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Examination of Yield Determination by the Magnetic Bubble Effect

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Examination of Yield Determination by the Magnetic Bubble Effect

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1. Overview

This report surveys the preliminary work of P. J. Ebert, a proponent of the "Magnetic Bubble" Effect, as well as conclusions reached by L. F. Wouters and others. The idea behind the Magnetic Bubble Effect is that an exploding fireball from a surface or subsurface nuclear explosion releases electrons which circulate around the earth's magnetic field lines and create an expanding magnetic dipole which persists underground for ≈ 0.01 sec. This dipole radiates a low-frequency electromagnetic field through the earth in a diffusive manner because the earth's conduction current overwhelms the displacement current at VLF and ELF frequencies. By using this concept of diffusive fields it might be possible to estimate the yield of an underground nuclear test from transient low-frequency field measurements on or near the ground in the range 1 – 5 km from ground zero.

We closely examine the diffusive-field description of the transient fields by L. W. Miller (1973). Miller's work enables us to derive rather easily the "Green-function" fields in space and

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time of a spatially and temporally impulsive vertical magnetic dipole, m_z , and horizontal dipole, m_x , in a homogeneous earth of scalar electrical conductivity σ , magnetic permeability μ_0 , and relative dielectric constant, ϵ_r , and account for the earth-air interface. The fields of an event would be the convolution of these Green-function fields and the temporally evolving m_x - and m_z -dipoles integrated over space. The spectra of the Green-function fields are readily calculated, and multiplied by the magnetic-dipole spectrum to give the event-field spectra.

From Miller's formulas we estimate the quantitative fields to be expected in situations discussed by Ebert and Wouters. Then we estimate the early peak fields measured in several events reported by Zablocki, who, in the writer's opinion, made a qualitatively incorrect analysis. We comment on the rather brief report of Malik, Fitzhugh, and Homuth, who dismiss the Magnetic Bubble source in favor of the electroseismic effect based on analysis of one event. Our estimate of $(E_\phi)_{\max}$ indicates that the Magnetic Bubble would account for features seen in the field data. We next review measurements of ELF fields from underground events at NTS by Sweeney (1989). Comparison between magnitude estimates and measurements on three events is inconclusive, but does not rule out the Magnetic Bubble as the primary field source.

A review of data summarized by Wouters (1980) indicates that estimates of fields differed in four measurements by factors of 10 – 100. Perhaps the inclusion of ground surface reflections would improve these estimates.

The conclusions from this study are as follows. We have compared *measured* early-time peak surface fields in various underground nuclear events with *predictions* from Miller's infinite-earth field expressions using Wouter's expression for the maximum magnetic dipole moment (with $W^{3/4}$ changed to W , yield) and its temporal history. Those comparisons sometimes differ within an order of magnitude, and sometimes by more, of measurements reported by Zablocki, Sweeney, and Malik et. al., for reasons not fully understood. Miller's expressions allow us to correct for the earth-air interface; use of these more accurate field formulas might result in better agreement with

measurements. Past instrumentation has sometimes lacked the required bandwidth, which can be estimated from the event-dipole time history and the spectra of Miller's Green-function fields.

The recommendations from this study are as follows. For an advanced concept project we propose field experiments, coupled with modeling, to (a) confirm the diffusion theory, (b) optimize and calibrate the measurement systems, and (c) understand better the nature of the EM source fields of an underground event. As an initial study to achieve goals (a) and (b) we propose to carry out a series of experiments at NTS with a downhole controlled source to simulate the magnetic-dipole source of an underground explosion. We will use surface sensors to measure the fields produced by the downhole source. Depending on the design criteria, we might also be able to create a downhole or surface-horizontal electric dipole source and study its fields with suitably modified diffusive-field theory. This will allow us to develop ways to better decouple the effects of multiple sources.

The diffusion theory will be used to design the experiment and resolve such details as the necessary distances and orientation of the sensors and required bandpass of the measurement system. We will locate the magnetic dipole source at a depth of a few tens of meters in a borehole which has been logged so that the medium characteristics are well known. In this way we can see if the measurements made in the NTS EM environment confirm the theoretical predictions.

Once we have completed the initial calibration and field testing of the EM measurement system, we will be ready to field the system during underground tests. Sensors will be deployed at distances less than 5 km from GZ, and will be designed to decouple optimally the fields created by magnetic or electric dipoles. The data will then be used to predict yield from EM measurements. Based on the results of one or two tests, provided that the data collection is successful, we will be in a position to recommend whether the technique is successful enough to warrant further study as an alternative system for yield measurement.

The personnel and funding requirements are stated at the end of this report.

2. Work of P. J. Ebert

In 1989 Ebert strongly advocated the "Magnetic Bubble" effect as a means of non-intrusive verification of underground nuclear yield [1]. We summarize his salient results (in rationalized MKS units) below.

Following Raizer's theoretical approach [2],

$$r_c(\text{meter}) = 17 W_{\text{kTon}}^{1/3}, \quad r_c = \text{final cavity radius; } W, \text{ yield} \quad (1)$$

$$V_c(\text{meter}^3) = \frac{4}{3} \pi r_c^3 = 2.06 \times 10^4 W, \quad V_c = \text{cavity volume} \quad (2)$$

$$E_m(\text{Joule}) = (B_o^2/2\mu) V_c, \quad E_m = \text{energy magnetic in the cavity} \quad (3)$$

$$B_o = \text{earth's field} = 0.5 \times 10^{-4} \frac{\text{web}}{\text{m}^2}$$

$$\mu = 4\pi \times 10^{-7}$$

$$\text{The Bubble is produced in } \Delta t \simeq 0.01 \text{ sec (CORRTEX measurements).} \quad (4)$$

$$M, \text{ magnetic dipole moment} = \frac{2\pi B_o}{\mu} r_c^3 \text{ Amp m}^2 \quad (5)$$

(Found by substituting $B_o/\mu = H_o = I_o/2r_c$, $I_o(\text{Amp})$ being the ring current circling the fireball, into $M = I_o \pi r_c^2$)

Then ΔB , change in B a distance R away is

$$\Delta B = B_o r_c^3 e^{-kR} \sin\beta/2R^3 \quad \left(\frac{\mu M}{2\pi} = B_o r_c^3 \right) \quad (6)$$

where k , attenuation $= \sqrt{\omega\mu\sigma/2}$, (7)

in which σ , earth conductivity in mho/m, and ω is the primary event frequency, assumed by Ebert to be $(2\pi/\Delta t)$.. (8)

Then the peak EMF (volt) induced in a coil at R is estimated as

$$\text{EMF (volt)} \simeq AN (\Delta B/\Delta t) \quad (9)$$

where A = coil area (m^2), N = number of turns.

For the following data: $W = 1 \text{ kTon}$, $r_c = 17 \text{ m}$, $E_m = 20 \text{ J}$, $\Delta t = 0.01 \text{ s}$., $k = 0.004 \text{ m}^{-1}$ (NTS), $\sin\beta = 0.8$, $R = 500 \text{ m}$, $A = 10^2 \text{ m}^2$, $N = 10^2$ we obtain

$$\text{EMF} = 1.1 \times 10^{-4} \text{ volt} \quad (10)$$

(This agrees closely with an estimate from Wouter's work to follow,

$$\text{EMF} \cong NAB = NA\mu_o\omega_o H_o r_c^3 \frac{e^{-\beta l}}{l^3} \sqrt{\cos^2\psi + \sin^2\psi/4} \approx 1.4 \times 10^{-4} \quad (10')$$

Ebert deduces from an error analysis that the principal errors are in attenuation k and in a factor K defined by $r_c^3 = KW$ (K is 17^3 in (1)).

The advantages claimed by Ebert for this method of yield measurement are: (1) non-intrusive, (2) VLF frequencies predominate in the range 3 – 30 kHz, requiring no high frequency rf instrumentation, (3) no access to emplacement hole is required, (4) no zero-time signal is needed, (5) a nearby satellite hole is unnecessary, (6) it is less costly than hydrodynamic-yield (H-Y) techniques, (7) it is potentially more accurate than H-Y techniques, [one reason being it measures W not $W^{1/3}$], (8) one can detect seismic decoupling, (9) it is insensitive to small anisotropies/inhomogeneities, (10) it is insensitive to test instrumentation downhole because of its small mass relative to the earth mass displaced, and (11) there is a potential for measuring yields down to 1 kTon.

