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3-D SOLUTION OF FLOW IN AN INFINITE SQUARE ARRAY OF CIRCULAR TUBES BY USING BOUNDARY-FITTED COORDINATE SYSTEM

by

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Heat transfer and fluid flow over circular tubes have wide applications in the design of heat exchangers and nuclear reactors. However, it is often difficult to accurately calculate the detailed velocity and temperature distributions of the flow because of the complex geometry involved in the analysis, and a lack of an appropriate coordinate system for the analysis. Boundary conditions on the surfaces of the tubes are often interpolated. This interpolation process introduces inaccuracy. To overcome this difficulty, the present study used the technique of the boundary-fitted coordinate system . In this technique, all the physical boundaries are transformed into constant coordinate lines in the transformed coordinates. Therefore, the boundary conditions can be specified on the grid points without interpolation.

The coordinate transformation technique used for the present analysis is based on the numerical solution of a set of elliptic partial differential equations (PDE)[1-3]. The transformed coordinates (ξ, η, ζ) are independent variables; the physical coordinates (x, y, z) are dependent variables. Constant values of one of the curvilinear coordinates (ξ, η, ζ) are specified as Dirichlet boundary conditions on each boundary. Values of the other curvilinear coordinates are either specified by a monotonic variation over a boundary as Dirichlet boundary conditions, or determined by Neumann boundary conditions. In the latter case, the curvilinear coordinate lines can be made to intersect the boundary according to some specified conditions, such as being normal or parallel to some given directions. Also, the spacings of the curvilinear coordinate lines can be controlled. The specific PDE used for coordinate generation in the present analysis of an infinite square array of tubes with parrallel flow is

$$\xi_{xx} + \xi_{yy} = P(\xi, \eta)$$

$$\eta_{xx} + \eta_{yy} = Q(\xi, \eta),$$

subject to boundary conditions,

$$\begin{bmatrix} \xi \\ \\ \eta \end{bmatrix} = \begin{bmatrix} \xi_1(x, y) \\ \\ \eta_1 \end{bmatrix}, \ (x, y) \in \tau$$

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where ξ_1 is a specified monotonic function of x and y, η_1 is a specified constant, P and Q are functions for controlling spacings and τ is the physical boundary. P and Q were set equal to zero for the analysis presented here. The computational meshes so generated are shown in Fig. 1 for the case of pitch-to-diameter ratio (S/R) of 1.05. The inlet conditions of the flow and the geometry are also given in Fig. 1. All the physical properties are assumed to be constant. Once the curvilinear coordinates are generated, the conservation of mass, momentum, and energy equations in terms of the transformed coordinates are solved.

A computer code (BODYFIT-1FE)[3] based on this procedure was developed at Argonne National Laboratory. BODYFIT-1FE is a three-dimensional, steady-state/transient single-phase thermal-hydraulic code for rod-bundle applications. It solves the complete Navier-Stokes and energy equations by a cell-by-cell numerical procedure. It uses a modified staggered-cell arrangement where velocity, energy, and mass-balance cells are all staggered at different locations. Detailed descriptions of the code are given in Ref. 3.

Several cases of different S/R ratios with the same hydraulic diameter and inlet conditions were studied. Water with the uniform velocity of 1 cm/s is flowing parallel to the axis of the cylindrical tubes arranged in a square pitch. Uniform heat flux was used to simulate the reactor fuel rod-bundle. Both the velocity and the temperature profiles were developing along the tubes. Since the analytical solution in the developing region for this configuration was not available, only the velocity and the temperature in the fully developed region were compared. For the axial velocity, Figs. 2(a) and 2(b) give the comparison between the BODYFIT results and the analytic solutions by Sparrow and Loeffler[4]. The agreements are in general very good. The total number of computational grid lines between tubes is fixed to be nine for all cases of different S/R ratios. In the case of large S/R ratio where tubes are far apart, the number of grid lines used in the present analysis may not be fine enough to resolve the detailed velocity profile. This slight inaccuracy can be seen in Fig. 2(a) for the case of S/R = 4.

For the comparison of temperature distributions, Table 1 gives the BODYFIT-calculated Nussult number as a function of the dimensionless $\bar{Z} = (Z/D_e)/(RePr)$ for various S/R ratios. The Nusselt number at $Z = \infty$ is given by the analytic solution[5] for the case of constant heat flux. Reference 6 gives the similar analytic solution for the case of constant peripheral temperature. In the case of large S/R ratios, the two cases are very similar. In the case of small S/R ratios, the two cases differ quite a bit. However, the constant heat flux case is closer to the condition in reactor application than the constant peripheral temperatures case. The same information in Table 1 is plotted in Fig. 3. It is observed that the temperature profile reaches fully developed profile more slowly as the S/R ratio gets smaller. For the case of S/R = 1.05, the temperature profile did not fully develop at the length of 156 times of hydraulic diameter, D_e. This phenomena also effects the comparisons shown in Fig. 2.

