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OF THIN INTENSE ELECTRON BEAMS**

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Small Signal Gain Based on Analytic Models of Thin Intense Electron Beams

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We develop the free-electron laser theory of the effective energy distribution and the small signal gain for a thin electron beam. The assumption of thinness allows us to treat various transverse locations and electron beam trajectory angles as introducing phase shifts that have the same effect as those introduced by a change in energy of the electron. These ideas extend previous work of Colson et al, Dattoli et al, Scharlemann and others in five important ways. The first is the ability to treat electron beams with three different classes of matching or symmetry conditions: (i) electron beams with separate betatron matching in each plane, (ii) those with aspect ratio matching, and (iii) crossed matched beams. Manifestations of these symmetries include elliptical crosssections and electron beams that have modulated spatial profiles. For these we derive analytical expressions for effective energy distributions. Second, two emittance parameters for the electron beam are shown to consolidate into a single parameter that describes most of the energy variation of the effective energy distributions. Thus, the effective energy distribution for a 1:4 ribbon electron beam is nearly equivalent to a distribution for a beam of circular crosssection. Third, these calculations extend to energy distributions, angular distributions, and spatial distributions that all follow gaussian profiles. Fourth, this model incorporates the description of the incident gaussian optical beam and the above electron beam dynamics into a single influence function kernel. Emittance, energy spread, diffraction, and gain may be interpreted as limiting the length over which the bunching contributions of the propagating electric fields downstream are important. Fifth, three-dimensional profiles of the optical fields are computed. This work is complementary to the recent work of Yu, Krinsky, and Gluckstern in that ours always describes the *transition* from low gain to high gain for a thin beam and not necessarily the high gain regime itself. Therefore in this work the parameters of the incident optical beam are included, whereas, their work is not concerned with these parameters. The resulting transverse dependence of the fields may be characterized by an optical beam radius. This optical beam width starts out large compared to the thin electron beam and then, in the example given, contracts to a size that becomes so small that the thin beam assumption is violated.

I. INTRODUCTION

The free-electron laser has produced light from $0.24\mu\text{m}$ to $800\mu\text{m}^1$, and much work is taking place to extend oscillation into the XUV² and with compact systems. To easily evaluate such systems, very general gain formulas are required. Dattoli et al³ has given very useful expressions. Colson et al⁴ has developed 1-D integral equations. Recently Yu, Krinsky, and Gluckstern⁵ have applied variational techniques to computation of asymptotic gain coefficients when the gain is very high. Our work complements that of Yu, Krinsky and Gluckstern for thin electron beams by considering the transition region leading to high-gain and it applies in the example given until the gain drives the approximate size of the optical field smaller than the e-beam.

II. THE ELECTRON BEAM MODEL

The distribution of electrons at the entrance to the wiggler determines the effective energy distribution throughout the interaction region. The particles are distributed in two positions, two angles, and energy. We develop an analytical model when the electron distribution functions are gaussian and possess additional symmetries.

The electron beam spatial model is an initial spatial gaussian profile in x and y and an angular gaussian profile in the angles x' and y' . The number of electrons at the entrance to the wiggler in a volume element $d\tau = dx dx' dy dy'$ is thus given by

$$f(x, y, x', y') d\tau = \frac{N_e}{\pi^2 x_0 y_0 x'_0 y'_0} \exp\left(-\frac{x^2}{x_0^2} - \frac{y^2}{y_0^2} - \frac{x'^2}{x'_0{}^2} - \frac{y'^2}{y'_0{}^2}\right) d\tau \quad (1)$$

