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A RECOIL STUDY OF THE REACTION $C^{12}(p,pn)C^{11}$

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ABSTRACT

Recoil ranges of C^{11} from the reaction $C^{12}(p,pn)C^{11}$ are presented for incident proton energies from 0.25 to 6.2 Gev. From these data it is concluded that a neutron evaporation mechanism cannot be the major mechanism. The results for incident energies of 3 and 6.2 Gev are consistent with a fast reaction consisting of a single inelastic nucleon-nucleon collision. Assuming this mechanism, an average kinetic energy of 19 Mev can be deduced for the struck neutron (before the collision) in the C^{12} nucleus.

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I. INTRODUCTION

The usual theoretical approach to high-energy nuclear reactions rests on considerations of nucleon-nucleon collisions inside nuclei.[1,2] Calculations of most experimental observables involve the consideration of a complex spectrum of various kinds of collisions. One of the most direct studies of these collisions is the observation of products of the so-called simple reactions (p,pn), (p,2p), ($p,p\pi^+$), etc. These reactions involve only a small number of collisions, and result in residual nuclei with small energies of excitation. Therefore the complexities of the interactions are minimized. These simple reactions are, however, sensitive to the individual properties of the target nuclei. Nuclear shell structure, for example, appears to have a significant effect on cross sections for (p,pn) reactions.[3]

At present, the experimental information concerning simple reactions consists mainly of excitation-function measurements for (p,pn) reactions. A few studies of (p,2p) and ($p,p\pi^-$) reactions have been made. In order to gain a more detailed picture of the kinematics of these reactions, measurements of angular and energy distributions are needed. It is very difficult to obtain velocity measurements for protons and neutrons ejected in these simple reactions, because of the occurrence of many reactions that are more complex. However,

radiochemical techniques are suitable for observations of the recoil properties of the heavy residual nuclei.

Many different kinds of recoil measurements can be made — each having its own particular experimental difficulties.[4] Most of the experimental difficulties arise from the fact that the recoil energies and ranges of (p,pn) products are expected to be very small (region of kev to a few Mev). We have chosen the very simple thick-target integral-range technique in order to get an initial survey of some features of the recoil properties of (p,pn) reactions.[5,6] The reaction $C^{12}(p,pn)\,C^{11}$ has been selected because a very simple experimental method is possible for this case.

The experimental method consists of irradiating a foil stack of thick plastic targets and thick Be catcher foils. The fraction of the C^{11} atoms that recoil from the target into the Be catchers was measured by direct observation of the beta radiation from target and catcher foils. From these measurements we obtain the average components of the recoil range: (a) along the beam direction, (b) opposite to the beam direction, and (c) perpendicular to the beam direction. These measurements are sensitive to the combined effects of the angular and energy distributions of the C^{11} products. Quantitative conclusions can be reached only with the aid of a detailed theory of the (p,pn) reaction. Nevertheless, several important qualitative conclusions can be obtained from these initial experiments.

In the course of this study we have performed auxiliary experiments to test the experimental method and establish the range-energy relationship. The effect of diffusion of C^{11} from the plastic targets has been investigated. It has been established that this diffusion effect is very small for the polystyrene targets. In order to establish the range-energy relationship we have measured the range of N^{13} formed in the $C^{13}(p,n)N^{13}$ reaction. The

kinematics of this reaction have been determined by other workers,[7] and our measured average range has been correlated with the known distribution of recoil velocities.

II. EXPERIMENTAL METHOD

We have performed a number of thick-target recoil experiments with plastic targets and Be catcher foils. The basic target diagram is shown in Fig. 1. One or more of these stacks of foils were clamped together and exposed to proton beams from the Berkeley Bevatron, 184-inch cyclotron, and 60-inch cyclotron. After irradiation the foils were separated, and the relative amounts of C^{11} (N^{13} in the 60-inch-cyclotron experiments) in the targets and catchers were determined by end-window β proportional counting. The direction of the proton beam was parallel to the normal to the target plane for "forward-backward" experiments, and at an 80-deg angle for "perpendicular" experiments. The target holders have been described elsewhere.[8]

The targets were polystyrene and polyethylene foils of 2 to 3 mg/cm^2. Beryllium foils (from Brush Beryllium, Cleveland, Ohio) of ≈ 5 or ≈ 10 mg/cm^2 were used. Targets and catchers were cut to known areas by using stainless steel templates. For 60-inch cyclotron studies of the $C^{13}(p,n)\ N^{13}$ reaction, both target and catcher foils were cut to an area of 3.62 cm^2. In these studies a collimated external beam was used. For Bevatron and 184-inch cyclotron studies of the $C^{12}(p,pn)\ C^{11}$ reaction, targets and catchers were cut to areas of 2.33 and 3.00 cm^2, respectively. The larger areas of the catcher foils ensured that no recoils were lost from the target edges in these internal beam exposures.

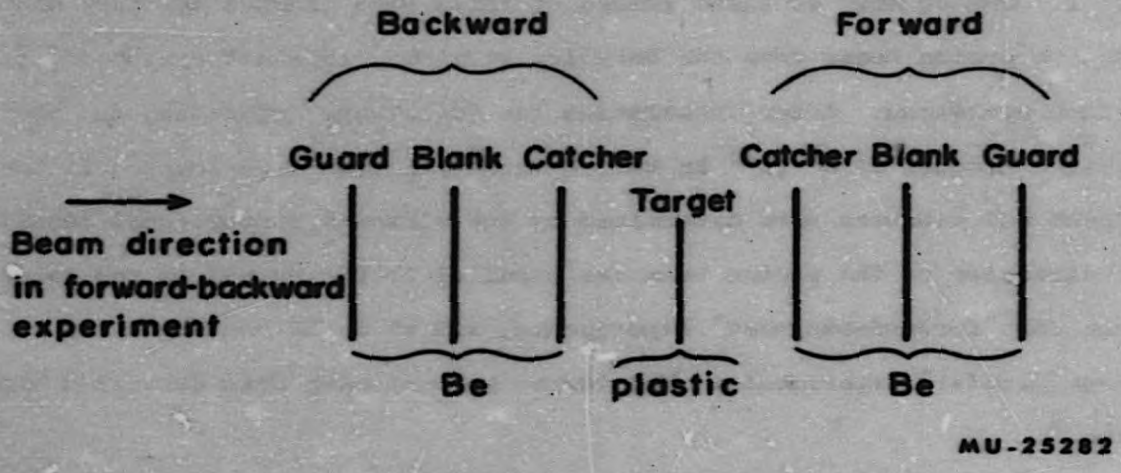

Fig. 1. Target diagram.