From the study, it is concluded that BODYFIT-1FE can provide detailed velocity and temperature distributions with good accuracy. This information is valuable for designing a mechanical heat transfer component. Furthermore, the code is very flexible and can provide analysis of the complicated geometries in most nuclear reactor applications.

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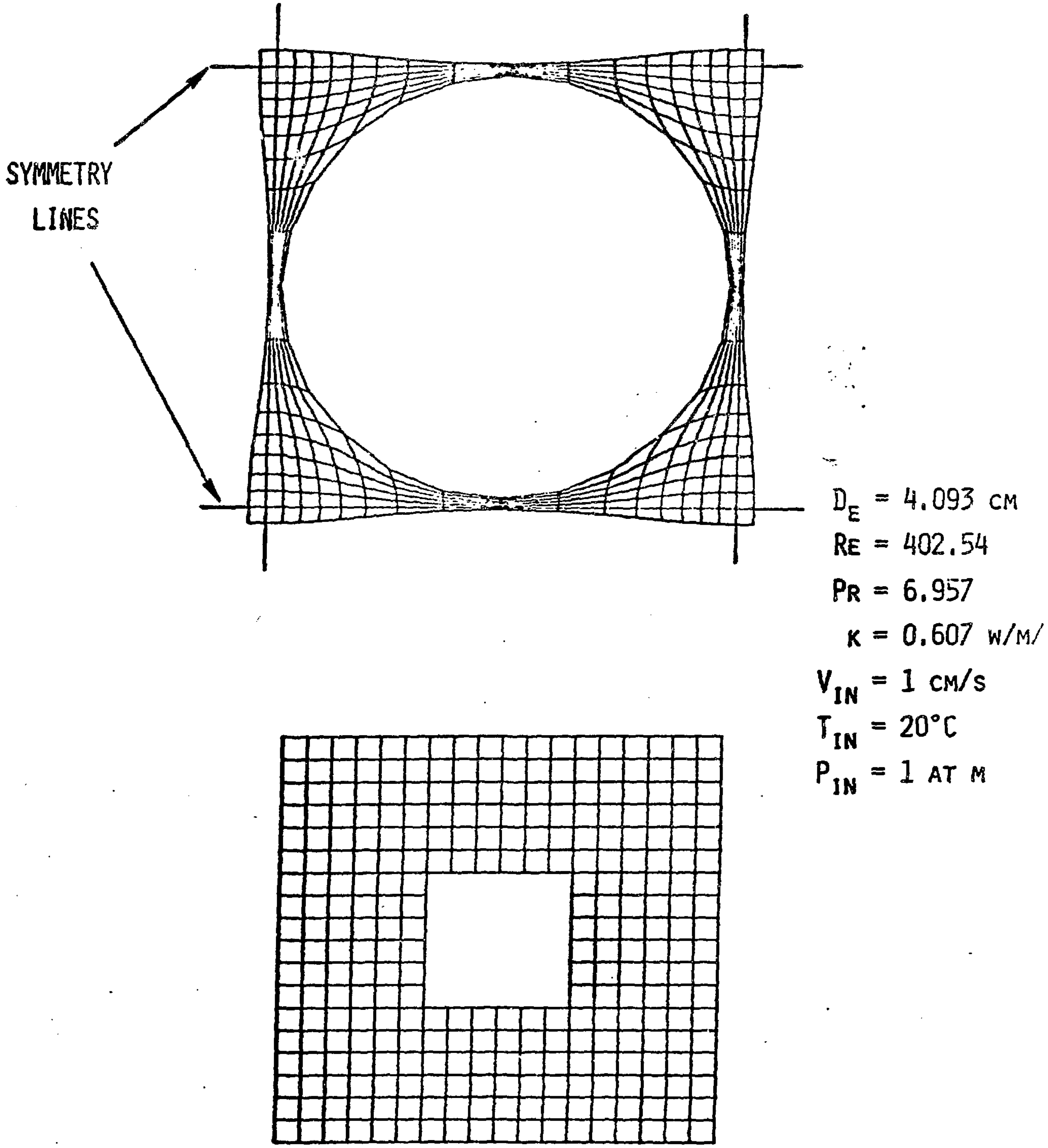