The disadvantages, according to Ebert, are: (1) high- σ attenuates the signals more strongly, (2) high- μ volumes would upset measurements, (3) the rapid decrease of cavity σ toward the end of the fireball, (4) unusual propagation paths would upset the R^{-3} dependence of the pickup-coil EMF.

Ebert says that ~4 kHz magnetic dipole transmission was successfully demonstrated at NTS in 1966 but that neither this demonstration nor a later one of Malik, et-al [3] contains data amenable to quantitative interpretation. We shall see that the magnetic dipole frequency spectrum is considerably lower than 4 kHz.

To conclude this summary of the earliest features of Ebert's work in 1989 we quote a table of EMF (μV) vs W (kTon) following the analysis of (1) – (9) except that R is chosen to be $244 W^{1/3}$ m for $\beta = 60^\circ$, $\Delta t = 0.6 \times 10^{-3} W^{1/3}$ and $r_c = 4.93 W^{1/3}$ (both from a formalism developed by Moss [4] for determining hydrodynamic yield from particle velocity together with the empirical strong shock relation $r_c/W^{1/3} = 6.29 \times (\Delta t_{msec}/W^{1/3})^{0.475}$, valid up to a scaled time $\Delta t = 0.6 \times 10^{-3} W^{1/3}$ s, assumed by Ebert to be the Bubble lifetime) and k of (7) obtained with $\omega = 2\pi f_c = 2\pi/\Delta t$. Eqs. (6) and (10') then give $EMF = AN\dot{B} = 3. \times 10^{-7} AN e^{-kR} W^{1/3}$, $kR = 2.0 W^{1/6}$.

For $AN = 10^4 \text{m}^2$ Ebert obtains

W (KTon)	R (m)	EMF (μV)	Bevensee (check)
8	488.	92.	88.
27	732.	32.	(not computed)
64	976.	14.	(not computed)
125	1220.	7.	6.8

The Δt Bubble lifetime just assumed is considerably shorter than Wouter's value.

3. Work of L. F. Wouters

In 1989 Wouters examined four representative models of the underground electromagnetic pulse (EMP) from a fireball, including the diamagnetic plasma ("Magnetic Bubble") [5]. We now summarize his computations of the $\dot{B}$ (weber/(m^2sec)) — field generated by the "Bubble".

Wouters assumes a somewhat different dependence of final cavity radius r_c relative to yield W , namely

$$r_c = R_o (\rho d_o)^{-1/4} [W (\text{kTon}) \times 10^3]^{1/4} \approx 32 \text{ meter} \quad (11)$$

where $R_o = 70 \text{ m}$, ρ , earth density = $1.8 \text{ g/cm}^3 = 1.8 \times 10^3 \text{ kg/m}^3$, $d_o = 125 \text{ m}$, relative to the MERLIN data ($W = 10. \text{ kTon}$).

The maximum magnetic dipole moment M from (5) is

$$\begin{aligned}
M &\simeq 2\pi \frac{B_o}{\mu} r_c^3 = 2\pi \frac{B_o}{\mu} [R_o (\rho d_o)^{-1/4}]^3 W(\text{kTon})^{1/4} \times 10^{3/4} \\
&\simeq 1.48 \times 10^6 W(\text{kTon})^{3/4}
\end{aligned} \tag{12}$$

which checks closely his Fig. 4-4 for M vs $W(\text{kTon})$. He takes

$$\Delta H_{\text{peak}} = h_s \sqrt{\cos^2 \psi + \sin^2 \psi/4} \quad , \quad h_s = H_o r_c^3 e^{-\beta l} / l^3 \tag{13}$$

where ψ is the (azimuthal) angle relative to the earth's $\bar{H}_o$ -vector, $\beta = \sqrt{\omega \mu \sigma / 2} \text{ m}^{-1}$, and l , meter, is the slant distance to the detector. Eq. (13) is essentially (6) without a factor 2 in the denominator.

Then $\dot{B} = dB/dt$ at the detector (receiver coil is taken

$$\dot{B} = \mu_o \omega_o \Delta H \quad , \tag{14}$$

where $\omega_o = 2\pi(10 \text{ Hz}) \simeq 63$ is the frequency of a half sinusoid (15)

fitted to the cavity growth-collapse history.

Combining (11) and (13),

$$\dot{B} \left(\frac{\text{web}}{\text{m}^2 \text{sec}} \right) = \mu_o \omega_o H_o r_c^3 (e^{-\beta l} / l^3) \sqrt{\cos^2 \psi + \sin^2 \psi/4} \tag{16}$$

we find for $r_c = 32 \text{ m}$, $l = 282 \text{ m}$, $e^{-\beta l} = 0.9$, $\sqrt{\cos^2 \psi} \dots \simeq 0.8$,

$$\dot{B} \simeq 3.3 \mu\text{V/m}^2 \quad , \quad \left(B \cong \frac{\dot{B}}{\omega_o} = 52 \text{ nT} \right) \tag{17}$$

which is close to the mean value in Wouter's Table 4.2.

Eq. (17) represents a large value of field, for the (peak) EMF induced in a coil of $NA = 10^4$ would be 33. mVolt. It is larger because the slant distance l to the receiver is only 282 m. If l

were 5 km $e^{-\beta l/l^3}$ would decrease from $0.9/(282)^3 = 4.0 \times 10^{-8}$ to $(.9)^{17.7}/(5 \times 10^3)^3 = 1.2 \times 10^{-30}$ (!). We shall see that this gives far too pessimistic a value for $\dot{B}$.

4. Work of L. W. Miller

In 1973 Miller in a brilliant report [6] analyzed the radiation from a point magnetic dipole impulsive in time through an earth of high electrical conductivity and obtained expressions for the *diffusive* fields in space-time. He also showed how to treat reflection and transmission at the earth's surface, which depends on the orientation of both the earth's magnetic field and the source dipole. Below we will convert his equations to rationalized MKS form, apply them to the vertical component of the Magnetic Bubble dipole, neglect reflection from the earth's surface, and predict the magnitude of the EMF in a pickup coil 1 – 5 km from the fireball.

4.1 Equations

Miller starts with Maxwell's equations in CGS units for a medium of uniform conductivity σ and relative dielectric constant ϵ_r (his ϵ),

$$\nabla \times \bar{E}(\bar{r},t) + \frac{1}{c} \frac{\partial \bar{B}}{\partial t}(\bar{r},t) = 0, \quad \nabla \cdot \bar{B} = 0 \quad (18)$$

$$\nabla \times \bar{B} - \frac{\epsilon_r}{c} \frac{\partial \bar{E}}{\partial t} = \frac{4\pi}{c} (\sigma \bar{E} + \bar{J}_o(\bar{r},t)), \quad \nabla \cdot \bar{E} = 4\pi\rho(\bar{r},t) \quad (19)$$

Here c = light velocity in vacuo, 3×10^{10} cm/s, and charge density ρ is *defined* by the second of (19).

Since $\bar{J}_o$ of the fireball is assumed to be solenoidal ($\nabla \cdot \bar{J}_o = 0$) it is written as

$$\bar{J}_o = c \nabla \times \bar{M}(\bar{r},t) \quad , \quad \bar{M} = \bar{m} \delta(\bar{r}) \delta(t) \quad , \quad (20)$$

which introduces magnetic dipole moment $\bar{M}$ as a spatial-temporal impulse.

$$\text{Writing } \bar{B} = \bar{H} + 4\pi\bar{M} \quad (21)$$

we obtain Maxwell's equations [6, (18) – (19)] in the forms

$$\nabla \times \vec{E} + \frac{1}{c} \dot{\vec{H}} = -\frac{4\pi}{c} \vec{M} \quad (22)$$

$$\nabla \times \dot{\vec{H}} - \frac{\epsilon_r}{c} \ddot{\vec{E}} - \frac{4\pi\sigma}{c} \vec{E} = 0 \quad (23)$$

These are solved within an infinite medium via a Hertz vector $\vec{\Pi}(\vec{r}, t)$. $\vec{\Pi}$ must be integrated over circular frequency ω , where each ω -component propagates as $e^{i\Gamma r}$, Γ complex ($e^{-i\omega t}$ time dependence). Because $\omega \epsilon_r \ll 4\pi\sigma$ (CGS units) at all frequencies of interest, which implies negligible displacement current compared to conduction current, the evaluation of $\vec{\Pi}(\vec{r}, t)$ by integration in the complex $-\omega$ plane simplifies at all points r such that

$$r \gg q \text{ defined as } \frac{c}{4\pi\sigma} \text{ (CGS)} \quad \text{or } r \gg \frac{\sqrt{\epsilon_r/\mu}}{\sigma} \text{ (MKS)}. \quad (24)$$

Note that for $\sigma = 10^{-2}$ mho/m or $9 \times 10^7 \text{ sec}^{-1}$ CGS q is only 26 cm.; even for $\sigma = 10^{-4}$ q is only 26 m.