Plastic foils were washed with methyl alcohol and distilled water. The Be foils were cleaned in various ways— always including washes with petroleum ether, distilled water, and acetone.

After irradiation the blank, catcher, and target foils were rigidly mounted on Al plates for counting. Samples were fixed to the counting plate with double-faced adhesive tape, and covered with thin plastic (about 0.5 mg/cm^2). The most active areas of the foils were centered on the counting plates, and the side of the Be catcher foils that faced the target was mounted toward the counter. Simultaneous counting was usually done on a group of end-window β proportional counters gated by a single off-on switch. In some experiments the samples were rotated from one counter to the next, but this was found to be unnecessary because the relative efficiencies of the various counters differed by less than 3%. In a few experiments γ counters were used.

The usefulness of these experiments as a measure of recoil properties of the nuclear reaction depends on a knowledge of the relative importance of thermal diffusion and recoil phenomena. It is known that some C^{11} diffuses out of plastic foils in the form of hydrocarbons.[9] We will refer to loss by diffusion effects as hot-atom loss. It is essential to evaluate the following effects on these experiments: (a) a hot-atom loss from the plastic targets, (b) retention of activity on Be catchers as a result of hot-atom loss from the plastics, (c) hot-atom loss from the Be catchers, and (d) the dependence of the above effects on irradiation conditions such as beam intensity.

The hot-atom loss from polystyrene targets has been measured both by absolute measurement of gaseous C^{11} and by relative measurements of retained C^{11}. In two separate experiments a stack of polystyrene foils was exposed to neutrons produced from 48-Mev α bombardment of thick Be. The plastic foils were mounted in an evacuated glass tube. After irradiation the gaseous activity

of C^{11} was measured by sweeping it into a proportional counter with inactive methane carrier.[10] The C^{11} retained in the plastic foils was measured by end-window β proportional counting. A similar tube not containing plastic was simultaneously irradiated as a blank. The blank activity was about 10% that of the sample.

A group from the Brookhaven National Laboratory has made similar measurements of the hot-atom loss from polystyrene and polyethylene foils.[9] We have measured the specific activity of C^{11} activity retained in the plastic used in this work, relative to some plastic foils from the Brookhaven group. The polyethylene and polystyrene foils from Brookhaven were about 7 to 10 mg/cm^2 thick; in our work we used thinner foils. The dependence of counting efficiency on sample thickness was measured as described in the Appendix. The hot-atom loss from Brookhaven polystyrene was taken to be 3±1%, and from the Brookhaven polyethylene was taken to be 14±3%.[11] With these values as reference standards, the hot-atom loss from our plastics has been calculated from measurements of relative amounts of C^{11} activity retained in stacks of plastic foils exposed to 6.2-Gev proton beams. Various methods were used in the alignment of the different plastics and both β and γ counting were used for the relative activity measurements.

The results of all measurements of the hot atom loss are shown in Table I. In the last column appears the measured hot atom loss. Most measurements were for duplicate foils and the error shown is the standard error of these determinations. The hot atom loss from our polystyrene was only about 3% but for polyethylene it was about 12%. Thus it is possible to correct the observed target activities for this effect.

The results of the high-energy (> 250 Mev) recoil experiments show that the ratio of the observed C^{11} activity in the forward Be catcher divided

by that in the backward catcher is the same for polyethylene and polystyrene targets (see Table III in Section III). The amount of C^{11} activity observed in the Be catchers, was only about 5% of the total produced. It is clear that if any appreciable fraction of the hot-atom activity lost from the polyethylene or polystyrene targets was retained by the Be, then the observed forward-backward ratios would differ for the two materials. However, the observed forward-backward ratios are essentially the same for polystyrene and polyethylene targets. Thus we conclude that essentially no C^{11} observed in the Be catchers is from hot-atom effects in the plastic targets.

The possibility exists that some C^{11} was lost from the Be catchers by hot-atom effects. Since no volatile compounds of C and Be are known, this possibility seems unlikely. It is known, however, that some Beryllium oxide must be present on the surface of the Be foils. Thus some possibility exists for hot-atom loss of CO or CO_2. We have made a preliminary search for C^{11} activity in the form of CO_2. From experiments using neutron irradiation of plastic targets and Be catchers, it has been possible to set an upper limit on the C^{11} as CO_2. Less than 30% of the C^{11} activity in the Be foils escaped as CO_2. This limit does not rule out the possibility of significant loss of oxides of carbon. Nevertheless we have proceeded in analysis of the data with the assumption that this effect can be neglected.

At most bombarding energies, experiments were performed with quite different beam intensities. In every case the results were independent of beam intensity.

To summarize the effects of hot-atom loss, we conclude that polystyrene targets lose a negligible fraction ($\approx$3%) of the C^{11} activity produced. This hot-atom activity is not retained by the Be catcher foils, and therefore does not appreciably affect the range measurements. All measurements have been found to be independent of beam intensity.

Table I. Measurements of hot-atom loss of C^{11} from plastic foils.

Experiment number	Type of counting	Material	Hot-atom loss (%)
s-7	β(absolute)[a]	polystyrene	3.8
s-4	β(absolute)[a]	polystyrene	3.4
16[b]	β	polystyrene	0.9±2.0
16	γ	polystyrene	1.6±3.5
17	β	polystyrene	-0.7±2.5
17	γ	polystyrene	2.0±1.5
18	β	polystyrene	1.5±1.0
18	γ	polystyrene	3.2±1.0
16	β	polyethylene	10.1±2.5
16	γ	polyethylene	11. ±2.5
17	β	polyethylene	14.7
17	γ	polyethylene	14.8
18	β	polyethylene	12.8±5.0
18	γ	polyethylene	13.5±6.0

[a] The C^{11} activity in the gas phase was observed.

[b] In experiments 16 to 18, C^{11} activity retained in plastic foils was measured relative to standard plastic foils.

III. ANALYSIS OF EXPERIMENTAL RESULTS

Experimental observations for a typical high-energy (> 250 Mev) irradiation are given in Table II. The first column gives the foil designation (see Fig. 1), and the second, the material. The third and fourth columns give the results of the first and last counting. These data show that the counting rates of the Be foils are almost equivalent after the C^{11} in the target has decayed away. This activity is attributed to activation of impurities in the Be foils. However, the first counting shows that the Be recoil catcher foils have significantly more activity than the blanks. We attribute this additional activity to recoil atoms of C^{11} from the target that have come to rest in the catchers. (See the discussion of hot-atom effects above.)