Fig. 1. Computational Mesh for a Unit Cylindrical Tube Arranged in a Square Pitch

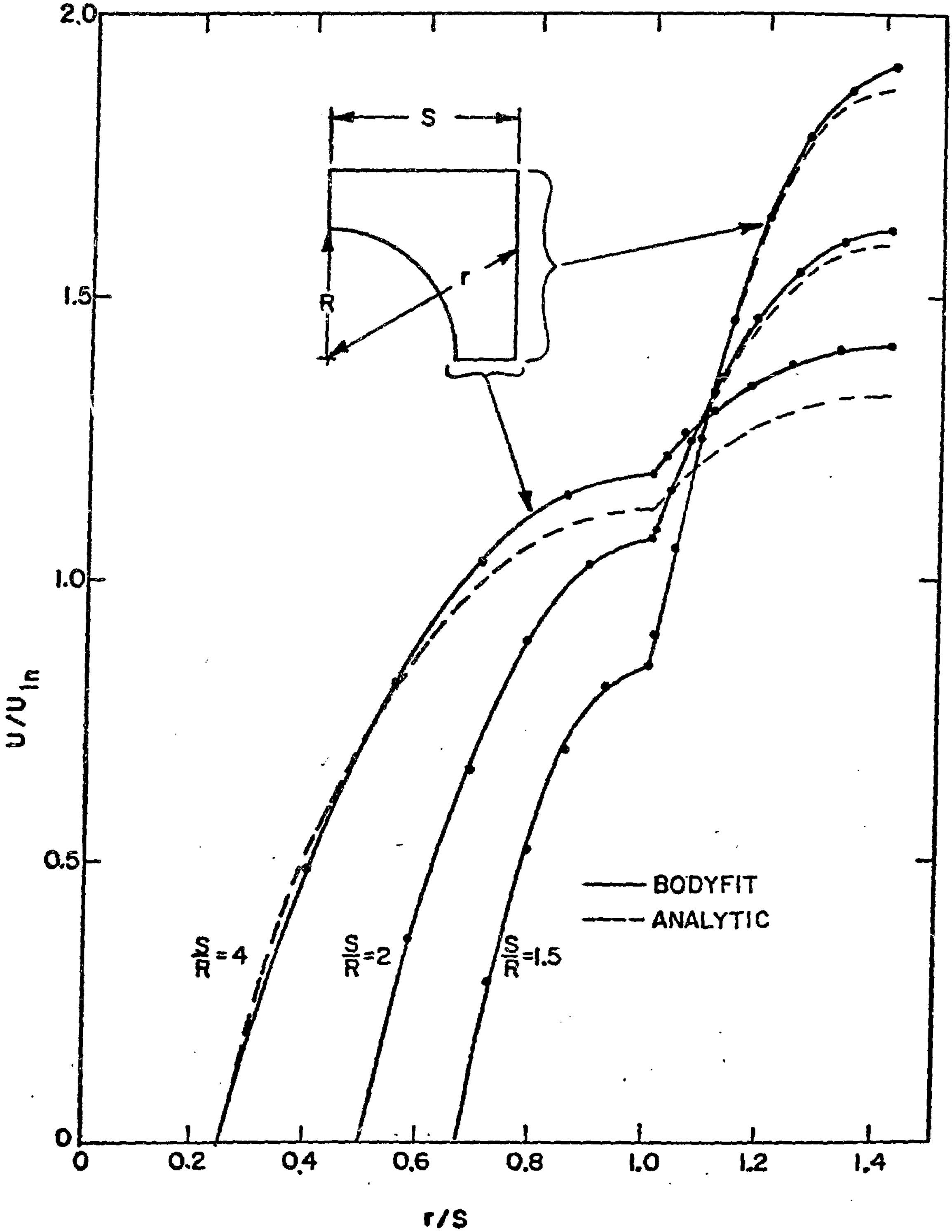

Fig. 2(a). Comparison of Axial Velocities between BODYFIT and Analytic Calculations for S/R = 4, 2, and 1.5

