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AN S-MATRIX THEORY OF THE VERTEX $\rho \to N\bar{N}$ BASED ON THE STRIP APPROXIMATION*

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Abstract

The $\pi\pi \to N\bar{N}$ amplitudes are studied in the neighborhood of the ρ-meson mass and a simple dynamical calculation based on a consistent use of the strip approximation is proposed to study the vertex $\rho \to N\bar{N}$. In particular, the vertex $\rho \to N\bar{N}$ is expressed in terms of a "potential" (assumed to get its strongest contributions from low mass baryon states exchanged in the πN channels) and the $J = 1$, $I = 1$ $\pi\pi$ phase shifts. We can therefore generate the vertex $\rho \to N\bar{N}$ and compare this prediction with experimental information about the nucleon electromagnetic (EM) structure. If there is agreement, this will provide a posteriori justification of the strip approximation in bootstrap calculations in the πN and NN systems. In both cases the vertex $\rho \to N\bar{N}$ plays a non-trivial role.

The ρ-meson can couple to the $N\bar{N}$ system in two ways: first ρ can couple to the charge of the nucleon (call this coupling γ_N) and second to the magnetic moment (call this coupling γ_N'). If we denote the coupling of $\rho \to \pi\pi$ as γ_π the results of this analysis can be summarized as follows:

$$\frac{\gamma_N'}{\gamma_\pi} = \not\simeq \frac{6}{\gamma_\pi^2}(2.3) \quad ; \quad \frac{\gamma_N}{\gamma_\pi} = \not\simeq \frac{6}{\gamma_\pi^2}(0.85) \quad ; \quad \frac{\gamma_N'}{\gamma_N} = +2.7$$

The first two numbers depend on the details of the strip approximation used for $\pi\pi$ scattering. The ratio $\frac{\gamma_N'}{\gamma_N}$ is relatively independent of these details. It is also possible to calculate γ_N'/γ_π, γ_N/γ_π and γ_N'/γ_N from the nucleon EM structure. The results are

$$\frac{\gamma_N'}{\gamma_\pi} = \neq \frac{6}{\gamma_\pi^2}(2.6) \quad ; \quad \frac{\gamma_N}{\gamma_\pi} = \neq \frac{6}{\gamma_\pi^2}(0.7) \quad ; \quad \frac{\gamma_N'}{\gamma_N} = + 3.7$$

Again the first two numbers depend on the details of the strip approximation used for $\pi\pi$ scattering and the third is relatively independent of these details.

I. Introduction

The present study was motivated by an attempt to understand low energy πN scattering within the framework of the bootstrap principle and the un-Reggeized version of the strip approximation.[1] This work (which is in progress) attempts to generate low energy πN scattering in the p(1,1) and p(3,3) states assuming the potential operating in these states is generated by the exchange of low mass meson states in the crossed t-channel ($\pi\pi \to N\bar{N}$) and low mass baryon states in the crossed u-channel ($\pi N \to \pi N$). In particular, the ρ-meson is kept in channel t; the ρ mass (m_ρ) and the coupling of $\rho \to \pi\pi$ (γ_π) and $\rho \to N\bar{N}$ (γ_N, γ_N') appear as parameters. The parameters of the nucleon and (3,3) poles are taken as the elements to be determined by self-consistency.

In principle, one should determine all the parameters in the analysis self-consistently making systematic use of a particular approximation scheme. However, what happens in practical calculations is that some parameters in the analysis are introduced as "known" and others are calculated self-consistently. In the πN strip calculation described above m_ρ and γ_π are assumed known and an attempt is made to calculate γ_N and γ_N' from a strip approximation to the $\pi\pi \to \pi\pi$ and $\pi\pi \to N\overline{N}$ amplitudes. The present work discusses in detail the calculation of γ_N and γ_N'. It differs from past studies[2] of the vertex $\rho \to N\overline{N}$ by making consistent use of the strip approximation and avoiding the introduction of phenomenological parameters.

II. Notation

The channels for the 2π-$2N$ problem are defined in Figure 1. The kinematics for the $\pi\pi \to N\overline{N}$ channel has been discussed by Frazer[3] and Fulco and the equations used here are taken from their work.

The T-matrix for the process $\pi\pi \to N\overline{N}$ can be written in terms of the invariant amplitudes (A,B).

$$T = -A + i\gamma \cdot \left(\frac{q_1 + q_2}{2}\right) B \qquad (1)$$

The amplitudes (A,B) are functions of the Lorentz invariant variables (s,t,u) and are assumed to satisfy a Mandelstam Representation.

Following Jacob and Wick[4] the differential cross section for $\pi\pi \to N\overline{N}$ is expressed in terms of helicity amplitudes $F(\lambda,\overline{\lambda})$

$$\frac{d\sigma}{d\Omega} = \sum (p/q)\,|F(\lambda,\overline{\lambda})|^2 \qquad (2)$$

where $F(\lambda,\overline{\lambda})$ is the amplitude for production of a nucleon with helicity $\lambda(\pm)$ and an anti-nucleon with helicity $\overline{\lambda}(\pm)$. The decomposition of $F(\lambda\overline{\lambda})$ into partial wave helicity amplitudes is given by

$$F_{++} \equiv + F_{--} = \frac{1}{q} \sum_J (J + 1/2)\, T^J_{++}(t)\, P_J(z) \tag{3}$$

$$F_{+-} \equiv + F_{-+} = \frac{1}{q} \sum_J \frac{J+1/2}{[J(J+1/2)]^{1/2}}\, T^J_{+-}(t)\, P'_J(z) \left(-\frac{dz}{d\theta}\right) \tag{4}$$

The scattering angle in channel t is defined in terms of the invariant variables for this channel[5] as follows:

$$\left.\begin{aligned} t &= 4(q^2+1) = 4(p^2+M^2) \\ s &= M^2-1-2q^2 + 2pq\, z_t \quad ; \quad z_t = \cos\theta_t \\ u &= 2(M^2+1) - s - t = M^2-1-2q^2 - 2pq\, z_t \end{aligned}\right\} \tag{5}$$

where

q = magnitude of the CM momentum for the 2 pions

p = same for the 2 nucleons

M = nucleon mass

θ_t = CM scattering angle

The partial wave helicity amplitudes with simple analytic properties are

$$f^J_{++}(t) \equiv f^J_{+}(t) = \frac{p}{q}\left[\frac{E}{(pq)^J}\right] T^J_{++} \equiv + f^J_{--}(t) \tag{6}$$

$$f^J_{+-}(t) \equiv f^J_{-}(t) = \frac{p}{q}\left[\frac{1}{(pq)^J}\right] T^J_{+-} \equiv + f^J_{-+}(t) \tag{7}$$