Miller's final expression for the diffusive fields in CGS units in spherical coordinates, due

to $\vec{m} = \bar{a}_z m_z \delta(t)$, at all r satisfying (24) are (note $\vec{m} = \int \vec{M} dV$)

$$h_r(r, \theta, t) = \frac{16m_z qc}{\sqrt{\pi}} \frac{\exp(-r^2/4qct)}{(4qct)^{5/2}} \cos\theta \quad (25)$$

$$h_\theta(r, \theta, t) = -\frac{16m_z qc}{\sqrt{\pi}} \left(1 - \frac{r^2}{4qct}\right) \frac{\exp(-r^2/4qct)}{(4qct)^{5/2}} \sin\theta \quad (26)$$

$$e_\phi(r, \theta, t) = \frac{80m_z q^2 c}{\sqrt{\pi}} \left(1 - \frac{2}{5} \frac{r^2}{4qct}\right) \frac{r \exp(-r^2/4qct)}{(4qct)^{7/2}} \sin\theta \quad (27)$$

These satisfy (22), (23). We have replaced Miller's capital letters by small letters e and h to denote Green-function fields from an impulsive $\delta(t)$ - source. The fields are conveniently written in terms of a characteristic time t_r which depends on r,

$$t_r = r^2/4qc \quad (28)$$

In terms of a normalized time $T = t/t_r$ (25) - (27) are

$$h_r(r, \theta, T) = \frac{16m_z qc}{\sqrt{\pi} r^5} [T^{-5/2} e^{-1/T}] \cos \theta \quad (29)$$

$$h_\theta(r, \theta, T) = \frac{16m_z qc}{\sqrt{\pi} r^5} \left[\left(1 - \frac{1}{T}\right) T^{-5/2} e^{-1/T} \right] \sin \theta \quad (30)$$

$$e_\phi(r, \theta, T) = \frac{80m_z qc}{\sqrt{\pi} r^6} \left[\left(1 - \frac{2}{5} \frac{1}{T}\right) T^{-7/2} e^{-1/T} \right] \sin \theta \quad (31)$$

Miller has plots [6] of the behaviors of these bracketed quantities; each one reaches a peak value of nearly 1 or ± 2 in about $T = t/t_r \simeq 1/3$ or $1/4$. Note that the fields decay as r^{-5} or r^{-6} !

We now want to convert (25) – (31) to (rationalized) MKS form. One straightforward way to do this is to rearrange the terms of the starting Eq. (18) – (23) so they are in MKS form. If certain groupings of CGS terms look like MKS terms in these equations of *MKS* form it follows that the same groupings can be replaced by their MKS terms in the answers. This is a way to convert CGS *functions* to MKS functions, *independent* of the *numerical* relations between CGS and MKS quantities.

The CGS groupings and MKS correspondences are

$$\begin{aligned}
E_{\text{MKS}} &= \sqrt{\mu_0/\epsilon_0} E_{\text{CGS}}, & H_{\text{MKS}} &= H_{\text{CGS}}, & \mu_0 \epsilon_0 &\text{ are MKS, } \epsilon_r \text{ remains } \epsilon_r \\
\sigma_{\text{MKS}} &= \epsilon_0 4\pi\sigma_{\text{CGS}}, & B_{\text{MKS}} &= \mu_0 B_{\text{CGS}} \\
J_{\text{MKS}} &= \frac{4\pi}{c} J_{\text{CGS}}, & M_{\text{MKS}} &= 4\pi M_{\text{CGS}}, & c &= \frac{1}{\sqrt{\mu_0\epsilon_0}} = 3 \times 10^8 \frac{\text{m}}{\text{s}}
\end{aligned} \tag{32}$$

$\bar{M}$ was defined relative to $\bar{J}_0$ in (20). The relation $\bar{J}_0 = c\nabla \times \bar{M}$ in CGS and $\bar{J}_0 = \nabla \times \bar{M}$ amp/m²

in MKS imply the conversion factor for $m = \int M dv$ of

$$\left(\frac{1 \text{ m}_{\text{MKS}}}{10^3 \text{ m}_{\text{CGS}}} \right)$$

i.e. $10^3 \text{ m}_{\text{CGS}} \text{ units} = 1 \text{ m}_{\text{MKS}} \text{ unit}$.

Now the conditions (24) for applicability of the diffusive-field expressions becomes

$$r \gg q_{\text{MKS}} = \frac{c\epsilon_0}{\sigma_{\text{MKS}}} = \frac{2.65 \times 10^{-3}}{\sigma_{\text{MKS}}} \text{ meter} \tag{33}$$

The CGS fields of (25) – (27) converted to MKS units are

$$h_r(r, \theta, t) = \frac{4m_z}{\pi\sqrt{\pi}} \frac{1}{\sigma\mu_0} \frac{\exp(-r^2\mu_0\sigma/4t)}{(4t/\mu_0\sigma)^{5/2}} \cos\theta \left(\frac{\text{amp}}{\text{m}\cdot\text{sec}} \right) \tag{34}$$

$$h_\theta(r, \theta, t) = -\frac{4m_z}{\pi\sqrt{\pi}} \frac{1}{(\sigma\mu_0)} \left(1 - \frac{r^2\mu_0\sigma}{4t} \right) \frac{\exp(-r^2\mu_0\sigma/4t)}{(4t/\mu_0\sigma)^{5/2}} \sin\theta \left(\frac{\text{amp}}{\text{m}\cdot\text{sec}} \right) \tag{35}$$

$$e_\phi(r, \theta, t) = -\frac{20m_z}{\pi\sqrt{\pi}} \frac{1}{(\sigma\mu_0)} \frac{1}{\sigma} \left(1 - \frac{2}{5} \frac{r^2\mu_0\sigma}{4t} \right) \frac{r \exp(-r^2\mu_0\sigma/4t)}{(4t/\mu_0\sigma)^{7/2}} \sin\theta \left(\frac{\text{volt}}{\text{m}\cdot\text{sec}} \right) \tag{36}$$

with m_z of units amp.m². (The coefficients are checked following (79)).

4.2 The Event Fields as Convolutions of (34) – (36) and the Magnetic Dipole Time History

The dimensions of (34) - (36) remind us that these are the fields of a per-unit-time source $m_z(t)$ at $t = 0$. If $m_z(t)$ has a general time variation for $t \geq 0$ the fields are a convolution of $m_z(t)$ with (34) – (36). For example

$$H_\theta(r, \theta, t)_{\text{net}}, \left(\frac{\text{amp}}{\text{m}}\right) = -\frac{4}{\pi\sqrt{\pi}\sigma\mu_0} \sin\theta \int_0^t m_z(t-t') \left(1 - \frac{r^2\mu_0\sigma}{4t'}\right) \frac{\exp(-r^2\mu_0\sigma/4t')}{\left(\frac{4t'}{\mu_0\sigma}\right)^{5/2}} dt' \quad (37)$$

In terms of the reference time $(t_r)_{\text{MKS}} = r^2\mu_0\sigma/4$ and $T = t/t_r$, (37) is

$$H_\theta(r, \theta, T) = -\frac{4t_r}{\pi\sqrt{\pi}\mu_0\sigma} \frac{\sin\theta}{r^5} \int_0^T m_z(T-T') \left(1 - \frac{1}{T'}\right) \frac{e^{-1/T'}}{(T')^{5/2}} dT' \left(\frac{\text{amp}}{\text{m}}\right) \quad (38)$$

For $r = 1$ km, $\sigma = 0.001$ (low) $t_r = 0.3$ ms; the Bubble lasts many tens of millisec. [5]. H_θ peaks at $T' \approx 1/3$. Hence, to good approximation we can approximate (38) by

$$H_\theta(r, \theta, T) \approx -\frac{4t_r}{\pi\sqrt{\pi}\sigma\mu_0} \frac{\sin\theta}{r^5} m_z\left(T - \frac{1}{3}\right) \underbrace{\int_0^\infty \left(1 - \frac{1}{T'}\right) \frac{e^{-1/T'}}{(T')^{5/2}} dT'}_{\approx 0.7} \quad (39)$$

and $B_\theta = \mu_0 H_\theta$ is nearly

$$B_{\theta}(r, \theta, t) \simeq -\frac{4(0.7)t_r}{\pi\sqrt{\pi}\sigma} \frac{1}{r^5} m_z \left(t - \frac{1}{3}t_r\right) = -\frac{2.8\mu_o}{4\pi\sqrt{\pi}} m_z \left(\frac{\text{web}}{m^2}\right) \quad (40)$$

$$\simeq -\frac{1.6 \times 10^{-7} m_z}{r^3} \quad (\sigma \text{ has cancelled})$$

σ has cancelled only because $t_r = r^2 \mu_o \sigma / 4 \ll \text{duration of the Bubble}$.