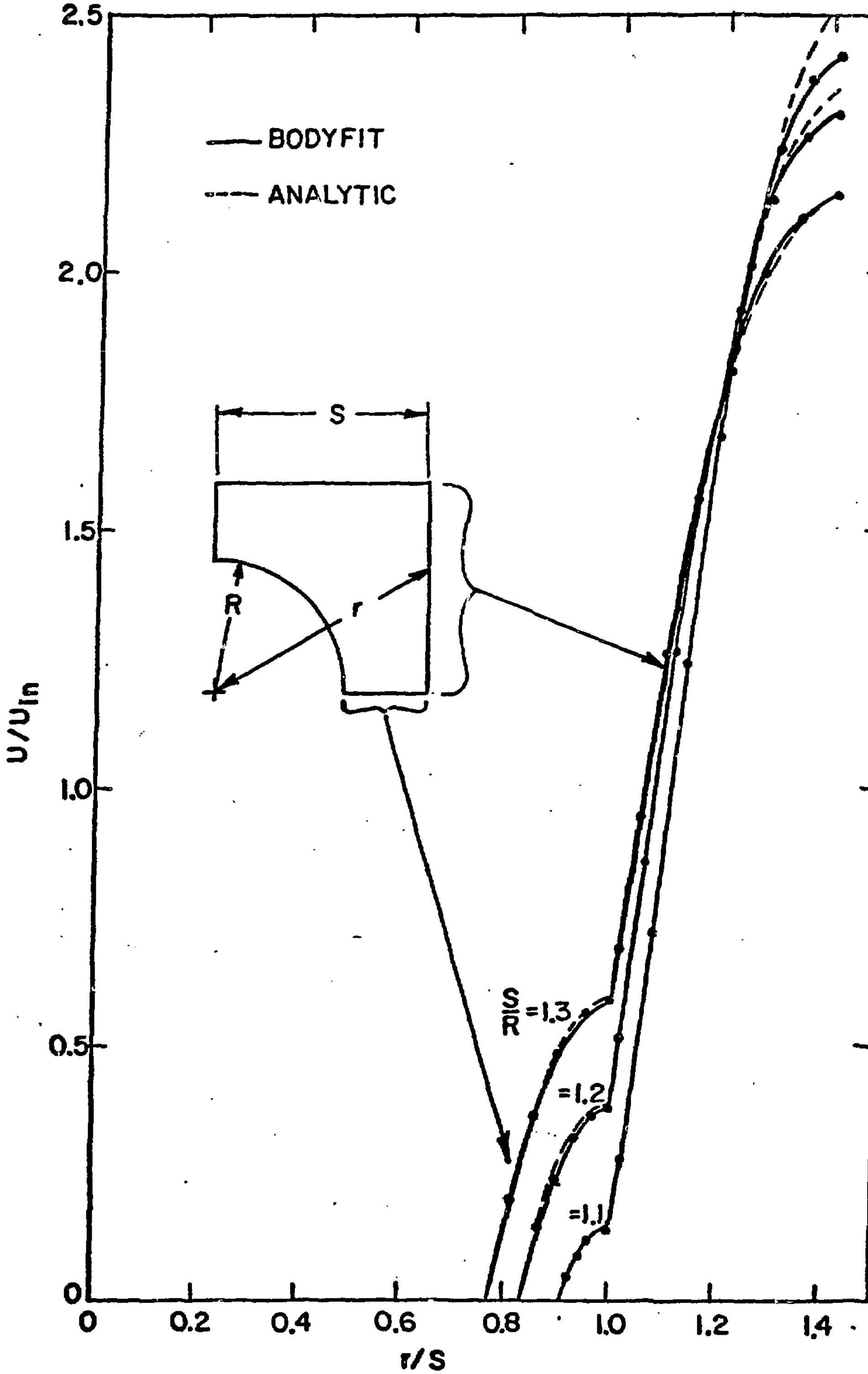

Fig. 2(b). Comparison of Axial Velocities between BODYFIT and Analytic Calculations for S/R = 1.3, 1.2, and 1.1