where E = total nucleon energy in CM and $f^J_{\alpha\beta}$ → constants at the physical threshold ($p^2 \to o$) and the unphysical threshold ($q^2 \to o$). The $f^J_{\alpha\beta}$ can be related to partial wave projections of the invariant amplitudes. The result is

$$f_+^{J,I}(t) = \frac{1}{8\pi}\left[-\frac{p^2}{(pq)^J} A_J^I + \frac{M}{(2J+1)(pq)^{J-1}}\{(J+1)\ B^I_{J+1} + J\ B^I_{J-1}\}\right] \quad (8)$$

$$f_-^{J,I}(t) = \frac{1}{8\pi}\frac{[J(J+1)]^{1/2}}{(2J+1)}\frac{1}{(pq)^{J-1}}\left[B^I_{J-1} - B^I_{J+1}\right] \quad (9)$$

where

$$[A^I_J(t);\ B^I_J(t)] = \int_{-1}^{+1} dz\ P_J(z)\ [A^I(t,z);\ B^I(t,z)] \quad (10)$$

III. Expressions for γ_N and γ_N'

Let $f_\pm$ be the partial wave helicity amplitude for the state J = 1, I = 1. In the usual way an N/D solution for $f_\pm$ can be formulated.

$$f_\pm(\nu) = \frac{N_\pm(\nu)}{D_\pm(\nu)} \quad ; \quad \nu = q^2 \quad (11)$$

Unitarity in channel t — in the approximation where only the 2π intermediate states are kept — reads

$$\mathrm{Im}\ f_\pm = \rho\ A_\pi\ [f_\pm]^* \quad (12)$$

where A_π = the elastic ππ scattering amplitude for the state J = 1, I = 1.

Since ρ is a real phase space factor for $t > 4$, it follows that A_π must carry the phase of $f_\pm$. In this approximation we can therefore write

$$f_\pm(\nu) = \frac{N_\pm(\nu)}{D_\pi(\nu)} = V_\pm(\nu) + R_\pm(\nu) \tag{13}$$

where $D_\pi(\nu)$ is the denominator function in the N/D solution for A_π. In detail we have

$$A_\pi(\nu) = \frac{N_\pi(\nu)}{D_\pi(\nu)} = \frac{e^{i\delta}\sin\delta}{\rho_\pi} \quad ; \quad \rho_\pi = \sqrt{\frac{\nu}{\nu+1}} \tag{14}$$

The quantity $V_\pm$ is the potential operating in channel t. We assume the strongest contributions to $V_\pm$ come from low mass baryon states exchanged in the crossed s- and u-channels (in particular, the nucleon and (3,3) resonance). The quantity $R_\pm$ carries the channel t poles (which correspond to Re $D_\pi = 0$) and the residual integral over the channel t strip ($0 < \nu < \nu_2$).

Define $N_\pm$ to carry the singularities of $V_\pm$ and to be real on the channel t strip ($0 < \nu < \nu_2$). Normalizing $D_\pi(\pm\infty) = 1$ we have

$$D_\pi(\nu) = 1 - \frac{1}{\pi}\int_0^{\nu_1} d\nu' \, \frac{\rho_\pi(\nu')\, N_\pi(\nu')}{\nu' - \nu} \tag{15}$$

where the strip for the $\pi\pi \to \pi\pi$ amplitude is ($0 < \nu < \nu_1$). At first sight there seems to be no connection between ν_1 and ν_2, but making consistent use of the strip approximation requires $\nu_1 \approx \nu_2$. With this in mind we can write

$$N_\pm(\nu) = V_\pm(\nu)\, D_\pi(\nu) + \frac{1}{\pi}\int_0^{\nu_1} d\nu' \, \frac{\rho_\pi(\nu')\, V_\pm(\nu')\, N_\pi(\nu')}{\nu' - \nu} \tag{16}$$

In writing down (15) and (16) elastic unitarity has been used over the strip. i.e.

$$\text{Im } D_\pi(\nu) = -\rho_\pi N_\pi(\nu) \qquad (0 < \nu < \nu_1) \tag{17}$$

It then follows that

$$N_\pm(\nu_\rho) = \frac{\underline{P}}{\pi} \int_0^{\nu_1} d\nu' \frac{\rho_\pi(\nu') V_\pm(\nu') N_\pi(\nu')}{\nu' - \nu_\rho} \tag{18}$$

where $\underline{P}$ implies a principal value integral.

Next consider (13) for $\nu \approx \nu_\rho$. In the narrow resonance approximation we find

$$f_\pm(\nu) \approx \frac{N_\pm(\nu_\rho)/D_\pi'(\nu_\rho)}{\nu - \nu_\rho} \tag{19}$$

where $\quad D_\pi'(\nu_\rho) = (\frac{d}{d\nu} \text{Re } D_\pi)\nu = \nu_\rho$

From (8) and (9) $f_\pm$ can be written

$$f_+(\nu) = \frac{1}{8\pi}\left[-\frac{p}{q} A_1^\rho + \frac{M}{3} (2B_2^\rho + B_0^\rho)\right] \tag{20}$$

$$f_-(\nu) = \frac{1}{8\pi}\left[\frac{\sqrt{2}}{3} (B_0^\rho - B_2^\rho) \right] \tag{21}$$

where the partial wave projections of the invariant amplitudes are made with reference to channel t. The contributions[6] of the ρ-meson to (A,B) in the pole approximation are

$$A^{\rho}(t,z) = \not{}\left(\frac{8\pi\gamma_\pi\gamma_N'}{t - m_\rho^2}\right)\left[\frac{m_\rho^2/2 + s - (M^2+1)}{M}\right] \tag{22}$$

$$B^{\rho}(t,z) = -\frac{16\pi\gamma_\pi(\gamma_N + \gamma_N')}{t - m_\rho^2} \tag{23}$$

where s is given by (5). We first make the appropriate partial wave projections of (22) and (23) and substitute these into (20) and (21). Comparing with (19) we find

$$\frac{\gamma_N}{\gamma_\pi} + \left(1 + \frac{p_\rho^2}{M^2}\right)\frac{\gamma_N'}{\gamma_\pi} = \frac{-3}{\gamma_\pi^2 D_\pi'(\nu_\rho)}\left(\frac{N_+(\nu_\rho)}{M}\right) \tag{24}$$

$$\frac{\gamma_N}{\gamma_\pi} + \frac{\gamma_N'}{\gamma_\pi} = \frac{-3}{\gamma_\pi^2 D_\pi'(\nu_\rho)}\left(\frac{N_-(\nu_\rho)}{\sqrt{2}}\right) \tag{25}$$

where $p_\rho^2 = (\nu_\rho + 1) - M^2$

Solving (24) and (25) for γ_N'/γ_π and γ_N/γ_π we find

$$-\left(\frac{p_\rho^2}{M^2}\right)\frac{\gamma_N'}{\gamma_\pi} = +\frac{-3}{\gamma_\pi^2 D_\pi'(\nu_\rho)}\left(\frac{N_-(\nu_\rho)}{\sqrt{2}} - \frac{N_+(\nu_\rho)}{M}\right) \tag{26}$$

$$\frac{\gamma_N}{\gamma_\pi} = +\frac{-3}{\gamma_\pi^2 D_\pi'(\nu_\rho)}\left(\frac{N_-(\nu_\rho)}{\sqrt{2}}\right) - \frac{\gamma_N'}{\gamma_\pi} \tag{27}$$

