E} \ \tilde{\propto}\ E_o^2 .$$

In fact, when E_o is greater than in the Geiger rule range, the rule breaks down by R varying more rapidly than $E_o^{3/2}$, and tending toward E_o^2. On the other hand, when E_o is smaller, the rule fails by a slower variation.

(c) Aside from these rough rules, we can always do the theory more completely, integrate more accurately, and by comparison with experiment plot quite good range-energy curves. (See, e.g., Bethe and Livingston.)

The range-energy relation is the best way to get energies of heavy particles, because (due to their mass) they are difficult to deflect in magnetic fields unless their energies are low.

(d) Relation to mass:

$$-\frac{dE}{dx} = -\frac{d}{dx}\left(\tfrac{1}{2} M v^2\right) = -M v \frac{dv}{dx} = F .$$

$$R = \int_0^{v_o} \frac{dv}{dv/dx} = \int_0^{v_o} \frac{M v \, dv}{F(v)}$$

since $F(v)$ is independent of M, we have $\frac{Z^2 R}{M} = f(v_o)$.

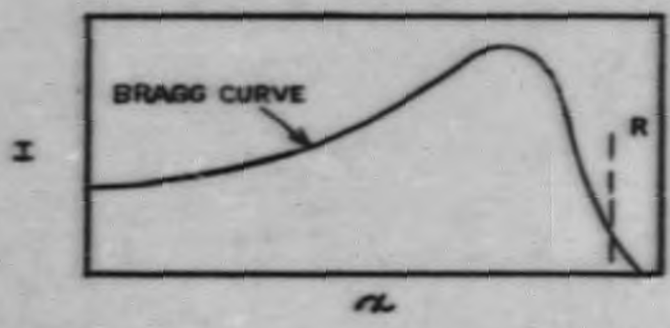
Momentum is $p_o = \frac{M v_o}{\sqrt{1 - \frac{v_o^2}{c^2}}}$. Thus, mass of meson can be measured in cyclotron, since p is known from $H\rho$, and range in emulsion can be measured. The function $f(v_o)$ can be determined for protons. Then for any particular track, only one value of M gives same value of v_o from both formulas (i.e., that for p_o and that for R).

Note how the use of this theory to find masses depends only on the fact that <u>$F(v)$ is independent of M and varies as Z^2</u>. Thus, only velocity dependence gives trouble, but can be gotten rid of by calibration against a particle of known Z and M. Note v is a handier parameter than E.

3. <u>Specific and Total Ionization</u>.

(a) The mean energy required to ionize is $k Z_o R \hbar$. Ionization chamber measurements show, however, that the number of ions produced in air is perhaps three times the number $E_o/k Z_o R \hbar$. This is explained by secondary effects; an electron produced in the primary process produces an average of two more ion pairs before stopping. (Especially fast primary electrons are "δ -rays") The result is, however, a proportionality between the energy loss and the ionization. We thus have:

(1) $I = KF$; (2) $M = KE_o$.



(b) Ionization chambers and proportional counters.

Due to their much greater total ionization, heavy particles are readily distinguished from light ones. Thus, it is possible to count α's in presence of β background. (How about vice versa?) Also, slow neutron counters can be constructed with BF_3. The reaction $B^{10}(n, \alpha) Li^7$ yields heavily ionizing α and Li^7, which can be detected in presence of a β background.

(c) Early measurements of meson mass.

(α) Brode and Fretter. $H\rho$ could be measured, but not R. However, I could be measured (or at least a quantity proportional to it) by counting drops in cloud chamber tracks. Since $I \propto F$, this means F (except for a constant factor) is measured. The constant can be gotten by calibration with protons. In fact, the whole function can be gotten so.

F(v) is independent of mass, so velocity is determined. From $H\rho$, know ρ. From $M = \frac{p}{v}\sqrt{1 - \frac{v^2}{c^2}}$, get M.

Since F(v) is slowly varying near minimum, this method requires slow mesons. Thus, counting rates are low.

(β) Powell, etc. $H\rho$ couldn't be measured. But we know:

$$\frac{R}{M} = f(v_0) \quad \text{and} \quad I = KF(v) \; ;$$

thus, the "residual range" at a place where I is measured is $\frac{R}{M} = \phi(I)$, where ϕ is a universal function. With I in arbitrary units (the grain density, measured somewhat subjectively, all right if reproducible for a given observer), ϕ is gotten for protons (for which M is known) and then M for mesons. The method requires meson and proton tracks to be formed in the emulsion at nearly the same time, as I (observed) is affected by the age of the tracks.

D. Large Particles: (1) Fission Fragments.
(2) Particles of Large Z Seen in Cosmic Rays by the Minnesota Group.

Here it is possible for the classical b_{min} to be larger than the quantum b_{min}. A more important effect is possibility of capture of electrons by the particle, reducing its effective charge, and thus its specific ionization. As a result, I decreases toward the end of the range, instead of increasing as in the Bragg curve. (See Teller, p. 32 of LA-24. Also, Rasetti, p. 51, for gain and loss of electrons by α's.)

Tracks of fission particles have small branches, which, at end of track, increase to form a tuft like the feather of an arrow. This is due to collisions with nuclei. See Teller, p. 32 of LA-24, for account of this.

It was by their very dense ionization (proportional to Z^2) and by the decrease in I at the ends of tracks that the Minnesota group recognized the large Z of the particles seen by them in cosmic rays at great heights.

E. <u>Scattering and Straggling</u>.

1. Due to the nature of the ionization process—namely, many <u>small</u> energy losses—the tracks of α's are quite straight, and the ranges uniform. It is this more than anything else which suggests that the stopping power of matter is due primarily to its electrons.

2. Nevertheless, due to fluctuations in number of collisions, and also to capture and loss of electrons, α's don't have a perfectly uniform range, but straggle a little. The effect is small for α's (See Rasetti), but presumably larger for mesons.

3. The same holds for scattering. The multiple scattering is a distinctive feature of meson tracks in emulsion, and has been used by Powell to estimate their mass. On the other hand, it imposes a definite limit on $H\rho$ measurements in cloud chamber mass determinations.

4. Occasionally, an α scatters off a nucleus. (Rutherford scattering.)