THE CONTINUING DEVELOPMENT
OF THE DEROB SYSTEM

MASTER

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for Period April 1, 1981 - June 30, 1981

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ABSTRACT

The daylighting sky factors have been calculated for an arbitrary point. The point represents a "light meter" that can be located anywhere in the simulated structure, and it can be oriented in any direction desired. It reads the "sky factor" and it can disaggregate the first incidence component from the contributions due to reflections from surfaces, and transmissions from other spaces.

The reflected and trasmitted components show a marked spatial dependence. The results are compared with standard calculation methods. These standard calculation methods are not capable of discerning the spatial dependence, and they have no capability to account for transmissions from other spaces.

The fact that DEROB can describe in detail multispatial interactions allows it to calculate the effect of light-wells, courtyards, solariums on the illumination pattern of simulated spaces.

The report includes a description of the results of the first set of comparisons, as well as the mathematical theory for multispace coupling.-

Overall Objectives

The overall objective of the project is to develop additional capabilities of the public domain version of the DEROB (Dynamic Energy Response of Buildings) program. This program is to be used as a research and analysis tool for the investigation of the energy response of buildings.

The objectives of this phase of the project are:

- a) to modify the load calculation module (DRBTL) so the solution convergence tolerance is more easily specified by the user; and also build the logic into this module so that it can self-diagnose the tolerance required for a given analysis. This task has been completed and was reported in the September 1980, and December 1980 Technical Reports.
- b) to incorporate daylighting calculations with the existing thermal load calculations. The work toward the completion of this objective is underway now. This technical report contains the details of the tasks accomplished since the previous report of March, 1981 toward this end. The additional tasks to be carried out toward achieving daylighting objectives are expected to be completed over the next two reporting periods. Thus by September 1981, there should be a version of DEROB with daylighting capabilities available for technical evaluation.
- c) to incorporate specular reflection algorithms into the DRBSOL module. The tasks toward achieving this objective will be undertaken during the September-December 1981 quarter.
- d) to incorporate a multispace cross ventilation algorithm. The tasks will be undertaken during the course of the 1982 calendar year.

The execution of any given task may have to be postponed whenever a correction needs to be implemented in any of the modules.

The schedule of completion of the tasks may be protracted if the level of funding falls below the grant level.

Tasks Completed During the April 1 - June 30, 1981 Quarter

Task 1 Re-organize the tape transfer logic to permit DRBSOL to be run before DRBLUM.

This change in the sequence is needed in order to use the direct solar radiation data for the calculation of its contribution to the day-lighting calculations.

Task 2 Write and code the multispace radiation coupling algorithm.

This algorithm allows the calculation of radiation distribution among spaces that are visually connected. This capability is mandatory if lightwells, courtyards, solariums or other light transmitting configurations are to be simulated with credibility.

The mathematical details are presented in appendix II.

Task 3 Write and code the multi-space luminance matrix.

This is needed in order to calculate the diffuse day-lighting using the split beam model.

The mathematical detail is included in appendix II.

Task 4 To compare the results of these daylighting calculations with the standard methods appearing in the pertinent literature.

See appendix I.

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This portion of the report
was written by Jin-Jong Kim

THE CALCULATION METHODS OF INDOOR ILLUMINATION LEVEL

Indoor illumination levels are firstly dependent on outdoor illumination conditions. The outdoor illumination level varies with time of day, time of year, orientation of the window towards the sky, local weather conditions and latitude of the position of the investigation.

Since the luminance distributions of the sky are subject to many complicated variables, the formulation of mathematical description of the outdoor illumination level are based on assumptions and empirical data taken under the typical criteria of the sky condition categorized, as uniform sky, standard overcast sky and standard clear sky.

Uniform sky

The sky is considered as a perfectly diffusing surface having a unit intensity of illumination focused to the point of interest on the ground. Thus the direct illumination from the sun is ignored(see figure 2-1). The illumination level falling on a surface on the ground varies with the angular relation between the surface and

Figure 2-1: Uniform sky. The unit intensity of illumination focused to to the center of hemisphere base (P) is distributed over the entire surface of the sky. The brightness of the ground is assumed to be the product of its reflectance and illumination intensity of the sky.

Figure 2-2: The orientation of receiving surface. The illumination from the uniform sky on the surface A-A' is the integration of the sky illumination component (S-s') over the angle ($\theta_2 - \theta_1$)

the sky(see figure 2-2).

Standard overcast sky

The sky luminance distribution under overcast conditions does not depend on the position of the sun, but only varies with the angle above the horizon. Standard¹ overcast sky luminance distribution is

$$L_{\theta} = \frac{1}{3} L_z (1 + 2 \sin \theta)$$

where L_{θ} = luminance of the sky at an angle above the horizon

L_z = luminance at the zenith

θ = angle above the horizon

This, CIE(Commission Internationale De'Eclaire) recommended formula for overcast sky is derived from the analysis of the results obtained under empirical observations made by Moon and Spencer, England. The implication of this formula is that the luminance at the horizon is 1/3 that of the zenith. Thus the illumination contribution from the same area of the sky seen at a reference point indoors varies-with the angle of elevation above the horizon(see figure 2-3).

¹"Standard" stands for that the governing formula was recommended by CIE for the international use.

Figure 2-3: Standard overcast sky luminance distribution. The Luminance Distribution of the sky is a function of the angle above the horizon, as

$$L_{\theta} = L_z (1 + 2 \sin \theta) / 3$$

Standard clear sky

The luminance distribution of clear sky is the function of the sun position and the scattering of light by the particles in the atmosphere. The CIE recommended formula for clear sky luminance distribution, known as the Kitter formula, is

$$\frac{B_{\theta}}{B_z} = \frac{(1 - e^{-.32/\sin\theta})(.91 + 10e^{-3a} + .45\cos^2 a)}{.274(.91 + 10e^{-3z} + .45\cos^2 z)}$$

where B_{θ} = sky luminance at a given point at an angle of elevation above horizon

B_z = luminance at zenith

θ = zenith angle of the sun

a = angular distance of a particular point from the sun

z = zenith distance of the sun (zenith to the sun)

Calculation methods

The light reaching the reference point indoors is the function of the sky condition, the geometry of the window, the position of the reference point, the transmittance of glazing medium, the geometry of the walls.

There are many methods developed to calculate the daylight level in a given space each considering the variables in their own way. Some tools, such as the daylight

Figure 2-4: Standard Clear Sky Luminance Distribution.
Luminance distribution of the sky calls some extra variable by the presence of the sun. The point P which is 90° away from the sun along the arc of the great circle going through the solar position exhibits the least luminance

protractor, have also been developed to calculate with speed and ease rather than with accuracy.

The methods can be classified into two categories. These are the lumen method and the daylight factor method.

Lumen method

The lumen method is a highly simplified method whose basic concept is to treat the windows as extended sources of light similar to artificial lighting calculations. Basically, this method is an absolute value method which gives actual illumination level, while the ratio methods(daylight factor method) give the ratio of indoor and outdoor illumination level. It, firstly, determines available daylight incidence on windows from the sun(excluding direct portion), the sky and the ground. Based on the available daylight on the window, the actual transmitted amount of illumination is determined by multiplying available daylight by transmittance and by a maintenance factor, which is determined by the actual surface conditions of the window. Finally, based on the room geometry, the distribution of light flux within the room is determined.

The formula for sidelighting is

$$E_p = E_i \times A_w \times K_u \times K_m$$

where E_p = work plane illumination at point p
(lumen/sq ft)

E_i = illumination from the sky or ground incident on the window(lumen/sq ft)

K_m = light loss coefficient

K_u = utilization coefficient

The formula for top lighting is

$$E_{tl} = \frac{E_h \times A_{tl} \times K_u \times K_m}{A_r}$$

where E_{tl} = average illumination produced on the work plane(lumen/sq.ft)

E_h = incident illumination on exterior of top lighting element(lumen/sq.ft)

A_{tl} = gross area of toplight element(sq.ft)

K_u = utilization coefficient

K_m = light loss coefficient

A_r = room area(sq.ft)

The lumen method has the advantage of rapid calculation based on the information generated by other sources. The dependence on this information limits the calculation to cases with the conditions(such as room geometry or reflectance) that are known and tabulated.

Daylight factor method

The discussion of the daylight factor method requires a definition of daylight factor. The CIE definition of daylight factor is as follows:

The daylight factor is "a measure of the daylight illumination at a point on a given plane expressed as a ratio of the illumination on the given plane at that point and simultaneous exterior illumination on a horizontal plane from the whole of unobstructed sky of assumed or known luminance distributions. Direct sunlight is excluded from both exterior and interior value of illumination."