For future reference we introduce the following notation

$$\frac{N_-(\nu_\rho)}{\sqrt{2}} - \frac{N_+(\nu_\rho)}{M} = \frac{P}{\pi}\int_0^{\nu_1} d\nu' \, \frac{\rho_\pi(\nu')\, N_\pi(\nu')}{\nu' - \nu_\rho}\,[v(\nu')] \tag{28}$$

$$\frac{N_-(\nu_\rho)}{\sqrt{2}} = \frac{P}{\pi}\int_0^{\nu_1} d\nu' \, \frac{\rho_\pi(\nu')\, N_\pi(\nu')}{\nu' - \nu_\rho}\,[\omega(\nu')] \tag{29}$$

where $v(\nu) = \dfrac{V_-(\nu)}{\sqrt{2}} - \dfrac{V_+(\nu)}{M}$ (30)

and $\omega(\nu) = \dfrac{V_-(\nu)}{\sqrt{2}}$ (31)

IV. Calculation of the Potential ($V_\pm$)

The potential is just the J = 1 projection of the Born approximation to the scattering amplitude. We define

$$V_\pm = V_\pm^N + V_\pm^{N^*} \quad f_\pm^N + f_\pm^{N^*} \tag{32}$$

where

$f_\pm^N$ = contribution to the partial wave helicity amplitudes $f_\pm$ arising from nucleon exchange in the crossed πN channels.

$f_\pm^{N^*}$ = same arising from exchange of the (3,3) resonance (Δ - particle).

For nucleon exchange the invariant amplitudes take the form

$$\left.\begin{aligned} A_{I=1}^N &= 0 \\ B_{I=1}^N(t,z) &= -\left(\frac{16\pi\gamma_{11}}{s - M^2}\right) \beta_{1,1/2}^{ts} \end{aligned}\right\} \tag{33}$$

for a nucleon in channel s and

$$\left.\begin{aligned} A^{N}_{I=1} &= 0 \\ B^{N}_{I=1}(t,z) &= +\left(\frac{16\pi\gamma_{11}}{u - M^2}\right)(-\beta^{ts}_{1,1/2}) \end{aligned}\right\} \quad (34)$$

for a nucleon in channel u ($\beta^{ts}_{1,1/2} = +2/3$) .

For Δ - particle exchange the invariant amplitudes take the form

$$\left.\begin{aligned} A^{N^*}_{I=1}(t,z) &= \left(\frac{d+bu}{s-M^2_{33}}\right)\beta^{ts}_{1,3/2} \\ B^{N^*}_{I=1}(t,z) &= \left(\frac{e+fu}{s-M^2_{33}}\right)\beta^{ts}_{1,3/2} \end{aligned}\right\} \quad (35)$$

for a Δ - particle in channel s and

$$\left.\begin{aligned} A^{N^*}_{I=1}(t,z) &= \left(\frac{d+bs}{u-M^2_{33}}\right)(-\beta^{ts}_{1,3/2}) \\ B^{N^*}_{I=1}(t,z) &= -\left(\frac{e+fs}{u-M^2_{33}}\right)(-\beta^{ts}_{1,3/2}) \end{aligned}\right\} \quad (36)$$

for a Δ - particle in channel u ($\beta^{ts}_{1,3/2} = -2/3$). We further notice that $s \longleftrightarrow u$ interchanges the 2 pions and this requires[7]

$$\left.\begin{aligned} A^{I=1}(s,u) &= -A^{I=1}(u,s) \\ B^{I=1}(s,u) &= +B^{I=1}(u,s) \end{aligned}\right\} \quad (37)$$

The invariant amplitudes in this approximation must then have the form

$$\left.\begin{aligned}
A^{I=1}(t,z) &= \beta^{ts}_{1,3/2}\left[\frac{d+bu}{s-M_{33}^2} - \frac{d+bs}{u-M_{33}^2}\right] = -A^{I=1}(t,-z) \\
B^{I=1}(t,z) &= \beta^{ts}_{1,1/2}\left[-16\pi\gamma_{11}\left(\frac{1}{s-M^2}+\frac{1}{u-M^2}\right)\right] \\
&\quad + \beta^{ts}_{1,3/2}\left[\frac{e+fu}{s-M_{33}^2} + \frac{e+fs}{u-M_{33}^2}\right] = +B^{I=1}(t,-z)
\end{aligned}\right\} \quad (38)$$

The explicit calculation of $V_\pm$ now involves the tedious chore of calculating the appropriate partial waves of (38) and substituting these projections into (8) and (9). The final result is

$$\left.\begin{aligned}
v(\nu) &= \frac{V_-(\nu)}{\sqrt{2}} - \frac{V_+(\nu)}{M} = \left(\frac{V_-^N}{\sqrt{2}} - \frac{V_+^N}{M}\right) + \left(\frac{V_-^{N^*}}{\sqrt{2}} - \frac{V_+^{N^*}}{M}\right) \\
w(\nu) &= \frac{V_-(\nu)}{\sqrt{2}} = \left(\frac{V_-^N}{\sqrt{2}} + \frac{V_-^{N^*}}{\sqrt{2}}\right)
\end{aligned}\right\} \quad (39)$$

where $v(\nu)$ and $w(\nu)$ are defined in terms of $N_\pm(\nu_\rho)$ through (28) and (29) and

$$\frac{V_-^N}{\sqrt{2}} - \frac{V_+^N}{M} = -\frac{8}{3}\gamma_{11}\frac{Q_2(x_t)}{pq} \quad (40)$$

$$\frac{V_+^N}{M} = +\frac{4}{3}\gamma_{11}\,(1+2q^2)\,\frac{Q_1(x_t)}{p^2\,q^2} \quad (41)$$

