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THREE LECTURES ON CONTROLLED THERMONUCLEAR POWER PRODUCTION

Herbert F. York

August 29, 1952

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THREE LECTURES ON CONTROLLED THERMONUCLEAR POWER PRODUCTION

Herbert F. York

Radiation Laboratory, Department of Physics
University of California, Berkeley, California

August 29, 1952

This report was written from the notes used by Dr. York in delivering his three lectures on considerations pertinent to the problem of utilizing controlled thermonuclear reactions for the production of power. Suggestions, additions, and corrections have been made by H. Brown, J. Hadley, R. Jastrow, C. Leith, R. LeLevier, E. Martinelli, and J. Roberts.

The first lecture discussed fundamental properties of the DD and DT reactions and the reacting substances. The second lecture described the stellarator, a proposed machine for the controlled production of power from the above reactions. This lecture was essentially an abstract of a report written by Lyman Spitzer[1] and primarily dealt with the utilization of the DT reaction. The third lecture described the pinch effect and considered its possible application to the thermonuclear power problem.

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THREE LECTURES ON CONTROLLED THERMONUCLEAR POWER PRODUCTION

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First Lecture

FUNDAMENTALS

The Reactions

The substances that we are considering for our source of power are deuterium and a deuterium-tritium mixture. The use of nuclei of higher Z does not seem desirable for a couple of reasons. The power lost from the reaction mixture by radiation varies as Z^3, and the presence of the higher coulomb barriers would greatly diminish the nuclear reaction rate. These effects would require much higher operating temperatures.

The chief reaction of the deuterium-tritium mixture is

$$D + T \longrightarrow He^4 + n + 17.6 \text{ Mev.}$$

The n produced can presumably be used for making more T.

With pure deuterium one has two primary reactions of approximately equal cross section taking place:

$$D + D \xrightarrow{\sim 50\%} T + P + 3.98 \text{ Mev}$$

$$D + D \xrightarrow{\sim 50\%} He^3 + N + 3.25 \text{ Mev,}$$

and so for each DD reaction one has essentially

$$D + D \longrightarrow 1/2\ T + 1/2\ He^3 + 1/2\ N + 1/2\ P + 3.62 \text{ Mev.}$$

For each deuteron consumed, then, one obtains 1.8 Mev directly. By using the neutron produced to induce a fission in the container walls or in some similar location one has the possibility of liberating an additional 45 Mev per deuteron consumed.

The products of the primary reactions are capable of further reaction:

$$D + T \longrightarrow He^4 + N + 17.6 \text{ Mev}$$

$$D + He^3 \longrightarrow He^4 + P + 18.3 \text{ Mev}$$

$$N + He^3 \longrightarrow T + P + 0.73 \text{ Mev.}$$

The extent to which these reactions contribute to the total energy production depends on the temperature of the system and the concentrations of T and He^3. The cross section of the DHe^3 reaction is such that it is probably inactive under most operating conditions. On the other hand the DT reaction should contribute significantly, as discussed later in connection with the power production, leading to a liberation of 12.4 Mev per DD reaction.

Reaction Cross Sections

The cross sections for the DD and DT reactions as functions of energy are plotted in Figure 1. Figure 1 was plotted using the Gamow low energy formulas

$$\sigma_{DD} = \frac{280}{E(\text{kev})} e^{-\frac{44.3}{E}}$$

up to 10 kev and

$$\sigma_{DT} = \frac{2.376 \times 10^4}{E_D} e^{-\frac{44.555}{E_D}}$$

up to 6 kev.

At higher energies the following values, taken from LA-1190[2] were used:

E_D, kev	σ_{DD}, barns	σ_{DT}, barns
10		8.6×10^{-4}
20	5.4×10^{-4}	3.4×10^{-2}
40	6×10^{-3}	0.4
60	1.6×10^{-2}	1.4
100	3.7×10^{-2}	4.2
200	8×10^{-2}	2.9
400	0.116	0.69
600	0.134	0.37
1000	0.154	0.20

One can see that the cross sections rise very rapidly from almost insignificant values around one kilovolt to large values at 100 kev. As a result, at relatively low temperatures the bulk of the reacting will be accomplished by the gas particles on the high energy end of the Maxwellian distribution.

Thermal Language

Before continuing, it might be advantageous to introduce some of the thermal language that is commonly used in connection with thermonuclear reactions.

Temperature, θ, is usually spoken of in energy units, specifically kev. θ is set equal to kT and therefore one kilovolt equals 1.16×10^7 °K. Consequently,

$$C_v = \frac{3}{2}\frac{kev}{kev} = \frac{3}{2} \times 1.6 \times 10^{-9}\frac{erg}{kev} = 2.4 \times 10^{-9}\frac{erg}{kev}$$

per particle for a completely ionized gas.

To get C_v per unit mass of completely ionized gas, we note that one atom ionized into Z + 1 particles of total mass A, and so

$$C_v = \frac{2.4 \times 10^{-9} \times 0.6 \times 10^{24} \times (Z+1)}{A} \; \frac{1.44 \times 10^5}{\bar{A}} \frac{erg}{kev\ gm} \; \frac{3.4 \times 10^7}{\bar{A}} \frac{cal}{kev\ gm}$$

where $\bar{A} = \frac{A}{Z+1}$.

The radiation energy density as a function of kilovolts is

$$E = 1.37 \times 10^{14}\ \theta^4 \frac{erg}{cm^3\ kev} = 1.31 \times 10^7\ \theta^4 \frac{cal}{cm^3\ kev} ,$$

and the heat capacity of the vacuum becomes

$$C_{vac} = 5.48 \times 10^{14}\ \theta^3 \frac{erg}{cm^3\ kev} = 1.31 \times 10^7\ \theta^3 \frac{cal}{cm^3\ kev}$$

Similarly, the perfect gas law takes the form

$$P_{atm} = 1.58 \times 10^{-15}\ n\ \theta$$

where n is the particle density in particles per cc.

Power Production

If some deuterons were to move in a beam with a velocity, v, through an assembly of stationary deuterons, the reaction rate per incident particle would be given by

$$R = n\ \sigma\ v$$

where n is the density of the stationary deuterons. In the case of a confined deuteron gas the particles are moving relative to each other with varying velocities, and, because the reaction cross section is a function of the relative particle energies, σv must be averaged over all v. Likewise, to get the reaction rate per unit volume we must sum over all the deuterons, obtaining, since this procedure counts each particle twice,

$$R_{DD} = 1/2\ n_D^{\ 2}\ \overline{\sigma v_{DD}}\ \frac{reactions}{cc\ sec} .$$

Since two deuterons are consumed per reaction, we can also note that

$$-\dot{n}_D = n_D^{\ 2}\ \overline{\sigma v_{DD}} .$$

Similarly for the DT system the reaction rate is

$$R_{DT} = -\dot{n}_D = n_D n_T \overline{\sigma v}_{DT} \frac{\text{reactions}}{\text{cc sec}} .$$

The values of $\overline{\sigma v}$ at various temperatures are given in Table I for both the DD and DT reactions and are plotted against temperature in Figure 2. These values were supplied by J. L. Tuck in a letter dated June 7, 1952. They were computed from measurements made at Los Alamos and are considered to supersede those appearing in LA-1190. The DT values were characterized as being "final" and the DD values as being "semifinal". Also included at the bottom of Table I are an expression for $\overline{\sigma v}_{DHe^3}$ given by Fermi[3] and an expression for $\overline{\sigma v}_{DD}$. The $\overline{\sigma v}_{DD}$ expression is also from Fermi's draft, but the leading coefficient has been changed to reproduce Tuck's values at 2 and 3 kev. This expression was used to extend the $\overline{\sigma v}_{DD}$ curve of Figure 2 to below 2 kev. Values of $\overline{\sigma v}_{DHe^3}$ calculated from Fermi's expression are also plotted in Figure 2.

The mean reaction time for a deuteron is

$$\tau = \frac{1}{n\overline{\sigma v}} .$$

In the DD reaction, with $\theta = 30$ kev and $n = 6 \times 10^{14}$, τ is approximately 240 seconds.

One atmosphere pressure at about room temperature corresponds to 2.50×10^{19} molecules/cm^3. We usually deal with small particle densities in order to avoid excessive pressures at the temperatures of thermonuclear interest.