The daylight factor at a reference point is composed of three components. These three components are:

- 1) Sky component
- 2) Externally reflected component
- 3) Internally reflected component

Among these, at present, the calculation of the sky component is emphasised rather than the other components and the methodology to derive the sky component is clear. It is vitally important to calculate the reflected components accurately, as they often contribute significantly to the illumination level at a point when the reflectance of the surface the point facing is high. In addition, when the reference point is located in a room where the sky is not directly visible, the reflected component is the only way

illumination can reach the point.

Sky component

The CIE definition of sky component is as follow.

Sky component is "the ratio of that part of illumination at a point on a given plane which is received by direct reflection from external surface, to the simultaneous exterior illumination on a horizontal plane from unobstructed sky of assumed or known illumination distribution. Direct sunlight is excluded from both interior and exterior value of illumination".

By this definition sky component is a function of the geometric factor¹ at a reference point from any openings facing directly sky, the luminance distribution of the sky and the characteristics of the glazing medium.

Sky factor

When the transmittance of glazing is 100% and the illumination distribution of the sky is assumed to be uniform, the sky component is called the sky factor. Consequently the sky factor depends on the geometric relation between the point of interest and the openings facing directly the sky, ie, the sky factor represents how much portion of the hemisphere sky can be seen at a given point inside the room through the openings.

¹View or angle factor

Many formulae are developed with slightly different expressions and limitations of their applications.

The Higbie-Levin formula is the one generalized up to the angle between the window plane and the plane on which the reference point is placed.

The Higbie-Levin formula is

$$E_p = \frac{b}{2} \left[\tan^{-1} \frac{B}{A} + \frac{\cos \phi - A}{\sqrt{A^2 + 1 - 2A \cos \phi}} \tan^{-1} \frac{B}{\sqrt{A^2 + 1 - 2A \cos \phi}} \right. \\ \left. + \frac{B \cos \phi}{\sqrt{A^2 \sin^2 \phi + B^2}} \tan^{-1} \frac{\sqrt{A^2 \sin^2 \phi + B^2}}{A^2 + B^2 - A \cos \phi} \right]$$

where E = illumination at a given point (foot candle)
 b = brightness of the surface source (foot candle)
 m = width of the window (feet)
 f = height of the window (feet)
 ϕ = angle between work plane and window plane
 A = a/f
 B = m/f
 a = distance of the point from the intersection of window plane and work plane (feet)

The Higbie-Levin formula, though, can be applied only to rectangular windows.

The formula derived by Arumi, which is a recast form of radiation exchange equation of two surfaces, is

generalized to be applied to any shape of window with variable angle between the reference plane and the window plane.

The Arumi formula is

$$g = \frac{1}{2\pi} \hat{s} \sum_k (n^{(k)} \times \delta^{(k)}) [Q_{(lk)} - Q(0)]$$

where $\hat{s}$ = vector normal to the surface on which measuring point is located

$\hat{n}$ = orientation unit vector of the edge

$\delta^{(k)}$ = distance vector between measuring point and K^{th} corner of the surface

Here $Q(p)$ is defined as

$$Q(p) = \frac{1}{q} \tan^{-1} \left[\frac{p - \delta^{(k)} \hat{n}^{(k)}}{q} \right]$$

where

$$q = \sqrt{(\delta^{(k)})^2 - (\delta^{(k)} \hat{n}^{(k)})^2}$$

l_k = length of the K^{th} edge

$\underline{r}$ = position vector of the measuring point

$\underline{r}^{(k)}$ = position vector of the K^{th} corner of the surface

Reflected component

The light entering a room through a window is subject to infinitive number of repeated reflections by the walls, ceiling and floor. Through these series of

reflections, the light from the opening is redistributed to each surface facing into the space. Thus each wall (including floor and ceiling) act as secondary sources of illumination to the reference point.

During the process of infinitive reflections, some portions of light will be transmitted out through the transmittable walls(i.e., opening or glazing) to another space and outdoors(see figure 2-5).

A fraction of light will be absorbed by the surface at the time of each repeated reflection.

Since the reflectance of the surface is less than unity, except the ideal case of a perfect reflecting surface, the sum of infinitive number of reflections results in a finite value of light illumination at the surface. For instance, if the initial incidence of light is I , and surface reflectance are is equally r , the sum of reflection is

$$\sum_i r^{(i)} \cdot I = I + rI + r^2I + \dots$$

$$= I(1 - r)^{-1}$$

This logic can be summarized by the following two equations. The radiation emitted by a surface is the sum of each intercepted portion by the encircling surfaces.

This can be expressed as $\sum_i G_{ij} = 1$

Figure 2-5: Light Reflection, Absorption and Transmission in Multi-space.

where G_{ij} = geometric factor
 i = receiving surface
 j = emitting surface

The radiation arriving at a surface can be divided into three components, transmittance, reflectance and absorptance maintaining a sum of unity. This relation can be expressed as

$$\underline{U} = \underline{t} + \underline{r} + \underline{a}$$

where $\underline{U}$ = unitary vector
 $\underline{t}$ = transmittance vector
 $\underline{r}$ = reflectance vector
 $\underline{a}$ = absorptance vector

Combining these two equations and considering infinite number of repeated reflections, the equations can be recast as

$$\underline{U} = (\underline{a} + \underline{t}) \cdot \underline{J}$$

where $\underline{J} = (\underline{I} - \underline{P})^{-1}$
 $\underline{P} = r_j G_{ij}$

The matrix $\underline{J}$ stands for the illumination tensor at the surface where the reflection is considered.

Similarly, the illumination tensor of the surfaces in a system of surfaces which have visually connecting transparent surfaces, to another system of surfaces under-

Figure 2-6: Transmission Coupling of Light between two visually connecting spaces.

goes the radiation coupling through the transparent wall between the two spaces(see figure 2-6).

The illumination tensor, considering transmission recursion, can be expressed as

$$\underline{U} = \underline{a} \cdot \underline{J}(\underline{I} - \underline{H})^{-1}$$

where $\underline{a}$ = absorptance vector

$\underline{J}$ = illumination tensor taken into account the reflectances

$\underline{I}$ = identity vector

and $\underline{H}$ is defined as

$$\underline{H} = \sum_i (\underline{e}_j) \times (t_i \underline{e}_i \underline{I})$$

where $\underline{e}_j$ = j^{th} base vector

$\underline{e}_i$ = i^{th} base vector

t_i = transmittance of surface

$\underline{J}$ = illumination tensor taken into account the reflectances

Thus the term $\underline{J} (\underline{I} - \underline{H})^{-1}$ represent generalized illumination tensor taken into account the transmissions and the reflections.

Split flux method

A highly simplified method to calculate the internal reflection component is the split flux method. The formula is

$$IRC = \{ t_g \cdot W / A(1 - R_{avg}) \cdot (S_1 R_f + S_2 R_c) \}$$

where t_g = transmittance of glass
 W = window area
 A = total surface area
 R_{avg} = average reflectance of total surfaces
 R_f = average reflectance of floor area
 R_c = average reflectance ceiling area

The average reflectance of floor or ceiling stands for the average reflectance of surfaces above or below the middle height of the window.

This formula is based on the assumption that

1) the light reflected by the surfaces in a space is evenly distributed over the entire plane of the surface, thus the internally reflected component is the same regardless of the position inside the space.

2) the luminance of the ground is a certain ratio of that of the sky(usually 10%) depending on the reflectance of the ground surface.

Obviously these assumption are not true in reality, but can be justified by the simplification of the calculation.

THE COMPARISON OF CALCULATED VALUES BY CURRENT PRACTICING METHOD AND THE PROPOSED METHOD

Based on the methods and associated factors for calculating indoor illumination level introduced in the proceeding chapter, it is desired to compare the values calculated by current methods and the proposed method. The intent of these comparisons are not only to validate the proposed methods, but also to find the limitations of the current method.

The methodology of comparisons

Among the currently available methods of calculating indoor illumination levels, the approach of the lumen method as indicated is an approximated method different from others as well as from the proposed method. Thus this comparison is focused on the current daylight factor method and the proposed method. The comparison of these two methods is approached by breaking the values into each component - the sky component and the reflected component, in order to determine any differences and the causes of the differences.

The comparison of the sky components

The Higbie-Levin formula is the most generalized form of formula among the currently available methods for calculation of the sky component. This formula and the Arumi formula which is used in the proposed method for calculating the geometric factor at a given reference point are analytically similar expressions, except the Higbie-Levin formula is only applicable to rectangular shapes of windows, while the Arumi formula is applicable to any shape of windows.

Because of the limitation of the Higbie-Levin formula, the model for the comparison is confined to rectangular shapes of windows, varying the angle between the reference plane and the window plane(see below illustration).

Figure 3-1. Calculation of Sky Factor as a Function between Window Plane and measuring plane.

The results of the calculations by both formula are exactly the same at any distances and angles.(see figure 3-1)

By this comparison, it is concluded that any differences in daylight factors would be due to the differences in the reflected component calculations.

