ANNUAL PROGRESS REPORT

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ABSTRACT

A number of topics in nonlinear and linear instability theory are covered in this report. The nonlinear saturation of the dissipative trapped electron instability is evaluated and its amplitude compares well with existing experimental observations. The nonlinear saturation of the drift cyclotron loss-cone mode is carried out for a variety of empty loss-cone distributions. The saturation amplitude is predicted to be small and stable. An improved linear theory of the collisionless drift instability in sheared magnetic fields yields the surprising result that no instability occurs for a wide range of parameters. Finally, the bump-on-tail calculation is shown to be unchanged by some recent results of Case and Siewart, and a rough time scale is established for the transition from the O'Neil trapping regime to the final time-asymptotic result.
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INTRODUCTION

The past year's research, under this contract, covered a number of topics in nonlinear and linear instability theory.

The calculation of the nonlinear saturation of the dissipative trapped-electron instability was completed, submitted to Physics of Fluids and has been accepted for publication. The calculated saturation amplitude is in good agreement with experimental observations in a linear mirror geometry carried out at Columbia University. This is for a model in which resonant circulating electron effects are absent, as would be expected in a mirror machine. The theory predicts a much smaller saturation amplitude when resonant effects of circulating electrons are present, as would be the case for a toroidal device.

Calculations of the nonlinear saturation of the drift cyclotron loss-cone mode are well along. Stable small-amplitude saturation is predicted for a variety of empty loss-cone distributions. Extension of the model to more realistic mirror geometries, including end losses, is now under way.

A linear calculation of the collisionless drift wave in sheared magnetic fields, using both an improved eigenvalue equation and a numerical solution of the relevant differential equation, established the surprising result that there was no instability at all for a wide range of parameters. This result, also found by the Texas group, and since confirmed by a number of other investigators, was one of the more interesting results in the plasma theory field this year.

Finally, some calculations relevant to the bump-on-tail nonlinear asymptotic theory are reported. In one, it is shown that an additional linear solution, occurring when there is a discrete eigenmode embedded in the real axis, does not change any of the previously reported results. In addition, an estimate
has been obtained for the time-scale necessary to reach the asymptotic solution. This will help in the design of computer experiments which might show the transition from the O'Neil trapping limit to the final asymptotic solution.

1. Nonlinear Saturation of the Dissipative Trapped Electron Instability

This work, which was described at some length in the previous Annual Report\textsuperscript{1}, was completed during this year. Previous experiments at Columbia University\textsuperscript{2} had established the existence of the trapped electron instability in linear mirror geometry. A stability threshold was observed and measurements were made of the saturated electrostatic field amplitude as a function of the electron collision frequency.

We calculated the time-asymptotic saturation amplitude of the dissipative trapped electron instability for threshold conditions with parameters chosen to resemble those of the experiments in Ref. 2. The square amplitude of the oscillating potential was predicted as a function of the small increase in the electron effective collision frequency above its critical onset value. The results were in good agreement with the experimental observations for a model in which the “circulating” electrons are either represented by a Boltzmann response or by a drift-kinetic equation with neglect of Landau damping. Inclusion of Landau damping results in a saturation amplitude which is much smaller than seen in the experiment. A model with Landau damping would not be expected to be appropriate in the linear mirror geometry, but could be relevant in closed geometries. Density and temperature flattening of the trapped electron profile are the principal nonlinear saturation mechanisms.

Details may be found in Stefano Migliuolo's Ph.D. Thesis\textsuperscript{3}. A condensed version has been accepted by the Physics of Fluids\textsuperscript{4}. 

2. Nonlinear Saturation of the Drift Cyclotron Loss-Cone Instability

Earlier experiments on mirror machines (1973 - 1975), particularly on PR-6\textsuperscript{5}, PR-7\textsuperscript{6}, and 2XII\textsuperscript{7}, demonstrated the onset of the drift cyclotron loss-cone instability (DCLC)\textsuperscript{8}. In these observations, there was a threshold regime, near onset, where the electrostatic oscillation spectrum was very narrow. A particularly interesting set of observations on PR-6\textsuperscript{9} showed a nearly "universal" relation between the time-averaged plasma potential and the oscillation amplitude. Observations like these, in the threshold regime, may be amenable to fairly accurate theoretical interpretation using Boguliubov-type single-mode threshold theories.