TABLE I

$\overline{\sigma v}_{DD}$ and $\overline{\sigma v}_{DT}$ at Various Temperatures

θ, kev	$\overline{\sigma v}_{DD}$, cm^3/sec	$\overline{\sigma v}_{DT}$, cm^3/sec
0.05		6.8×10^{-35}
0.1		3.2×10^{-30}
0.5		6.1×10^{-23}
1.0		7.0×10^{-21}
2	5.4×10^{-21}	2.9×10^{-19}
3	2.8×10^{-20}	
5		1.4×10^{-17}
6	2.7×10^{-19}	
10		1.1×10^{-16}
12	1.4×10^{-18}	
20	3.6×10^{-18}	4.3×10^{-16}
40	1.0×10^{-17}	7.9×10^{-16}
60		8.7×10^{-16}
80	2.3×10^{-17}	8.5×10^{-16}
100	3.0×10^{-17}	8.1×10^{-16}

<u>**Empirical Functions:**</u>

$$0 \le \theta \le 5: \quad \overline{\sigma v}_{DT} \times 10^{19} = 7.332327078541502 \times 10^{-5}\,\theta - 2.7600781 \times 10^{-3}\,\theta^2 + 3.2510296 \times 10^{-2}\,\theta^3 - 1.4324404 \times 10^{-1}\,\theta^4 + 2.1106837 \times 10^{-1}\,\theta^5 - 2.808987 \times 10^{-2}\,\theta^6$$

$$5 \le \theta \le 100: \quad \overline{\sigma v}_{DT} \times 10^{16} = -7.35644244 \times 10^{-2}\,\theta + 2.0446127 \times 10^{-2}\,\theta^2 + 7.423756 \times 10^{-5}\,\theta^3 - 3.1783311 \times 10^{-5}\,\theta^4 + 8.0905986 \times 10^{-7}\,\theta^5 - 7.9792719 \times 10^{-9}\,\theta^6 + 2.8037971 \times 10^{-11}\,\theta^7$$

$$2 \le \theta \le 20: \quad \overline{\sigma v}_{DD} = 0.42770 - 0.7157\theta^2 - 0.3674\theta^3 - 0.02570\theta^4 + 0.0005651\theta^5$$

$$20 \le \theta \le 100: \quad \overline{\sigma v}_{DD} = -5.865 \times 10^{-2} + 9.00 \times 10^{-3}\theta - 7.746 \times 10^{-4}\theta^2 - 9.964 \times 10^{-6}\,\theta^3 + 4.38 \times 10^{-8}\theta^4$$

$$\overline{\sigma v}_{DHe^3} = \frac{60 \times 10^{-14}}{\theta^{2/3}} \times 10^{-\frac{13.74}{\theta^{1/3}}}$$

$$\overline{\sigma v}_{DD} = \frac{248}{\theta^{2/3}} \times 10^{-16} \times 10^{-\frac{8.14}{\theta^{1/3}}}$$

In the pure deuterium system Figure 2 indicates that unless the concentration of He^3 is allowed to build up to near the deuteron concentration the DHe^3 reaction contributes rather little to the total power release. At higher temperatures the DHe^3 reaction becomes more important, and perhaps this property can be used to advantage. Since the cross section for the DT reaction is much larger than that for the DD reaction, the T concentration will build up to only a small value, of the order of one half to one atom percent, before its rate of consumption will equal its rate of production. In this steady state condition, then, every DD reaction will be accompanied by half of a DT reaction, and the energy released per DD reaction will then be

$$3.62 + 1/2 \times 17.6 \text{ Mev} = 12.4 \text{ Mev} = 1.98 \times 10^{-12} \text{ joules.}$$

On this basis the power production is

$$P_{DD} = 0.99 \times 10^{-12} \, \overline{\sigma v}_{DD} \, n_D^2 \, \frac{\text{watts}}{\text{cm}^3}$$

The power produced by the DT reaction is

$$P_{DT} = 2.82 \times 10^{-12} \, \overline{\sigma v}_{DT} \, n_D n_T \, \frac{\text{watts}}{\text{cm}^3}$$

Some values of P_{DD}/n_D^2, P_{DD}, $P_{DT}/n_D n_T$, and P_{DT} are given for various temperatures in Table II.

Power Loss

Energy will be lost from the hot gases by radiation and by heat conduction.

The diffuseness of the gases coupled with the dimensions of the reactors we have considered are such that thermal equilibrium of the radiation with the particles is not attained. In fact practically all of the photons produced flow from the gas without interacting with it at all. As a consequence, there is virtually no black body radiation, and the radiant energy is lost as bremsstrahlung. At the temperatures of interest the heat capacity of the radiation is very large compared with that of the gaseous particles, and the immediate exodus of the photons is desirable to prevent undue cooling of the reacting mixture.

The power radiated per unit volume by bremsstrahlung is given by

$$P_{brems} = \frac{-dE}{dt} = n_Z\, n_e\, \frac{16}{3}\frac{Z^2 e^6}{mc^3 \hbar}\, \bar{v}_e\, \frac{ergs}{sec\ cm^3},^{4}$$

where $\bar{v}_e$ is the mean velocity of the electrons.

$$\bar{v}_e = \sqrt{\frac{8\,kT}{\pi m}} = \sqrt{\frac{8 \times 1.6 \times 10^{-9}\,\theta}{\pi\, m}} = \frac{1}{14.1}\, c\theta^{1/2}.$$

For deuteron and tritons $Z = 1$, and $n_Z = n_e = n_D + n_T$. For pure deuterium, of course, $n_D + n_T = n_D$.

$$P_{brems} = 0.538 \times 10^{-30}\, (n_D + n_T)^2\, \theta^{1/2}\ \text{watts/cm}^3.$$

Some value of $\frac{P_{brems}}{(n_D+n_T)^2}$ and of P_{brems} for $n_D + n_T = 6 \times 10^{14}$ are given in Table II. One will note that between 10 and 20 kilovolts P_{DD} becomes larger than P_{brems}, and that between 2 and 3 kilovolts P_{DT} becomes larger than P_{brems}. The temperature at which $P_{\text{nuclear reaction}}$ equals $P_{\text{radiation}}$ is called the ideal ignition temperature. If appropriately insulated against heat loss by conduction and if warmer than their ideal ignition temperatures, reacting gases will maintain or increase their temperatures and consequently will sustain their reactions. One might note that although this statement is rather commonly made without any further qualifications, that its truth requires a completely closed system for the particles produced by the reactions and a completely open system for the bremsstrahlung.

During the ensuing discussion on heat conductivity we will picture the gases as being contained in cylindrical vessels of infinite length. Of course real vessels cannot be that long and giving them finite dimensions remains a foremost problem, although one for which a number of tentative and perhaps workable solutions have been advanced.

At first blush the heat conductivity appears to be overwhelmingly large, but inasmuch as we are dealing with completely ionized gases, we may diminish the thermal diffusion by applying a magnetic field along the axis of the cylinder. The field constrains the particles to move in spirals around the lines of force, and if strong enough, can thereby decrease the heat loss by conduction to even a negligible value.

The immediately preceding remarks are based on the following figuring:

The quantity of heat flowing per unit area per unit time in a gas is given by

$$h = 1/3\, n\, \lambda\, v \frac{d\bar{E}}{dX},$$

Where $\lambda = \frac{1}{n\sigma}$ is the mean free path, σ being the transport cross section, $\bar{E} = 3/2\ kT$ is the mean energy per particle, and X is length.

The transport cross section, σ, is derived by applying a $1 - \cos\alpha$ factor to the coulomb angular cross section and then integrating over all possible angles. The $1 - \cos\alpha$ factor takes into account the fact that the small angle scatterings are not as effective in stopping the advance of a particle as are the large angle scatterings.