The motion of these particles near the axis in the absence of the FEL interaction is governed by a magnetic field $\mathbf{B}$ given by

$$\mathbf{B} = -\nabla \chi, \quad (2)$$

$$\chi = -K \frac{k_w}{k_y} \cosh(k_x x) \sinh(k_y y) \cos(k_w z). \quad (3)$$

Here, K is the usual on-axis amplitude of the vector potential. This field gives rise to the well known averaged betatron motion⁶ when $\gamma^2 \ll 1$, where $\gamma = 1/(1 - \beta^2)^{1/2}$, β being the electron velocity in units of c ,

$$x'' = -k_{tx} x, \quad (4)$$

$$y'' = -k_{ty} y, \quad (5)$$

where

$$k_{tx} = K k_x / \sqrt{2\gamma^2}, \quad (6)$$

and

$$k_{\beta y} = K k_y / \sqrt{2\gamma^2}. \quad (7)$$

The betatron motion can be integrated and the distribution function established at all locations down the wiggler. In particular, the angular coordinates can be integrated over to give the marginal distributions f_x, f_y where

$$C_x^2(z) = x_0^2 x_0'^2 \left[\frac{\sin^2(k_{\beta x} z)}{k_{\beta x}^2 x_0^2} + \frac{\cos^2(k_{\beta x} z)}{x_0'^2} \right], \quad x \rightarrow y. \quad (8)$$

Now $f_x(x; z) \equiv \int_{-\infty}^{\infty} f_x(x, x', z) dx'$ may be expressed as

$$f_x(x; z) = \frac{1}{\pi \sqrt{C_x}} \exp\left(-\frac{x^2}{C_x^2}\right) \quad (9)$$

The distribution f_y can be obtained by letting $x \rightarrow y$ in the Eqs. (8)-(9). When $\Delta\gamma/\gamma \ll 1$, the effective energy is given by

$$\gamma_e = \gamma - \gamma_0 - \gamma_0^3(x'^2 + y'^2 + k_{\beta x}^2 x^2 + k_{\beta y}^2 y^2)/(2 + K^2). \quad (10)$$

We use a gaussian distribution of energies⁸ of width $\Delta\gamma$. In this case the unity normalized energy distribution f_γ is

$$f_\gamma(\gamma) = \frac{1}{\sqrt{\pi}\Delta\gamma} \exp[-(\gamma - \gamma_0)^2/\Delta\gamma^2], \quad (11)$$

and the number of particles in the volume $d\tau d\gamma$ is $f_\gamma(\gamma)f(x, y, x', y')d\tau d\gamma$. Thus, the effective energy distribution has a number of electron in an interval $d\gamma_e$ given by

$$f_e(\gamma_e) = N_e \int \exp(-\mathcal{Z}) d\tau, \quad (12)$$

$$\mathcal{Z} = (\Delta\gamma)^{-2} \left[\gamma_e + \frac{\gamma_0^3}{2 + K^2} (x'^2 + y'^2 + k_{\beta x}^2 x^2 + k_{\beta y}^2 y^2) \right]^2$$

$$- \frac{x^2}{x_0^2} - \frac{y^2}{y_0^2} - \frac{x'^2}{x_0'^2} - \frac{y'^2}{y_0'^2},$$

$$N_e = \frac{N_s}{\pi^{5/2} x_0 y_0 x_0' y_0' \Delta\gamma}.$$

For a single electron N_e is taken to be unity. Although the general integration of Eq. (12) is *not* readily performed analytically, there are a number of special cases that are both physically interesting and tractable. First consider the overly special condition of rotational symmetry of the electron beam with $x_0 = y_0$ and $x_0' = y_0'$, and a corresponding symmetry in the pole shaping⁶ of the magnetic field, $k_{\beta x} = k_{\beta y}$ (as seen below, this condition can be relaxed slightly). Then the mathematics simplify because x^2 and y^2 combine to $\alpha = r^2$, and x' and y' combine to $\beta = r'^2$; also the volume element becomes $\pi^2 d\alpha d\beta$. The integral is then a quadratic exponential in α and β and is well known. In addition, other symmetries exist in which the same technique applies. The requirement is that variables pair up by having the same ratio

of coefficients in the $\Delta\gamma$ term as in the x_0, x'_0, y_0, y'_0 term. Thus, there are three symmetry cases where x pairs with either y, x' , or y' , and the unselected two variables make the remaining pair.