The amount of C^{11} activity in the recoil catchers has been determined by correcting the observed counting rates for activation of impurities. The relative activities of the Be blank foils were essentially independent of decay time. The variation in the magnitude of these count rates is attributed to imperfect alignment of the Be foils, and to variations in the quantity of impurities.

The last counting was taken as a measure of the relative activities due to impurities. For each counting time, t, the activity of each Be-catcher foil due to impurity activation, $A_i(t)$, was taken to be the average blank activity $\langle B(t) \rangle$ normalized by the final counting rates:

$$A_i(t) = \langle B(t) \rangle \frac{A_i(\text{final counting})}{\langle B(\text{final counting}) \rangle} .$$

The activity due to impurities, $A_i(t)$, was subtracted from the gross activity of each catcher for each counting. After subtraction the 20-min decay period of C^{11} was observed in all but one experiment. This one experiment was rejected.

Table II. Activation correction in a typical C^{11} recoil experiment.[a]

Sample	Type	Count rate after ≈15 min (cpm)	Count rate after ≈7 hr (cpm)	Count corrected for activation
Blank	Be	3198	30	
Backward catcher	Be	7063	27	3916
Target	Polyethylene[b]	194753	< 3	194753
Forward catcher	Be	12654	26	9575
Blank	Be	3789		
Backward catcher	Be	7694	31	4047
Target	polystyrene[b]	189273	< 3	189273
Forward catcher	Be	14330	28	11024
Blank	Be	2917	23	

[a] This particular experiment was for 6.2-Gev protons.

[b] The thicknesses of the polyethylene and polystyrene were 2.40 and 2.08 mg/cm^2, respectively.

The fractions (F) of the activity in each catcher were taken as the average result of the first several counts. The precision of the activation correction is reflected by the reproducibility of the measurements (see Table III).

The average component of the range in the forward direction is defined as the effective forward range. It is given by the product ($F_F W$), in which F_F is the fraction of the total C^{11} activity observed in the forward Be catcher, and W is the target thickness.[5,12] Similarly, the effective backward range is given by $F_B W$. We define the effective perpendicular range as ($2F_P W$), in which F_P is the average fraction of the total C^{11} activity observed in the Be catchers for exposures with the target plane at 10 deg to the beam direction. ($2F_P W$) is the average component of the range on a line perpendicular to the beam. It can be shown that this effective perpendicular range ($2F_P W$) is $2/\pi$ times the average component of the range on a plane perpendicular to the beam.

A summary of the experimental data for the reaction $C^{12}(p,pn)C^{11}$ is shown in Table III. The first column shows the nominal beam energy. Beam energies of 0.25, 0.40, and 0.70 Gev were obtained from different radial positions in the 184-inch cyclotron. The other irradiations were performed at the Bevatron. The second column gives the number of experiments. The third through the fifth columns give the effective ranges, and the last column gives the ratio of the forward to backward ranges (or fractions), F_F/F_B. The quoted errors are the standard deviations of the mean (or standard error). Figure 2 shows the dependence of the measured effective ranges on proton energy.

A very similar procedure was used for the analysis of 60-inch cyclotron irradiations. In these experiments C^{11} activity could not be produced because of the high threshold for the reaction $C^{12}(p,pn)C^{11}$. In these experiments 10 min. N^{13} was observed from the reactions, $C^{12}(p,\gamma)N^{13}$ and $C^{13}(p,n)N^{13}$.

Table III. Thick-target recoil data for the $C^{12}(p,pn)C^{11}$ reaction

Incident energy (Gev)	Number of experiments	Effective forward range($F_F W$) (mg/cm^2)	Effective backward range($F_B W$) (mg/cm^2)	Effective perpendicular range($2F_P W$) (mg/cm^2)	Forward backward ratio (F_F/F_B)
		Polystyrene Targets[a]			
0.25	4	0.144±0.004[b]	0.0309±0.003		4.76±0.44
0.25	2			0.185±0.011	
0.40	4	0.141±0.002	0.0368±0.001		3.81±0.05
0.40	2			0.193±0.005	
0.70	3	0.132±0.006	0.0364±0.001		3.73±0.08
0.70	2			0.185±0.011	
3.0	1	0.118	0.0431		2.75
3.0	2			0.179±0.007	
6.2	4	0.115±0.004	0.0469±0.003		2.50±0.13
6.2	2			0.155±0.001	
		Polyethylene Targets[a]			
0.25	2	0.135±0.008	0.0290±0.006		4.81±0.7
0.40	1	0.137	0.0342		4.02
0.40	2			0.175±0.006	
0.70	1	0.142	0.043		3.3
0.70	1			0.160	
3.0	1	0.112	0.0382		2.88
3.0	2			0.162±0.002	
6.2	3	0.109±0.002	0.042±0.002		2.60±0.08
6.2	3			0.153±0.009	

[a] These data have not been corrected for hot-atom loss from the plastic targets, or for counting efficiency. These combined effects are estimated to multiply the tabulated ranges by 1.00 for polystyrene and by 0.91 for polyethylene.

[b] The errors are the standard deviation of the mean.