TABLE II

Power Production and Power Radiation at Various Temperatures

θ	P_{DD}/n_D^2	P_{DD} for $n_D = 6 \times 10^{14}$	$1/4\ P_{DT}/n_Dn_T$ for $n_D = n_T = 1/2\ (n_D + n_T) = P_{DT}/(n_D+n_T)^2$	P_{DT} for $n_D = n_T = 3 \times 10^{14}$	$\frac{P_{brems}}{(n_D + n_T)^2}$	P_{brems} for $n_D + n_T = 6 \times 10^{14}$
—	(watts cm^3)	(watts/cm^3)	(watts cm^3)	(watts/cm^3)	(watts cm^3)	(watts/cm^3)
30	8.5×10^{-30}	3.07	5×10^{-28}	180	2.94×10^{-30}	1.06
20	4.65×10^{-30}	1.62	3.03×10^{-28}	109	2.41×10^{-30}	0.87
10	1.30×10^{-30}	0.465	7.75×10^{-29}	27.9	1.71×10^{-30}	0.62
5	2.06×10^{-31}	0.0740	9.85×10^{-30}	3.54	1.21×10^{-30}	0.44
3	3.81×10^{-32}	0.0138	1.38×10^{-30}	0.50	0.93×10^{-30}	0.33
2	7.80×10^{-33}	2.41×10^{-3}	2.04×10^{-31}	0.074	0.76×10^{-30}	0.27
1	2.51×10^{-34}	9.03×10^{-5}	4.93×10^{-33}	0.0018	0.54×10^{-30}	0.19

$$d\sigma = \left(\frac{ZZ'e^2}{2\mu V_{12}^2}\right)^2 \frac{1}{\sin^4 \frac{\alpha}{2}} \; 2\pi \; \sin\alpha \; d\alpha \; (1 - \cos\alpha)$$

Making the substitutions $\sin^4 \frac{\alpha}{2} = \frac{(1 - \cos\alpha)^2}{4}$ and $\sin\alpha \; d\alpha = -d\cos\alpha$, and integrating, we get,

$$\sigma = \left(\frac{ZZ'e^2}{2\mu V_{12}^2}\right)^2 8\pi \int_{\alpha_1}^{\alpha_2} \frac{-d\cos\alpha}{1 - \cos\alpha}$$

$$= \left(\frac{ZZ'e^2}{2\mu V_{12}^2}\right)^2 8\pi \; \ln\left(\frac{1 - \cos\alpha_2}{1 - \cos\alpha_1}\right).$$

The upper limit, α_2, must be π, corresponding to head-on collisions. The lower limit, α_1, will be small corresponding to a large collision parameter, and so

$$\sigma \doteq \left(\frac{ZZ'e^2}{2\mu V_{12}^2}\right)^2 8\pi \; \ln\left(\frac{4}{\alpha_1^2}\right).$$

Inasmuch as we have an ionic gas, though, the collision parameter, b, cannot be infinite, because for large enough b the plasma hosting the colliding particles will adjust continuously to their motion in such a way that they will have no effect on each other. The maximum value for b will then be that for which b/v corresponds to the relaxation time of the plasma. Electron plasma oscillations are much more rapid than positive ion oscillations and so the former should determine the effective relaxation time here. The electron plasma angular oscillation frequency is given approximately by[5]

$$\omega_e^2 = \frac{4\pi \eta_e e^2}{m}.$$

Since

$$\frac{b_{max}}{v} \approx \frac{1}{\omega_e},$$

$$\sigma_1 = \frac{2ZZ'e^2}{b_{max} m v^2} = \frac{2e^3 (4\pi n_e)^{1/2}}{m^{3/2} v^3} = 2.98 \times 10^{-15} \sqrt{\frac{n_e}{\theta^3}} \,,$$

and we get for the effective collision cross section, putting $\mu v_{12}^2 = 3 \times 1.6 \times 10^{-9} \theta$,

$$\sigma = \left(\frac{e^2}{2 \times 4.8 \times 10^{-9} \theta}\right)^2 \; 8\pi \ln \left(\frac{4\,\theta^3}{8.85 \times 10^{-30} n_e}\right)$$

$$= \frac{1.453 \times 10^{-20}}{\theta^2} \left[68.4 + \ln \left(\frac{\theta^3}{n_e}\right)\right]$$

$$= \frac{9.94 \times 10^{-19}}{\theta^2} + \frac{0.334 \times 10^{-19}}{\theta^2} \log \left(\frac{\theta^3}{n_e}\right).$$

The term in the square brackets is rather insensitive to changes in θ or n_e. Taking θ as 10 kev and n_e as 6×10^{14} cm^{-3} we obtain an approximate expression,

$$\sigma = \frac{6.00 \times 10^{-19}}{\theta^2} \text{ cm}^2$$

Now, substituting back into our heat flow equation, for deuterium gas we find

$$h = -\frac{1}{3} n \,\lambda\, \bar{v} \frac{dE}{dX}$$

$$= -\frac{1}{3} (n_D + n_e) \times \frac{1}{(n_D + n_e)} \times \frac{1}{2} (\bar{v}_D + \bar{v}_e) \times \frac{3}{2} (1.6 \times 10^{-9}) \frac{d\theta}{dX}$$

$$= -\frac{1}{3} \times \frac{\theta^2}{6 \times 10^{-19}} \times \frac{1}{2} \left(\frac{1}{14.1} c\, \theta^{\frac{1}{2}}\right) \times \frac{3}{2} (1.6 \times 10^{-9}) \frac{d\theta}{dX}$$

$$= -1.42 \times 10^{18}\, \theta^{5/2} \frac{d\theta}{dX} \; \frac{\text{ergs}}{\text{cm}^2 \text{ sec}} = -3.39 \times 10^{10}\, \theta^{5/2} \frac{d\theta}{dX} \frac{\text{cal}}{\text{cm}^2 \text{ sec}}$$

$$= -1.42 \times 10^{11}\, \theta^{5/2} \frac{d\theta}{dX} \frac{\text{watts}}{\text{cm}^2} .$$

To take into account that we are considering a cylindrical container we note that there are $2\pi r$ cm^2 area normal to the temperature gradient per cm length of the cylinder. The radial heat flow per unit length is then

$$h = -8.91 \times 10^{11}\, r\, \theta^{5/2} \frac{d\theta}{dr} \frac{\text{watts}}{\text{cm}} .$$

If one had a 1 cm radius cylinder of deuterium gas at 30 kev in the center of a 30 cm radius cylinder whose walls were at room temperature, one would estimate that the heat loss by thermal diffusion was roughly 10^{15} times the heat produced by the thermonuclear reaction.

To look at the insulating effect of a magnetic field applied along the axis of the reaction cylinder, we must modify somewhat the no-field equation,

$$h = -\frac{1}{3} n \lambda \bar{v} \frac{d\bar{E}}{dX} .$$

In the following, and throughout this report, we assume that we have a quiescent plasma. Plasma oscillations lead to heat conductivities of the order of the geometric mean between the field free case and the magnetic field case with a quiescent plasma.

The mean distance of the movement of the particles up and down the temperature gradient between collisions is no longer the mean free path, λ. At each collision the axis of rotation of a spiraling particle is moved a distance on the average about equal to R, the radius of the spiral, and so we replace λ by R in the equation for h. The rate at which collisions take place is $\frac{\bar{v}}{\lambda}$, and so we replace $\bar{v}$ by $\frac{\bar{v}R}{\lambda}$, the effective velocity with which the particle move normal to the magnetic field. The equation for h is consequently

$$h = -\frac{1}{3} n R \frac{\bar{v}}{\lambda} R \frac{d\bar{E}}{dX} = -\frac{1}{3} n \lambda \bar{v} \frac{d\bar{E}}{dX}\left(\frac{R}{\lambda}\right)^2$$

and the heat conducted by the deuterium gas normal to a magnetic field is

$$h = h_D + h_e = \frac{1}{3} n_D R_D \left(\frac{\bar{v}_D}{\lambda} R_D\right) \frac{d\bar{E}}{dX} \frac{1}{3} n_e R_e \left(\frac{\bar{v}_e}{\lambda} R_e\right) \frac{d\bar{E}}{dX}$$

$$= \frac{1}{3} n_D \frac{1}{\lambda} \frac{d\bar{E}}{dX} (\bar{v}_D R_D{}^2 + \bar{v}_e R_e{}^2) \ . \quad (n_e = n_D)$$

To express h in terms of the particle density, temperature, and magnetic field strength, H, we must evaluate the elements of the above equation.

$$R = mv \frac{c}{eH}$$

$$R^2 = \frac{1}{2} mv^2 \cdot \frac{2mc^2}{e^2H^2} = \frac{3}{2} (1.6 \times 10^{-9}) \theta \cdot \frac{2mc^2}{e^2H^2}$$

$$= 3.10 \times 10^7 \frac{M\theta}{H^2}$$

where m is the mass of the particle and M is its atomic weight.