The comparison of reflected component

The current method of calculating the reflected component is based on the split flux method(see discussion in page I-18). As discussed before, this method is based on the assumption that the reflected component is the same over the entire surfaces in a space.

The proposed method for calculating daylight factor is based on the radiation equilibrium in the spaces. Thus the geometry of the walls influences the value of reflected components, on the other hand, the split flux method does not take into account the geometric relations of the walls.

Since the split flux method does not take into account the wall geometry, while emphasising the reflectances of the floor and the roof, the test model for this stage of the comparison started with a structure which has only a roof and a floor and all the vertical walls are assumed to be made of 100% transparent walls(see below

illustration).

The reflectances of the floor and the roof were given equal values.

Under the uniform sky condition and with 100% transparent vertical window, the split flux method equation can be reduced as follows.

$$IRC = \frac{W(50R_f + 50 \cdot R_g \cdot R_c)}{A(1 - R_{avg})}$$

where W = window area(sq.ft)

R_c = reflectance of the surfaces above the middle height of the window

R_c = reflectance of the surfaces below the middle height of the window

R_g = reflectance of the ground surface

A = total surface area of the space(sq.ft)

R_{avg} = average reflectance of the total surfaces
of the space

The constant 50 stands for the geometric (factor view or angle) of the vertical window intercepted by the sky or by the ground, which is sometimes called the sill factor of the window. This factor varies as a function of the angle between the window plane the ground plane as follows.

$$\text{Sill factor} = \frac{1}{2}(1 + \cos\theta)$$

where θ = angle between the window plane and the ground plane

In figure 3-2, the parallel straight lines are the ones calculated by using the split flux method and the curves are the ones calculated using the proposed method. These set of lines which are the values of indential surface reflectances cross each other. The maximum deviation occurs when the surface reflectance is 1.0(highest). The deviation appear to be reduced as the surface reflectances are reduced. In the central zone, the values by the proposed method are higher than those by the split flux method. While, in perimeter zone, the opposite situation has occurred. The parabolic curvatures of values by the proposed method are due to the symmetry with respect to

Figure 3-2. A Comparison of Internal Reflected Component by DEROB Method and Split Flux Method (case with 2-surface model)

Figure 3-3. Internal Reflected Component vs. Room Surface Reflectance (case with 2-surface model)

the vertical windows.

Figure 3-3 is a plot of the reflected components by both methods, as the function of the surface reflectance locating the reference point at the center of floor. At this particular location, the values by the proposed method are higher than those by the split flux method.

The case with 3 surfaces

The next step of the comparison was investigated by substituting a vertical window with an opaque wall (see below illustration). The procedures of the calculation are the same as before.

The model is not symmetric with respect to the inclusion of the opaque wall, the values calculated by the proposed method are not symmetric along the center line. Figure 3-4

Figure 3-4. A Comparison of Internal Reflected Component by DEROB Method and Split Flux Method (Case with 3-surface model)

Figure 3-5. Internal Reflected Component vs. Room Surface Reflectance (case with 2-surface model)

shows that the values increase as the calculated points are moved near the back wall(opaque wall).

The crossing of the lines by using the split flux method and proposed method occurs around the distance range of 7 feet to 12 feet away from the window. In front of those crossing points, the proposed method produces lower value than the other. Considering the fact that the split flux method is based on the normalization of values, and that the reflected component is higher when the reference points are located near the walls, this result is to be expected.

Figure 3-5 shows a plot taken by both methods at the center of the floor. This case also shows that the proposed method results in higher values.

The case with five opaque walls

A further similiar set of comparisons were investigated by substituting two more vertical windows by opaque walls. Thus the model structure has only one window(see below illustration). All the other condition remain the same as before.

Figure 3-6. A Comparison of Internal Reflected Component by DEROB Method and Split Flux Method (case with 5-surface model)

Figure 3-7. Internal Reflected Component vs. Room Surface Reflectance (case with 5-surface model)

The results are similar to the case with only one opaque wall, however, when the points were moved deep into the room (near the back wall), the differences were not as severe. This is due to the tendency of levelling of the values as the point are located in the zone near the back wall (see figure 3-6).

The case of a court

For the windows placed on the roof, the sill factor of the split flux method formula are subject to change as follow. Thus the formula for split flux method is

$$IRC = \frac{W(100 + R_f)}{A(1 - R_{avg})}$$

where W = window area

R_f = average of reflectance the surface below
the window

R_{avg} = average reflectance of the total surface

A = total surface area of the space

Here the constant 100 stands for the sill factor of the skylight. Thus the reflectance of the ground does not affect the values of the reflected component anymore.

The model structure for this stage of comparison has a skylight covering all the area of the roof, and the other walls are made of opaque material (see below illustration).

Figure 3-8. A Comparison of Internal Reflected Component by DEROB Method and Split Flux Method (case with a court model)

Figure 3-9. Internal Reflected Component vs. Room Surface Reflectance (case with a court model)

The effect of ground reflectances

The reflectance of the ground surface affects the reflected component as the amount of available illumination outdoor increases as the ground reflectance increases. Consequently, the reflectance of the ground surface influences the amount of initial incident light arriving on the surfaces above the sill height. This idea implicitly forms the bases of the split flux method. In the split flux formula can be recast as

$$IRC = \frac{50T \cdot W \cdot R_g \cdot R_f}{A(1-R_{avg})} + \frac{50T \cdot W \cdot R_g \cdot R_c}{A(1-R_{avg})}$$

Thus variations of the ground reflectance result in variations of the second term of the above formula. Since this term is a linear function with respect to the parameter R_g (ground reflectance) the whole term varies linearly.

Figure 3-11 shows a set of graphs calculated by using both methods, varying the reflectance of the ground from .1 to 1.0 with the increment of .1. Increases of the reflected component are monotoneous. At the same location of the reference point, the increase is by similar increments. Figure 3-12 shows this linearity explicitly.

Another noticable point of this plot, figure 3-15, is that the values with a ground reflectance of 1.0 are exactly twice that of those with the ground reflectance of

Figure 3-10. Internal Reflected Components by DEROB Method and Split Flux Method as a Function of the Ground Reflectance

Figure 3-11. Internal Reflected Component vs. the Ground Reflectance

O(zero) in both methods of calculation. This can be explained by examining the governing formulae. In the split flux method formula, the first term

$$\frac{50 \cdot T_g \cdot W \cdot R_f}{A(1 - R_{avg})}$$

is equal to the second term

$$\frac{50 \cdot T_g \cdot W \cdot R_g \cdot R_c}{A(1 - R_{avg})}$$

when R_c and R_f are identical and R_g is 1..

Thus the internal reflected component is twice of either terms. While when R_g is 0, the second term is 0. Consequently, the values with the ground reflectance of 0, are half of the value with the ground reflectance of 1. The same reasoning is applied to the proposed method, which produced the same results.

The comparison review

Reviewing these series of comparison, it can be seen that the proposed method for calculation of the daylight factor gives similar results to the approximation by current practicing methods.

The deviation of the values by these two methods are only due to the internal reflected component.

Considering the fact that the split flux method is based on averaging assumption, it is suggested that the proposed method represents the illumination levels at all point in the spaces more accurately.

THE MODEL BUILDING SIMULATION

In the preceding chapters, the discussion only dealt with the question of which factors are involved in indoor illumination levels.

Based on the logic discussed in the preceeding chapters, the investigation of how these factors influence the total daylight available, i.e., the indoor illumination level under a certain lighting conditions, are dealt in this chapter.

A series of results, obtained by the parametric studies of a model building, are presented and reviewed.

The simulated model building and surrounding conditions

The simulated structure is a ractangular parelle-piped shape of two volumes(space) connected by a wall which has a window in it. The dimensions of volume 1 which simulates an indoor space, are 30 ft. wide, 30 ft. deep and 15 ft. high. The window demension is initially set to the whole area of the connecting wall, i.e., 30 ft. wide and 15 ft. high.

Since the proposed method is based on the radiation exchanges between a system of surfaces which define a space, and on the incidental radiation coupling through the trans-

Figure 4-1. The Simulated Model Building

parent walls(windows) between multi-spaces, the outdoor space between the window and the external objects(walls) was regarded as a volume.

Thus the volume 2 which is a simulated form of the outdoor space, between the window and the objects outside of the window, is partially made of an imaginary material which transmits all the radiation perfectly. The objects outside of volume 1, which reflect light into the room are referred to as the external reflecting wall hereafter.

According to the purpose of the simulation run, the dimensions and materials of walls in volume 2 are modified. The reflectance of the ground which is the floor of volume 2 in the model building is held fixed to .1(see figure 4-1). The sky luminance distribution is assumed to be uniform in all calculations.