The three transformations and corresponding conditions are tabulated in (13).

Case	Transformation	Condition
1	$\alpha = \frac{\gamma_0^3}{(2+K^2)\Delta\gamma}(k_{\beta x}^2 x^2 + k_{\beta y}^2 y^2)$ $\beta = \frac{\gamma_0^3}{(2+K^2)\Delta\gamma}(x'^2 + y'^2)$	$k_{\beta x}/k_{\beta y} = y_0/x_0$ $x'_0 = y'_0$
2	$\alpha = \frac{\gamma_0^3}{(2+K^2)\Delta\gamma}(k_{\beta x}^2 x^2 + x'^2)$ $\beta = \frac{\gamma_0^3}{(2+K^2)\Delta\gamma}(k_{\beta y}^2 y^2 + y'^2)$	$k_{\beta x} = x'_0/x_0$ $k_{\beta y} = y'_0/y_0$
3	$\alpha = \frac{\gamma_0^3}{(2+K^2)\Delta\gamma}(k_{\beta x}^2 x^2 + y'^2)$ $\beta = \frac{\gamma_0^3}{(2+K^2)\Delta\gamma}(k_{\beta y}^2 y^2 + x'^2)$	$k_{\beta x} = y'_0/x_0$ $k_{\beta y} = x'_0/y_0$

(13)

Case (1) is a generalization of the rotational symmetry, it requires the angular widths to be initially equal. Furthermore, the betatron length $2\pi/k_\beta$ in each plane must be proportional to the initial spatial width in that plane; this we refer to as the *aspect ratio matching* condition. Case (2) has separate betatron matching conditions⁶ in x and y ; in each plane the maximum betatron angle developed from a particle placed at the initial width is equal the initial angular width and the spatial profile is independent of z . Jerby⁷ treats this case in the limit of $y'_0 \rightarrow 0, y_0 \rightarrow 0$, and $k_{\beta y} = 0$. Case (3) is a cross-matched beam, with the peak angle $x_0 k_{\beta x}$ in x from the width x_0 matched to the initial angular width in y , namely y'_0 , (and likewise with $x \leftrightarrow y$). To achieve this condition, the ratio of the betatron lengths must be matched to the emittance ratio, where the two emittances are $\epsilon_x = \pi x <_0 x'_0$ and $\epsilon_y = \pi y_0 y'_0$. A special limit of case (3) is with $k_{\beta x} = k_{\beta y}$ (balanced emittances), but $x_0 \neq y_0$. This gives rise to a condition in which the spatial cross-section is an ellipse that remains quiescent to variations in z as in case (2). There are two cases, (1) and (3), whose spatial profile is changing in z , and these have identical properties as the quiescent case, providing, as we next establish, the two distribution parameters that we introduce C'_x and C'_y are the same. The three cases are identical when $C'_x = C'_y$ where rotational symmetry in angle space, aspect ratio matching, and betatron matching occurs.

FIG. 1. Two views of the $1/\epsilon$ of peak density surface from Eq.(9) corresponding to case (3) with $x_0 = 2$, $y_0 = 0.5$, $k_{\beta x} = 1$, $k_{\beta y} = 2$, $x'_0 = 1$, $y'_0 = 2$. The z -axis is upward.