Now if we use Wouter's expression (12) for $M_{\max} = (m_z)_{\max}$ (amp m^2) but change the $W^{3/4}$ -dependence to a W -dependence according to Ebert's (5) and use (1) and take $\Delta t = 30$ msec for the nearly-linear rise to the maximum we get

$$(m_z)_{\max} \simeq 1.48 \times 10^6 W \text{ (kTon)} \quad \text{amp} - m^2 \quad (41)$$

and (40) yields

$$B_{\theta}(r, \theta, t)_{\max} \simeq -\frac{0.24}{r^3} W \text{ (kTon)} \frac{\text{Web}}{m^2} \quad (\simeq -(B_z)_{\max}) \quad (42)$$

At 1 km we would expect a peak (B_{θ}) -field of

$$\begin{aligned} |(B_{\theta})_{\max}| \text{ at } r = 1 \text{ km} &\simeq 0.24 \times 10^{-9} W \text{ (kTon)} \text{ web}/m^2 \\ &\simeq 0.24 W \text{ (kTon)} \text{ nT} \end{aligned} \quad (43)$$

For $r = 5$ km, $\sigma = 0.001$, $t_r \simeq 25 \times 0.3 = 7.5$ ms < 30 ms and

$$\begin{aligned} (B_{\theta})_{\max} \text{ at } r = 5 \text{ km} &\simeq 0.24 \left(\frac{1}{5}\right)^3 W \text{ (kTon)} \quad \text{nT} \\ &\simeq 1.9 W \text{ (kTon)} \quad \text{pT} \end{aligned} \quad (44)$$

However if $r = 5$ km, $\sigma = 10^{-2}$, $t_r = 78$ ms > 30 ms and we would have to evaluate (37) more carefully.

Now we assume a "worst case" of $r = 5 \text{ km}$, $\sigma = .04 \text{ mho/m}$. $t_r = 310 \text{ ms} > \text{Bubble duration of } 50 \text{ ms}$ according to Wouter's [6, Fig 4.3; the $m(t)$ -shape is a half-sinusoid, shortened on the decreasing side]. The Green-function $h_\theta(T)$ is shown in Miller's Fig. 3 [6]. Now we must evaluate the integral in (37) more carefully in order to estimate $\dot{H}_\theta(r, \theta, t)_{\max}$. First we take

$\frac{\partial}{\partial t} \int_0^t \dots$. There is no contribution from differentiating the limit because $m_z(0) = 0$.

Differentiation of $m_z(t - t')$ in the integrand yields a $\cos(60 t_{\text{sec}})$ -behavior — see below. From the plot of $h_\theta(T = t/t_r)$ we approximate it by a sinusoid. From (35),

$$\left(1 - \frac{r^2 \mu_o \sigma}{4t'}\right) \frac{\exp(-r^2 \mu_o \sigma / 4t')}{(4t' / \mu_o \sigma)^{5/2}} \approx \frac{1}{r^5} h_\theta(T' = t'/t_r) \approx \frac{1}{r^5} 1.75 \sin\left[\pi \frac{T' - 0.1}{.32}\right] \quad (45)$$

over $0.1 \leq T' \leq 0.42$. We approximate the $m_z(t)$ Bubble history by a sinusoid, as Wouters did [5, Fig. 4.3],

$$m_z(t) = (m_z)_{\max} \sin\left(\pi \frac{t(\text{ms})}{52.4}\right) \quad (46)$$

so $\dot{m}_z(t) = 60 (m_z)_{\max} \cos(60 t_{\text{sec}})$ over $0 \leq t \leq 52.4 \text{ ms}$. Then, using (41), we write (37) with $\sin \theta \approx 1$ as

$$\dot{B}_\theta(r, \theta, t)_{\text{net}} = - \frac{4(60)(1.75)(m_z)_{\max}}{\pi \sqrt{\pi} \sigma} \frac{1}{r^5} \int_{.031}^{t \leq .25 t_r = .077} \cos[60(t - t')] \sin\left[\pi \frac{t' - .1 t_r}{.32 t_r}\right] dt' \quad (47)$$

in which $\sigma = .04$, $r = 5 \times 10^3 \text{ m}$. There is an additional constant of $t - t' < .05 \text{ sec}$, the Bubble duration.

The integral of (47) maximizes at $t \approx .25 t_r$ and has the value $\approx 1.87 \times 10^{-2}$. This makes

$$\dot{B}_0 (r = 5 \text{ km}, \theta = 90^\circ, t)_{\max} = 1.7 \times 10^{-11} \text{ W(kTon)} \frac{\text{T}}{\text{sec}} \quad (48)$$

From an estimate analgous to (47) we would expect a peak B_θ -field of about

$$(B_\theta)_{\max}, r = 5 \text{ km}, \sigma = .04, \simeq 0.5 \text{ W(kTon)} \text{ } \mu\text{T} \text{ ,} \quad (49)$$

due to m_z , neglecting reflection from the earth-air interface. m_x would not contribute to this EMF directly near the interface but it induces an m_z which would contribute, albeit a small amount (see later discussion).

Eq. (49) represents a measurable EMF for $W \geq 10 \text{ kTon}$ for this "worst case"!

The direct, infinite-medium fields due to the m_x -component of the Bubble can often be estimated using the Green-functions (34)-(36) but for a source rotated from alignment with the Z-axis (origin at the fireball) to alignment with the X-axis. The coordinate transformations to the $h_{x,y,z}$ - and $e_{x,y,z}$ -fields are readily performed.

4.3 The Fields Including the Earth-Air Interface

Miller [6] has expressed the Π_x - and Π_z -functions, from which the fields are obtained by differentiation, as double integrals over frequency ω and cylindrical radial propagation constant, α . The m_z -fields are given by three Π_z -functions, Π_{z0} for the source-driven field incident on the interface, Π_{zx} for the field reflected from the interface at $z = h$, and Π_{zt} for the transmitted field. These are formally, in cylindrical coordinates,

$$\Pi_{z0}(\rho, z, t) = \frac{im_{z0}}{2\pi} \int_{-\infty}^{\infty} d\omega e^{-i\omega t} \int_0^{\infty} \alpha d\alpha J_0(\alpha\rho) \frac{e^{i\gamma z}}{\gamma}, \quad z > 0 \text{ .} \quad (50)$$

Here $\gamma = \sqrt{i\omega/qc - \alpha^2}$ CGS, $i = \sqrt{-1}$, and q is defined in (24).

$$\Pi_z(\rho, z, t) = -\frac{im_{zo}}{2\pi} \int_{-}^{+} d\omega e^{-i\omega t} \int_0^{\infty} d\alpha \frac{\alpha}{\gamma} \frac{\beta - \gamma}{\beta + \gamma} J_0(\alpha \rho) e^{-i\gamma z} e^{i2\gamma h}, \quad z \leq h \quad (51)$$

and

$$\Pi_z(\rho, z, t) = \frac{im_{zo}}{2\pi} \int_{-}^{+} d\omega e^{-i\omega t} \int_0^{\infty} d\alpha \frac{2\alpha}{\beta + \gamma} J_0(\alpha \rho) e^{i\beta z} e^{i(\gamma\beta)h}, \quad z \geq h \quad (52)$$

where $\beta = \sqrt{(\omega/c)^2 - \alpha^2}$.