$$\frac{V_-^{N^*}}{\sqrt{2}} - \frac{V_+^{N^*}}{M} = +\frac{1}{8\pi M}\frac{p}{q}A_1^{N^*} - \frac{B_2^{N^*}}{8\pi} \quad (42)$$

$$\frac{1}{8\pi M}\frac{p}{q}A_1^{N^*} = -\frac{2b}{3\pi M}\frac{Q_1(x_t^*)}{q^2}\left(q^2 + \frac{1}{2}\left(1 + \frac{M_{33}^2}{2} - M^2\right) - \frac{d}{4b}\right) \quad (43)$$

$$\frac{B_2^{N^*}}{8\pi} = -\frac{2f}{3\pi}\,\frac{Q_2(x_t^*)}{pq}\left\{q^2 + \frac{1}{2}\left(1 + \frac{M_{33}^2}{2} - M^2\right) - \frac{e}{4f}\right\} \tag{44}$$

$$\frac{V_+^{N^*}}{M} + \frac{1}{8\pi M}\,\frac{p}{q}\,A_1^{N^*} = \frac{[2B_2^{N^*} + B_0^{N^*}]}{3(8\pi)} \tag{45}$$

$$\frac{[2B_2^{N^*} + B_0^{N^*}]}{3(8\pi)} = \frac{f}{9\pi} - \frac{2f}{3\pi}\left[q^2 + \frac{1}{2}(1 + M_{33}^2 - M^2)\right]\frac{Q_1(x_t^*)}{p^2\,q^2}\left\{q^2 + \frac{1}{2}\left(1 + \frac{M_{33}^2}{2} - M^2\right) - \frac{e}{4f}\right\} \tag{46}$$

Other quantities needed to evaluate (39) are

$$b = +\,8\pi\gamma_{33}\left[\frac{3(M_{33}+M)(E^*-M)}{2M_{33}q^{*2}}\right]$$

$$d = a + c$$

$$a = -\,8\pi\gamma_{33}\left[\frac{3(M_{33}+M)(E^*-M)}{M_{33}}\left\{1 + \frac{2(M^2+1) - M_{33}^2}{2q^{*2}}\right\}\right]$$

$$c = -\,8\pi\gamma_{33}\left[\frac{(M_{33}-M)(E^*+M)}{M_{33}}\right]$$

$$f = +\,\frac{8\pi\gamma_{33}}{M_{33}}\left[\frac{3(E^*-M)}{2q^{*2}}\right]$$

$$e = -\,\frac{8\pi\gamma_{33}}{M_{33}}\left[3(E^*-M)\left\{1 + \frac{2(M^2+1) - M_{33}^2}{2q^{*2}}\right\} - (E^*+M)\right]$$

$$E^* \pm M = \frac{(M_{33} \pm M)^2 - 1}{2M_{33}}$$

$$q^{*2} = (E^{*}+M)\,(E^{*}-M) = \frac{[(M_{33}-M)^2-1][(M_{33}+M)^2-1]}{4M_{33}^2}$$

$$x_t = \frac{1 + 2q^2}{2pq}$$

$$x_t^{*} = \frac{M_{33}^2 + 2q^2 + 1 - M^2}{2pq}$$

$$Q_1(x) = \frac{x}{2}\ln\left(\frac{x+1}{x-1}\right) - 1$$

$$Q_2(x) = \frac{1}{4}(3x^2-1)\ln\left(\frac{x+1}{x-1}\right) - \frac{3}{2}x$$

The quantities $v(\nu)$ and $\omega(\nu)$ are plotted in Figures 2 and 3 ($\nu \equiv q^2$) with $M = 6.7$, $M_{33} = 8.9$, $\gamma_{11} = 15$ and $\gamma_{33} = 7.5$.

V. Numerical Results

To facilitate the discussion in this section we recall that γ_N'/γ_π and γ_N/γ_π are given by (26) and (27), namely,

$$\left.\begin{aligned} -\left(\frac{p_\rho^{\,2}}{M^2}\right)\frac{\gamma_N'}{\gamma_\pi} &= \frac{-3}{\gamma_\pi^{\,2}}\left\{\frac{1}{D_\pi'(\nu_\rho)}\left(\frac{N_-(\nu_\rho)}{\sqrt{2}} - \frac{N_+(\nu_\rho)}{M}\right)\right\} \\ \frac{\gamma_N}{\gamma_\pi} &= \frac{-3}{\gamma_\pi^{\,2}}\left\{\frac{1}{D_\pi'(\nu_\rho)}\left(\frac{N_-(\nu_\rho)}{\sqrt{2}}\right)\right\} - \frac{\gamma_N'}{\gamma_\pi} \end{aligned}\right\} \qquad (50)$$

We must calculate (using (18))

$$J_1(\nu_\rho) = \frac{1}{D_\pi'(\nu_\rho)}\left(\frac{N_-(\nu_\rho)}{\sqrt{2}} - \frac{N_+(\nu_\rho)}{M}\right) \qquad (51)$$

$$J_2(\nu_\rho) = \frac{1}{D'_\pi}\left(\frac{N_-(\nu_\rho)}{\sqrt{2}}\right) \tag{51}$$

To carry out the analysis a strip model for $\pi\pi$ scattering in the state $J = 1$, $I = 1$ is needed. Such a model has recently been proposed by Balázs.[1] It is admittedly a crude model and the main source of error in the present analysis.[8] However, there is no other strip model available which calculates m_ρ, $\Gamma_\rho = -\frac{N_\pi(\nu_\rho)}{D'_\pi(\nu_\rho)}$, and ν_1 self-consistently. In the Balázs model one can see how these 3 numbers are interrelated; it does not appear possible at this time to construct a 1-channel strip model for $\pi\pi$ dynamics which produces the <u>experimental</u> ρ-meson mass and width with a reasonable strip width.