We have been carrying out a series of calculations of this type, starting with extremely simple models of the equilibrium and progressing to more realistic models. During this year, we have used a model in which the equilibrium plasma varied only in the directions perpendicular to the magnetic field and with the particles having various empty loss-cone distributions. The plasma perturbations were treated in the local approximation and also did not vary in the field direction. For each of these velocity distributions, linear theory was used to derive stability boundaries in $\epsilon, n$ space, where $\epsilon$ represents the reciprocal of the density e-folding length and $n$ is the central density. Nonlinear calculations using methods similar to those of Simon and Rosenbluth\textsuperscript{10} then yielded analytic formulas for the time-averaged modifications to the distribution function, for the time-averaged potential, and finally for the squared amplitude of the time-asymptotic oscillating electrostatic potential. The formulas were then integrated and summed numerically to yield values of the saturation amplitude in the vicinity of any given point on the stability boundary. It is interesting to note that the nonlinear frequency shift plays no role in this
particular calculation, and in fact is indeterminate.

The analysis and numerical calculations were carried out for the following empty loss-cone distributions:

(a) \( \delta(v_1 - v_{10}) \)

(b) \( \frac{1}{v_{10}} \exp\left(-\left(v_1 - v_{10}\right)^2/v_{10}^2\alpha^2\right) \quad (\alpha \ll 1) \)

(c) \( \frac{1}{v_H - v_H^2} \left(e^{-v_H^2/v_H^2} - e^{-v_L^2/v_H^2}\right) \)

The distribution of type (c) is that proposed by Tang, Pearlstein and Berk\textsuperscript{11}.

In all cases, the absolute square of the saturation amplitude was found to be small near the onset-point. Further, the saturated state is stable and thus onset is predicted to be of the "soft" variety. The saturation mechanism is nonlinear broadening of the velocity spectrum in all cases.

While there is a relationship between the nonlinear potential and the oscillation amplitude, it is not universal and has the wrong sign. This is no surprise, since z-variation and end-loss of electrons play a key role in actual mirror devices. Work just beginning will include these essential features, including nonlocal radial variation. Some details of the results mentioned above will be presented at Colorado Springs (Nov. 1978).

3. Stability of the Collisionless "Instability"

Previous investigations of the collisionless drift wave in a sheared magnetic field employed perturbation theory\textsuperscript{12} or approximate numerical solutions\textsuperscript{13} of the perturbation-theory solution near marginal stability\textsuperscript{14}. Recently Catto and Tsang\textsuperscript{15} obtained an improved eigenvalue equation for all even and odd radial eigenmodes and were able to find the limit in which the
perturbation-theory results could be recovered from more exact expressions valid for arbitrary growth rates. There emerged the possibility that the perturbation-theory form of the dispersion relation is inadequate because it can only be recovered in the limit in which small corrections are important.

We have compared the improved analytic result of Refs. 14 and 15 with the perturbation form and with a numerical solution of the relevant differential equation. The agreement between the improved analytic result and the numerical solution was found to be remarkably good. To our surprise, in contrast to the perturbation form, both the analytic result and the numerical solution show no instability at all for a wide range of parameters.

Details showing the result of numerically solving the eigenmode equation for the numerical drift, including retaining the full ion $z$-function, may be found in the Physical Review Letter of Tsang, Catto, Whitson, and Smith. In this same publication, comparison is made with the eigenvalues obtained from an equation obtained by solving the eigenmode equation by the method of matched asymptotic expansions. The agreement is remarkable. Similar results have been obtained by Ross and Mahajan.

Both methods predict stability for the ratio of the shear length to the density length, $L_S/L_N$, varying from 8 to 32 and $k_\rho$ from 0 to 1.25. It should be noted that in the more strongly damped region, the perturbation theory agrees quite well with the numerical result. In conclusion, the reader should be cautioned that stability in the slab limit treated here does not imply that stability will persist when trapped electrons and other toroidal effects such as ion drift are retained.
4. Influence of the Siewart-Case Solutions on the Bump-on-Tail Result

Siewart\(^{18}\) and Case\(^{19}\) have shown that when there is a discrete mode embedded in the real axis, an extra set of solutions of the linearized Vlasov equations exist, and are necessary for completeness. We have reviewed our previous calculation of the bump-on-tail nonlinear saturation amplitude\(^{10}\) in the light of these results and found that no modification of the results are necessary.