Despite Miller's arguments that $|\beta| \ll |\gamma|$ for all ω except in a narrow range such that $|\omega/qc| \simeq \alpha^2$ — which implies that on the surface $z = h$

$$E_{x,y}^{\text{net}} \simeq 2E_{x,y}^{\text{inc}}, E_z^{\text{net}} \equiv 0, |H_{x,y}^{\text{net}}| \ll |H_{x,y}^{\text{inc}}|, H_z^{\text{net}} \simeq 2H_z^{\text{inc}} \quad (53)$$

it will be "safer" to evaluate the double integrals for Π_z and Π_t over the significant narrow ranges of ω and α and then evaluate the r- and t-fields near the interface, using Π_z and Π_t , respectively and the above, from

$$E_x(m_z) = -\frac{1}{c} \frac{\partial}{\partial t} \frac{\partial \Pi_z}{\partial y}, \quad E_y(m_z) = +\frac{1}{c} \frac{\partial}{\partial t} \frac{\partial \Pi_z}{\partial x}, \quad E_z \equiv 0 \quad (54)$$

$$H_x(m_z) = \frac{\partial}{\partial x} \frac{\partial \Pi_z}{\partial z}, \quad H_y = \frac{\partial}{\partial y} \frac{\partial \Pi_z}{\partial z}, \quad H_z = -\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \Pi_z. \quad (55)$$

Of more interest are the m_x -fields. (At NTS, $m_z = m \cos 30^\circ$) $m_x = + m \sin 30^\circ$). Miller distinguishes two cases: (i) $|\alpha^2| \ll |\omega/qc|$ and (ii) $\alpha^2 \gtrsim |\omega/qc|$. Let us conservatively examine his criteria. First, let $h \geq 100$ m (depth of burial); $\sigma \geq 9 \times 10^6$ esu = 10^{-3} mho/m; then

$$\left(\frac{h}{q}\right)_{\min} = \frac{4\pi}{c} (h\sigma)_{\min} \simeq 40. \text{ (CGS)} \quad (56)$$

Let $\text{Im}(\gamma) \cdot h \leq 5$, the range where the incident fields are significant on the interface. Then

$$\sqrt{\frac{\omega}{qc^2}} h = \frac{h}{q\sqrt{2}} \sqrt{\frac{\omega q}{c}} \leq 5 \text{ and}$$

$$\sqrt{\frac{\omega q}{c}} \leq \frac{5\sqrt{2}}{(h/q)_{\min}} \simeq 0.18, \quad \frac{\omega q}{c} \leq .032 \quad (57)$$

For case (i) $|\beta| \geq \frac{\omega}{c}$ (except for a narrow range $|\alpha|$ about $\left|\frac{\omega}{c}\right|$) and

$$|q\gamma| \simeq \sqrt{\frac{\omega q}{c}}, \quad \left|\frac{\omega}{c} q\gamma\right| \simeq \frac{\omega}{c} \sqrt{\frac{\omega q}{\gamma}} \text{ and} \quad (58)$$

$$(i) \quad \left|\frac{\omega}{c} q\gamma\right| \ll \beta \text{ as } \sqrt{\frac{\omega q}{c}} \leq 0.18 \ll 1 \text{ (somewhat marginal)} \quad (59)$$

$$\text{For case (ii) } \alpha^2 \gg \left(\frac{\omega}{c}\right)^2 \text{ and } |\beta| \simeq |\alpha| \text{ and } |\gamma| \leq |\alpha| \quad (60)$$

so

$$(ii) \quad |\beta| \gg \left|\frac{\omega q}{c} \gamma\right| \text{ because } \left|\frac{\omega q}{c}\right| \leq .032 \quad (61)$$

For both these cases $\left|\frac{\omega}{c} q\gamma\right| \ll |\beta|$ except in the narrow ω -range

$$\begin{aligned}
& \left. \begin{aligned}
|\beta| &\simeq \sqrt{2\left|\frac{\omega}{c}\right|} \sqrt{\left|\frac{\omega}{c} - \alpha\right|} \simeq \left|\frac{\omega}{c} q \gamma\right| \simeq \left|\frac{\omega}{c} \sqrt{\frac{i\omega q}{c}}\right| \leq 0.18 \frac{\omega}{c} \\
\text{or} \quad & 2\left|\frac{\omega}{c} \left(\frac{\omega}{c} - \alpha\right)\right| \leq \left(\frac{\omega}{c}\right)^2 (.032) \\
\text{or} \quad & \left|\frac{\omega}{c} - \alpha\right| \leq 0.016 \left|\frac{\omega}{c}\right|
\end{aligned} \right\} \quad (62)
\end{aligned}$$

Therefore we can assume in Miller's (34) [6] that

$$a \simeq -1, \quad b \simeq 2\gamma/\beta \quad (63)$$

and that his (44), (45) are *good* approximations. Π_{xt} above ground is

$$\Pi_{xt} \simeq \frac{im_{xo}}{\pi} \int_{-\infty}^{\infty} d\omega e^{-i\omega x} \int_0^{\infty} \alpha d\alpha J_0(\alpha \rho) \frac{e^{i\beta(z-h)}}{\beta} e^{i\gamma h}, \quad z \geq h \quad (64)$$

and $\Pi_{zt}(m_x)$ *induced* by m_x is

$$\Pi_{zt}(m_x) = -\frac{\partial}{\partial x} \frac{m_{xo}}{\pi} \int_{-}^{+} d\omega e^{-i\omega x} \int_0^{\infty} \alpha d\alpha J_0(\alpha \rho) \frac{e^{i\beta(z-h)}}{\beta(\beta + \gamma)} e^{i\gamma h}, \quad z \geq h \quad (65)$$

With Π_{xt} of (64) the transmitted Green-function e_z -field just above ground at $z \geq h$ due to m_x is

$$e_z(m_x) = -\frac{16 m_x q c}{\sqrt{\pi} r' h^4} \left(8 \frac{q^2}{h^2}\right) \sin \theta \sin \phi \left(\frac{t_h}{r'/c} \frac{\partial}{\partial T} + \frac{\partial^2}{\partial T^2}\right) U(T) \quad (66)$$

$$\left. \begin{aligned} r' &= \sqrt{(z-h)^2 + \rho^2} & \sin \phi &= y/\rho, \quad \cos \phi = x/\rho \\ T &= \tau/t_h, \quad t_h = h^2/4qc, \quad \tau = t - r'/c \\ U(T) &= T^{-3/2} e^{-1/T} \end{aligned} \right\} \quad (67)$$

Miller implies (at least to this writer) that just above the interface E_z^{net} is finite, $H_{x,y}^{\text{net}} \rightarrow 0$ as $z \rightarrow h^+$, ($E_x \equiv 0$) and $E_y^{\text{net}}, H_z^{\text{net}}$ are twice their incident field values. He also says $|\Pi_z(m_x)| \ll |\Pi_{xt}|$ for $r' \gg h$, implying $\Pi_z(m_x)$ -generated fields are small.

However these assertions remain to be checked. The Green function $h_x(\Pi_{xt})$ -field evaluates, by Miller's procedure, to

$$\begin{aligned} h_x(\Pi_{xt}) &= - \left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \Pi_{xt} = \frac{8m_x qc}{\sqrt{\pi} h^2 r'} \left(\frac{4q}{h^2} \right)^2 \left((2 - 3 \sin^2 \theta \sin^2 \phi) \right. \\ &\quad \cdot \left. \left[\left(\frac{h^2}{4q} \right)^2 \frac{1}{(r')^2} U(T) + \frac{h^2}{4q} \frac{1}{r'} \frac{\partial}{\partial T} U(T) \right] - \sin^2 \theta \sin^2 \phi \frac{\partial^2}{\partial T^2} U(T) \right) \quad (68) \end{aligned}$$

where $\theta = 90^\circ$ at $z = h^+$, and

$$h_y (\Pi_{xt}) = \frac{\partial^2}{\partial x \partial y} \Pi_{xt} = \frac{8m_x qc}{\sqrt{\pi} h^2} \frac{1}{r'} \left(\frac{4q}{h^2} \right)^2 \left\{ 3 \left(\frac{h^2}{4q} \right)^2 \frac{1}{(r')^2} U(T) + \right. \\ \left. + 3 \frac{h^2}{4q} \frac{1}{r'} \frac{\partial}{\partial T} U(T) + \frac{\partial^2}{\partial T^2} U(T) \right\} \sin^2 \theta \sin \phi \cos \phi \quad (69)$$

$$h_z (\Pi_{xt}) = \frac{\partial^2}{\partial x \partial z} \Pi_{xt} = 0 \quad \text{at } z = h^+ \text{ because } \frac{\partial r'}{\partial z} = 0 \text{ there} \quad (70)$$

It is certainly not obvious that these $h_{x,y}$ -transmitted fields are $\simeq 0$ as Miller apparently claims. We must add to them the fields generated by $\Pi_z (m_x)$ of (65), and the relevant integrals probably should be evaluated numerically because of the non-trivial factor $(\beta + \gamma)$ in the denominator.