The effect of the orientation of the reference plane

Although there are several methods of daylight calculation available at present, the orientation of the reference plane on which measuring points are placed, are assumed to be parallel to the horizontal plane in all calculations. Higbie-Levin formula generalized the angle between the reference plane and the window plane, however, this formula is confined only to the calculation of sky factor and to rectangular windows.

Figure 4-2.
Illumination of Reference Point

Figure 4-3. Rotation of the Orientation Vector

Figure 4-4. Calculation of the Daylight Factor as a Function of the Orientation of the Measuring Plane

After the illumination distribution inside of the system of surfaces through infinitive interreflection of radiation, each surface facing to the reference point act as secondary sources of light. Therefore, as the orientation of reference plane varies, the amount of secondary sources of illumination contributable to that particular orientation of reference plane also varies(see figure 4-2)

Since the calculations of daylight factor presented here are based on the algorithm of the illumination tensor of each surface by the interreflection exchange equations and integration of available sources at the reference point, daylight factor calculations with varying orientation of the reference plane are made possible. In addition, as the orientation of the reference plane varies, the sky component also varies(see discussions on uniform sky in chapter 2).

Figure 4-3 shows an example of rotating the angle of the reference plane along the plane normal to the window and floor plane. The location of the reference point is held fixed at the center of the floor

Figure 4-4 shows that as the reference plane faces towards the window, the values of the daylight factor with a constant internal surface reflectance increase, while

the reference plane is rotated away from the window the values decrease up to the limit value which happens when the reference plane can not see the window at all. The levelling at the limit value is because the reflectance of the opaque walls is the same.

The effect of indoor surface reflectance

The reflectance of surfaces facing into the volume is the key factor for the distribution of light entering the volume. With no external reflecting wall outside, by varying only the entire reflectance of indoor wall surfaces from 0 to 1., the effects of reflectances on the indoor illumination level are investigated as shown in figure 4-5.

The glazing material of this stage of simulation, is assumed to have 100% transmittance, i.e., the window has no glazing medium and is an opening. Consequently, the daylight factor, when the reflectances of the indoor surfaces are zero(perfect absorbing surface), correspond to the CIE definition of sky factor.

The points of investigation are placed along the center line of the room on the floor, varying the distance from the window. The reference points are facing straight up, i.e., normal to the floor.

The daylight factor appears to increase as the re-

reflectance of the surface increase (ie as the absorptance of the surface decrease). In general, the results show that as the reflectance of the surface increases, the daylight factors increase significantly. The rate of increase is high at the location of a point in the internal zone (near the back wall) rather than in the external zone (near the window). A detailed review of each curve's behavior shows that, at the lower values of surface reflectance, daylight factor values decrease more steeply. As the location point moves farther away from window, daylight factor decreases as the reference point moves further away from the window in the internal zone. As the surface reflectance increases this phenomena is not so pronounced.

The reasons of these results can be explained from two points of view, 1) The sky component is getting smaller as the reference point moves further away from the window due to the variation of the solid angle (see figure 4-6). 2) At the location near the back wall, the reflected component contributes more to the daylight factor values than the location near to the window (far from the back wall). This behavior is getting more severe as the reflectance increase.

The detailed analysis of the governing formula of the internal reflected component is desirable at this point.

Figure 4-5. Daylight Factors as a Function of the Room Reflectance (case with 100% transparent window)

Figure 4-6. The Location of the Measuring Point and Solid Angle of the Window

Figure 4-7. Internal Reflected Component as a Function of the room reflectance (cases with 100% transparent window)

The formula of the split flux method can be recasted as

$$IRC = \frac{T_g \cdot W \cdot (50R_f + 5R_c)}{A(1 - R_{avg})}$$

when the window is vertical and the ground reflectance is .1. It is analytically expected the floor reflectance contribute larger portion of the internal reflected component than the ceiling reflectance by the coefficients of each parameter R_f and R_c . Thus the increase of the same extent of the floor reflectance will result in greater return than that of the roof reflectance. This is also true for the cases of artificial lighting

Due to the consideration for the psychology in a space, in most case of buildings, the reflectance of the ceiling is much higher than that of floor. Solving this problems in some other architectural solution, it is commendable to increase the floor reflectance so reasonably as to utilize daylight, or artificial lighting efficiently.

The effect of transmittance of glazing medium

The presence of glazing mediums and their light transmitting characteristics are often important factors in a real situation. The transmittance of the glass, the absorption and redistribution of light by dirt on the glass,

Figure 4-8. Daylight Factors as a Function of Room Reflectance (case with regular glass window)

Figure 4-9. Daylight Factor vs. Room Surface Reflectance

window casement and sash structure are factors which influence the amount of light entering the room in a real situation.

To calculate the daylight factor considering all these factors, however, takes laborous efforts and time, though it is possible. The results after all these calculations may not give correspondingly better returns. Thus some simplification of the calculations are needed.

Figure 4-8 shows the results obtained when the window is glazed with a single pane of regular glass, transmittance of .76, absorptance of .17, and reflectance of .07. Overall values of daylight factors are decreased when compared with the result of figure 4-5 which is the result with no glazing medium. Figure 4-9 shows a plot of daylight factor as the function of inside surface reflectance. The ratios of the values between glazed cases and unglazed case do not correspond to the ratio of the transmittances of two cases. This is due to the other characteristics, as reflectance and absorptance of the glass.

The effects of external reflecting walls

In most urbanized environments, the daylight within a building is impacted by external reflecting walls. In addition, the intensity of illumination falling on the ground from the sky is higher than that on a vertical surface.

Thus when the reflectance of the ground is high, large amounts of illumination will bounce into the indoor space from the ground surface. A room facing a neighboring external reflecting wall is subjected to high reflectance effects from it and in reality, because of the presence of direct sunlight reflected, higher effects will be experienced than the calculated value which does not count the direct sunlight.

As a model for this purpose of simulation, the outdoor space(volume 2 in model) has an opaque wall against the window and the adjacent walls and roof are assumed to be made of 100% radiation transmitting materials. The reflectance of the ground is held fixed to .1. A single pane of regular glazing covers the window opening.

The effects of the reflectance of the external wall

Walls or objects in front of the window block direct illumination from the sky to some extent depending on the solid angle from the reference point, while affecting the reflected component depending on the reflectance of the wall or object. Therefore the total impact of external reflecting walls on indoor illumination levels will be the sum of these two contradicting terms, ie the reduction of the sky component plus the increase of the reflected component.

Figure 4-10. Daylight Factor with Varied External Wall Reflectances (case with room surface reflectance = .1)

Figure 4-11. Daylight Factor with Varied External Wall Reflectances
(case with room surface reflectance = .5)

Figure 4-12. Daylight Factor with Varied External Wall Reflectances
(case with room surface reflectance = .9)

The effect of the reflectance of external wall on daylight factor indoors was investigated by varying the reflectance of the wall located 30ft away from the window.

Three sets of conditions are calculated varying the indoor space(volume 1) wall reflectances by .1, .5 and .9. For the case when the external wall reflectance is 1. (perfect reflecting surface) the highest value of daylight factor resulted. However, this value is lower than the case with no external reflecting wall.

The comparison of the results varying the room surface reflectance, shows that the deviation of values due to the variation of external wall reflectance are sensitive as the model room reflectances go up(brighter), and loose sensitivity as the reflectances go down(darker). (see figure 4-10, 11, 12) These results imply that the room wall reflectances are the factor to be considered in order to maximize the daylight utilization scheme.

The distance between the window and the external reflecting wall

The variation of the distance between external reflecting wall and window means a variation of the geometric relation of the reference point and the wall or the amount of sky available at that point(see figure 4-13).

The effect of distance between the window and the external wall on the room illumination level was investigated by changing the distance in between. Three sets of calculation were done by changing the external reflecting wall reflectance 0, .5 and .9 respectively holding other conditions constantly. The reflectance of the opaque walls are .5.

Figure 4-14, 15, 16 show that the daylight factors inside of the room vary sensitively when the external object moves nearer the distance range. The returns of moving the wall away from the window are diminishing as the distance gets longer. This result can be explained by the solid angle variation due to the increase of distance from the window and the external reflecting wall at the reference point (see figure 4-13). Note the increase of angle as the distance increases.

Figure 4-17 is the plot of the daylight factor at the center of the room as the function of the wall distance. The values increase rapidly up to the distance of 20ft., then the rates of increase slow down beyond that distance. Such behavior is even significant when the reflectance of the exterior reflecting wall is high.

Figure 4-13. The Locations of the External Object and Corresponding Solid Angle.

