ULTRAPRECISE MAGNET DESIGN AND SHIMMING*

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Abstract

Computer studies of pole design and magnet shimming techniques are discussed for a very precise 14.72 kG iron core storage ring magnet to be used for the proposed measurement of the muon anomalous magnetic moment. The experiment requires knowledge of the field in the 7m radius storage ring dipole to approximately 0.1 ppm (1x10^-7). The goal is to produce field uniformity of approximately 1 ppm. Practical and mathematical limitations prevent obtaining such accuracy directly with a computer code such as POISSON, which is used in this study. However, this precision can be obtained for perturbations of the magnetic field. Results are presented on the internal consistency of the computations and on the reliability of computing perturbations produced by Fe shims. Shimming techniques for very precise field modification and control are presented.

I. Introduction

This report, limited in its scope to computer studies by the authors, discusses a part of the ongoing design effort for an ultraprecise 3 GeV/c storage ring. The g-2 experiment proposal has been approved as part of the future physics program at the high intensity, post-Booster, Alternating Gradient Synchrotron (AGS).

principal error was control of each of the 40 magnet sections by correction coils. These used feedback from a single point NMR measurement in each section. With extra space much more elaborate control can be used. (ii) A "trolley" capable of moving around the circumference inside the beam aperture carrying a matrix of NMR probes is being constructed. This can be "parked" out of the way without breaking vacuum. This "online," albeit intermittently, coexistence of complete mapping and physics running is a new feature. (iii) The "end effects" of the CERN 40 magnet blocks, although continuous at the pole, contributed significant field and measurement errors between blocks. The new ring will be constructed with 45° sectors machined to be close fitting at their ends to approximate a continuous ring. (iv) More elaborate use of field shimming by adjustment to the iron cross section remote from the pole faces is planned. A large air gap between the poles and the return yoke will be used as part of this strategy. (v) Superconducting coils improve $B_0$ stability and reduce the need for magnet cycling. (Power saving.)

The goal of the computer simulations has been to develop techniques to control the dipole field and lower order multipoles so that $\Delta B/B_0 < 1x10^{-5}$ over the necessary "good field" 9 cm diameter can be relatively easily obtained. The error would be reduced to $< 1x10^{-6}$ by special local static shimming or active current control such as pole face windings. The final factor...
of the future physics program at the high intensity, post-Booster, Alternating Gradient Synchrotron (AGS). An international collaboration is involved in detailed design of the storage ring and detection apparatus.

The computer studies are of general interest because of the precision required. Most accelerator magnets perform at a $\Delta B/B_0 > 1 \times 10^{-4}$ field uniformity, for which the computer codes—in this instance POISSON$^2$ can, if carefully used, reliably predict the field within the beam aperture. For example, the AGS Booster dipoles agreed with computations to $\Delta B/B \sim 1 \times 10^{-4}$ over the "good field" aperture. High field superconducting magnets designed by the authors had similar agreement.

The experiment and the storage ring design are solidly based on a highly successful CERN design.$^3$ The third of a series of muon g-2 experiments, it resulted in a knowledge of the magnetic field integral appropriately averaged over the muon orbits to $\Delta B/B_0 = 1$ to $2 \times 10^{-6}$. This, plus other smaller systematic errors were less than the statistical uncertainty of 7 PPM obtained in the experiment. The result stands as the state of the art.

Operation at $5 \times 10^{13}$ protons in the AGS using the Booster, should permit a statistical uncertainty of 0.3 PPM in the new experiment, assuming the same pion decay injection technique as at CERN. Other injection possibilities might further reduce this error. To carry out this very fundamental measurement, it is desirable that systematic errors be $<0.1$ PPM. These are dominated by magnetic field uncertainty, which involves the error in knowledge of the magnetic field, averaged over space and time in relation to the muon distribution.

Figure 1 taken from the 1986 update$^4$ of the proposal shows the general layout of the experiment. Figures 2 and 3 show the magnet cross section.

The improvements in precision anticipated for the new experiment come from several areas.

(i) The gap increase from 14 to 18 cm allows more elaborate field monitoring and feedback. For CERN the

* Work performed under the auspices of the U.S. Dept. of Energy.