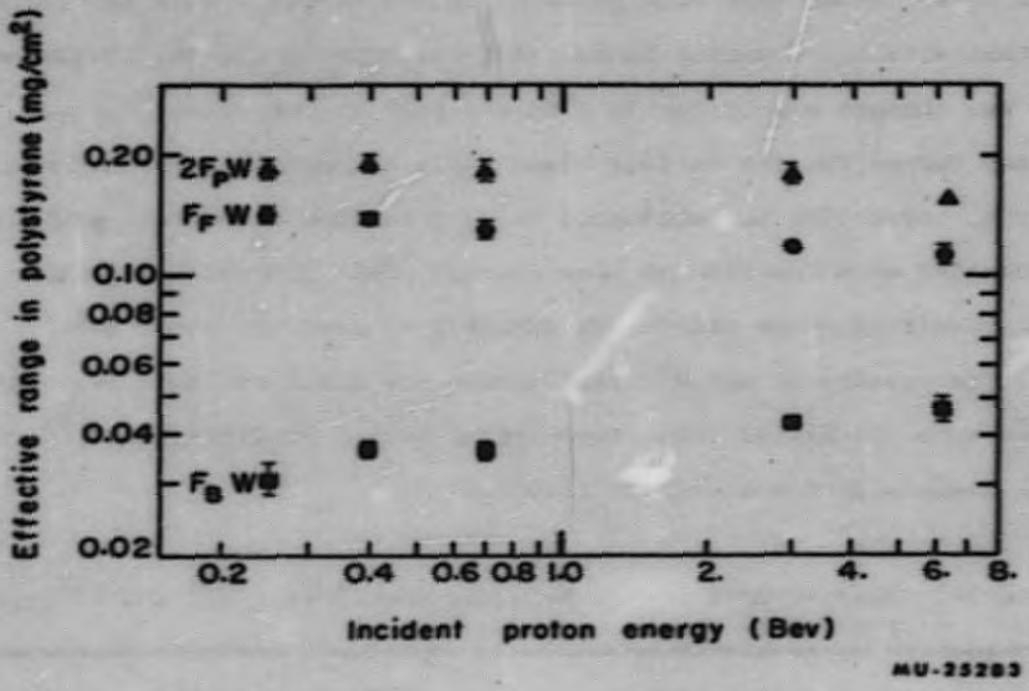

Fig. 2. Effective ranges as a function of bombarding energy. The effective forward range is F_pW, the effective backward range is F_BW, and the effective perpendicular range is $2F_pW$.

No studies of hot atom effects were performed; the assumption is made that these effects are negligible. In these reactions the momenta of the ejected neutrons or photons are such that all N^{13} recoil atoms must be directed forward in the laboratory system. Thus the Be foil just behind the target should contain no N^{13} atoms that were produced in the target. This expectation is consistent with the counting data. In these experiments the proton-beam energy was changed significantly from one foil to the next. For this reason the decay curves for the various blank foils changed in a regular manner. The activation correction was estimated by plotting the blank foil activities from each counting as a function of beam energy. This procedure is rather crude, and the resulting range values are probably in error by about 10%.

The results of the N^{13} experiments are given in Table IV. The beam energies were calculated from range-energy tables of Sternheimer,[13] and the nominal maximum proton energy of 12.0 Mev.

Table IV. Thick-target recoil data for the $C^{13}(p,n)N^{13}$ and $C^{12}(p,\gamma)N^{13}$ reactions.

Incident energy (Mev)	Effective forward range (mg/cm^2)
4.86	0.14
4.86	0.12
5.65	0.14
6.54	0.14

IV. DISCUSSION

A. Scattering and the Range-Energy Relation

The keys to the interpretation of any recoil range study are (a) the effects of scattering of the recoil atoms, and (b) the knowledge of the range-energy relationship.

Concerning scattering effects, very little quantitative information is available.[14,15] Some experiments with fission products have demonstrated that scattering effects cause errors of as much as 5% in effective ranges measured by the thick-target method.[15] These errors are caused by preferential scattering of recoils out of a target of heavy atoms (namely U) and into a catcher of much lighter atoms (namely Al). In our experiments the masses of the target and catcher atoms are so similar that we feel justified in neglecting scattering. We have analyzed the results as if the recoils followed a straight path. However, the actual assumption made is that deviations from a straight path are identical in all experiments — range-energy experiments and nuclear reaction studies. The whole analysis depends only on the relative ranges measured in different experiments.

No quantitative theory of the stopping process is available for the energy region of interest here. However, some range-energy data for N^{14} are available in the literature, and these data are shown in Fig. 3.[16] These data can be adequately represented by an empirical relationship between average range, R, and energy E:

$$R = kE^{\alpha}. \qquad (1)$$

The values of α are about 0.8 for all stopping materials, and thus we assume that this value is appropriate for N^{13} and C^{11} atoms in plastic targets. It has been shown empirically that α is not extremely sensitive to velocity or stopping material.[12,14]

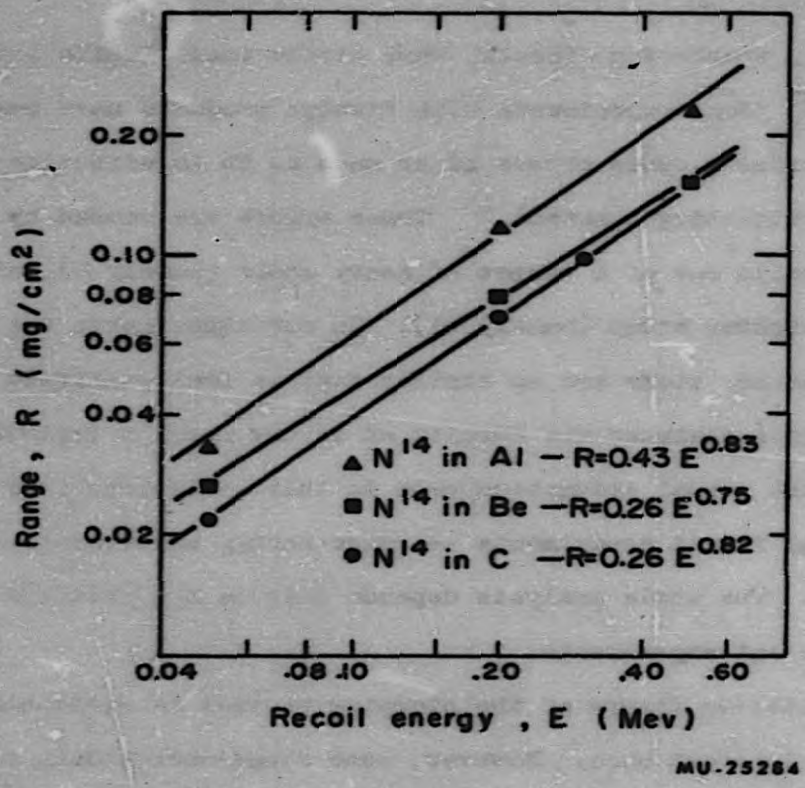

Fig. 3. The measured average ranges of N^{14} in various materials as a function of energy. The data are from Ref. 16.

Some method of converting range data for a given recoil atom and stopping material to another recoil or stopper is needed. The Bohr theory of the very-low-energy stopping process predicts that average range is proportional to energy for $A_R \gg A_S$:[17]

$$R_o = BE, \tag{2}$$

where

$$B = 0.600 \frac{A_S(A_S+A_R)(Z_S^{2/3}+Z_R^{2/3})^{1/2}}{A_R Z_S Z_R} . \tag{3}$$

Here, Z and A are atomic and mass numbers with subscripts S for stopping atoms and R for recoiling atoms. In a previous paper it has been shown empirically that this relationship gives reasonably accurate conversions of range-energy data.[12] This observation held true for $A_R \approx A_S$ and for recoil energies greater than those appropriate to the equation. Thus we will assume for conversion purposes that k values in Eq. (1) stand in the ratio of B values in Eq. (3).