III. THREE DIMENSIONAL E-BEAM EFFECTS

The normalization gives a condition on $C_\alpha C_\beta$. Also C_α can always be determined from the x term that is the same in all cases. Hence,

$$C_\alpha^{-1} = \frac{\gamma_0^3}{(2 + K^2)\Delta\gamma} \frac{k_{\beta x}^2 x_0^2}{2}, \quad (14)$$

$$C_\beta^{-1} = \frac{\gamma_0^3}{(2 + K^2)\Delta\gamma} \frac{k_{\beta y} y_0}{k_{\beta x} x_0} \frac{x'_0 y'_0}{2}. \quad (15)$$

Here the C_α , C_β terms are ratios of the energy width $\Delta\gamma$ to effective energy widths associated with (α) the x -motion only and (β) depending on case: (i) both betatron angles, (ii) the y betatron motion, (iii) the x -angle. The distribution is then

$$f_e(\gamma_e) = 2K_e \int \exp \left[-\left(\frac{\gamma_e}{\Delta\gamma} + \alpha + \beta \right)^2 - 2C_\alpha \alpha - 2C_\beta \beta \right] d\alpha d\beta, \\ K_e = 2 \frac{N_e C_\alpha C_\beta}{\Delta\gamma \sqrt{\pi}}. \quad (16)$$

The integral for $C_\alpha \neq C_\beta$ is

$$f_e(\gamma_e) = \frac{N_e}{\Delta\gamma} F \left(\frac{\gamma_e}{\Delta\gamma}, C_\alpha, C_\beta \right), \\ F(x, C_\alpha, C_\beta) = \frac{C_\alpha C_\beta}{(C_\beta - C_\alpha)} \left\{ \exp \left((C_\alpha^2 + 2xC_\alpha) \operatorname{erfc}(C_\alpha + x) \right) \right. \\ \left. - \exp \left((C_\beta^2 + 2xC_\beta) \operatorname{erfc}(C_\beta + x) \right) \right\}. \quad (17)$$

and for $C_\alpha = C_\beta \equiv C$, either direct evaluation or l'hospital's rule gives

$$\begin{aligned} \mathcal{F}(x, C, C) = & 2 \frac{C^2}{\sqrt{\pi}} \exp(-x^2) \\ & - \frac{2N_e C^2}{\Delta\gamma} (x + C) \times \\ & \exp(C^2 + 2xC) \operatorname{erfc}(C + x). \end{aligned} \quad (18)$$

There are, thus, two shape parameters $C^{-1} = 1/2(C_\alpha^{-1} + C_\beta^{-1})$ and $\mathcal{R} = C_\alpha/C_\beta$ and an energy scale $\Delta\gamma$ that characterizes these distributions. When $\mathcal{R} = 1$, the symmetrical case, the parameter $C = \mathcal{C}$ gives the distribution by Eq.(18). Variation of the distribution, $\tilde{\mathcal{F}}(x, \mathcal{C}, \mathcal{R}) \equiv \mathcal{F}(x, C_\alpha, C_\beta)$ with respect to $\mathcal{C}$ is shown in Fig. (2). Note that $\tilde{\mathcal{F}}(x, \mathcal{C}, \mathcal{R}) = \tilde{\mathcal{F}}(x, \mathcal{C}, 1/\mathcal{R})$. All these curves are normalized to have unity area. The variation of the height and width of the curves changes rapidly with variations of $\mathcal{C}$. The height, width and displacement of the curve varies little with the $\mathcal{R}$ parameter, unless variations in $\mathcal{R}$ or $1/\mathcal{R}$ are greater than order unity.

FIG. 2. The lowest chain-dot curve that goes through $x = 0$ is the distribution $-2Cx \times \exp(-2Cx)$ at a value of $C = 0.25$. And this compares to the lowest solid curve $\mathcal{F}(x, 0.25, 0.25)$. The other three solid curves from lowest peak to highest peak are for $c = 1/2, 1, 2$. The highest curve, the second chain-dot curve, is for no emittance, or $c = \infty$. It is gaussian.