$$R_D^{\,2} = 6.24 \times 10^7 \frac{\theta}{H^2} \; ; \; R_e^{\,2} = 1.70 \times 10^4 \frac{\theta}{H^2}$$

$$\bar{v} = 1.66 \times 10^{-3} \, c \, \frac{\theta}{M} \; ; \; \bar{v}_D = 1.17 \times 10^{-3} \, c \, \theta^{1/2} \; ; \; \bar{v}_e = \frac{1}{14.1} \, c\theta^{1/2}$$

$$\frac{d\mathcal{E}}{dX} = \frac{3}{2} (1.6 \times 10^{-9}) \frac{d\theta}{dX}$$

$$\frac{1}{\lambda} = n\sigma = (n_D + n_e) \frac{6.00 \times 10^{-19}}{\theta^2} = 1.2 \times 10^{-18} \frac{n_D}{\theta^2}$$

One will note that $\bar{v}_D R_D^{\,2}$ is $\sqrt{\frac{m_D}{m_e}} = 60.6$ times the size of $\bar{v}_e R_e^{\,2}$. In spite of the slower velocity of the deuterons, their greater radii of gyration make them the prime carriers of the heat in the magnetic field in contradistinction to the field free case.

We can now write for the heat flow

$$h = -\,2.10 \times 10^{-12} \frac{n_D^{\,2}}{\theta^{1/2} H^2} \frac{d\theta}{dX} \frac{\text{ergs}}{\text{cm}^2\ \text{sec}} = -\,5.02 \times 10^{-20} \frac{n_D^{\,2}}{\theta^{1/2} H^2} \frac{d\theta}{dX} \frac{\text{cals}}{\text{cm}^2\ \text{sec}}$$

$$= -\,2.10 \times 10^{-19} \frac{n_D^{\,2}}{\theta^{1/2} H^2} \frac{d\theta}{dX} \frac{\text{watts}}{\text{cm}^2} ,$$

and the heat flow per unit length of the cylinder is now

$$h = -\,1.32 \times 10^{-18} \frac{r\, n_D^{\,2}}{\theta^{1/2} H^2} \frac{d\theta}{dr} \frac{\text{watts}}{\text{cm}} .$$

To see how much the application of a magnetic field insulates the system against thermal diffusion we can divide $h_{\text{field free}}$ by h_{magnetic}, obtaining

$$\frac{h_{\text{field free}}}{h_{\text{magnetic}}} = \frac{-8.91 \times 10^{11}\, r\theta^{5/2} \frac{d\theta}{dr}}{-1.32 \times 10^{-18} \frac{r\, dD^2}{\theta^{\frac{1}{2}} H^2} \frac{d\theta}{dr}}$$

$$= 6.76 \times 10^{29} \frac{\theta^3 H^2}{n_D^2}$$

For the case we considered earlier where $\theta = 30$ kev, $\frac{d\theta}{dX} \sim 1$, and if $n_D = 6 \times 10^{14}$ and $H = 20{,}000$ gauss,

$$\frac{h_{\text{field free}}}{h_{\text{magnetic}}} = 2.03 \times 10^{13} ,$$

which is just about an adequate insulation factor. By changing the temperature, particle density, or magnetic field, one can increase the insulation factor to the point where the heat loss by thermal diffusion is less than that produced by the DD reaction.

The magnetic field does not come out of the clear blue sky and the power to generate it must be supplied by the thermonuclear reaction. To estimate the power required we may consider the field produced by a current in the reaction vessel walls circulating perpendicular to the axis of the cylinder. The field strength in the vessel is

$$H = \frac{4\pi}{10} j\, (r_2 - r_1)$$

where r_1 and r_2 are respectively the inner and outer radii of the vessel wall and j is the current density. The power per unit length to create the field is then

$$P_H = j^2 \cdot (r_2 - r_1)^2 \cdot R$$

$$= \frac{100\, H^2}{16\pi^2 (r_2 - r_1)^2} \cdot (r_2 - r_1)^2 \cdot \frac{2\pi \frac{(r_2 - r_1)}{2}}{(r_2 - r_1)}$$

$$= \frac{25}{4\pi} \left(\frac{r_2 + r_1}{r_2 - r_1}\right) \frac{H^2}{\sigma} \frac{\text{watts}}{\text{cm}}$$

where R is the resistance per unit length of the current carrying cylinder and σ is its conductivity.

One can see that increasing the outer radius of the wall beyond a certain point does little toward reducing the power requirement of the magnetic field and also that for a given r_1 to r_2 ratio the power requirement for a given field strength is independent of the radius of the vessel. Consequently, to sustain the magnetic field by means of power from the thermonuclear reaction one need only take a vessel of sufficiently large diameter.

If one presumes that half the volume of the cylinder wall is copper, the remaining portion being occupied by cooling liquid, insulation, etcetera, then, at 100°C, using one half the conducitivity of copper, i.e., $\frac{1}{2}\sigma = 2.2 \times 10^5$ ohm^{-1} cm^{-1}, and taking r_2/r_1 equal to two and a field of 20,000 gauss, one calculates

$$P_H = 11,000 \frac{\text{watts}}{\text{cm}} .$$

The temperature at which the copper wall can be kept is certainly a still open question but at least in principle the copper could be cooled to liquid nitrogen temperature with a concomitant factor of ten increase in its conductivity. However, to run at liquid nitrogen temperature one would have to expend about as much power in liquifying the nitrogen as in supporting the magnetic field, and so in this case one would have

$$P_H \sim 2,200 \text{ watts/cm.}$$

Earlier we noted the mean reaction time for a deuteron. One can compare with it the times required for the particles to diffuse out of the reaction vessel. The time for an electron to make a collision is

$$\tau = \frac{1}{n\sigma v} = \frac{1}{2n_e \sigma v} = 3.92 \times 10^8 \frac{\theta^{3/2}}{n_e} ,$$

and for a deuteron,

$$\tau = 2.37 \times 10^{10} \frac{\theta^{3/2}}{n_e} .$$

Ignoring the variation of temperature between the central region and the walls, one gets for the time for an electron to diffuse out in the case of no magnetic field,

$$t = \left(\frac{r}{\lambda}\right)^2 \tau = r^2 n^2 \sigma^2 \tau = \frac{r^2 n \sigma}{v}$$

$$= 5.64 \times 10^{-28} \frac{r^2 n_e}{\theta^{5/2}} ,$$

and for a deuteron to diffuse out,

$$t = 3.42 \times 10^{-26} \frac{r^2 n_e}{\theta^{5/2}} .$$

When a magnetic field is applied the times to diffuse out becomes

$$t = \left(\frac{r}{R}\right)^2 \tau$$

$$= 2.30 \times 10^4 \frac{r^2 H^2 \theta^{1/2}}{n_e} \quad \text{for electrons,}$$

and $$t = 380 \frac{r^2 H^2 \theta^{1/2}}{n_e} \quad \text{for deuterons.}$$

One then obtains for an insulation factor,

$$\frac{t_{D\ \text{field}}}{t_{e\ \text{no field}}} = \frac{380\ r^2 H^2 \theta^{1/2}}{n_e} \times \frac{\theta^{5/2}}{5.64 \times 10^{-28}\ r^2\ n_e}$$

$$= 6.75 \times 10^{29} \frac{\theta^3 H^2}{n_e^2} .$$

For the conditions we have been considering, $\theta = 30$ kev, $n_e = 6 \times 10^{14}$, $H = 20{,}000$ gauss, $r = 30$ cm, we obtain for an insulation factor,

$$\frac{t_{D\ \text{field}}}{t_{e\ \text{no field}}} = 2.02 \times 10^{13} ,$$

with t_D field = 1.25 seconds and $t_{e\ \text{no field}} = 6.17 \times 10^{-14}$ seconds.

The foregoing has been the substance of the first lecture which presented the ideas and computations that seemed basic to the thermonuclear power problem. Admittedly some things were not considered or were not treated very accurately. For instance, no discussion was made of the ranges of the products of the nuclear reactions. These ranges are large compared to the dimensions of the reaction vessels that we have so far considered and would lead to additional large energy losses from the reactors. Similarly, the field free heat conduction equation does not apply to the vessel sizes and particle densities that we have talked about, inasmuch as the mean free path is several orders of magnitude greater than the vessel diameters. Even in the magnetic field case no account has been taken of the fact that the particles get cold out near the vessel walls and recombine to form neutral atoms, vastly increasing the heat conducitivity in that region.