The purpose of the range measurements for N^{13} was to get a calibration measurement for the range-energy relationship. It was necessary to determine the main reaction responsible for N^{13} production — $C^{12}(p,\gamma)N^{13}$ or $C^{13}(p,n)N^{13}$.

The relative cross section for N^{13} production in polystyrene was determined in a separate series of experiments. This excitation function follows closely the shape of the $C^{13}(p,n)N^{13}$ excitation function determined by neutron detection.[7] This is evidence that the $C^{12}(p,\gamma)N^{13}$ reaction does not contribute appreciably to the N^{13} radioactivity we observed. The excitation function for N^{13} that we observed showed a shift in the energy scale from experiment to experiment. We attribute this variation to fluctuations in the initial energy of the proton beam. From the fluctuations, and from the comparison of our excitation function to that of Dagley et al.,[7]

we can estimate errors for the incident energies in Table IV to be about ±0.3 Mev.

As previously stated, it is likely that the 10-min N^{13} activity that we observed is mainly from the $C^{13}(p, n)N^{13}$ reaction. The kinematics of this reaction are known from experimental data. There are no excited states of N^{13} that decay by photon emission to the ground state. Thus the kinetic energies of the emitted neutrons as a function of angle are specified by the Q value of the reaction. Dagley et al., have measured the angular distribution of the emitted neutrons for many incident proton energies.[7] From these data, the energy and angular distributions of the N^{13} recoil atoms can be calculated.

Consistent with the data in Fig. 3, we assume that the recoil distance is proportional to the initial energy, E_L, in the laboratory system to the power 0.8. In these experiments we have measured the effective forward range or the average $\langle R_F \rangle$ of the components of the range along the beam direction. The average quantity $\langle E_L^{0.8} \cos\theta_L \rangle$, where $\cos\theta_L$ is the laboratory angle of recoil with respect to the beam, can be evaluated from the kinematics of the reaction. From the relationship

$$\langle R_F \rangle = k \langle E_L^{0.8} \cos\theta_L \rangle \qquad (4)$$

we have determined the value of 0.176 (mg cm^{-2} $Mev^{-0.8}$) for k of N^{13} recoils in polystyrene. (If 20% of the N^{13} observed was produced by $C^{12}(p,\gamma)N^{13}$ reaction, the value of k would be changed by less than 10%.) This determination was made only for the range datum at 5.65 Mev, because the neutron angular distribution is not sharply dependent on proton energy in this region.[7] From the value of 0.176 for k of N^{13} in polystyrene a value of 0.213 for k of C^{11} recoils in polystyrene was obtained by the conversion procedure previously described.

Thus the range-energy relationship used for C^{11} in polystyrene is $R=0.213E^{0.8}$, with range expressed in mg/cm^2 and energy in Mev. This relationship has been obtained from recoils of up to ~ 1/2 Mev. We use the relationship for recoils of ~2 Mev.

This range-energy relationship can be tested by two comparisons:
(a) The measured ratio of ranges of C^{11} in polyethylene (CH_2) to polystyrene (CH_1) is about 0.83 (see Table III). The calculated ratio, using Eqs. (2) and (3) to estimate the relative stopping effectiveness, is 0.84.
(b) From the data for N^{14} in carbon,[16] using the conversion method previously described, a value of 0.25 for k of C^{11} in polystyrene has been calculated. This value differs from ours by only about 20%.

B. Recoil Consequences of Several Reaction Mechanisms

Let us consider four possible mechanisms for the (p,pn) reaction: (a) a low-deposition (p,p') process consisting of an elastic nucleon-nucleon collision followed by neutron evaporation on a slow time scale, (b) a single inelastic collision with a neutron having an isotropic momentum distribution, (c) a fast reaction consisting of an elastic proton-neutron collision, and (d) a single inelastic collision with a neutron having an anisotropic momentum distribution. For each mechanism the limiting case of high incident energy ($E_b \gg 931$ Mev) is discussed. Neutron evaporation (a) is found to be inconsistent with the recoil data at all energies. Various complications prevent a detailed consideration of the role of elastic collisions (c). It is concluded that for the highest energies a fast reaction consisting of a single inelastic collision is consistent with these data. By using this mechanism, the average kinetic energy of the struck neutron is found to be 19 Mev. Detailed discussions of the above four possible mechanisms for the (p,pn) reaction follow.

Neutron evaporation following (p,p') reaction. Consider a process in which the incident proton strikes a C^{12} nucleus — imparting some excitation energy and leaving an excited C^{12} nucleus. Then, on a much slower time scale, imagine that a neutron is evaporated. According to the Serber model of high-energy nuclear reactions, the initial impact is between the incident proton and one (or possibly more) of the nucleons of the C^{12} nucleus.[1] Let us assume that an incident proton strikes a nucleon in the C^{12} nucleus, and then the incident proton passes out of the nucleus. (We will consider only elastic collisions because it seems unlikely that a low energy nucleon can result from an inelastic collision,) The struck nucleon does not escape from the C^{12} nucleus, but its energy is taken up by the whole nucleus and converted into excitation energy. The excitation energy is then dissipated by nucleon evaporation and photon emission. The separation energy of a neutron from C^{12} is 18.3 Mev. Thus the initial impact must deposit more than 18.3 Mev if the final product is to be C^{11}. Also, the initial impact cannot impart much more energy than about 30 Mev, because the probability of evaporating only one neutron from a highly excited nucleus is small.