In conclusion, the temporal field components just above the earth's surface should be evaluated carefully by the above formulas when designing an experiment to measure yield of an underground explosion at distances of $1 \leq \rho \leq 5$ km.

4.4 Spectra of the Fields

For the spectra of the Green-functions "direct" or incident fields (25) – (27) or (34) – (36) due to $m_z \delta(t)$ and the Π_{xt} -induced total fields of (66), (68) – (70) we need the Fourier transform of $e^{-\gamma/t} t^{-5/2}$, $e^{-\gamma/t} t^{-7/2}$, $e^{-\gamma/t} t^{-9/2}$, where $\gamma = t_r$ or t_h . To evaluate

$$F_{5/2} = \int_0^\infty t^{-5/2} e^{-\gamma/t} e^{-i\omega t} dt \quad (71)$$

substitute $t = 1/x$ so it becomes

$$F_{5/2} = \int_0^{\infty} dx x^{1/2} e^{-\gamma x} e^{-i\omega/x} \quad (72)$$

From the Tables of Integrals, Series and Products, I. S. Gradshteyn and I. M. Ryzhik, p 340, #9

$$\int_0^{\infty} x^{\nu-1} e^{-\gamma x} e^{-\beta/x} dx = 2 \left(\frac{\beta}{\gamma} \right)^{\nu/2} K_{\nu} (2\sqrt{\beta\gamma}) \begin{cases} \operatorname{Re}\beta > 0 \\ \operatorname{Re}\gamma > 0 \end{cases} \quad (73)$$

Let $\nu = 3/2$, $\beta = i\omega$ so

$$F_{5/2} = 2 \left(\frac{i\omega}{\gamma} \right)^{3/4} K_{3/2} (2\sqrt{i\omega\gamma}) \quad (74)$$

Let $z = 2\sqrt{i\omega\gamma}$; from p 967 of the same reference

$$K_{n+1/2}(z) = \sqrt{\frac{\pi}{2z}} e^{-z} \sum_{k=0}^n \frac{(n+k)!}{k! (n-k)! (2z)^k} \quad (75)$$

This gives

$$F_{5/2} = \sqrt{\pi} (i\omega)^{1/2} \frac{1}{\gamma} e^{-2\sqrt{i\omega\gamma}} \left[1 + \frac{1}{z} \right] \quad (76)$$

$F_{7/2}$ and $F_{9/2}$, obtained by replacing $t^{-5/2}$ by $t^{-7/2}$ or $t^{-9/2}$ in (71), are evaluated similarly.

With these the spectra of (34) – (36) in MKS units with $\gamma = t_r = r^2 \mu_o \sigma / 4$ are

$$h_r(r, \theta, \omega) = \frac{1}{2\pi} \frac{m_z}{r^2} \sqrt{i\omega \mu_o \sigma} e^{-2\sqrt{i\omega\gamma}} \cos \theta \left[1 + \frac{1}{2\sqrt{i\omega\gamma}} \right] \quad (77)$$

$$\xrightarrow{\omega \rightarrow 0} \frac{m_z}{2\pi r^3} \cos \theta$$

$$h_\theta(r, \theta, \omega) = \frac{1}{4\pi r} i\omega\mu_0 \sigma e^{-2\sqrt{i\omega\gamma}} \sin \theta \left[1 + \frac{3}{2\sqrt{i\omega\gamma}} + \frac{3}{4i\omega\gamma} \right]$$

$$\xrightarrow{\omega \rightarrow 0} \frac{m_z}{4\pi r^3} \sin \theta \quad (78)$$

and

$$e_\theta(r, \theta, \omega) = \frac{5m_z}{4\pi} \frac{i\omega\mu_0}{r^2} e^{-2\sqrt{i\omega\gamma}} \left[\left(1 + \frac{3}{2\sqrt{i\omega\gamma}} + \frac{3}{4i\omega\gamma} \right) + \right.$$

$$\left. - \frac{2}{5} \sqrt{i\omega\gamma} \left(1 + \frac{3}{\sqrt{i\omega\gamma}} + \frac{15}{4i\omega\gamma} + \frac{15}{8(i\omega\gamma)^{3/2}} \right) \right]$$

$$\xrightarrow{\omega \rightarrow 0} 0 \quad (79)$$

The limiting values of (77) – (79) as $\omega \rightarrow 0$ are the well-known expressions for the MKS fields of a static magnetic dipole. These results check the transformation formulas for conversion of the CGS (25) – (27) of Miller to MKS forms (34) – (36).

The spectrum of an event field like (38) is the product of the spectra of the Green-function and magnetic dipole.

5. A Summary of Zablocki's Report [7]

In his summary he states that the electric field components recorded from eleven underground events at NTS are only a few microvolts/m at distances < 10 km from the shot point, and that they decay approximately as r^{-3} . The magnitudes are proportional to $W(\text{kTon})^{1/4}$. However Miller [6] gives an r^{-6} dependence for E_ϕ and implies magnitudes proportional to W .

Zablocki assumes final cavity radius r_c varies as $17 W^{1/3}$ but takes magnetic dipole $m \simeq H_0 r_c^3$ instead of $2\pi H_0 r_c^3$ (H_0 = earth field). Thus he obtains $m_{\max} = 2.5 \times 10^5 W$ instead of $1.48 \times 10^6 W$ in our (41).

The theory in Zablocki's report is based on J. R. Wait's work [8] for a single-frequency magnetic dipole at the earth's surface. This predicts magnetic field $H_r \sim r^{-4}$ in [7, (9)] and $E_\phi \sim r^{-4}$ in [7, (11)], whereas Miller's space-time analysis shows $H_r, H_\theta \sim r^{-5}$ and $E_\phi \sim r^{-6}$. Because the theory is single-frequency the dipole is not at the event point, and the fields differ qualitatively from Miller's, I regard all of Zablocki's analysis as suspect.

Now we will summarize the characteristics of several explosions discussed by Zablocki and make field amplitude estimates using Miller's MKS formulas.

5.1 Nougat Test Series (1961 – 1963). Peak amplitudes accurately recorded.

1. **Fisher.** $r \approx 4$ km from fireball to instruments, Alluvium of $\sigma = 0.02$ mho/m. $t_r = r^2 \mu_o \sigma / 4 = 100$ ms. Spectral analysis of the Green-function fields shows $f_{\max} \approx \frac{6}{2\pi t_r} = 9$ Hz, much smaller than the spectral width of the Bubble, ≤ 200 Hz.

We need a formula for $E_\phi(r, \theta, t)_{\text{net}}$ analogous to (37). From (33) and (36) it is

$$E_\phi(r, \theta, t) = \frac{20}{\pi \sqrt{\pi} \mu_o \sigma^2} \frac{1}{r^6} \sin \theta \int_0^t m_z(t-t') \left(1 - \frac{2}{5} \frac{t_r}{t'}\right) \frac{e^{-t_r/t'}}{(t_r/t')^{7/2}} dt' \quad (80)$$

We estimate $\left(\int_0^t dt' \right)_{\max} \approx (m_z)_{\max} 2 \int_0^{.05} \sin^2\left(\pi \frac{t'}{.05}\right) dt' \approx .05 (m_z)_{\max}$ and this gives

$$E_\phi(r, 90^\circ, t)_{\max} \approx 1.7 \times 10^{-6} \text{ V/m} \quad (81)$$

which is *ten times* higher than Zablocki's value [7, Table 2]. Perhaps σ was closer to .04 than .02.