Second Lecture

THE STELLARATOR

As mentioned earlier, this lecture was essentially an abstract of a report by Lyman Spitzer[1], and consequently it was intended to be more a description than a critique.

An axial magnetic field in a cylinder is capable of decreasing the heat conductivity of an ionic gas only normal to the field. Along the lines of force the heat conductivity remains huge, and therefore any practical thermonuclear power producing device must in some way diminish the otherwise excessive heat losses at its ends. Suggestions such as holding the plasma back from the ends with microwaves or with suitably shaped magnetic fields aided by microwaves perhaps will prove workable.

One suggestion for eliminating end effects is to eliminate the ends, such as by bending the cylinder around into a torus. The torus, however, turns out to be an unworkable shape. The magnetic field is stronger at the inner wall than at the outer wall, and as Figure 3 illustrates for velocities normal to the field, this inhomogeneity causes the charged particles of one sign to drift toward the top and those of the other sign to drift toward the bottom of the torus. Charged particles moving along the lines of force in a torus drift up and down in a like manner. These drifts are quickly stopped by the space charge they produce, but this space charge in turn causes the particles of both signs to drift to the outer wall of the torus. In a tube the size of the one used in the standard stellarator, described below, the particles reach the wall in the order of 10^{-3} seconds, which, since the mean reaction time for the standard stellarator is about 30 seconds, turns out to be at least twenty times too rapid.

The stellarator is a device proposed by Dr. Spitzer to eliminate both the ends and their concomitant heat losses and likewise to minimize the excessive drifts across the field that occur in the torus. It consists of a cylindrical tube that is closed upon itself but is in the form of a figure 8, as illustrated in Figure 4. The basic feature of the stellarator is its reverse curvatures which cause particles moving around the stellerator to drift one way in one loop of the 8 and the other way in the other loop, thereby decreasing the net drift to the wall to an acceptable value. Dr. Spitzer has investigated many facets of the problem of the ability of the stellarator's magnetic field to constrain the ionic gas. The problem is a complicated one and although he has looked into it in considerable detail, his inquiry is still not complete. However, at the present time, he has concluded that (1) if the ratio of material pressure, 1.6×10^{-9} n θ, to magnetic pressure, $H^2/8\pi$, is small compared with $5r_1/L$, where r_1, and L are the inside radius and length of the stellarator tube, (2) if the stellarator is symmetric with respect to rotations about the Z axis (see Figure 4), (3) if it is designed with end loops of slowly varying radius of curvature and likewise sufficiently removed from the cross-over loops by the straight tubes, and finally, (4) if there is a radial electric field of 1000 to 10,000 volts per centimeter to cause the particles to circulate around the tube normal to the magnetic field, that the ionic gas will be satisfactorily constrained.

The operating conditions and dimensions of the stellarator were arrived at in a straightforward manner. Dr. Spitzer based most of his considerations on the DT reaction because it produces much more power at the temperatures of interest than does the DD reaction. One will note in

Table II that the ideal ignition temperature for the DT reaction lies between one and ten kilovolts. It seems reasonable then to adopt ten kilovolts as the operating temperature of a "standard" stellarator.

For obvious reasons connected with ordinary construction problems the operating pressure of the stellarator must not be too high. If one assumes an operating pressure of ten atmospheres he gets a particle density of 6×10^{14} cm^{-3} at a temperature of ten kilovolts, which corresponds to a pressure of about 5×10^{-3} mm at room temperature when account is taken of the recombination of the charged particles.

The material energy density at the above temperature and pressure is

$$P = 1.6 \times 10^{-9} \times 6 \times 10^{14} \times 10 \approx 10^{7} \frac{\text{erg}}{\text{cm}^3} .$$

Originally it was believed that a magnetic pressure of approximately twice the material pressure would be adequate for the confinement of the ionized gas. On that basis if one solves

$$P_{\text{magnetic}} = \frac{H^2}{8\pi} \approx 2 \times 10^{7} \frac{\text{erg}}{\text{cm}^3} ,$$

he gets

$$H \approx 22,400 \text{ gauss.}$$

A magnetic field strength of 20,000 gauss has usually been taken for the standard stellarator although that field strength is much too low compared with the other ordinarily adopted stellarator dimensions if the ratio of material pressure to magnetic pressure must be less than $5r_1/L$ as mentioned earlier. The problem is resolvable, though, even if one must adopt the most pessimistic value for the field strength. Inasmuch as the power required for a particular field strength can be independent of the tube diameter, a larger tube with a lower particle density could get around any difficulty arising here.

One would like the mean reaction time of a particle to be not too much longer than the time for it to drift to the wall of the tube lest there should be more energy delivered to the wall than is produced by the thermonuclear reaction. With a magnetic field strength of 20,000 gauss the time for the triton to diffuse out of a 50 cm radius vessel is about 33 seconds as compared with the 30 second mean DT reaction time. Dr. Spitzer adopted 50 cm as the radius of the stellarator tube.

Adopting the foregoing as elements of the design of the standard stellarator and presuming the reaction fills the entire volume of the stellarator one calculates that about 55 kilowatts per centimeter length of the tube are produced. About 1.2 kilowatts per centimeter are lost in bremsstrahlung, of the order of 10 kilowatts per centimeter are lost by heat conduction, and for $r_2 = 2r_1$ two to eleven kilowatts per centimeter go into producing the magnetic field. One also might note at this point that this stellarator design will require about 0.3 ton of copper per inch of tube length or about 1200 tons of copper altogether.

Without a detailed analysis of the temperature distribution, which does not as yet exist, one cannot make a complete quantitative description of the particle and material pressures as functions of the distance from the center of the tube. However, in view of Lenz's law one can be sure that any currents circulating normal to the magnetic field in the stellarator tube will be in such a direction as to tend to cancel the magnetic field, and consequently one can see that the magnetic pressure will be less in the center than at the edge. As shown in the derivation below the sum of the particle and magnetic pressures is constant, and so one can be sure that the particle pressure will be greater in the central region than at the outside.

To show that the sum of the particle and magnetic pressures is constant we note that the force per unit volume acting outward is the gradient of the particle pressure and that it is balanced by the inward force of the magnetic field's interaction with the Lenz's law current, i.e.,

$$\frac{dP}{dr} + i \times H = 0 ,$$

where i is the current density. We would like to eliminate i and we may note that the magnetic field strength, H in a solenoid is given by

$$H = 4\pi I,$$

where I is the total current per unit length of the solenoid circulating about the position at which H is measured.

Inasmuch as

$$i = \frac{dI}{dr} ,$$

if we differentiate H, we get

$$\frac{dH}{dr} = 4\pi \frac{dI}{dr} = 4\pi i .$$

Now, substituting

$$i = \frac{1}{4\pi} \frac{dH}{dr}$$

into the force balance equation, we have

$$\frac{dP}{dr} + \frac{1}{4\pi} H \frac{dH}{dr} = 0 ,$$

and integrating, we get

$$P + \frac{H^2}{8\pi} = \text{constant}.$$

Now, even though we have seen that the presence of the magnetic field tends to increase the particle pressure in the center of the tube, the heat conductivity of the gas will cause it to be cool in the vicinity of the wall, with a concomitant large particle density near the wall. As we saw in the preceding lecture the heat conductivity varies as $\frac{n^2}{\theta^{1/2}}$ from which we can infer that the low temperature-high particle density region extends well into the stellarator tube from its wall. Such a temperature-particle density distribution is not desirable both from the fact that it means that only a small fraction of the gas is warm enough to sustain the thermonuclear reaction and also because the heat that is produced will be conducted too rapidly to the wall of the tube.

Dr. Spitzer has suggested placing ports, which he illustrates as in Figure 5, to draw off the cold gas and thus permit the hot region of the thermonuclear reaction to fill virtually the whole tube. The design of the ports is to be such that the magnetic field strength decreases from the main stellarator tube into the ports and continues to decrease along the ports. Since the spiraling ions behave diamagnetically, they will be driven down the port tubes by the magnetic field until the particles are cooled and pumped out of the system. The gas pumped out may be purified of reaction products and enriched with reactants and then blown through jets back into the stellarator.