The basic structure of the Balázs model is as follows:

$$A_\pi(\nu) = \frac{N_\pi(\nu)}{D_\pi(\nu)} = \frac{e^{i\delta}\sin\delta}{\rho_\pi} \quad ; \quad \rho_\pi = \sqrt{\frac{\nu}{\nu+1}} \tag{52}$$

$$\left.\begin{aligned} &N_\pi(\nu) = \beta\nu \\ &D_\pi(\nu) = 1 - \frac{1}{\pi}\int_0^{\nu_1} d\nu' \, \frac{\rho_\pi(\nu')\, N_\pi(\nu')}{\nu'-\nu} \\ &\text{Re } D_\pi(\nu) = 1 - \beta h(\nu)\ . \quad \text{In the linear approximation} \\ &h(\nu) = \frac{1}{\pi}\left[\left(\nu_1 - \frac{1}{2}\ln 4\nu_1\right) + \nu \ln 4\nu_1\right] \end{aligned}\right\} \tag{53}$$

Balázs found the self-consistent values $\nu_\rho = 3$, $\Gamma_\rho = 2$ and $\nu_1 = 26$. These are to be compared with the experimental values $\nu_\rho = 6$, $\Gamma_\rho = 1$ and $\nu_1 \approx 26$. Of course, to make consistent use of the strip approximation we must use the self-consistent values found by Balázs. Using (53) it follows that

$$D'_\pi(\nu_\rho) = -\frac{8\ln 4\nu_1}{\pi} \tag{54}$$

Then (51) becomes

$$\left.\begin{aligned} J_1(\nu_\rho) &= -\left(\frac{1}{\ln 4\nu_1}\right)\left[P\int_0^{\nu_1} d\nu' \frac{\rho_\pi(\nu')\,\nu'\,v(\nu')}{\nu'-\nu_\rho}\right] \\ J_2(\nu_\rho) &= -\left(\frac{1}{\ln 4\nu_1}\right)\left[P\int_0^{\nu_1} d\nu' \frac{\rho_\pi(\nu')\,\nu'\,\omega(\nu')}{\nu'-\nu_\rho}\right] \end{aligned}\right\} \tag{55}$$

Referring to (50) with $p_\rho^{\,2}/M^2 \approx -1$ we find

$$\frac{\gamma_N}{\gamma_\pi} \approx \frac{-3}{\gamma_\pi^{\,2}}\,[J_2(\nu_\rho) - J_1(\nu_\rho)] \tag{56}$$

We now notice that for $1/2 \lesssim \nu' \leq \nu_1$

$$\omega(\nu') = v(\nu') + 0.26 \tag{57}$$

The failure of (57) for $\nu' \lesssim 1/2$ is of no consequence since both J_2 and J_1 get their strongest contributions for ν' large. Using (55) - (57) one finds

$$\frac{\gamma_N}{\gamma_\pi} = +\frac{6}{\gamma_\pi^{\,2}}\,(0.85) \tag{58}$$

We can now calculate $\frac{\gamma_N'}{\gamma_\pi}$ from (50) by noting that for $1/2 \lesssim \nu' \leq \nu_1$ a fairly good linear approximation can be made for $v(\nu')$. i.e.

$$v(\nu') = m_1\nu' + \alpha_1 \text{ where } \begin{cases} m_1 = -0.043 \\ \alpha_1 = +1.26 \end{cases} \tag{59}$$

Again the failure of (59) for $\nu' < 1/2$ is of no consequence. Carrying out the appropriate integrations we find

$$\frac{\gamma_N'}{\gamma_\pi} = + \frac{6}{\gamma_\pi^{\,2}} (2.3) \tag{60}$$

From (58) and (60) it follows that

$$\frac{\gamma_N'}{\gamma_N} = + 2.7 \tag{61}$$

Equation (61) is to be compared with the corresponding ratio used in recent π-N and N-N phase shift analyses. In particular, Hamilton[9] and co-workers have shown that $\gamma_\pi \gamma_N$ and $\gamma_\pi \gamma_N'$ must be positive in order to be consistent with the nucleon EM structure and S-wave πN scattering at low energies. Bryan-Arndt[10] and Scotti-Wong[11] have fit the p-p phase shifts from the Livermore Group with

$$\frac{\gamma_N'}{\gamma_N} = + 3 \quad .$$

We conclude this section with a discussion of the sensitivity of γ_N'/γ_π, γ_N/γ_π, and γ_N'/γ_N on the details of the strip model for $\pi\pi$ scattering. Suppose we force $\nu_\rho = 6$, $\nu_1 = 26$ in the Balázs strip model. This requires $\Gamma_\rho = 4$ (as compared to the experimental value $\Gamma_\rho = 1$). It is clear that γ_N'/γ_π and γ_N/γ_π will be sensitive to this change ($\gamma_\pi^{\,2} = 4\Gamma_\rho$). To find out if these numbers are sensitive to a large change in the ρ-mass we calculated γ_N'/γ_π, γ_N/γ_π and γ_N'/γ_N using $\nu_\rho = 6$ (instead of 3). The results are

$$\gamma_N'/\gamma_\pi = \neq \frac{6}{\gamma_\pi^{\ 2}}(2.1) \; ; \; \frac{\gamma_N}{\gamma_\pi} = \neq \frac{6}{\gamma_\pi^{\ 2}}(0.9) ; \; \frac{\gamma_N'}{\gamma_N} = +2.3$$

The sensitivity is therefore buried in $\gamma_\pi^{\ 2}$, for a given strip width ν_1.

VI. Calculation of γ_N'/γ_π, γ_N/γ_π and γ_N'/γ_N from the Nucleon EM Structure

The nucleon EM structure is described by 4 form factors usually denoted[12]

$$G_i^s(t) = \frac{1}{\pi}\int_9^\infty dt' \, \frac{g_i^s(t')}{t'-t} \quad ; \quad i = 1,2 \tag{62}$$

$$G_i^V(t) = \frac{1}{\pi}\int_4^\infty dt' \, \frac{g_i^V(t')}{t'-t} \quad ; \quad i = 1,2 \tag{63}$$

The $G_i^{V,s}(t)$ are normalised so that

$$\left.\begin{array}{ll} G_1^s(0) + G_1^V(0) = e & G_2^s(0) + G_2^V(0) = \mu_p \\ G_1^s(0) - G_1^V(0) = 0 & G_2^s(0) - G_2^s(0) = \mu_n \end{array}\right\} \tag{64}$$

This means that

$$G_1^V(0) = +\frac{e}{2} = \frac{1}{\pi}\int_4^\infty dt' \, \frac{g_1^V(t')}{t'} \tag{65}$$

$$G_2^V(0) = \left(\frac{\mu_p - \mu_n}{2}\right)\left(\frac{e}{2M}\right) = \frac{1}{\pi}\int_4^\infty dt' \, \frac{g_2^V(t')}{t'} \tag{66}$$

where e = electric charge

μ_p = + 1.79 = anomalous magnetic moment of the proton

μ_n = - 1.91 = same for the neutron

$\frac{e}{2M}$ = 1 nuclear magneton in natural units

Equations (65) and (66) can be used to predict γ_N'/γ_π, γ_N/γ_π, and γ_N'/γ_N once a model for $\pi\pi$ scattering is specified. To be consistent with our previous work the Balázs model shall be used again.