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>COMMENTS</th>
<th>ERROR (PPM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetic field</td>
<td>Includes absolute calibration of Wien Function and averaging over space, time, and high distribution.</td>
<td>0.07</td>
</tr>
<tr>
<td>Electric field correction</td>
<td>0.7 ppm correction</td>
<td>0.01</td>
</tr>
<tr>
<td>Pitch correction</td>
<td>0.9 ppm correction</td>
<td>0.02</td>
</tr>
<tr>
<td>Particle losses</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>Timing errors</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td></td>
<td>0.08</td>
</tr>
</tbody>
</table>

Spin motion -
In a magnetic field $B$,
$$\gamma = \frac{eB}{mc^2} + \frac{1}{\gamma^2} - \frac{eE}{mc^2}$$

In a magnetic field $B$ and electric field $E$,
$$\gamma = \frac{eB}{mc^2} \left( aB + bE \right)$$

$\gamma_{mag} = 29.3$, $\gamma = \frac{E}{mc^2} aB$

Fig. 1. AGS Muon g-2 Experiment.

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II. Design Optimization

During 1986 the computer calculations were used to reduce the cross section and weight of the magnet to 2/3 that in the Proposal. The use of 1 cm "air" gaps between each pole and return yoke facilitated this, since the flux return reluctance is significantly decoupled from the behavior of the poles. (Table I.)

<table>
<thead>
<tr>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base*</td>
<td>W=55cm</td>
<td>W=55cm</td>
<td>W=55cm</td>
</tr>
<tr>
<td>(W=65cm)</td>
<td>+4 corners off</td>
<td>+4 corners off</td>
<td>+10 cm off</td>
</tr>
<tr>
<td>ANI/NI(base)</td>
<td>0</td>
<td>+2.16%</td>
<td>+2.40%</td>
</tr>
<tr>
<td>$\frac{A B_n}{B_0}$ (Normalized)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n=1$ (quad)</td>
<td>0</td>
<td>-1.3 PPM</td>
<td>-2.6PPM</td>
</tr>
<tr>
<td>$n=2$ (sext)</td>
<td>0</td>
<td>-0.5</td>
<td>-2.5</td>
</tr>
<tr>
<td>$n=3$</td>
<td>0</td>
<td>-0.1</td>
<td>-1.1</td>
</tr>
<tr>
<td>$n=4$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$n=5$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$n=6$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$n=7$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$n=8$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Col. I: the 1985 Proposal Magnet Cross Section, with 1 cm air gap behind each pole.

Col. II: for a 10 cm (18%) reduction in width of the return yoke block, centered on the horizontal midplane.

Col. III: also cut four corners off magnet.

Col. IV: also reduced thickness of top and bottom yoke member by 10 cm. (This increased reluctance by ~2%.)

In all cases in this Report, multipoles are expressed at $R = 4.5$ cm, $y = 0$; $B_0 = 14.7$ kG.

TABLE III. Perturbing Air Gap Behind Pole to Remove Quadrupole.

<table>
<thead>
<tr>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{A B_n}{B_0}$</td>
<td>&quot;Standard&quot;</td>
<td>Pole gap</td>
</tr>
</tbody>
</table>

The C-magnet return produces a very large systematic gradient. Three perturbations have been explored: (i) tilt the pole faces, (ii) larger bumps on the inside pole edges than on the outside, (iii) shim in the air gap at the rear of the poles to induce more flux on the inside. While (i) and (ii) are possible for refined shimming, they are too local to the "good field" aperture and generate significant octupole. Method (iii) can give a large almost pure quadrupole so the magnet can start off with the systematic C-magnet gradient removed. See Table III.
The result of very large weight (and cost) reduction is an appreciable increase in reluctance and ampere turns requirement, but no significant change in multipole field errors. The magnetic and dimensional tolerances of the yoke flux return are not unusually tight and are relevant mainly to the dipolar term. For example, scaling from Col. II, a 0.65 mm change in width of the HMP block would produce dipolar change of 1.4 x 10^-5; equivalent to a 25 μm change in the 18 cm gap.