A suggestion made here is that such ports may not be necessary due to the fact that if the ions are allowed to diffuse to the wall they will cool and recombine to form neutral atoms and molecules. The rate of diffusion of the neutral particles back into the hot region will be much greater than the outward diffusion rate of the charged particles inasmuch as the neutrals can move in straight lines across the magnetic field.

As a consequence the region near the wall should be evacuated to some extent and the hot reaction region should again fill a large fraction of the stellarator tube. No calculations have been made yet with regard to substantiating this suggestion.

Many of the ingredients of the stellarator have not yet been mentioned. For instance, to get the thermonuclear reaction started one would have to warm the gas in some manner. Exciting a glow discharge by induction appears to be a reasonable solution to the heating problem. By placing lithium between the wall of the tube and the coils of the magnet one could regenerate by neutron capture the tritium consumed in the DT reaction. Heavy water could be used for slowing down the neutrons, cooling the lithium, and for subsequent generation of electrical power with the heat received.

The stellarator we have described, though containing only a few hundredths of a gram of tritium at any given time would consume some ten to a hundred kilograms of tritium per year and would produce 10^5 to 10^6 kilowatts of power in steady operation.

A DD stellarator could produce as much power or more than the DT stellarator but would have to be either a ten times larger device or one with a magnetic field strength of the order of 10^5 gauss. The DD machine would also have the advantage that it could be used to manufacture tritium.

Before leaving the stellarator we should make some mention of Dr. Spitzer's "System B", which is based on the idea that one might be able to make the current creating the magnetic field circulate in the ionized gas itself rather than in external coils. If that could be done, then the operation of the stellarator would go as follows. With cold, unionized gas in the stellarator the current would be turned on in the external coils, creating a strong magnetic field. Then the gas would be heated to operating temperature, at which point it is of course fully ionized, and the current in the outside coils would be turned off. At ten kilovolts temperature, due to the high conductivity of the gas, the magnetic field should require about two minutes to drop to 1/e of its initial value, for a tube radius of 50 cm. At forty kilovolts, due to the yet higher conductivity of the gas, the decay time for the magnetic field becomes closer to fifteen minutes. A difficulty with System B, however, is that the magnetic pressure is greatest at the center of the tube where one wants the particle density to be as high as possible. At the present time no computations regarding the performance of a System B stellarator exist.

Third Lecture

THE PINCH EFFECT

This lecture was based largely on discussions with J. L. Tuck at Los Alamos

If a current passes in the same direction through two freely suspended parallel wires the wires are drawn toward each other. One might figure then that a current discharging through a gas would cause the various elements of current carrying volume to exert attractive forces on each other. In fact one might figure that if the current density was great enough the attractive forces would be sufficient to cause a compression of the gas along the axis of the discharge. Such a discharge could be used to insulate thermally a hot gas from the walls of its container. The name "pinch" has been given to this compression. W. Bennett,[6] for instance, has made more involved calculations than we will present here to arrive at radial particle density distributions such as

$$n_r = \frac{n_o}{(1 + br^2)^2} ,$$

where n_o is the particle density along the axis of the discharge and n_r is the particle density at a distance r. This expression does show a bunching of the particles along the discharge axis with a small but always finite particle density at large radial distances.

We will now make a simple derivation that only crudely approximates the above result but should nevertheless serve to enhance the plausibility of the argument for a pinch effect and from which we may form some idea of the anatomy of the pinch. We presume the temperature is constant throughout the pinch and that the gradient of the particle pressure is opposed by the attractive forces produced by the current:

$$kT\frac{dn}{dr} + iH = 0 \ ,$$

where i is the current density. We would like to have H expressed in terms of i, and so we will digress a moment to investigate i, the current density.

$$i = ne\bar{V}_e = \sigma_e E$$

where V_e is the drift velocity of the ions in the direction of the electric field and σ_e is the electrical conductivity of the gas.

By making successive substitutions (and including conversion factors to evaluate σ_e in $ohm^{-1}\ cm^{-1}$ for later use) we next get an expression for the electrical conductivity:

$$\sigma_e\,(ohm^{-1}cm^{-1}) = \frac{i(amps/cm)}{E(volts/cm)} = \frac{ne\left(\frac{10}{c}\right)V_e}{E(300)} = \frac{ne\left(\frac{10}{c}\right)}{E(300)}\frac{1}{2}\,at$$

$$= \frac{ne\left(\frac{10}{c}\right)}{E(300)}\frac{1}{2}\frac{Ee}{m}\,t = \frac{ne\left(\frac{10}{c}\right)}{E(300)}\frac{1}{2}\frac{Ee}{m}\frac{\lambda}{\bar{v}}$$

$$= \frac{ne\left(\frac{10}{c}\right)}{E(300)}\frac{1}{2}\frac{Ee}{m}\frac{1}{n\sigma\bar{v}} = \frac{e^2}{2m\sigma\bar{v}}\left(\frac{10}{300c}\right)$$

where σ is the collision cross section and $\bar{v}$ is the mean thermal velocity.

Substituting the expressions for σ and $\bar{v}$ that we got in the first lecture, we get

$$\sigma_e = 1.10 \times 10^5\ e^{3/2}\ ohm^{-1}\ cm^{-1} \ .$$

We see that σ_e is independent of the particle density and consequently both σ_e and i will be constant over most of the pinch for a given driving potential, although of course they will have to drop to zero where the particle density is zero. We can now note that

$$H = \frac{2I}{r} = \frac{2\pi r^2 i}{r} = 2\pi ri,$$

where I is the total current, and by substitution into our force balance equation get

$$kT \frac{dn}{dr} + 2\pi i^2 r = 0 \, .$$

Integration of this equation leads us to

$$n = n_o - \frac{\pi i^2}{kT} \; r^2 \; .$$

This result is only qualitatively similar to the more refined calculation by Bennett in that it gives a parabolic drop off of particle density, reaching zero at a certain value of r which we will call the pinch radius, R.

$$R = \frac{1}{i} \sqrt{\frac{n_o kT}{\pi}} = \frac{I}{\sqrt{\pi n_o kT}}$$

Integrating our equation for n, we can get an expression for N, the total number of particles per unit length.

$$N = \int_o^R 2\pi n r dr = 2\pi \int_o^R \left(n_o - \frac{\pi i^2}{kT} r^2\right) r dr$$

$$= n_o \pi R^2 - \frac{\pi^2}{kT} i^2 \frac{R^4}{2}$$

$$= n_o \pi R^2 - \frac{\pi^2}{kT} \frac{n_o kT}{R^2 \pi} \frac{R^4}{2}$$

$$= \frac{n_o \pi R^2}{2} \, .$$

Now we can also write

$$R^2 = \frac{\sqrt{2NkT}}{\pi i} \, .$$

It is interesting to note that on this model the pinch current is independent of the pinch radius if the temperature and the number of particles per unit length are known.

$$I = \sqrt{2NkT}$$

If we consider a sample situation of a gas at ten kilovolts temperature and at a particle density of 10^{14} per cc, then to just pinch the gas off the walls of a 20 cm radius tube would require a current of

$$I\ (\text{amperes}) = 10\sqrt{2NkT} = 10\sqrt{2N\ (1.6 \times 10^{-9})\ \theta}$$

$$= 10\sqrt{2 \times 10^{-4} \times \pi \times 400\ (1.6 \times 10^{-9})\ 10} = 6.3 \times 10^5\ .$$

To get the power required to sustain a unit length of this pinch we first note that

$$I = \pi R^2\ \sigma_0\ E = 1.10 \times 10^5\ \pi R^2\ E\ \theta^{3/2}$$

$$= 6.3 \times 10^5\ \text{amperes}$$

and

$$E = \frac{6.3 \times 10^5}{1.10 \times 10^5 \times \pi \times 400 \times 31.6} = 1.44 \times 10^{-4}\ \frac{\text{volts}}{\text{cm}}\ .$$

Then the power per unit length is

$$\frac{P}{l} = EI = 91\ \text{watts/cm.}$$

One might note that for a given temperature and total number of particles that the applied voltage and consequently the power requirement for a pinch varies inversely as the square of the pinch radius.