Figure 4-14. Daylight Factor as a Function of Distance Between the Window and the External Wall (case with external wall reflectance = 0.0)

Figure 4-15. Daylight Factor as a Function of the Distance Between the Window and the External Wall (case with external wall reflectance = .9)

Figure 4-16. Daylight Factor as a Function of the Distance Between the Window and the External Wall (case with external wall reflectance = .5)

Figure 4-17. Daylight Factor vs. the Distance Between Window and the External Wall.

The daylight factor in the room next to the open court

When the view of the window is surrounded by external reflecting walls, the room situation can be referred as the room next to the court. For instance, a room facing high rise buildings or a room facing an open shaft inside of a large building are such cases. In such situations, the illumination levels of the room are mainly dependant upon the light reflected by the court walls. Thus, the expected factors for the illumination level of the room are height, size, transmittance of the court opening and the wall surface reflectance. Here, the effect of court wall height and that of the reflectance are investigated.

The dimensions of the court were initially set to 30ft deep, 30ft side and 15ft high. The court has a unglazed toplight opening which covers the whole area of the roof. The window glazing of the room is used with a single pane of regular glass which covers whole area of that connecting wall. The reflectance of the court ground is held fixed .1 (see figure 4-18).

The height of court walls

The increase or decrease of the court wall height means increase or decrease of the amount of sky available. This, as discussed in the height of external reflecting wall,

Figure 4-18. The Court Model

Figure 4-19. The Height of Court Wall and Solid Angle

Figure 4-20. Daylight Factor as a Function of Height of the Court Wall
(case with court wall reflectance = .1)

Figure 4-21. Daylight Factor as a Function of Height of the Court Wall
(case with court wall reflectance = .5)

Figure 4-22. Daylight Factor as a Function of Height of the Court Wall
(case with court wall reflectance = .9)

Figure 4-23. Daylight Factor vs. Court Wall Height

is a factor related to the solid angle subtended to the solid angle subtended to the opening of the court at the reference point, which eventually affect the sky component (see figure 4-19).

Three sets of calculation results are presented in figure 4-20, 21, 22, which correspond to entire court wall surface reflectance of .1, .5 and .9 respectively. The results show that the higher the court wall, the lower value or daylight factors inside of the room. The decrease in the room daylight factor due to the increase of height of court wall are fast when the reflectance of court walls are low. Figure 4-23 is a plot of daylight factor as the function of court height at the center of the floor of the room. When the court wall reflectance is .9 and the height is 60 ft. above the ground, the daylight factor calculated was 4.6, while court wall is 20 ft. above the ground, the daylight factor calculated was 4.3%. These results can be understood when we consider the reflected factor in conjunction with the surface reflectance.

The effect of court wall reflectances

Similar to the effect of external wall reflectances, the reflectances of court walls will affect, initially, the illumination distribution by the reflections inside of the court volume, which incidentally affecting the radiation coupling

court volume, which incidentally affecting the radiation coupling between connected spaces.

The effect of court wall reflectance on the illumination level of the room next to the court was investigated in two cases of the court wall height.

The results show that the daylight factor in the room increase monotoneous as the reflectance increase in both cases. When the court wall is 50ft., the rate of increase is much sensitive than the case with 15ft. high court wall, especially in high reflectance range(see figure 4-24, 25, 26

The effect of the location of the window with identical dimension

One feels that the window facing south gives more illumination level than one facing north. In uniform sky luminance distribution concept, and the elimination of the effect due to the direct sun beam, the orientation of the window has no effect on the calculated value of daylight factor. When standard cloudy sky luminance distribution is adopted the window orientation still has no effect on the daylight factor, however, the altitude angle of window effects the values. In standard clear sky condition, the daylight factor varies with the orientation and altitude

Figure 4-24. Daylight Factor as a Function of Reflectance of the Court Wall
(case with court wall height = 15 feet)

Figure 4-25. Daylight Factor as a Function of Reflectance of Court Wall
(case with court wall height = 50 feet)

Figure 4-26. Daylight Factor vs. Court Wall Reflectance

angle of the window for identical window shapes and dimensions.

Here, the effect of the location of window with identical dimensions are investigated. The window dimension of the room is held constant 5ft. high and 10ft. wide. The location of window was varied as shown in figure 4-27.

Two sets of calculation are investigated varying the reflectance of the simulated room. Generally, the results show that, in the zone near the window, the variation of window location resulted in variation of the value i.e., the higher the window was located, the lower illumination in that zone. In the zone near back wall only small variation of daylight factor was detected. However, in this case, the higher the window was located, the higher illumination level was available.

The analysis of these results shows that where the outdoor illumination level is typically high, then higher locations of window are recommended, which will result in enough illumination level to the entire space avoiding extreme contrasts of illumination level.

Figure 4-27. The Location of Window with Identical Size

Figure 4-28. Daylight Factor as a Function of Location of the Window
(case with room inside surface reflectance = .1)

Figure 4-29. Daylight Factor as a Function of Location of the Window
(case with room inside surface reflectance = .5)

CONCLUSION

The prime issue of this thesis was an accurate calculation of the indoor illumination level. Although the proposed method generalized two limitations of the current practicing calculation methods - the shape of window in the sky factor calculation and the normalized estimation of the reflected component in single space, it still has a challenging problem - the exclusion of the effect of the direct illumination from the sun.

At present, many on-going daylight utilization schemes are rather concern with the materialization of the direct beam light into the building than the diffused radiation. This tendency of the developement of daylight utilization scheme will be continued in future.

It is hoped, in future, to develop a theoretical background for taking into account the effect of the direct beam light from the sun on the illumination level of indoor.

THE GEOMETRIC FACTOR MATRIX

Cloture Requirement

If there are n surfaces facing into a set of spaces such that the spaces are completely enclosed by these surfaces, then the geometric factors must satisfy the cloture requirement.

$$\sum_{i=1}^n G_{ij} = 1 \quad 1 \leq j \leq n \quad (1)$$

Defining a vector space spanned by the base vectors

$$\sum_{i=1}^n |e_i\rangle \langle e_i| = \underline{I} \quad (2)$$

where $\underline{I}$ is the unit matrix, we can define the vector

$$\langle \underline{U} | = \sum_{i=1}^n \langle e_i | \quad (3)$$

Thus, the cloture requirement can be expressed in the form

$$\langle \underline{U} | \underline{G} = \langle \underline{U} | \quad (2)$$

where the tensor $\underline{G}$ is defined such that its G_{ij} component is given by

$$G_{ij} = \langle e_i | \underline{G} | e_j \rangle \quad (4)$$

Symmetry Requirement

From the definition of geometric factors, it follows that G_{ij} must satisfy the requirement

$$G_{ij} A_j = G_{ji} A_i \quad (5)$$

where A_i is the area of the i th surface. A_i can be considered to be the i th member of the vector $|A\rangle$

$$A_i = \underline{A} \cdot |e_i\rangle \quad (6)$$

or the i th diagonal member of the diagonal matrix

$$A_{ij} = A_i = \langle e_i | \underline{A} | e_j \rangle \quad (7)$$

Thus the symmetry requirement can be expressed in the form

$$\underline{\underline{G}} \cdot \underline{\underline{A}} = \underline{\underline{A}} \cdot \underline{\underline{G}}^T \quad (8)$$

where $\underline{\underline{G}}^T$ is the transpose of $\underline{\underline{G}}$. Since $\underline{\underline{A}}$ is diagonal $\underline{\underline{A}} = \underline{\underline{A}}^T$.

The Corollary Theorem

Combining the closure with the symmetry requirement

$$\langle \underline{\underline{U}} | \underline{\underline{G}} = \langle \underline{\underline{U}} | \quad (2)$$

$$\underline{\underline{G}} \cdot \underline{\underline{A}} = \underline{\underline{A}} \cdot \underline{\underline{G}}^T \quad (8)$$

we obtain the corollary theorem of geometric factors

$$\underline{\underline{G}} \cdot \underline{\underline{A}} |U\rangle = \underline{\underline{A}} |U\rangle \quad (9)$$

The Reflection Illumination Tensor for a Single Space

Consider the case where the n surfaces enclose a single space. Each surface has a radiation transmittance (t_i), reflectance (r_i), and absorptance a_i such that

$$a_i + t_i + r_i = 1 \quad (10)$$

Defining a_i , t_i , and r_i to be the i th elements of the vectors $|a\rangle$, $|t\rangle$, and $|r\rangle$ respectively, or the i th diagonal members of the diagonal matrices $\underline{\underline{a}}$, $\underline{\underline{t}}$, and $\underline{\underline{r}}$ respectively, condition 10 becomes

$$\underline{\underline{a}} + \underline{\underline{t}} + \underline{\underline{r}} = \underline{\underline{I}} \quad (11)$$