Another measurement at $r = 6.5$ km yielded $.06 \mu\text{V/m}$, whereas we estimate, from Zablocki's value for E_ϕ , $0.17 (4/6.5)^6 \approx .009 \mu\text{V/m}$, too low. So we don't estimate the Fisher E_ϕ -fields very well with (80).

2. **Hardhat.** The peak $|E_\phi|$ was $> 100 \mu\text{V/m}$, saturated. $t_r \approx .16$ msec, very small, according to $r = .61$ km, $\sigma = 0.00135$ for this event.
3. **Aardvark.** $W = 37$ kTon, $r = 3.4$ km, in Alluvium of $\sigma = .02$. $(E_\phi)_{\text{peak}}$ measured was $12 \mu\text{V/m}$. $t_r = 72$ ms, comparable to 100 ms for Fisher. So we estimate from Fisher

$$(E_\phi)_{\text{max}} \approx 1.7 \times 10^{-6} \left(\frac{4}{3.4} \right)^6 \left(\frac{37}{13.5} \right) \approx 12.3 \mu\text{V/m} , \quad (82)$$

coincidentally very close!

5.2 Hardtack, phase II (1958) [7, Fig. 10]

The peak E-fields measured $1 - 2 \mu\text{V/m}$, with E_ϕ predominant, suggesting a magnetic dipole source. σ -value is not quoted. The 5 kTon-curve of E_ϕ looks most like it was generated by a magnetic dipole; the 19 kTon-peak should be four times higher but measures only two times higher than the 5 kTon-peak. The E_ϕ -curve for 19 kTon is very ragged.

Estimating $\sigma = 0.01$, for $r = 6.72$ km, $t_r = 140$ ms. $\omega \geq \frac{2}{2\pi t_r}$ so $f \geq 2 - 3$ Hz; the 2 – 8 Hz spectral peaks could be due to a magnetic dipole.

There is a strong E_r -component for event V, perhaps due to an electric dipole source.

5.3 Gnome (1961)

Here $W = 3.1$ kTon, $r = 7.75$ km, $f < 100$ Hz. There was halite ($\sigma \approx 10^{-4}$) around the cavity but did not typify the ground along the direct transmission path. We estimate $\sigma = 0.01$, then $t_r = 180$ ms. From Fisher, we expect

$$(E_\phi)_{\text{peak}} \approx 1.7 \times 10^{-6} \left(\frac{4}{7.75} \right)^6 \left(\frac{3.1}{13} \right)^2 = 0.03 \mu\text{V/m} \quad , \quad (83)$$

whereas the measured value was $.21 \mu\text{V/m}$. Since $(E_\phi)_{\text{peak}} \sim \sigma^{-2}$ it seems σ is effectively $\sqrt{\frac{3}{21}} \times 10^{-2} \approx .38 \times 10^{-2}$ mho/m. This would reduce t_r to $180 \sqrt{\frac{1}{7}} = 68$ ms and increase the integral in (80). So perhaps $\sigma \approx .60 \times 10^{-2}$.

5.4 Bilby.

$W = 200$ kTon, $r = 7.62$ km, $\sigma = 0.01$ (estimate), $(E_\phi)_{\text{peak}}$ measured $= 3.6 \mu\text{V/m}$. $t_r = 180$ ms, and from Fisher

$$(E_\phi)_{\text{max}} \geq 1.7 \cdot 10^{-6} \left(\frac{4}{7.62} \right)^6 \frac{200}{13.5} 2^2 = 2.1 \mu\text{V/m} \quad , \quad (84)$$

rather close to the measured value.

We now summarize Zablocki's discussion. His Fig. 15 shows field relaxation time τ_R as yield-dependent, $\tau_R = 30 W^{1/3}$ msec, from 5 nuclear events in the range $5 < W < 200$ kTon. But this functional form could be due to the variation of $t_r = r^2 \mu_0 \sigma / 4$, which changes the convolution integrals. Since his analysis is based on Wait's work at single frequency rather than on Miller's expressions many conclusions may be wrong.

For example, he concludes in [7, Fig. 3] for the Nougat series that radial E_r is necessary to justify a magnetic dipole source. This is simply not so!

Atop [7, p. 3541] he says the polarization is incompatible with a magnetic dipole for events I – IV and Hardhat. But this is probably due to the positive direction of E_ϕ in [7, Fig. 1] and H_ϕ in [7, Fig. 4] — which are in the $-a_\phi$ spherical coordinate direction! (Note the earth's H_o -field points downward at an angle of 30° with the vertical at Las Vegas, so the magnetic dipole $\vec{m}(t)$ is *upward* at this angle. So $m_x > 0$, with x in the geographic *south* direction.)

The curve-fit $|E|$ expression [7, (43)]

$$|E| = 2.2 \times 10^{-2} W^{0.44} / r_{\text{km}}^3 \text{ } \mu\text{V/m} \quad (85)$$

was obtained from various events in which E_ϕ was $\propto W$ times the convolution integral in our (80), and the latter depends on t_r in an involved way. So this formula for $|E|$ is deceptive.

Similarly, the curve-fit [7, Fig. 15] relating field relaxation time τ_R to W is more subtle because of the involved dependence of the convolution integral on t_r in (80). Note for $W = 5$, Hardhat, $\tau_R = 50$ ms, just the Bubble duration (according to Wouters) as expected for the small t_r of 0.16 ms. The dot to the left of Fig. 15 probably represents a small- t_r event too. If $\sigma = .02$ in Fisher, $E_\phi(t)$ decreases rapidly for $t > 50$ ms so we would expect $\tau_R = 50^+$ ms, as plotted. But if $\sigma = .04$, $t_r = 200$ ms, and τ_R would be larger. And why weren't τ_R of Fisher and Bilby ($t_r = 180$ ms) comparable? Perhaps because the Fisher t_r was $\simeq 100$ ms ($\sigma \simeq .02$), which would put Fisher closer to Aardvark ($t_r = 72$ ms) than to Bilby

At the very end of the article Zablocki acknowledges the dispersive nature of the fields in a conductive medium.

6. Comments on the Report of Malik, Fitzhugh, and Homuth [3]

These authors say in their abstract and conclusion that the *late*-time signals are probably explained by the seismoelectric effect proposed by Zablocki and Keller (1961 report). They say in

their introduction [3]: "Despite over two decades of sporadic investigations the data are fragmentary and either incompletely or unconvincingly interpreted". The writer (RMB) heartily concurs.

Figures 2 and 3 of [3] show $E_r(t)$ and $E_\phi(t)$ are *not* due to a magnetic dipole because of their almost identical shapes!

Malik, et. al, recognize the diffusive nature of the fields but ignore the correct analysis of Miller [6] and instead refer to J. Wait's work, from which they quote an E-field space-time formula which disagrees with Miller's expression, our (36), and (80). Fig. 5 [3] for the radial electric field from a horizontal *electric* dipole at depth of 715 m shows we must include the field reflection and transmission from the earth interface. Space-time fields from a horizontal magnetic dipole, m_x , below the surface are quoted but they show only *one* Cartesian component, whereas Miller's (47), divided by $(-iy)$ to give a $\Pi_z(m_x)$, and (32) with $a = -1$ for $\Pi_x(m_x)$ clearly show the presence of all $e_{x,y,z}$ away from the interface.

For the Lowball event $r = 2.9$ km, the first pulse of E_ϕ with $55 \mu\text{V/m}$ maximum appears to come from magnetic dipole m_z . Figs. G-2 through G-9 refer to this event, and Malik, et. al., say the signals on the shorter time scale records (≤ 5 ms) appear to be diffusive and due either to a magnetic Bubble as to an electroseismic effect. They note no azimuthal E and so discount the Bubble. This contradicts the information in Appendix F containing the Lowball data — the "best data" of any of the events in [3] according to these authors. Fig. F-1 shows $(E_\phi)_{\max} = 55 \mu\text{V/m}$ at $r = 2.9$ km. There is a relevant static water level at 500 m. With $\sigma = 0.01$ (Table G-II) we have $t_r = 26$ ms.