For the case of high bombarding energies we need consider only a single collision event in the fast stage of the reaction. This is because the probability of transferring in one collision only a small amount of energy (< 30 Mev) is small.[2] Correspondingly, the probability of a two-collision event depositing the same amount of energy is given by a product of two even smaller probabilities, and so forth. It is convenient to resolve the resultant laboratory velocity of the final C^{11} nucleus into two parts: the velocity due to the initial impact, $\underset{\sim}{v}$, and the velocity due to the neutron evaporation $\underset{\sim}{V}$. A vector diagram is shown in Fig. 4. The initial impact velocity, $\underset{\sim}{v}$, may be described by a component along the beam direction, $v_{\parallel}$,

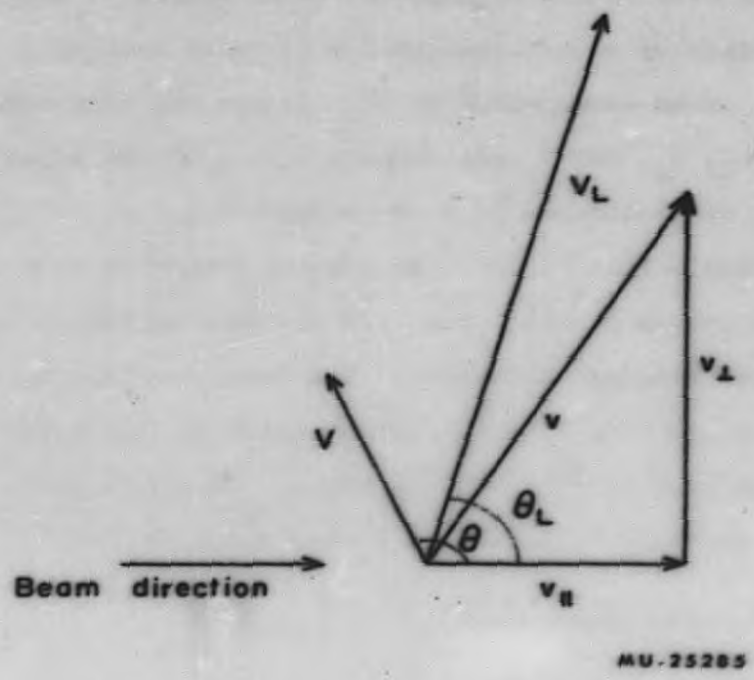

Fig. 4. Vector diagram. The vector $\underline{v}$ has component $v_\parallel$ parallel to the beam and component $v_\perp$ perpendicular to the beam. The vector $\underline{V}$ is directed at an angle θ with respect to the beam. The resultant laboratory system velocity V_L is the vector sum of $\underline{v}$ and $\underline{V}$, and is directed at an angle θ_L with respect to the beam.

and a component perpendicular to the beam direction, $v_\perp$. Let θ denote the angle between the direction of $\underline{V}$ and the beam. The angular distribution, $W(\theta)$, of $\underline{V}$ due to evaporation is expected to be symmetric about the 90-deg plane. For simplicity we will consider $W(\theta)$ to be isotropic.

With the above assumptions we can discuss the expected magnitude of the velocities $v_\parallel$, $v_\perp$, and V, and compare them with the experimental range data. Consider the collision of a very-high-energy ($E_b \gg 931$ Mev) incident proton and a nucleon in C^{12}. If only a small amount of energy, E(in Mev), is given to the struck nucleon, then the incident proton is deflected only slightly from its original direction. The struck nucleon is directed almost perpendicular to the beam with a total momentum of $(2\times931\times E)^{1/2}$ Mev/c and a forward momentum component of about E Mev/c. Therefore the struck nucleon is directed at an angle θ_1 such that $\theta_1 \approx \arccos\left[E(2\times931\times E)^{-1/2}\right]$. Thus

$$v_\parallel \approx E/12 \text{ (Mev/c)(amu)}^{-1} \tag{5}$$

and

$$v_\perp \approx (1/12)(2\times931\times E)^{1/2} \text{ (Mev/c)(amu)}^{-1}. \tag{6}$$

The maximum kinetic energy of an evaporated neutron would be (11/12)(E-18.3) Mev, and the minimum kinetic energy approximately (11/12)(E-8-18.3). (8 Mev is the separation energy plus the effective Coulomb barrier of an α particle of C^{11}.[3] The separation energy of a proton or neutron from C^{11} is larger.) Thus from this evaporation stage, momentum conservation requires that

$$V_{max} \approx (1/11)[2\times931(E-18.3)(11/12)]^{1/2} \text{ (Mev/c)(amu)}^{-1} \tag{7}$$

and

$$V_{min} \approx (1/11)[2\times931(E-8-18.3)(11/12)]^{1/2} \text{ (Mev/c)(amu)}^{-1}. \tag{8}$$

We see that this mechanism predicts that $v_\perp \gg v_\parallel$ and V. For this situation the measured effective ranges are related to the various velocities as follows:

$$W(F_p + F_B) \sim (k' v^{2x}/2)(V/v) \qquad (9)$$

$$2WF_p \sim (2/\pi)\, k' v^{2x} \qquad (10)$$

and

$$W(F_p - F_B) \sim (k' v^{2x})(v_\parallel/v). \qquad (11)$$

where k' is a constant.

These equations are correct only to first order in (V/v) and $v_\parallel/V$. If V is indeed less than v, then $4/\pi\, W(F_p+F_B)$ must be less than $(2W F_p)$. However, we see from Table III that this result is not observed for any bombarding energy. This evaporation mechanism leads to values of $v_\perp$ that are much too large with respect to v or V. This argument has been made for $E_b > 931$ Mev, but Eqs. (5) and (6) can be modified for $E_b \geq 250$ Mev, and a similar result is obtained. Evaporation processes induced by Coulomb excitation, or by interactions with clusters (such as α particles) in the nucleus, also predict the same qualitative result, namely $v_\perp \gg v_\parallel$ and V. Thus we conclude that neutron evaporation after elastic cascades does not account for the major part of the mechanism of the $C^{12}(p,pn)C^{11}$ reaction at any energy greater than 250 Mev.

A single inelastic collision with a neutron having an isotropic momentum distribution. Consider a process in which the incident proton strikes a neutron in C^{12}, and both nucleons — along with all mesons created — escape from the nucleus with no further interactions. In this section we consider that the struck nucleon has an isotropic angular distribution (before the collision). This situation would result from the participation of many quantum states. Later we consider the struck nucleon to have an anisotropic angular distribution. The residual nucleus is excited C^{11}. If the final

product is to be 20-min C^{11}, then the excitation energy must be less than about 8 Mev, otherwise particle evaporation is expected.

We may visualize the reaction in terms of the independent-particle model. The nucleons of the target nucleus (C^{12}) are in motion with an average kinetic energy, KE, inside a potential well of depth PE. The separation energy of a neutron is the minimum difference between its PE and its KE (18.3 Mev for C^{12}). The incoming nucleon enters the nucleus, collides with a nucleon, and the collision partners escape. The residual nucleus recoils as the incident particle enters the well, as the particles collide (if the potential energy is velocity dependent), and as the particles leave the well. For inelastic collisions the collision partners emerge from the nucleus in a narrow cone along the incident beam direction.