The spectral functions $g_i^V(t)$ have been written down by Frazer-Fulco.[2] The result is (with $t = 4(\nu+1)$)

$$g_i^V(\nu) = - \frac{e\, F_\pi^*(\nu)\, \nu^{3/2}}{2E}\, \Gamma_i(\nu) \tag{67}$$

where $F_\pi(\nu) = \frac{D_\pi(-1)}{D_\pi(\nu)}$ = pion form factor (68)

$$\Gamma_1(\nu) = \frac{M}{p_-^{\,2}}\left[\frac{E^2}{M\sqrt{2}}\, f_- - f_+\right] \tag{69}$$

$$\Gamma_2(\nu) = \frac{1}{2p_-^{\,2}}\left[f_+ - \frac{M}{\sqrt{2}}\, f_-\right] \tag{70}$$

and $t = 4E^2 = 4(\nu+1)$

$$p_-^{\,2} = M^2 - \frac{t}{4} = M^2 - (\nu+1)$$

For $4 < t < 16$ Γ_i has the same phase as F_π. In order to preserve the reality of g_i^V we shall assume this is true at least for $t \approx m_\rho^{\,2}$. Now insert (67) into (65) and (66) and carry out the integrals over the spectral functions in the approximation where

$$g_1^V(t) \approx \text{const.}\ \delta(t - m_\rho^{\ 2}) \tag{71}$$

The integrations[13] are then simple and the results are

$$G_1^V(0) = \frac{e}{2} = \left(\frac{e}{2}\right)\left[\frac{2\rho_\pi^{\ 2}\ D_\pi(-1)}{3D_\pi'(\nu_\rho)}\right]\left(\frac{\gamma_N}{\gamma_\pi}\right)\ [-2] \tag{72}$$

$$G_2^V(0) = \left(\frac{\mu_p - \mu_n}{2}\right)\left(\frac{e}{2M}\right) = \left(\frac{e}{2M}\right)\left[\frac{\rho_\pi^{\ 2}\ D_\pi(-1)}{3D_\pi'(\nu_\rho)}\right]\left(\frac{\gamma_N'}{\gamma_\pi}\right)\ [-2] \tag{73}$$

Equations (72) and (73) require

$$\frac{\gamma_N'}{\gamma_N} = \mu_p - \mu_n = +3.7;\ \frac{\gamma_N'}{\gamma_\pi} = \not+ \frac{6}{\gamma_\pi^{\ 2}}\,(2.6);\ \frac{\gamma_N}{\gamma_\pi} = \not+ \frac{6}{\gamma_\pi^{\ 2}}\,(0.7) \tag{74}$$

Equation (74) is to be compared with our previous analysis

$$\frac{\gamma_N'}{\gamma_N} = +2.7;\ \frac{\gamma_N'}{\gamma_\pi} = \not+ \frac{6}{\gamma_\pi^{\ 2}}\,(2.3);\ \frac{\gamma_N}{\gamma_\pi} = \not+ \frac{6}{\gamma_\pi^{\ 2}}\,(0.85) \tag{75}$$

Therefore, the analysis is internally consistent and the next improvement will come from a better strip approximation to $\pi\pi$ scattering.

VII. Conclusions

The vertex $\rho \to N\bar{N}$ has been studied making consistent use of the strip approximation. This approximation is consistent with information known about the nucleon EM structure. Quantitative estimates of γ_N'/γ_π and γ_N/γ_π must await a better strip approximation to $\pi\pi$ scattering. However, γ_N'/γ_N seems to be relatively insensitive to the details of the strip model used for $\pi\pi$ scattering and can therefore be used as a measure of the relative importance of magnetic/charge coupling of $\rho \to N\bar{N}$. It is clear that magnetic-type coupling plays the most important role[14] in bootstrap calculations where the ρ-meson helps to generate the force to produce the scattering.

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Footnotes and References

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1 L. Balázs Simple ππ Bootstrap Calculation Using the Strip Approximation (1964 preprint — to be published). The Reggeized version of the strip approximation has been studied by Chew and Jones (UCRL-10992).

2 W. R. Frazer and J. R. Fulco, Phys. Rev. 117 1609-14 (1960); J. S. Ball and D. Y. Wong, Phys. Rev. 130 2112-16 (1963).

3 W. R. Frazer and J. R. Fulco, Phys. Rev. 117 1603-08 (1960).

4 M. Jacob and G. C. Wick, Ann. of Phys. 7 404-28 (1959).

5 For future reference the invariant variables for the s- and u-channels are recorded here. For channel s: $s = \nu_s^2$; $t = -2q_s^2(1-z_s)$; $u = 2(M^2+1) - s - t$. For channel u, just replace $s \to u$. Natural units are used throughout this work: $c = 1$, $\hbar = 1$, $m_\pi = 1$. Then $\lambda = \frac{\hbar}{m_\pi c}$ ($\approx 3/2 \times 10^{-13}$cm) = 1.

6 D. Roper (MIT Thesis -- unpublished). It should be pointed out that (22) and (23) are obtained by using Feynman's rules for the graph $\pi\pi \to \rho \to N\bar{N}$. One could carry on the same discussion without this reference by defining $f_\pm(t) \approx \frac{\gamma_\pi \gamma_\pm}{t-m_\rho^2}$. The $\gamma_\pm$ are then linear combinations of γ_N and γ_N'. Roper's equations are used in order to have a basis for comparison with phenomenological work done in the πN and NN systems.

7 G. F. Chew S-Matrix Theory of Strong Interactions (Benjamin, 1962).

8 The numerical values of γ_N and γ_N' are quite sensitive to the details of the model but γ_N'/γ_N seems to be relatively insensitive. These matters will be discussed in more detail later.

9 See for example A. Donnachie, J. Hamilton and A. T. Lea; <u>Prediction of P-, D- and F-Wave N Scattering</u> (1964 preprint) and references given there. The connection between (C_1, C_2) and (γ_N, γ_N') is $-\frac{12C_2}{\pi} = +\frac{4\pi}{M}\gamma_\pi\gamma_N'$ and $+\frac{12C_1}{\pi} = -8\pi\gamma_\pi\gamma_N$ where $\frac{C_2}{C_1} = +0.27$ and $C_1 = -0.95$. It is not hard to show that for $\gamma_\pi\gamma_N > 0$ the exchange of the ρ-meson leads to an attractive force in the p(1,1) and p(3,3) states and a repulsive force in the p(3,1) and p(1,3) states. The attraction is weak in the p(3,3) state, moderate in the p(1,1) state and the repulsion is moderate in the p(3,1) and p(1,3) states. This agrees with observations made by Bowcock, Cottingham and Lurie (Nuovo Cimento <u>16</u> 918 (1960)) and Frautschi (Phys. Rev. Lett. <u>5</u> 159 (1960)). I am indebted to Professor Geoffrey Chew for a helpful discussion on this matter.