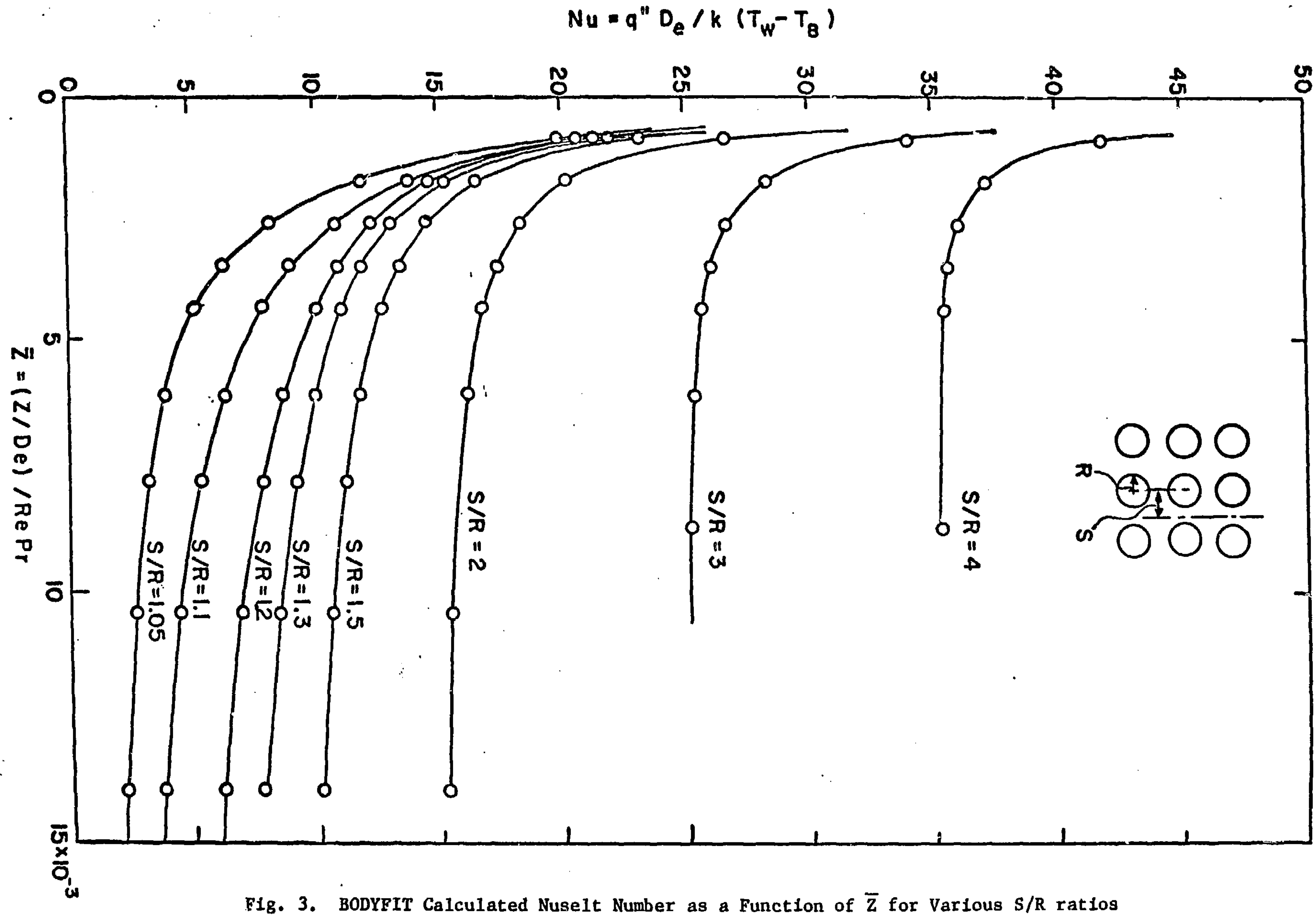

Fig. 3. BODYFIT Calculated Nusselt Number as a Function of $\overline{Z}$ for Various S/R ratios

Table 1. (BODYFIT Calculated) Nusselt Number

As a function of $\bar{Z}$ for Various S/R Ratios

$\bar{Z}x10^3$	S/R=1.05	S/R=1.1	S/R=1.2	S/R=1.3	S/R-1.5	S/R=2	S/R=3	S/R=4
0.873	19.95	20.74	21.47	22.10	23.31	26.69	34.07	41.92
1.75	11.89	13.84	14.72	15.35	16.65	20.33	28.34	37.21
2.62	8.23	10.89	12.36	13.16	14.59	18.43	26.78	36.12
3.49	6.31	9.11	11.00	11.94	13.49	17.49	26.10	35.70
4.36	5.15	7.89	10.05	11.12	12.77	16.92	25.73	35.52
6.11	3.88	6.30	8.75	10.01	11.86	16.24	25.39	---
7.85	3.20	5.33	7.86	9.27	11.27	---	---	---
10.47	2.62	4.44	6.95	8.49	10.68	15.50	---	---
13.96	2.19	3.74	6.16	7.79	10.16	15.27	25.17	35.34
27.92	1.58	2.61	4.73	6.47	9.26	---	---	---
55.84	1.19	2.02	3.89	5.72	---	---	---	---
∞*	0.92	1.68	3.68	5.82	9.29	15.05	25.23	36.64

* Analytic Solution (Ref. 5)