Let us first consider the case of a single inelastic collision at high energy ($E_b \gg 931$ Mev). The incident proton has a momentum of about E_b(Mev/c). The emerging nucleons and mesons move in almost the same direction as the incident proton but have slightly less total kinetic energy. The removal of the neutron requires an expenditure of 18.3 Mev, and residual excitation energy of the C^{11} nucleus can be as great as $\approx$ 8 Mev. Thus the emerging momentum is less than the incident momentum by 18.3 to about 26.3 Mev/c. In the vector diagram (Fig. 4) this corresponds to

$$v_{\parallel}=(1/11)(18.3 \text{ to } 26.3)(1+2\times931/E_b)^{1/2}(\text{Mev/c})(\text{amu})^{-1} \qquad (12)$$

and

$$v_{\perp}=0. \qquad (13)$$

After this neutron removal, the C^{11} nucleus finds itself with a "momentum hole" corresponding to the momentum of the struck neutron before the collision. This momentum hole gives rise to C^{11} recoil momentum equal in magnitude

$[(2\times931\times KE)^{1/2}$ Mev/c] and opposite in direction to the momentum of the nucleon before the collision. We first consider that this momentum has an isotropic angular distribution ($W(\theta)=1$) and corresponds to V in the vector diagram (Fig. 4) of $V=(1/11)(2\times931\times KE)^{1/2}(\text{Mev/c})(\text{amu})^{-1}$, and so this mechanism predicts that $V \gg v_{\parallel}$, $v_{\perp} \approx 0$, and that $W(\theta)$ is isotropic. The equations relating the effective ranges to the velocities for this situation are as follows:

$$W(F_F+F_B)=k/2(KE)^{0.8}[1+1.69(v_{\parallel}/V)^2], \tag{14}$$

$$2W(F_p)=k/2(KE)^{0.8}[1+0.195(v_{\parallel}/V)^2], \tag{15}$$

and

$$W(F_F-F_B)=k(KE)^{0.8}(v_{\parallel}/V)(1.20). \tag{16}$$

The relationships are correct to second order in $(v_{\parallel}/V)$. The values of $v_{\parallel}$ and KE have been calculated with these equations, and are listed in Table V. We note that the values of $v_{\parallel}$ are approaching the prediction of Eq. (12). The values of KE from forward-backward experiments [Eqs. (14) and (16)] and from perpendicular experiments [Eqs. (15) and (16)] are in general not quite consistent, indicating that an important effect has been omitted from the analysis. (This inconsistency is, in general, much greater than expected from the experimental errors.) However, the mean value of about 19 Mev for KE leads to an effective potential energy of 37 to 45 Mev, which is in reasonable accord with experimental fits to the optical model.[18]

Fast reaction consisting of an elastic collision. The case of a single elastic nucleon-nucleon collision leading to a (p,pn) reaction is much more complicated than the inelastic case previously discussed. The nucleon-nucleon collision data indicate that for elastic collisions low-energy transfers are most probable.[19] Thus, momentum transfers are directed

predominantly almost perpendicular to the beam (in the frame of the struck nucleon). The magnitude of the most probable momentum transfer for elastic collisions is indeed of the same order of magnitude as the intrinsic momentum of the nucleon inside the nucleus, therefore a large effect of the Pauli exclusion principle is expected. Only those collisions that result in increased lab-system momenta of the struck nucleon will be allowed. Also, only those struck nucleons that receive enough kinetic energy to overcome their separation energy will be able to escape the nucleus. Thus those elastic collisions that lead to (p,pn) reactions will be restricted to a certain class of nucleons in the nucleus. In general, velocity component $\underset{\sim}{V}$ (due to the momentum of the struck nucleon, as explained in the previous section) will not be isotropically oriented, and, in fact, the magnitude of V may vary with angle.

Table V. Impact velocity and energy of struck nucleon for isotropic distribution and $v_i = 0$.

Bombarding energy, E_b (Bev)	Impact velocity, $v_{\parallel}$ (Mev/c)(amu)$^{-1}$	KE struck nucleon (Mev)	
		forward-backward experiments	perpendicular experiments
0.25	4.9	18	22
0.40	4.6	19	23
0.70	4.3	18	22
3.0	3.4	16	21
6.2	3.1	18	18[a]

[a] This analysis has been made for the polystyrene experiments only. The polyethylene experiments give essentially the same results except for a higher value of the effective perpendicular range at 6.2 Gev.

A detailed calculation of the type performed by Winsberg and Clements is necessary to solve this problem.[20] At this time we are unable to assess the role of elastic collisions in the reaction $C^{12}(p,pn)C^{11}$.

A single inelastic collision with a neutron having an anisotropic momentum distribution. As presented by Benioff, there is strong evidence that (p,pn) reactions proceed by fast reactions occurring predominantly in the region of a surface band.[3] Benioff has presented a quantitative theoretical description of (p,pn) reaction cross sections for these high-energy surface reactions.[3] He has estimated that for incident energies above several Gev, inelastic nucleon-nucleon collisions are most important for (p,pn) reactions. No theoretical description of the recoil properties is available for this mechanism. It is possible, and even likely, that the results of a theory of these surface reactions will be somewhat different from the cases previously discussed. One possible difference centers on the angular distribution of the struck nucleon. If the reaction is restricted to a nuclear surface band and if a small number of quantum states are involved it may well be that these struck nucleons have certain preferred directions of motion.