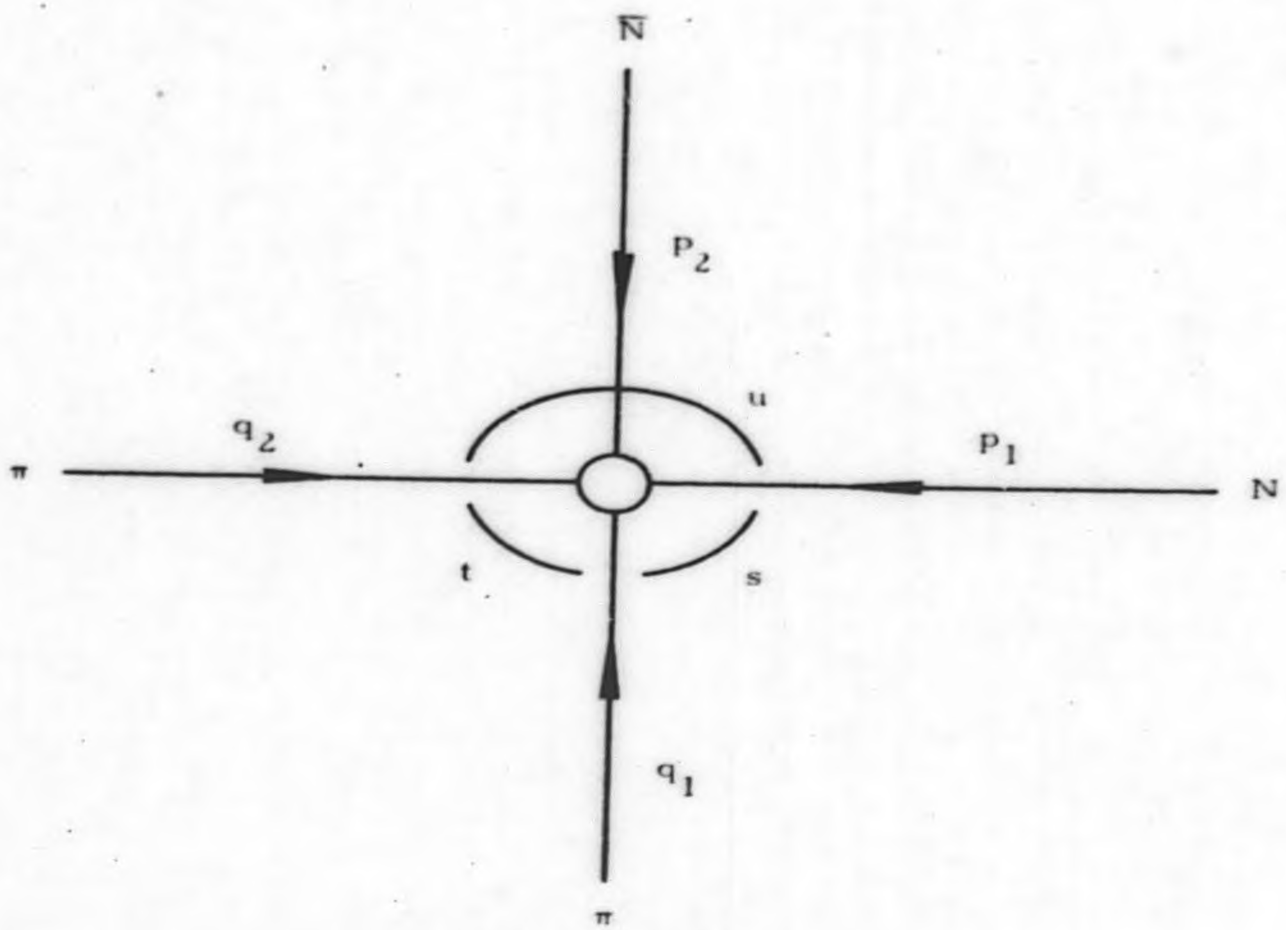
10 R. Bryan and R. Arndt <u>P-P Phase Shift Analysis Using the K-Matrix Formalism</u> (unpublished). I am indebted to Mr. Richard Arndt for a discussion of this work.

11 A. Scotti and D. Y. Wong Phys. Rev. Lett. <u>10</u> 142-46 (1963). I am indebted to Dr. David Wong for a helpful conversation about his work.

12 G. F. Chew, R. Karplus, S. Gasiorowicz, and F. Zachariasen, Phys. Rev. <u>110</u> 265.76 (1958). We have written the form factors without substractions. This implies $g_1^{V,s}(t) \xrightarrow{t \to \infty} O(\frac{1}{t^n})$ $n > 0$. The most recent data on the nucleon EM structure seems to justify this.

13 To calculate $D_\pi(-1)$ we have just made a linear extrapolation using (53).

14 This fact was not realized by Abers and Zemach, Phys. Rev. <u>131</u> 2305-17 (1963). In their work, $\gamma_N' \equiv 0$.



CHANNEL s: $\pi + N \rightarrow \pi + N$
CHANNEL t: $\pi + \pi \rightarrow N + \overline{N}$
CHANNEL u: $\pi + N \rightarrow \pi + N$

q_i and p_i are the 4-momenta for the pions and nucleons, respectively.