Inasmuch as the preceding description of the pinch shows the presence of no particles outside the pinch radius, we might feel a little better about the stability of our crude formulation if we could show that particles leaving the surface of the pinch will return to it rather than establishing a net outward and consequently pinch-disrupting particle flux.

We noted earlier that the magnetic field inside the pinch varies as

$$H = 2\pi ir.$$

The field consequently reaches a maximum value at the pinch radius and then drops off according to

$$H = \frac{2I}{r} \ .$$

Consequently, we might ask the specific question, "Is a particle that initially moves normally outward from the surface of a pinch returned by the $\frac{I}{r}$ dependent field to the surface?" The answer we get below is yes. We also arrive at a formula indicating how far the particles get from the pinch surface.

In the following derivation v_Z is the velocity in the direction of the pinch current, v_r is the velocity normal to the pinch current, p is the momentum of the particle, and is its radius of curvature, ρ_r being its radius of curvature at the edge of the pinch.

We can arrive at an expression for $v_{Z\ maximum}$ by the following series of evident maneuvers:

$$\frac{dv_Z}{dt} = \frac{ev_r H}{mc} = \frac{e\frac{dr}{dt}H}{mc} = \frac{e\frac{dr}{dt}2I}{mc10r} = \frac{dv_Z}{dr}\frac{dr}{dt}$$

$$\int_0^{v_{Z\ max}} dv_Z = \frac{2Ie}{10mc}\int_R^{r_{max}} d\ln r$$

$$v_{Z\ max} = \frac{2Ie}{10mc} \ln \frac{r_{max}}{R}$$

Now since p is constant along the trajectory of the particle, from

$$\frac{mv^2}{\rho} = \frac{Hev}{c} = \frac{pv}{\rho}$$

we can write

$$\frac{1}{m}\frac{pc}{e} = \frac{1}{m}H\rho = \frac{1}{m}\frac{2I}{10r}\rho = \frac{1}{m}\frac{2I}{10r}\rho_R$$

$$\frac{p}{m} = v_{Z\ max} = \frac{2Ie}{10mc} \ln \frac{r_{max}}{R} = \frac{2I}{10}\frac{\rho_R}{R}\frac{e}{mc}$$

and therefore

$$r_{max} = Re^{\frac{\rho_R}{R}} .$$

At r_{max} the returning acceleration is

$$\frac{dv_r}{dt} = \frac{ev_{z\,max}H}{mc} = \frac{e}{mc}\,\frac{2Ie}{10mc}\left(\ln\frac{r_{max}}{R}\right)\frac{2I}{10r_{max}} ,$$

which indicates that the particles do return to the pinch.

Since

$$\rho = \frac{pc}{He}$$

and

$$\rho_R = \frac{10pcR}{2Ie} ,$$

Now,

$$r_{max} = Re^{\left(\frac{5pc}{Ie}\right)}$$

$$p = \sqrt{2mE} = \sqrt{2m \times 3/2\,(1.6 \times 10^{-9})\,\theta} ,$$

and taking m equal to the mass of the deuteron we have

$$p = 1.27 \times 10^{-16} \sqrt{\theta} .$$

Consequently,

$$r_{max} = Re^{3.97 \times 10^{6}\frac{\sqrt{\theta}}{I}} .$$

For the conditions of the pinch we discussed earlier, that is, a temperature of ten kilovolts and a pinch current of 6.3×10^5 amperes we calculate r_{max} to be equal to 1.22 R.

A couple of devices for producing a pinch have been tried or suggested. Tuck at Los Alamos plans to use a gas filled torus as the secondary of a transformer made from a betatron magnet. He figures to produce for ten milliseconds a current of 2×10^5 amperes in the torus, with which, for a particle density of 10^{15} per cubic centimeter, he hopes to reach a gas temperature of one kilovolt, sufficiently warm to produce a large flux of neutrons. Baker at Berkeley tried discharging approximately three microfarads of capacity through a glass tube four feet long and three inches in diameter that was capped at each end by plane aluminum electrodes and was filled with hydrogen gas at 100 microns pressure. The condensers were charged to potentials of 2 to 115 kilovolts and were discharged in approximately two microseconds resulting in various currents up to 100,000 amperes. He took photographs of the discharges which showed in each case a luminous streak down the axis of the tube. However, the photographs indicate only that the current went down the tube axis -- one cannot tell whether the particles concentrated along the axis or not. On the basis of our crude formulation, we would calculate a pinch current of about 70,000 amperes for Baker's device. However, the short duration of Baker's discharges undoubtedly precluded the complete establishment of a pinch. We perhaps should note that if all the energy in Baker's condensers had gone into heating the gas, the temperature would have reached about 75 electron volts which is just about the point where a detectable number of neutrons would be produced.

As for any thermonuclear power machine, to operate on a paying basis the pinch device must be able to generate more power from the thermonuclear reaction than is required to sustain the reaction. In our formulation of the pinch there will be no heat loss by conduction except in devices with ends. The thermonuclear energy must still exceed the heat lost by radiation and the energy consumed in maintaining the pinch current.

We have tabulated below some sample situations. One will note that a pinch machine would operate on a paying basis with the DT reaction under a couple of the conditions given below but that with the DD reaction the machine would have to be slightly larger in diameter or would have to operate at a somewhat higher temperature.

R	1 cm	10 cm	10 cm
$N = \frac{n_0 \pi R^2}{2}$ ($n_a = 2n_e$: n_e = particle density before pinch current is applied)	3.14×10^{14} cm^{-1}	3.14×10^{17} cm^{-1}	3.14×10^{17} cm^{-1}
Θ	10 kev	10 kev	30 kev
I (amperes)	3.2×10^{4}	1.0×10^{6}	1.7×10^{6}
E (volts/cm)	2.9×10^{-3}	9.2×10^{-4}	3.0×10^{-4}
Pinch current power-watts per cm length of pinch	93	920	510
Radiation loss watts per cm length of pinch	0.02	180	330
Sum	93	1100	840
Thermonuclear power watts per cm { DT	83	8200	48.000
Thermonuclear power watts per cm { DD	01	84	740

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6 W. Bennett, Physical Review, 45, 890 (1934)