Consider the effect of raising the central field by 1% in two cases, the geometry of Col. I and of the Col. IV in Table I. This result is shown in Table II.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>ANI/NI (base)</td>
<td>1% + 0.16% 1% + 0.58%</td>
</tr>
<tr>
<td>AB_n/B_0 (4.5 cm)</td>
<td></td>
</tr>
<tr>
<td>n = 1 (quad)</td>
<td>-14.2 PPM</td>
</tr>
<tr>
<td>n = 2 (sext)</td>
<td>-6.6</td>
</tr>
<tr>
<td>n = 3</td>
<td>-0.3</td>
</tr>
<tr>
<td>n = 4</td>
<td>0</td>
</tr>
<tr>
<td>n = 5</td>
<td>0</td>
</tr>
<tr>
<td>n = 6</td>
<td>0</td>
</tr>
<tr>
<td>n = 7</td>
<td>0</td>
</tr>
<tr>
<td>n = 8</td>
<td>0</td>
</tr>
</tbody>
</table>

Note the effect on the multipoles of raising B_0 by 1% is almost independent of the very large changes in yoke geometry. The quadrupole is due to C-magnet yoke asymmetry. The 1% higher field reduces the permeability in the vicinity of the air gaps. The reduced permeability in the poles also affects the sextupole. Table II can also be used to establish tolerances on magnetization properties in the pole steel. A 1% change in saturation magnetization would produce roughly the change in Table II. The storage ring central field will always operate at 14.72 kG.
The effect of the coil motion is shown in Table IV.

<table>
<thead>
<tr>
<th>Multipole</th>
<th>Outer Coil</th>
<th>Outer Coil</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>(4.5 cm)</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>Dipole</td>
<td>-24.1 PPM</td>
</tr>
<tr>
<td>1</td>
<td>Quadrupole</td>
<td>+0.60</td>
</tr>
<tr>
<td>2</td>
<td>Sextupole</td>
<td>-0.09</td>
</tr>
<tr>
<td>3</td>
<td>Octupole</td>
<td>+0.02</td>
</tr>
<tr>
<td>4</td>
<td>Decapole</td>
<td>-0.01</td>
</tr>
</tbody>
</table>

**Notes:**
1. Outer coils are located at R=739 cm, y=±15 cm (Fig. 2 and Fig. 3.)
2. Inner coil-not tabulated but sensitivity less.
3. All multipole terms <1 PPM, except for dipole.

Table IV: Coil Position Tolerance

The approach of the g-2 design is to produce pole surfaces as flat as economically practical by machining plus possibly grinding or polishing the surface of sections to minimize "hill and dale" errors. Very homogeneous material will be used to minimize "pot holes."

For reference, consider simplified .001" (25 μm) errors in the gap and the parallelity of the pole surfaces: (i) a .001" systematic gap error gives 141 PPM dipole change, (ii) a .001" side-to-side tilt gives a quadrupole of 11 PPM at R = 4.5 cm, (iii) a .001 symmetric variation: the gap at the center .001" different than at the pole edges, gives ~3.6 PPM sextupole. These illustrate the incentive to make the dipole $\Delta B/B_0$ very small around the azimuth by shimming the reluctance or possibly by current loops remote from the pole surfaces. The present state of the design is shown in Col. II of Table III. A slight change to the symmetric pole profile will remove the 33 PPM sextupole. Touchup of radial asymmetry can take care of quadrupole and octupole in the computed magnet. The computations need only be credible to perform perturbations at the PPM level, i.e. to predict the necessary
The computations need only be credible to perform perturbations at the PPM level, i.e. to predict the necessary correction for the residual error measured in the magnet. The magnet will have both cylindrically symmetric and azimuthally varying field errors due to geometrical factors, magnetic forces, magnetization in iron, temperature control, etc. (Note that 1 PPM=0.18\( \mu \)m gap tolerance.) Careful operating control plus shimming perturbations can correct anything except the most local pole surface defects. A fundamental limit is the temporal stability and reproducibility of the magnet. Active feedback must be used beyond this limit. Dynamic and possibly also static corrections will be made with current loops applied in sections, possibly 1 meter long. Such coil corrections are analytically straightforward to compute, but should be small at least on pole surfaces. In addition to taking space and generating heat coils have "lumpy" current distributions which generate higher multipole errors as they correct. This will impact on the final < 0.1 PPM knowledge of the field.

References

1. AGS Proposal. A new Precision Measurement of the Muon g-2 Value at the Level of 0.35 PPM. V.W. Hughes et al. September 1985.
4. AGS Proposal 821, September 15, 1986 (Same title).
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