We now estimate $(E_\phi)_{\max}$ with (80), in which $m_z(t) \simeq (m_z)_{\max} \sin[\pi \cdot t/50 \text{ ms}]$, and the integral is approximately

$$\int_0^t dt' \approx \frac{0.8 (m_z)_{\max}}{r^6} \int_0^{t=t_r} \left(1 - \frac{2}{5} \frac{t_r}{t'}\right) \frac{e^{-t_r/t'}}{(t'/t_r)^{7/2}} dt' \quad (86)$$

With $T' = t'/t_r$ we have for (86)

$$\int_0^{t_r} dT' = -\frac{2}{5} t_r \left[T'^{5/2} e^{-1/T'} \right]_0^{T'=1} \quad (87)$$

Substitution of (87), $(m_z)_{\max} = 1.48 \times 10^6 \text{ W(kTon)}$ and the other data into (80) gives

$$(E_\phi)_{\max} \approx 0.22 \text{ W(kTon)} \mu\text{V/m} \quad (88)$$

From [3, Table G-II] we estimate $W = 56 \text{ kTon}$, so

$$(E_\phi)_{\max} \approx 12. \mu\text{V/m} , \quad (89)$$

fairly close to the measured $55 \mu\text{V/m}$.

7. Review of J. Sweeney's Measurements [9]

From FY1986 – 1988 he and his colleagues monitored ELF EM fields from underground events at NTS to determine whether they could be detected (yield estimation was not emphasized). Signals clearly related to underground tests were detected only when the sensors were $< 10 \text{ km}$ from surface ground zero. A major conclusion from the work was that ELF signals from underground tests of yields $< 150 \text{ kTon}$ are generally lower than background signals caused by worldwide lightning activity and hence not detectable via resonances in the earth-ionospheric cavity.

Below we compare measured peak fields with magnetic-dipole predictions for three tests discussed in [9] (The values of yields, W , are not quoted.). We assume $\sigma = 0.01$ and neglect fields reflected and transmitted from the earth-air interface; i.e., we estimate the fields at the interface by their infinite-medium values.

7.1 Hardin

We note the shape of all the signals is double-polarity, not characteristic of the *direct* fields of a magnetic dipole. H_n measured in the range $(2.4 - 4.0) \cdot 10^{-5}$ A/m [9, Fig. 7] at both A20 and 19X sites, where $t_r \approx 78$ ms. From (37), for H_θ of magnetic dipole m_x at $\theta = 90^\circ$ (in its coordinate system) we estimate

$$(H_\theta)_{\max} \approx 0.61 \times 10^{-6} W(\text{kTon}) \quad (90)$$

H_z was 13 pT at A20 and (26 – 38) pT at 19X measured [9, Fig. 9 and subsequent plots]. These translate to 1.04×10^{-5} A/m at A20 and $(2.1 - 3.0) \cdot 10^{-5}$ A/m at 19X. Our estimate is

$$(H_z)_{\max} \approx 1.05 \times 10^{-6} W(\text{kTon}) \quad \text{A/m} \quad (91)$$

at both sites, due to m_z .

E_ϕ measured in the range $(3 - 30) \mu\text{V/m}$ [plots], due to m_z . An estimate from (36), accounting carefully for the convolution integral, gives

$$(E_\phi)_{\max} \approx .035 \text{ W(kTon)} \mu\text{V/m, at both A20, 19X} \quad (92)$$

$B_e \approx B_r$ showed different character at A20 than 19X, for unknown reasons.

7.2 Kernville, site HR at 8.8 km

$H_r \approx H_n$ measured in the range $(2.0 - 2.4) 10^{-5} \text{ A/m}$ [9, Fig. 15 and subsequent plots]. However the double-polarity waveshape was not characteristic of the direct magnetic-dipole fields. The bandwidth of PDR-2 [9, Fig. 6(a)] was too limited; it should have been $\geq 50 \text{ Hz}$. The H_r -estimate was

$$(H_r)_{\max} \approx 2.1 \times 10^{-8} \text{ W(kTon)} \text{ A/m} , \quad (93)$$

too low.

B_z too, has the wrong waveshape. The measured H_z -peak [10, subsequent plots] is $21 \text{ pT} = 1.7 \times 10^{-5} \text{ A/m}$; the prediction due to m_z is

$$(H_z)_{\max} \approx 3.6 \times 10^{-8} \text{ W A/m,} \quad (94)$$

too low.

So the Kernville measurements are not predicted well by direct magnetic dipole fields; perhaps the inclusion of the effect of earth-air interface would improve matters.

7.3 Belmont. [9, subsequent curves]

(A) At site EG = 6.2 km the measured $(H_z)_{\max}$ was $\approx 9.6 \times 10^{-5} \text{ A/m}$. With $t_r = 120 \text{ ms}$ the estimate from (80) and remark following is

$$(H_z)_{\max} \simeq \frac{3.6 \times 10^{-5}}{92} W = 3.9 \times 10^{-7} \text{ W A/m} \quad (95)$$

(B) At site A20 = 7.4 km the measured $(E_\phi) = 18 \mu\text{V/m}$. With $t_r = 170 \text{ ms}$ the estimate from (80) and remark subsequent is

$$(E_\phi)_{\max} \simeq 1.1 \times 10^{-8} \text{ W V/m (low) .} \quad (96)$$

The discrepancy is not due to m_x or $(m_z)_{\text{electric dipole}}$, and interface reflection would probably not account for much.

8. Review of Classified Data for NTS Events

Much data was inconsistent relative to a simple magnetic dipole source. Missing data in some cases prevented us from comparing theory and measurement. For the calculations made, direct-field estimates differed from measured values by factors in the 10 – 100 range. The reasons for these discrepancies are not obvious.

9. Conclusions

We have compared *measured* early-time peak surface fields in various underground nuclear events with *predictions* from Miller's infinite-earth field expressions using Wouter's expression for the maximum magnetic dipole moment (with $W^{3/4}$ changed to W , yield) and its temporal history. Those comparisons sometimes differ within an order of magnitude of measurements reported by Zablocki, Sweeney, and Malik et. al., and sometimes by more, for reasons not fully understood. Miller's expressions allow us to correct for the earth-air interface, and the more accurate field formulas might agree better with measurements. Past instrumentation has sometimes lacked the required bandwidth, which can be estimated from the event-dipole time history and the spectra of Miller's Green-function fields.

10. Recommendations for Subsequent Work

For an advanced concept project we propose field experiments, coupled with modeling, to (a) confirm the diffusion theory, (b) optimize and calibrate the measurement systems, and (c) understand better the nature of the EM source fields of an underground event. As an initial study to achieve goals (a) and (b) we propose to carry out a series of experiments at NTS with a downhole controlled source to simulate the magnetic-dipole source of an underground explosion. We will measure the fields produced with surface sensors. Depending on the design criteria, we might also be able to create a downhole or surface-horizontal electric dipole source and study its fields with suitably modified diffusive-field theory. This will allow us to develop ways to deconvolve better the effects of multiple sources.

The diffusion theory will be used to design the experiment and resolve such details as the necessary distances and orientation of the sensors and required bandpass of the measurement system. We will locate the magnetic dipole source at a depth of a few tens of meters in a borehole which has been logged so that the medium characteristics are well known. In this way we can see if the measurements made in the NTS EM environment confirm the theoretical predictions.

Once we have completed the initial calibration and field testing of the EM measurement system, we will be ready to field the system during underground tests. Sensors will be deployed at distances less than 5 km from GZ, and will be designed to decouple optimally the fields created by magnetic or electric dipoles. The data will then be used to predict yield from EM measurements. Based on the results of one or two tests, provided that the data collection is successful, we will be in a position to recommend whether the technique is successful enough to warrant further study as an alternative system for yield measurement.

11. Personnel and Funding Requirements

Personnel needed to carry out this work are estimated to be: project manager, 0.1 FTE, EM field theory person, 0.25 FTE, field engineer, 0.25 FTE, field technician, 0.25 FTE, support from test designer to help us understand source characteristics, 0.15 FTE (optional). We will probably be able to design the measurement system using equipment available from previous ELF investigations and elements of the CSMS seismic system, but some additional funds will be needed for equipment. Additional funds will be needed for travel expenses for the field work. In summary, assuming 1 FTE total personnel expense and about \$50K for additional expenses, it will take about \$150K – \$200K to carry out the project as outlined above.

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