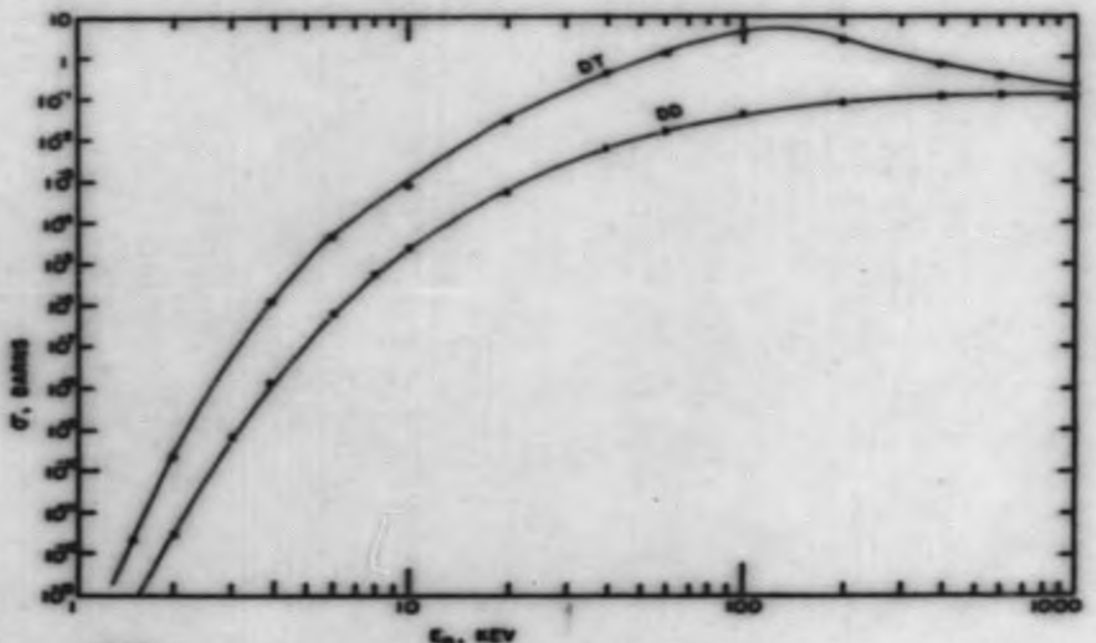

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FIG. 1

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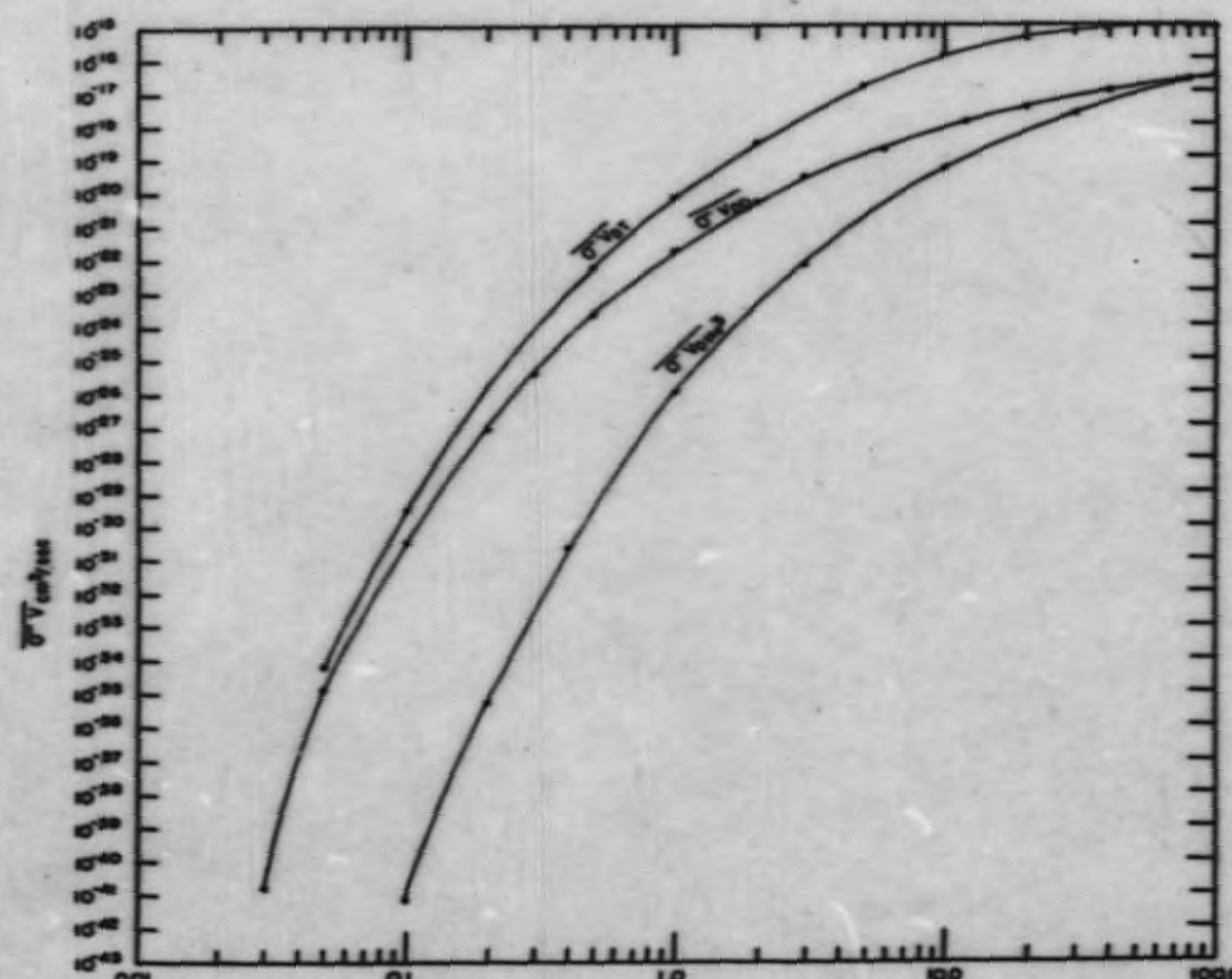

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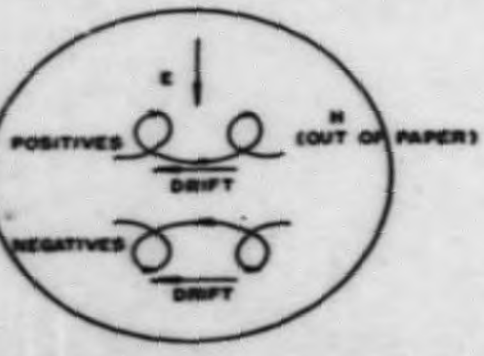
FIG. 2

THE VARIATION IN MAGNETIC FIELD STRENGTH IN THE TORUS LEADS TO A SEPARATION OF CHARGE AS ILLUSTRATED ABOVE.



THE SEPARATION OF CHARGE THEN LEADS TO A DRIFT OF BOTH POSITIVE AND NEGATIVE PARTICLES TO THE OUTSIDE WALL.

FIG. 5

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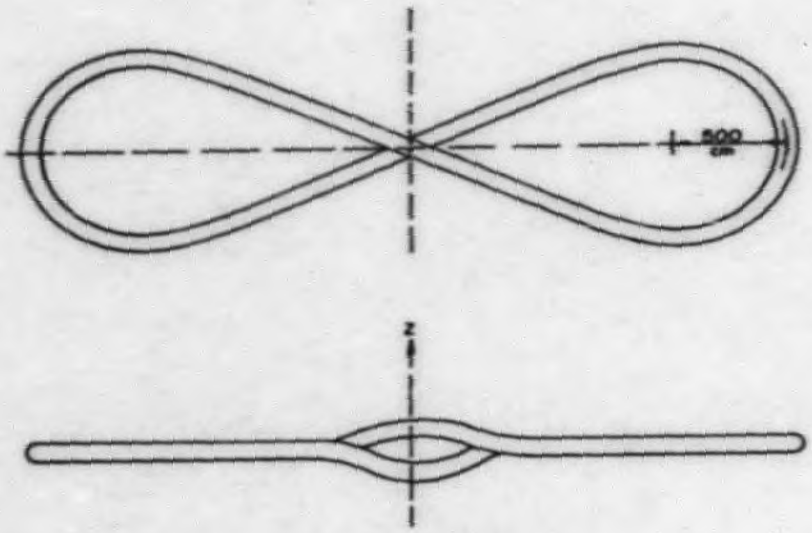

FIG. 4

DIAGRAM OF STELLARATOR TUBE

SECRET

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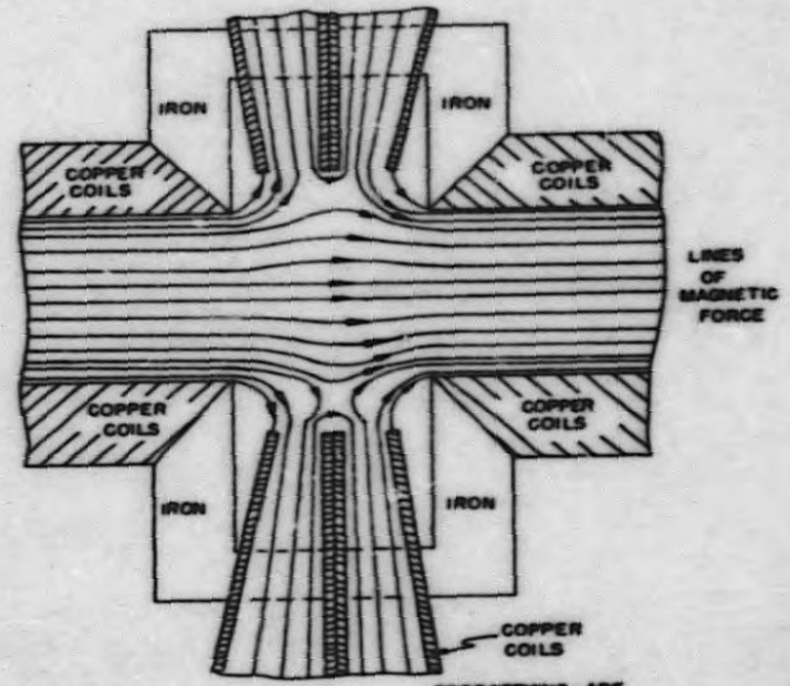

FIG. 5
PROPOSED ARRANGEMENT FOR EXTRACTING GAS

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END