We may fit the full-width at half-maximum to the formula

$$\Delta x = [1.665 + y_0(\rho)]e^{-1.176/2C} + \sigma(\rho)/(2C) + y_0(\rho) \quad (19)$$

where

$$\sigma(\rho) = \frac{1.4314 + 0.6867e^{-\rho/20}}{1.0 + 0.6825e^{-\rho/20}}, \quad (20)$$

$$y_o(\rho) = \frac{1.6317 - 1.646e^{-\rho/20}}{1 + 1.9888e^{-\rho/20}}, \quad (21)$$

$$\rho = R + \frac{1}{R} - 2. \quad (22)$$

This fit is good to 4% over a region considerably beyond that that of $C > 1$, $1/64 < \mathcal{R} < 64$.

Solution of the corresponding well known plane wave integral equation⁴ from $\tau = 0$ to $\tau = 1$ gives a gain, $|a(1)/a(0)|^2 - 1$ that is a function of five variables namely, $G(j, T, TC_\alpha, TC_\beta, \nu_0)$. [The required⁴ fourier transform is easily done using Eq.(16). Here T is given in Eq.(24); see Colson et al⁴ for j and ν_0 . This integral equation is obtained below in Eq.(24) by letting $\vec{r} = \vec{0}$, $g_\delta = 1$, $a_{0\delta}(\tau) = 1$, and $a_\delta(\tau) = a(\tau)$.] Alternately, we may express this function in the $\mathcal{C}$ - $\mathcal{R}$ representation with

$$\mathcal{G}(j, T, TC, \mathcal{R}, \nu_0) = G(j, T, TC_\alpha, TC_\beta, \nu_0). \quad (23)$$

Based upon the effective energy distribution dependence, we expect the $\mathcal{R}$ dependence of $\mathcal{G}$ to be quite weak compared to the TC dependence.

To explore the gain function, we introduce the maximum with respect to wavelength measure ν_0 and call this function $\mathcal{G}_>(j, T, TC, \mathcal{R})$. In Fig.(3) we show $\ln[\mathcal{G}_>(j, 1, C, 1) + 1]$.

FIG. 3. Contour of $\ln|a(1)/a(0)|^2$ at $\mathcal{R} = 1.0$, $T = 1$, maximized with respect to wavelength.

IV. THREE DIMENSIONAL EFFECTS

By solving the integral equation that results on axis with $\vec{r} = 0$ and $a_\delta(\tau) = a(\tau)$ the three dimensional effects can be subsequently generated

$$a_r(\tau) = a_0 r(\tau) + \frac{ij}{2} \int_0^\tau dr_1 a(r_1) h(\tau - r_1),$$

$$\begin{aligned}
h(x) &= \int_0^x ds g_T(x, s) \frac{C_\alpha C_\beta \exp[-(s/T)^2 - i\nu_0 s]}{(C_\alpha + is/T)(C_\beta + is/T)}, \\
g_T(\tau, \tau_3) &= \frac{\exp\left[-\frac{k_x^2}{2L_w} \frac{\tau^2}{\tau_r(\tau_3) + i(\tau - \tau_3)}\right]}{[1 + i(\tau - \tau_3)/\tau_r(\tau_3)]}, \\
\tau_r &= \pi x_0^2 / (\lambda_s L_w), \\
T &= 4\pi N_w \gamma / \Delta\gamma.
\end{aligned} \tag{24}$$

This expression is rather remarkable because it combines many effects in the determination of the single kernel function that admits a simple physical interpretation. The effects are (i) diffraction as τ_r (these results have been specialized to cylindrical symmetry here to illustrate the method) and the incident beam parameters, (ii) emittance in two planes as C_α and C_β (or $\mathcal{C}$ and $\mathcal{R}$), (iii) detuning as ν_0 , (iv) energy spread as T , and (v) length or current as j . All these effects simply change the complex influence function $ijh(x)/2$ of the downstream field at a distance x upstream on the field excited there.