The reasons are straightforward. In Ref. 10, the result was arrived at by two methods. The more rigorous argument was presented in Appendix A. An expansion of the final state is made around an equilibrium having two slightly complex discrete roots. In this case, it is obvious that no change occurs. In the main body derivation, the additional solution which should be considered is shown to correspond to a second continuum and hence to lead to damped behavior in time. Hence this approach, too, remains the same.

Some additional details will be presented at the Colorado Springs Meeting.

5. O'Neil Solutions and the Time Scale for Asymptotic Behavior

Work was begun on extending the theory derived in Ref. 10 to the classic O'Neil\(^{20}\) problem of a finite electrostatic wave damping in a Maxwellian plasma. It is of interest to see how the time-asymptotic solution differs from the particle-trapping results. The related problem of the growth of a single mode on a bump-on-tail distribution was treated (from initial values) by O'Neil, Winfrey and Malmberg\(^{21}\) and differs in essential ways from the results in Ref. 10. If both results are correct, the system must evolve from the O'Neil trapping solution, which sets in on a time-scale of order \(\tau, [\tau = (m/eE_k)^{1/2}]\), into the
asymptotic solution on some longer, but as yet unknown, time scale. Preliminary work has given some indication of the required time for this transition. It is considerably longer than any of the computer simulations or experimental runs, to date. We are attempting to improve the derivation of this time scale and then to specify the parameters for an optimum computer demonstration.
6. Personnel

FACULTY —

(a) Albert Simon (MAS and Physics)—Dr. Simon has devoted 20% of his time to this project during the academic year, and full time for two summer months. He supervised three graduate students, Stefano Migliuolo, Richard L. Myer and Randy Van Vranken.

(b) Peter J. Catto (MAS Department)—Dr. Catto devoted 20% of his time to this project during the past academic year. He supervised one graduate student, Tom Speziale, who has received his Ph.D. degree and is now working at the University of Arizona. It is with regret that I report that Dr. Catto resigned from the University of Rochester effective May 31, 1978. He will be devoting himself to research, full-time, at Science Applications Inc., Boulder, Colorado.

GRADUATE RESEARCH ASSISTANTS —

(a) Richard L. Myer (MAS Department)—Mr. Myer is now in his fourth year of graduate study and is well along on a thesis concerning the nonlinear saturation of the drift cyclotron loss-cone instability.

(b) Stefano Migliuolo (Physics Department)—Dr. Migliuolo successfully defended his dissertation on Dec. 7, 1977. He is now employed on the fusion project at M.I.T. and reports to Professor Bruno Coppi.

(c) Randy Van Vranken—Mr. Van Vranken completed his written preliminary examinations, and worked on some problems related to the bump-on-tail instability during the summer of 1978. He will be resuming his graduate studies in January 1979, after a leave during the Fall semester.
Other Federal Support None of the personnel working on this contract have other federal support.
REFERENCES


### A. WORK COMPLETED PREVIOUSLY, BUT PUBLISHED THIS YEAR

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<th>Authors</th>
<th>Journal/Status</th>
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<tr>
<td>Nonlinear Saturation of Two Unstable Modes: Survival Competition</td>
<td>A. Simon and S. Migliuolo</td>
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<tr>
<td>Collisional Effects on Trapped Electron Instabilities</td>
<td>K. T. Tsang, J. D. Callen and P. J. Catto</td>
<td>Phys. Fluids 20, 2113 (1977)</td>
<td>100%</td>
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<td>Adiabatic Modifications to Plasma Turbulence Theories</td>
<td>P. J. Catto</td>
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### B. WORK COMPLETED THIS YEAR, OR CONTINUING

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<td>Nonlinear Saturation of the Dissipative Trapped Electron Instability</td>
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