Applying this condition to the closure requirement (eq'n 2) we get

$$\begin{aligned} \langle U | &= \langle U | \underline{\underline{G}} \\ &= \langle U | \underline{\underline{I}} \cdot \underline{\underline{G}} \\ &= \langle U | (\underline{\underline{a}} + \underline{\underline{t}} + \underline{\underline{r}}) \cdot \underline{\underline{G}} \\ &= \langle U | (\underline{\underline{a}} + \underline{\underline{t}}) \cdot \underline{\underline{G}} + \langle U | \underline{\underline{r}} \cdot \underline{\underline{G}} \end{aligned} \quad (12)$$

Bringing the 2nd term on the right hand side to the left hand side we have

$$\begin{aligned} \langle U | - \langle U | \underline{\underline{r}} \cdot \underline{\underline{G}} &= \langle U | (\underline{\underline{a}} + \underline{\underline{t}}) \cdot \underline{\underline{G}} \\ \text{or } \langle U | (\underline{\underline{I}} - \underline{\underline{P}}) &= \langle U | (\underline{\underline{I}} - \underline{\underline{r}}) \cdot \underline{\underline{G}} \\ \text{where } \underline{\underline{P}} &\equiv \underline{\underline{r}} \cdot \underline{\underline{G}} \quad \& \quad \underline{\underline{a}} + \underline{\underline{t}} = \underline{\underline{I}} - \underline{\underline{r}} \end{aligned} \quad (13)$$

Thus the cloture requirement can be re-expressed in the form

$$\langle \underline{U} | = \langle \underline{U} | (\underline{I} - \underline{r}) \cdot \underline{J} \quad (14)$$

$$\text{where } \underline{J} = \underline{G} \cdot (\underline{I} - \underline{P})^{-1} \quad (15)$$

is the reflection illumination tensor for a single space.

The Interpretation of the Reflection Illumination Tensor

Re-express equation 15 in the form

$$\underline{J} = \underline{G} \cdot \underline{\sigma} \quad (16)$$

$$\underline{\sigma} \equiv (\underline{I} - \underline{P})^{-1} \quad (17)$$

The element G_{ij} represents the fraction of the diffuse radiation originating from surface j that is intercepted by surface i on a first incidence.

The element J_{ij} represents the fraction of the diffuse radiation originating from surface j that is intercepted by surface i after all reflections have been accounted for. Note that $\underline{J}$ can also be written in the form

$$\underline{J} = \underline{G} + \underline{G} \cdot \underline{P} \cdot \underline{\sigma} \quad (18)$$

Thus $J_{ij} > G_{ij}$. The difference, $\underline{G} \cdot \underline{P} \cdot \underline{\sigma}$, is the result of internal reflections.

The matrix $\underline{\sigma}$ is the illuminance term. σ_{ij} represents the relative source strength from surface i due to reflections from the radiation source at surface j .

The Symmetry Property of the Reflection Illumination Tensor

The corollary theorem of the geometric factors is given by eq'n 9.

$$\underline{G} \cdot \underline{A} \cdot | \underline{U} > = \underline{A} | \underline{U} > \quad (19)$$

Substituting 19 into 9 we obtain

$$\underline{J} \cdot \underline{A} \cdot | \underline{U} > - \underline{J} \cdot \underline{r} \cdot \underline{A} \cdot | \underline{U} > = \underline{A} | \underline{U} >$$

Since for a single space $\underline{r}$ is diagonal, the matrices $\underline{r}$ and $\underline{A}$ commute;

$$\underline{r} \cdot \underline{A} = \underline{A} \cdot \underline{r}$$

consequently

$$\underline{J} \cdot \underline{A} \cdot (\underline{I} - \underline{r}) \cdot |\underline{U}\rangle = \underline{A} |\underline{U}\rangle \quad (20)$$

Which is the statement of the corollary theorem in terms of the reflection illumination tensor.

Take the transpose to the closure statement for reflection illumination tensors (eq'n 14)

$$\underline{J}^T \cdot (\underline{I} - \underline{r}) \cdot |\underline{U}\rangle = |\underline{U}\rangle \quad (21)$$

and combine the result with 20 to obtain

$$(\underline{J} \cdot \underline{A} - \underline{A} \cdot \underline{J}^T) \cdot (\underline{I} - \underline{r}) |\underline{U}\rangle = 0 \quad (22)$$

which states that

$$\underline{J} \cdot \underline{A} = \underline{A} \cdot \underline{J}^T \quad (23)$$

or

$$J_{ij} A_j = J_{ji} A_i$$

which is the symmetry requirement in terms of J's

Multispace Coupling: The Transmission Illumination Tensor

Consider the two spaces shown in the figure

Each space is "enclosed" by 2 surfaces.
Space 1 by surfaces 1 and 2; space 2 by
surfaces 2 and 3.

A geometric factor tensor can be defined for each space

$$\underline{G}^{(1)} = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix} \quad \underline{G}^{(2)} = \begin{pmatrix} G_{33} & G_{34} \\ G_{43} & G_{44} \end{pmatrix} \quad (24)$$

For the special case shown, of course, $G_{11} = G_{22} = G_{33} = G_{44} = 0$ and $G_{12} = G_{21} = G_{34} = G_{43} = 1$.

Likewise, the reflection illumination tensor can be found for each space.

$$\underline{J}^{(1)} = (\underline{I} - \underline{r}_1 \underline{r}_2)^{-1} \begin{pmatrix} r_2 & 1 \\ 1 & r_1 \end{pmatrix} \quad \underline{J}^{(2)} = (\underline{I} - \underline{r}_3 \underline{r}_4)^{-1} \begin{pmatrix} r_4 & 1 \\ 1 & r_3 \end{pmatrix} \quad (25)$$

where r_i is the reflectance of the surface i

$J_{ij}^{(1)}$ represents the fraction of the radiation originating from surface j that arrives at surface i after all internal reflections within space 1 have been considered

Thus, if S_1 is the source flux from surface 1, the total radiation arriving at 2 is

$$\begin{aligned} R_{21} &= J_{21} S_1 \\ &= \frac{S_1}{(1-r_1 r_2)} \end{aligned} \quad (26)$$

The total radiation absorbed by surface 2 is

$$\begin{aligned} A_2 &= a_2 R_{21} \\ &= \frac{a_2 S_1}{1-r_1 r_2} \end{aligned} \quad (27)$$

while the radiation transmitted through surface 2 is

$$T_2 = t_2 S_1 / (1-r_1 r_2) \quad (28)$$

Likewise, the radiation absorbed by surface 1 is

$$A_1 = a_1 J_{11} S_1 = \frac{a_1 r_2}{(1-r_1 r_2)} S_1$$

and what was transmitted out through surface 1 is

$$T_1 = t_1 J_{11} S_1 = \frac{t_1 r_2}{(1-r_1 r_2)} S_1$$

The total energy absorbed and transmitted via the two surfaces:

$$\begin{aligned} A_1 + T_1 + A_2 + T_2 &= \frac{(a_1 + t_1) r_2 + (a_2 + t_2)}{(1-r_1 r_2)} \\ &= S_1 \end{aligned} \quad (29)$$

is, of course, the total radiation source (note that $a_i + t_i + r_i = 1$), which is of course, the interpretation of the closure requirement

$$\langle \underline{U} \rangle = \langle \underline{U} \rangle (\underline{a} + \underline{t}) \cdot \underline{J} \quad (14)$$

Now, the radiation transmitted out through surface-1 is assumed lost to the outdoors, while the radiation transmitted through surface 2 goes into space 2. Thus this transmitted radiation can be considered as the source strength originating from surface 3

$$S_3 = T_2 \quad (30)$$

$$S_3 = \frac{t_2 S_1}{(1-r_1 r_2)}$$

After internal reflections are considered within space 2, the total radiation arriving at surfaces 3 and 4 respectively are

$$A_3 = J_{33} S_3 = \frac{r_4 t_2 S_1}{(1-r_3 r_4)(1-r_1 r_2)} \quad (31)$$

$$A_4 = J_{43} S_3 = \frac{t_2 S_1}{(1-r_3 r_4)(1-r_1 r_2)}$$

Part of the radiation at surface 4 is transmitted to the outdoors, and part of the radiation arriving at surface is transmitted back into space 1. In this form, surface 2 behaves as a radiation source into space 1 with strength

$$\begin{aligned} S_2 &= t_3 A_3 \\ &= \frac{t_3 r_4 t_2 S_1}{(1-r_3 r_4)(1-r_1 r_2)} \end{aligned} \quad (32)$$

This process continues until all the radiation has either been absorbed or transmitted outdoors. In this limit, the radiation received by each surface is given by

$$\begin{aligned} R_1 &= (J_{11} + t_2 t_3 J_{12} J_{21} J_{33} / D) S_1 \\ R_2 &= (J_{21} / D) S_1 \\ R_3 &= (t_2 J_{21} J_{33} / D) S_1 \\ R_4 &= (t_2 J_{21} J_{43} / D) S_1 \end{aligned} \quad (33)$$