The two effective energy distribution shape constants C_α and C_β and the energy spread parameter T have similar effects when it comes to determining the asymptotic dependence of the kernel function. For instance, when $\tau_r \rightarrow \infty$ the kernel appears to similar to the plateau shown in Fig.(5) in the case $(T, \tau_r) = (3, 100)$. There the kernel functions starts off as $x^2/2$ and plateaus to a constant. When energy spread, emittance, and diffraction are not present, the upper curve develops. The addition of either emittance or energy spread causes the integrand in Eq.(24) to cutoff and results in the qualitative change from a parabolic to plateau behavior.

The inclusion of diffraction also makes a qualitative change. In a perfect beam with no emittance, energy spread, or diffraction the parabolic asymptotic behavior is lost to a more linear behavior upon introduction of diffraction. Furthermore, the broadened effective energy distribution case without diffraction is altered by diffraction in the asymptotic lowering of the plateau to a $1/x$ behavior at ∞ .

FIG. 4. Mapping of the magnitude of the influence function $|h(x)|$ for four pairs of values of (T, τ_r) at fixed $C_\alpha = C_\beta = 100, \nu_0 = 0$. Listed by upper to lower curve, they are $(100, 100), (100, 1), (3, 100), (3, 1)$. The upper curve is nearly $x^2/2$; the next is nearly $\tau_r^2 |ix/\tau_r - (1 + ix/\tau_r) \ln(1 + ix/\tau_r)|$. The 3rd curve ignores 3-D effects and plateaus to a constant as $\tau_r \rightarrow \infty$. This same behavior occurs even for finite C_α, C_β . The lowest curve includes 3-D effects and has a maximum at finite x .

The integral equation for $a(\tau)$, Eq.(24), gives a value for $a(\tau)$ that depends on $a_0(\tau)$, the incident wave, and the kernel h . The kernel function $h(\tau - \tau_1)$ is the relative weight coefficient of prior interactions of the field. It gives the ability of the field at location τ_1 to bunch the electron beam, emit the excited wave, and propagate the excited wave to the axial location, τ , under consideration in the wiggler. In short, it is the ability, per electron, of the axial field at τ_1 to produce axial field at τ . The argument of the kernel is the measure of the distance down stream from τ .

Only with the inclusion of diffractive effects, is the downstream influence function ever diminished. This fact has consequences at low to moderate gains. To understand these points, we need to consider a particular form for $a(\tau)$.

At low gain ($j \rightarrow 0$) we can take $a(\tau) = a_0(\tau)$. Without diffraction, a_0 is a constant independent of τ and the upper curve generalized for $\nu_0 \neq 0$ yields the usual⁴ maximum over ν_0 of $G_{\nu_0} = \Re\{ij \int_0^\tau ds (\tau - s)^2 / 2 \exp(i\nu_0 s)\} = 0.135j\tau^3$; i.e. $G_{max} = |a(1)/a(0)|_{max}^2 - 1 \approx 0.135j$. With energy spread and/or emittance, it starts off as τ^3 but asymptotes to an unbounded τ proportionality when $\nu_0 = 0$. For long amplifiers, because of the finite area under the $a \times h$ curve, addition of diffractive effects contributes to an upper bound for the gain no matter how long the system.

In Fig. (5) we show a calculation using the full off-axis diffractive solution. Here in the vicinity of $\tau = 0.3$ the optical beam necks down on the electron beam. Beyond this point, exponential gain occurs but the assumptions of the theory do not strictly hold. These calculations are simple to do and involve only a fraction of a second on a Cray making parameter searches quite feasible.

FIG. 5. The log of the intensity is shown at the five radial lanes: $(r/z_0)^2 = 0.0, 0.5, 2.0, 3.5, 5.0$. The incident filling factor, $(w_0/z_0)^2$, is 0.2 and the rayleigh range of the incident optical field is 4 wiggler lengths. The incident wave is centered on the middle of the wiggler. The electron distribution function is taken in the gaussian limit with $C_\alpha = C_\beta = 10^9$. The pulse is taken on resonance with $\nu_0 = 0$ and the scaled current j is 1000

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