Benioff's calculations of cross sections for (p,pn) reactions used harmonic oscillator independent-particle wave functions.[3] These wave functions are separable into a product of a radial function and an angular function. This separability property leads to the result that the speed V of the struck nucleon is independent of angle.[21] Nucleon-nucleon collision studies indicate that the exit particles from inelastic nucleon-nucleon interactions leave the nucleus at angles close to 0 deg. These approximations lead to the predictions that $v_\perp \approx 0$, and that V is independent of θ (see Fig. 4). With these assumptions, it is

possible to estimate $v_{\parallel}$, V, and the anisotropy $\frac{W(90)}{W(0)}$ from the effective-range measurements. We assume that the angular distribution of the struck nucleon is given by $W(\theta)=a+b\cos^2\theta$, and that $v_{\perp}=0$, and V and KE are the speed and kinetic energy, respectively, of the struck nucleon. The significance of $v_{\parallel}$ is the same as in the previous discussion of inelastic nucleon-nucleon collisions. The equations relating the effective ranges to $v_{\parallel}$, V, and b/a are as follows [to second order in $(v_{\parallel}/V)$]:

$$2W(F_P+F_B)=k\{(KE)^{0.8}/[1+(b/3a)]\}\ \{[1+(b/2a)]+(v_{\parallel}/V)^2\ [1.69+(0.31\ b/a)]\}, \quad (17)$$

$$W(F_P-F_B)=k\{(KE)^{0.8}/[1+(b/3a)]\}\ (v_{\parallel}/V)\ [1.20+(0.453\ b/a)], \quad (18)$$

and

$$W(F_P+F_B-2F_P)=k\{(KE)^{0.8}/[1+(b/3a)]\}\ \{(b/8a)+(v_{\parallel}/V)^2[0.749+(0.144b/a)]\}. \quad (19)$$

The results of the calculation of $v_{\parallel}$, KE, and b/a are given in Table VI. It is clear by comparing Tables V and VI that the values of $v_{\parallel}$ and KE (or V) are not very sensitive to the inclusion of anisotropy as approximated here, even though the values of the anisotropy so calculated are in general rather large. We conclude that it is possible to infer the average kinetic energy of the struck nucleon rather well by this method, even though the angular distribution of the struck nucleon can be estimated only roughly.

It is interesting that the values of KE deduced from these equations are almost independent of incident energy. One might expect the kinematics of the reaction to change decidedly between 250 Mev and 6.2 Gev. The relative probability of elastic and inelastic nucleon-nucleon collisions must change over this energy region. However the observed effective ranges are not extremely energy dependent. A detailed calculation of the kinematics of (p,pn) reactions following elastic nucleon-nucleon collisions would very desirable. The recoil properties of these (p,pn) reactions may indeed furnish a unique tool for studying the motions of nucleons inside nuclei.

From these data we conclude that for incident energies greater than 0.25 Bev, low-energy-transfer processes followed by neutron evaporation are not the major mechanism of the $C^{12}(p,pn)C^{11}$ reaction. For the higher bombarding energies, a fast reaction consisting of a single inelastic nucleon-nucleon collision is consistent with these data. Assuming that this is indeed the mechanism of the reaction (for $E_b \geq 3$ Gev), we estimate the kinetic energy of the struck nucleon to be 19 Mev, and thus its potential energy to be 37 to 45 Mev. The angular distribution of the struck neutron seems to be peaked perpendicular to the beam. We have not been able to calculate the recoil properties of an elastic nucleon-nucleon collision mechanism, but we hope that these data will be useful for such a comparison as the theory is developed.

Table VI. Impact velocity, energy of struck nucleon, and anisotropy of struck nucleon.[a]

Bombarding energy E_b (Mev)	Impact velocity, $v_{\parallel}$. (Mev/c)(amu)$^{-1}$	KE of struck nucleon (Mev)	Anisotropy of struck nucleon W(90)/W(0)
0.25	5.2	20	2.9
0.40	4.6	21	2.5
0.70	4.6	20	2.5
3.0	3.4	19	2.2
6.2	3.0	18	1.0[a]

[a] See footnote for Table V.

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APPENDIX

We have performed several experiments to determine the relative counting efficiency of C^{11} on the proportional counters as a function of sample thickness. This information was needed for the measurement of hot-atom loss of C^{11} from the targets (see section II). Also, we needed an estimate of the relative counting efficiences of Be catchers and plastic targets.

Stacks of about 15 polystyrene foils (≈2 mg/cm^2) were irradiated, and the relative activity of each foil was measured. Then samples of one, two, three, four, and five foils, respectively, were mounted and counted. In Experiment 15 the relative activities of these samples were measured by γ counting.

The results of these measurements for C^{11} are shown in Fig. 5. Similar measurements have been made for Na^{24} in Al that extend to sample thicknesses of about 0.15 mg/cm^2.[8] The data for Na^{24} indicate that counting efficiency is not drastically dependent on sample thickness down to 0.15 mg/cm^2. Thus we have drawn the dashed line in Fig. 5, and estimate that the Be catcher foils have a counting efficiency about 2.5% less than the plastic targets.

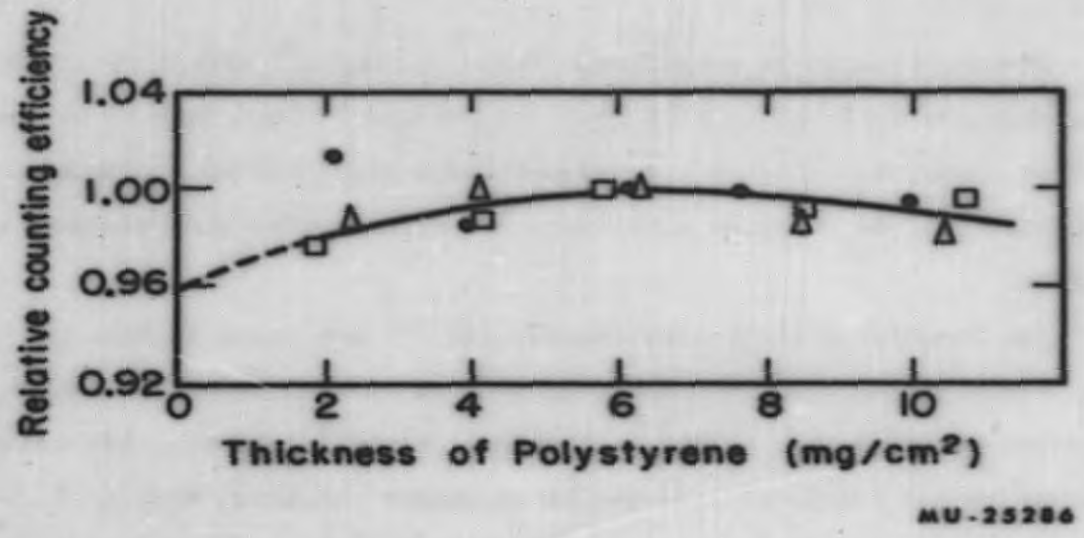

Fig. 5. Relative counting efficiency of C^{11} on the end-window proportional counters. These data were taken on Shelf 7 with different experiments denoted as follows: ⊙ Experiment 10, ⊡ Experiment 12, and △ Experiment 15.

FOOTNOTES AND REFERENCES

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