Note, of course, that energy is conserved since

$$(a_1 + t_1) R_1 + A_2 R_2 + A_3 R_3 + (A_4 + T_4) R_4 = S_1$$

Formally these results can be obtained by noting that the closure requirement for the reflected illumination tensors of each space can be combined into a single statement

$$(1,1,1,1) = (1,1,1,1) \begin{pmatrix} (a_1+t_1) & 0 & - & - & - \\ 0 & (a_2+t_2) & 0 & & \\ & 0 & (a_3+t_3) & 0 & \\ & & 0 & (a_4+t_4) & \end{pmatrix} \begin{pmatrix} J_{11} & J_{12} & & \\ J_{21} & J_{22} & & \\ & & J_{33} & J_{34} \\ & & J_{43} & J_{44} \end{pmatrix} \quad (34)$$

$$\langle U | = \langle U | \cdot (\underline{a} + \underline{t}) \cdot \underline{J} \quad (14)$$

The fact that the radiation transmitted through surface 3 serves as an extended source for surface 4 can be represented through the transposition operator $\underline{T}$.

$$\underline{T} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

note that $\underline{T} \cdot \underline{t} = \begin{pmatrix} t_1 & & & \\ & 0 & t_3 & \\ & t_2 & 0 & \\ & & & t_4 \end{pmatrix} \quad (35)$

and $\underline{T} |U\rangle = |U\rangle$

The Rule to Generate the Transmission Operation

Number the surfaces facing into interior spaces so that all the numbers are sequential for surfaces facing any given space. Use the unit matrix, dimensioned to the total number of surfaces. If surfaces i and j are opposite surfaces of the same wall so that surface i faces into one volume, and surface j into another, then zero out the diagonal terms (i,i) and (j,j) , and set to unity the elements (i,j) and (j,i) .

Properties of the Transposition Operator

Symmetric $\underline{T} = \underline{T}^T$

$$\text{Unitary} \quad \underline{T}^T = \underline{T}^{-1} \quad (36)$$

$$\text{Consequently} \quad \underline{T} \cdot \underline{T} = \underline{I}$$

$$\text{and} \quad \underline{T}|U\rangle = |U\rangle$$

The operator can be expressed as the sum of a diagonal matrix, and an off-diagonal matrix

$$\underline{T} = \underline{\theta} + \underline{\zeta} \quad (37)$$

$\underline{\theta}$ = diagonal components

$\underline{\zeta}$ = off-diagonal components

Since the surface area of one side of the wall is equal to the surface area of the other side

$$A_i = A_j \quad i \text{ \& } j \text{ opposite side of the same wall}$$

$$\underline{T} \cdot \underline{A} = \underline{A} \cdot \underline{T} \rightarrow \underline{\zeta} \cdot \underline{A} = \underline{A} \cdot \underline{\zeta}$$

The Transmission Illumination Tensor - Formal Argument

Start out with the closure requirement

$$\langle U| = \langle U| (\underline{a} + \underline{t}) \cdot \underline{J} \quad (14)$$

$$\text{express} \quad \underline{t} = \underline{T} \cdot \underline{T} \cdot \underline{t}$$

$$\underline{t} = \underline{T} \cdot (\underline{\theta} + \underline{\zeta}) \cdot \underline{t} \quad (38)$$

Substitute 38 into 14 and re-arrange

$$\langle U| = \langle U| (\underline{a} + \underline{\theta} \cdot \underline{t}) \cdot \underline{J} + \langle U| \underline{\zeta} \cdot \underline{t} \cdot \underline{J} \quad (39)$$

Making the assumption that $\langle U| \underline{\zeta} \cdot \underline{t} \cdot \underline{J}$ is the transmission source vector we can perform the series to obtain the closure requirement for the transmission illumination tensor

$$\langle U| = \langle U| \underline{\zeta} \cdot \underline{t} \cdot \underline{J} = \langle U| (\underline{A} + \underline{\theta} \cdot \underline{t}) \cdot \underline{J}$$

$$\langle U| = \langle U| (\underline{A} + \underline{\theta} \cdot \underline{t}) \cdot \underline{F} \quad (40)$$

where the transmission illumination tensor is given by

$$\underline{F} = \underline{J} \cdot (\underline{I} - \underline{\zeta} \cdot \underline{t} \cdot \underline{J})^{-1} \quad (50)$$

The element F_{ij} represents the fraction of the radiation originating from surface j that arrives to surface i after all reflections and transmissions through connected volumes have been considered.

The Symmetry Property of the Transmission Illumination Tensor

The corollary theorem expressed in terms of the reflection illumination tensor is given by equation 20

$$\underline{J} \cdot \underline{A} (\underline{a} + \underline{t}) |U\rangle = \underline{A} |U\rangle \quad (20)$$

From the equation defining the transmission illumination tensor (see equation 50), we have

$$\underline{J} = \underline{F} \cdot (\underline{1} - \underline{\zeta} \cdot \underline{t} \cdot \underline{J}) \quad (51)$$

Apply equation 51 to 20 to obtain

$$\begin{aligned} \underline{F} \cdot \underline{A} \cdot (\underline{a} + \underline{t}) |U\rangle - \underline{F} \cdot \underline{\zeta} \cdot \underline{t} \cdot \underline{J} \cdot \underline{A} (\underline{a} + \underline{t}) |U\rangle &= \underline{A} |U\rangle \\ &\quad \underline{A} |U\rangle \\ \underline{F} \cdot \underline{A} \cdot (\underline{a} + \underline{t}) |U\rangle - \underline{F} \cdot \underline{\zeta} \cdot \underline{t} \cdot \underline{A} |U\rangle & \end{aligned} \quad (52)$$

Since $\underline{T} |U\rangle = |U\rangle$ and $\underline{T} = \underline{\theta} + \underline{\zeta}$

$$|U\rangle = \underline{\theta} |U\rangle + \underline{\zeta} |U\rangle$$

$$\underline{F} \cdot \underline{A} \cdot (\underline{a} + \underline{t} \cdot \underline{\theta}) |U\rangle + \underline{F} \cdot (\underline{A} \cdot \underline{t} \cdot \underline{\zeta} - \underline{\zeta} \cdot \underline{t} \cdot \underline{A}) |U\rangle = \underline{A} |U\rangle \quad (53)$$

Since both $\underline{A}$ and $\underline{t}$ are diagonal; and since the 2 sides of a wall have the same surface area

$$\underline{A} \cdot \underline{t} \cdot \underline{\zeta} = \underline{t} \cdot \underline{\zeta} \cdot \underline{A}$$

Consequently equation 53 becomes

$$\underline{F} \cdot \underline{A} \cdot (\underline{a} + \underline{t} \cdot \underline{\theta}) |U\rangle + \underline{F} \cdot (\underline{t} \cdot \underline{\zeta} - \underline{\zeta} \cdot \underline{t}) \cdot \underline{A} |U\rangle = \underline{A} |U\rangle \quad (54)$$

Thus, if every wall has the same transmittance from either direction

$$\underline{t} \cdot \underline{\zeta} = \underline{\zeta} \cdot \underline{t} \quad (55)$$

then we can express the corollary theorem in terms of the transmission illumination tensor in the form

$$\underline{F} \cdot \underline{A} \cdot (\underline{a} + \underline{t} \cdot \underline{\theta}) |U\rangle = \underline{A} |U\rangle \quad (56)$$

Now, using the closure expression for the transmission illumination tensor

we have its transpose

$$|U\rangle = \underline{\underline{F}}^T \cdot (\underline{\underline{a}} + \underline{\underline{t}} \cdot \underline{\underline{\theta}}) |U\rangle \quad (56)$$

Now, using the cloture expression for the transmission illumination tensor we have its transpose

$$|U\rangle = \underline{\underline{F}}^T \cdot (\underline{\underline{a}} + \underline{\underline{t}} \cdot \underline{\underline{\theta}}) |U\rangle$$