Figure 1

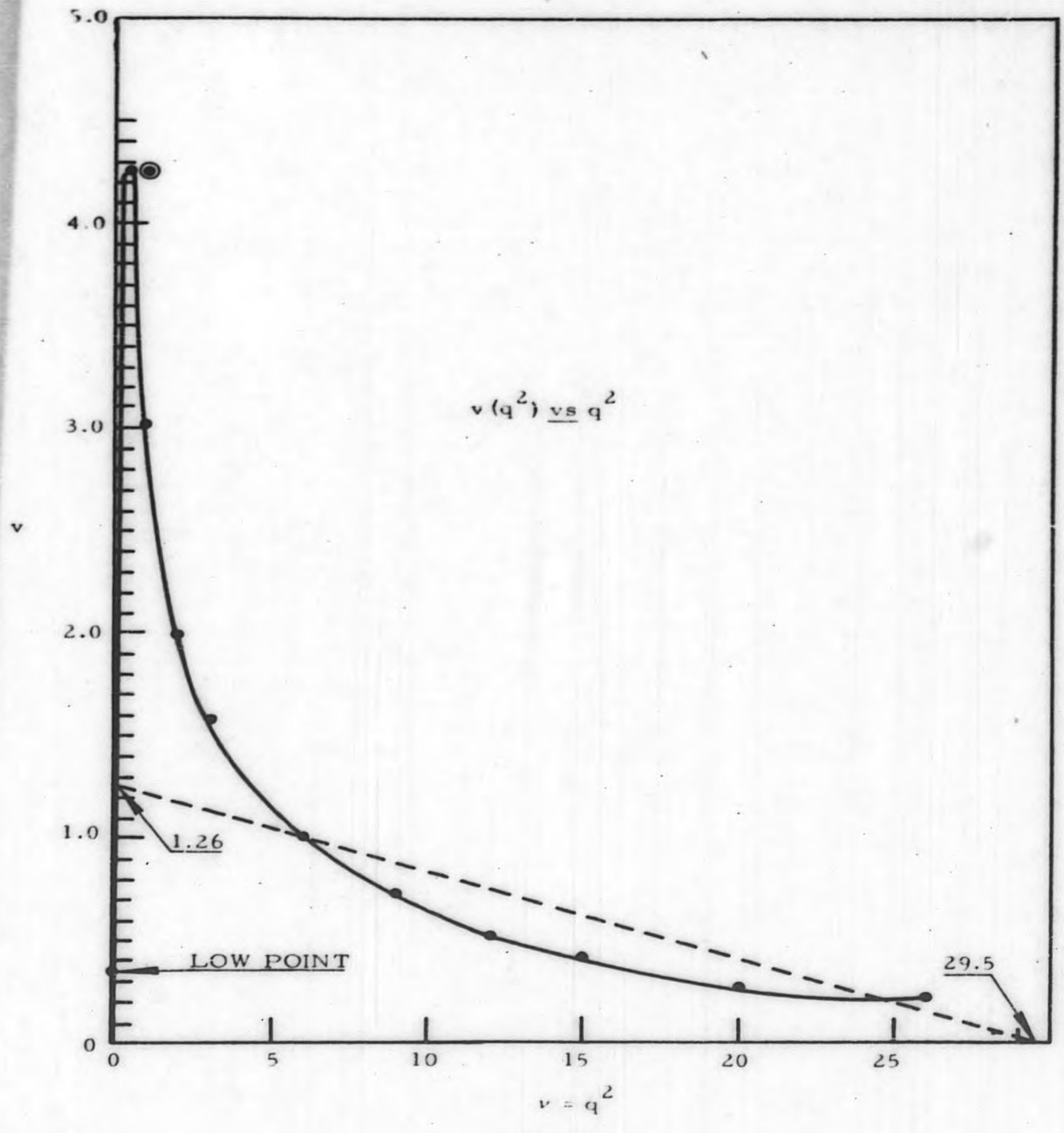


Figure 2

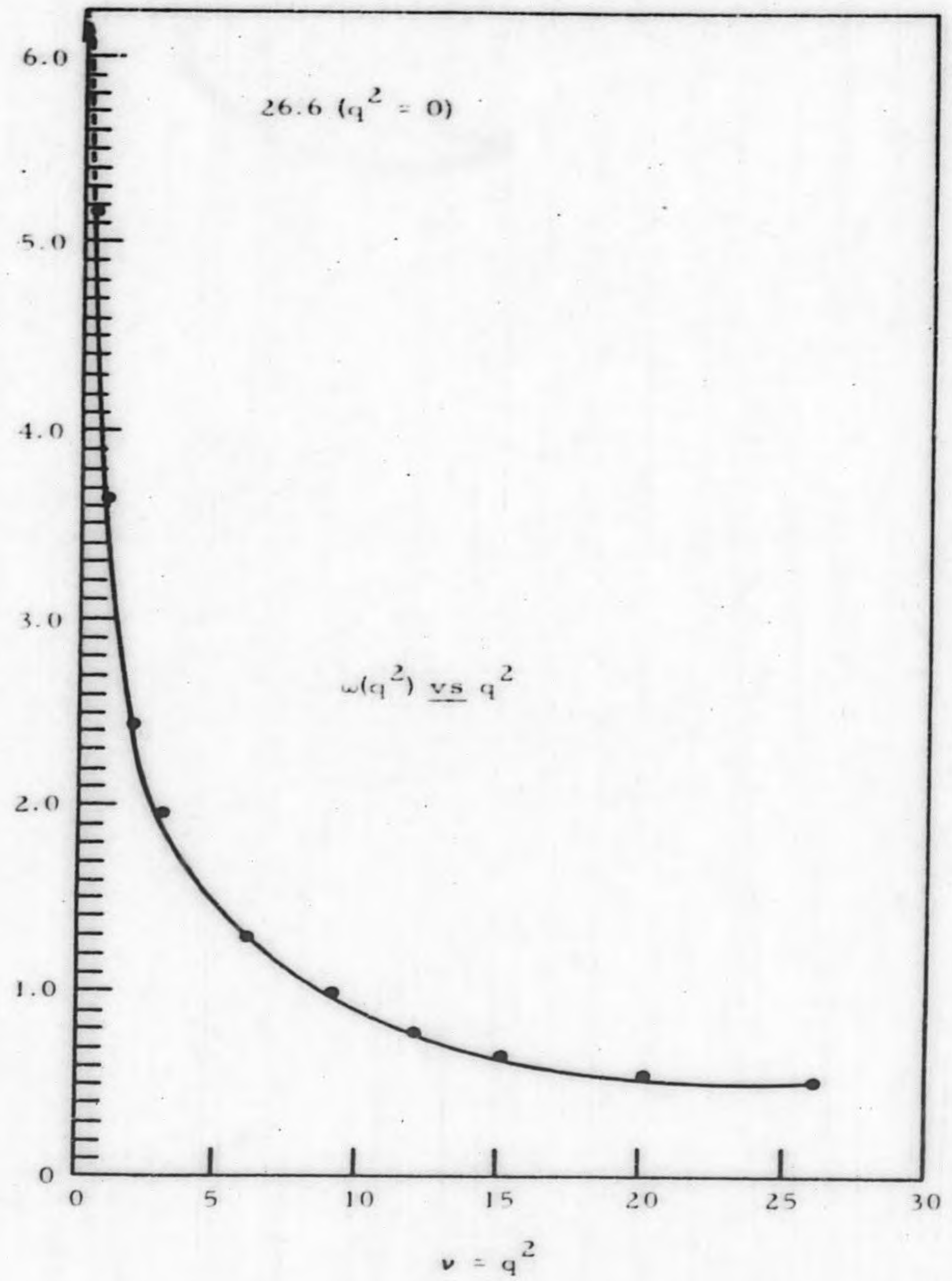


Figure 3

END