Applying it to the right hand side of 56 we obtain

$$(\underline{\underline{F}} \cdot \underline{\underline{A}} - \underline{\underline{A}} \cdot \underline{\underline{F}}^T) \cdot (\underline{\underline{a}} + \underline{\underline{t}} \cdot \underline{\underline{\theta}}) |U\rangle \quad (57)$$

which states that

$$\underline{\underline{F}} \cdot \underline{\underline{A}} = \underline{\underline{A}} \cdot \underline{\underline{F}}^T$$

$$\text{or } F_{ij} A_j = F_{ji} A_i$$

which is the symmetry requirement in terms of F's

Summary of Theorems

	Cloture	Symmetry ⁽¹⁾	Corollary
1st Incidence	$\langle \underline{\underline{U}} = \langle \underline{\underline{U}} \underline{\underline{G}}$	$\underline{\underline{G}} \cdot \underline{\underline{A}} = \underline{\underline{A}} \cdot \underline{\underline{G}}^T$	$\underline{\underline{G}} \cdot \underline{\underline{A}} U\rangle = \underline{\underline{A}} U\rangle$
Reflection	$\langle \underline{\underline{U}} = \langle \underline{\underline{U}} (\underline{\underline{a}} + \underline{\underline{t}}) \cdot \underline{\underline{J}}$	$\underline{\underline{J}} \cdot \underline{\underline{A}} = \underline{\underline{A}} \cdot \underline{\underline{J}}^T$	$\underline{\underline{J}} \cdot \underline{\underline{A}} (\underline{\underline{a}} + \underline{\underline{t}}) U\rangle = \underline{\underline{A}} U\rangle$
Transmission	$\langle \underline{\underline{U}} = \langle \underline{\underline{U}} (\underline{\underline{a}} + \underline{\underline{\theta}} \cdot \underline{\underline{t}}) \cdot \underline{\underline{F}}$	$\underline{\underline{F}} \cdot \underline{\underline{A}} = \underline{\underline{A}} \cdot \underline{\underline{F}}^T$	$\underline{\underline{F}} \cdot \underline{\underline{A}} \cdot (\underline{\underline{a}} + \underline{\underline{\theta}} \cdot \underline{\underline{t}}) U\rangle = \underline{\underline{A}} U\rangle^*$

* these hold only if the transmittance through opposite sides of a wall are the same

$$\underline{\underline{J}} = \underline{\underline{G}} \cdot (\underline{\underline{I}} - \underline{\underline{r}} \cdot \underline{\underline{G}})^{-1}$$

$$\underline{\underline{F}} = \underline{\underline{J}} \cdot (\underline{\underline{I}} - \underline{\underline{\zeta}} \cdot \underline{\underline{t}} \cdot \underline{\underline{J}})^{-1}$$

Primary, Reflected and Transmitted Sources

The most general statement of the illumination tensor contains all the transmitted and reflected component

$$\underline{\underline{F}} = \underline{\underline{J}} \cdot (\underline{\underline{I}} - \underline{\underline{\zeta}} \cdot \underline{\underline{t}} \cdot \underline{\underline{J}})^{-1} \quad (50)$$

(1) The symmetry properties allow us to define only the diagonal and one half of the array in DEROB thus reducing the demand for core memory.

Sometimes it is useful to express this quantity in terms of first incidence, reflected and transmitted components. To do so, we can re-arrange equation 50 in the form

$$\underline{F} = \underline{J} + \underline{F} \cdot \underline{r} \cdot \underline{t} \cdot \underline{J} \quad (58)$$

Using the defining equation of $\underline{J}$ (15)

$$\underline{J} = \underline{G} \cdot (\underline{1} - \underline{r} \cdot \underline{G})^{-1} \quad (15)$$

and re-arranging it in the form

$$\underline{J} = \underline{G} + \underline{J} \cdot \underline{r} \cdot \underline{G} \quad (59)$$

substitute this result into 58 to obtain

$$\underline{F} = \underline{G} + \underline{J} \cdot \underline{r} \cdot \underline{G} + \underline{F} \cdot \underline{r} \cdot \underline{t} \cdot \underline{J} \quad (60)$$

where each term can be interpreted as

1. $\underline{G}$ first incidence contribution
2. $\underline{J} \cdot \underline{r} \cdot \underline{G}$ reflected contribution
3. $\underline{F} \cdot \underline{r} \cdot \underline{t} \cdot \underline{J}$ transmitted contribution

Luminance Tensor

Using the defining equations of $\underline{F}$ and $\underline{J}$, $\underline{F}$ can be expressed in the form

$$\underline{F} = \underline{G} \cdot \underline{\sigma} \quad (61)$$

where
$$\underline{\sigma} = (\underline{1} - \underline{r} \cdot \underline{G})^{-1} \cdot (\underline{1} - \underline{r} \cdot \underline{t} \cdot \underline{J})^{-1} \quad (62)$$

is the luminance tensor. If element σ_{ij} can be interpreted to say that, σ_{ij} is the relative luminance strength from surface i due to a primary source from surface j .

Thus, if we know the view angle factor of surface i from some observation point, then the product of this angle factor with σ_{ij} will give us the effective view factor of the source surface j from the observation point.

The luminance tensor can also be expressed as a linear super-position of the first incidence, the reflected, and the transmitted terms. Use equations 60, 50, and 15 to obtain equation 61 once again, but $\underline{\sigma}$ is expressed in the form

$$\underline{\sigma} = \underline{1} + (\underline{1} - \underline{r} \cdot \underline{G})^{-1} \underline{r} \cdot \underline{G} + (\underline{1} - \underline{r} \cdot \underline{G})^{-1} (\underline{1} - \underline{r} \cdot \underline{t} \cdot \underline{J})^{-1} \underline{r} \cdot \underline{t} \cdot \underline{J} \quad (63)$$

primary term
reflection term
transmission term

Sky Diffuse Radiation

Every surface that belongs to a wall facing into the ambient air is treated as an extended source of diffuse radiation. In general the sky illuminance is a function of position, consequently the sky intensity perceived by each external wall is unique to the wall. Let K_i be the sky intensity perceived by wall i .

$$K_i = \langle \underline{e}_i | \underline{K} \rangle$$

$$|\underline{K}\rangle = \sum_i K_i |\underline{e}_i\rangle \quad (64)$$

The source strength of each external surface is determined by the transmittance of the wall, the area, and the perceived sky intensity

$$|\underline{S}\rangle = \underline{\theta} \cdot \underline{t} \cdot |\underline{K}\rangle \quad (65)$$

The illuminance of all the other surfaces due to these primary sources is

$$|\underline{Z}\rangle = \underline{g} \cdot |\underline{S}\rangle \quad (66)$$

while the total radiation received by a surface is

$$|\underline{R}\rangle = \underline{F} \cdot |\underline{S}\rangle \quad (67)$$

Therefore, the total radiation absorbed by the interior facing surfaces is

$$\begin{aligned} |\underline{\alpha}\rangle &= \underline{a} \cdot |\underline{R}\rangle \\ &= \underline{a} \cdot \underline{F} |\underline{S}\rangle \end{aligned} \quad (68)$$

while the total radiation transmitted out of the system of spaces is

$$|\underline{\beta}\rangle = \underline{\theta} \cdot \underline{t} \cdot \underline{F} \cdot |\underline{S}\rangle \quad (69)$$

The total radiation that entered the system of spaces is

$$\langle \underline{U} | (|\underline{\alpha}\rangle + |\underline{\beta}\rangle) = \langle \underline{U} | (\underline{a} + \underline{\theta} \cdot \underline{t}) \cdot \underline{F} \cdot |\underline{S}\rangle \quad (70)$$

which becomes after using the cloture requirement (equation 40)

$$\langle \underline{U} | (|\underline{\alpha}\rangle + |\underline{\beta}\rangle) = \langle \underline{U} | \underline{S}\rangle \quad (71)$$

The total radiation absorbed in the interior surfaces is

$$\langle \underline{U} | \underline{\alpha} \rangle = \langle \underline{U} | \underline{a} \cdot \underline{F} | \underline{S} \rangle \quad (72)$$

The total radiation transmitted back out is

$$\langle \underline{U} | \underline{\beta} \rangle = \langle \underline{U} | \underline{\theta} \cdot \underline{t} \cdot \underline{F} | \underline{S} \rangle \quad (73)$$

The albedo is the ration of the transmitted radiation to the total radiation that entered

$$\text{albedo} = \langle \underline{U} | \underline{\theta} \cdot \underline{t} \cdot \underline{F} | \underline{S} \rangle / \langle \underline{U} | \underline{S} \rangle \quad (74)$$

The Uniform Sky Approximation

If we assume that the diffuse sky intensity is uniform and equal to K, then

$$\begin{aligned} |\underline{K}\rangle &= K \sum_i |\underline{e}_i\rangle \\ &= K |\underline{U}\rangle \end{aligned} \quad (75)$$

The sky diffuse intensity is given by the weather tape, or it is calculated from weather information.

The effective area of diffuse insolation is given by

$$|\underline{d}\rangle = \frac{1}{K} |\underline{R}\rangle \quad (76)$$

$$|\underline{d}\rangle = \underline{F} \cdot \underline{\theta} \cdot \underline{t} |\underline{U}\rangle \quad (77)$$

and the effective area of absorbed diffuse radiation

$$|\underline{d}^{(a)}\rangle = \underline{a} \cdot \underline{F} \cdot \underline{\theta} \cdot \underline{t} |\underline{U}\rangle \quad (78)$$

This vector is calculated in subroutine DIFRAD in DRBLUM and it is passed to DRBTL to be used directly by the time loop solution.