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Energy and Environment Division

Reliability Planning in Distributed Electric Energy Systems

Edward Kahn

October 1978

Lawrence Berkeley Laboratory University of California/Berkeley

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1.0 INTRODUCTION

This paper is an exercise in strategic energy planning. The goal is to develop tools for technology evaluation which address questions involving the economics of large scale systems. Underlying this point of view is the assertion that the engineering constraints associated with different kinds of energy technologies are so different from one another that there are serious implications for costs in the large. Choosing one kind of energy conversion technology as opposed to another involves systems costs. This perspective is not marginalist. Rather it rests upon a claim that important large scale systems costs cannot easily be addressed by looking only at the next incremental project and its alternatives. The kind of cost we will be discussing usually involves some dynamic aspect of the energy system. In particular such properties as flexibility, stability, and resilience are features of entire systems.

The strategic perspective in energy technology evaluation has received most attention to date in the study of financial viability and stability of electric utilities. Ford has shown the financial advantage of small power plants compared to large units given uncertain future electricity demand.⁽³²⁾ The main feature in this study is the advantage of short lead time projects in tracking cyclic fluctuations of demand. Willey has shown the comparative financial advantages to utilities and their customers of an energy supply system based largely on solar, and conservation as opposed to large base load plants.⁽³³⁾ Kahn and Schutz have proposed a framework for financial risk assessment that distinguishes various kinds of financial risk associated with different technologies.⁽³⁴⁾

These analyses are all concerned in one way or another with the coupling between an energy system (i.e. a utility system) and the macro-economy in which it is embedded. These discussions do not look at the technical and economic consequences within the energy system itself of strategic technological choices. This is particularly significant where implementation of solar resources is anticipated. Attention must be paid to the question of

reliability; i.e., availability on demand. It is often claimed for example that solar alternatives will require so much storage to smooth out the mismatch of supply and demand patterns that these options are uneconomic.⁽³⁵⁾ The storage problem and the planning for reliability in utility systems are the subjects of this paper.

1.1 Reliability Planning: Preliminary Definitions

Electric energy systems consist of a variety of components which are designed into a system that provides service in response to time-varying demand. Fitting the various components together, integrating the system, is constrained by economics and service standards. Planning engineers seek to minimize the cost of providing electric service at specified levels of reliability. Without a reliability constraint, economies can always be achieved by degrading the quality of service delivered. Due in part to the complex nature of electric energy systems, there has not evolved a single measure of reliability. Instead there are indices which characterize the performance of subsystems. Ideally a power system planner would like to have models which show the relationship between reliability and cost components for the whole system. Unfortunately he would be lucky to have such models even for the major subsystems of generation, transmission and distribution.

In this exercise we will limit attention to the generation system. We will focus on the contrast between conventional power generation technologies with controllable output and intermittent resources such as wind and solar electric conversion devices. We will address the storage problem strategically rather than incrementally. That is we will imagine hypothetical power systems dominated by either conventional central station plants or wind generators and compare bulk storage requirements. This analysis is carried out in Section 3 after we develop a parametric characterization of the technologies studied in Section 2. The system studied is a stylized representation of California conditions.

1.2 Uncertainty, Resilience and Other Sensitivities

We are really, of course, interested in the basic relations among utility loads, resources, storage and reliability. This means that under-

standing the sensitivity of results to parameter changes is essential. We will examine the implications of various uncertainties and the importance of certain initial assumptions. One question of this kind is the ability of different systems to withstand large exogenous disturbances. The usual power system planning framework does not address itself to the occurrence of droughts, coal strikers or major inter-regional supply deficiencies. The ability to absorb such shocks gracefully has been called the "resilience" of a system. In Section 4 we formulate this property and compare the central station power system to the intermittent resource system. Our analysis supports the thesis associated with Lovins that the intermittent resource technologies produce a more resilient system.

We also examine the importance of our limited knowledge concerning the capacity factors of various generator types and the role of reliability standards. We will estimate trade-offs between storage capacity requirements for our two systems and the **service** standards, i.e., the design level of reliability, we will find that lower standards are relatively more favorable to the intermittent resource technologies.

Finally we address ourselves to the design of bulk power storage as a back-up to each of our two systems. We will argue that the quality of back-up requirements varies radically in the two systems. The magnitude and duration of expected outages; i.e., back requirements, are sufficiently different to require very different sizing of the storage unit. This logic will be applied to the case of hydroelectric facilities with large energy storage that are currently part of the California power system.

2.0 PARAMETRIC CHARACTERIZATION OF DISTRIBUTED AND CONVENTIONAL TECHNOLOGIES

We will be studying two different distinctions that characterize power plants; conventional vs. distributed and controllable vs. intermittent. These distinctions are related in the following way:

Drawing up this conceptual map requires that we invoke rather different criteria at each level. The notion of distributed vs. conventional technology is defined primarily by criteria which speak to the social functioning of engineering devices, rather than to technological parameters alone. The analysis of Lovins, for example, is based on qualities such as flexibility, thermodynamic appropriateness and relative scale.⁽⁵⁾ These concepts can only be defined of the interface of society and technology. However useful such criteria may be, they tend to obscure the basic technical difference between those generators with controllable output and those which are inherently intermittent. Thus there is a class of distributed technologies (fuel cells, co-generation, geothermal electric) which resemble conventional technology in their output characteristics. We can turn all of these qualitative distinctions into a formal framework by translating the output characteristics of each class into probabilistic language. The main concept used is the relative variance on availability.

2.1 The Small Variance Technologies

Power generators using conventional technologies with controllable output are modeled as independent random variables in system reliability analysis.⁽²⁷⁾ Each unit has a probability distribution of availability that is parameterized by the random forced outage rate. The mean $\bar{S}_i$ and the variance $\sigma_{s_i}^2$ on power availability of such technologies are given by the expressions below,

$$\bar{S}_i = \alpha_i (1 - L_i) \quad (1)$$

$$\sigma_{s_i}^2 = \alpha_i^2 (1 - L_i) L_i \quad , \quad (2)$$

where

α_i = capacity of unit i

L_i = random forced outage rate of unit i .

These expressions are discussed in references 2 and 12. It is clear from equation (2) that unit size has a significant impact on the variance of available power. A numerical example will illustrate this.

Let us compare two resource plans for adding 1000 MW of rated capacity to a power system. One option is the addition of a single 1000 MW plant. Another is the addition of 10 plants, rated at 100 MW apiece. An example might be nuclear versus geothermal. Assuming a forced outage rate (FOR) for each alternative will allow us to calculate total variance and compare. This involves an extra subtlety since FOR typically increases with unit size. To check this sensitivity we will also add a variation which includes an FOR 50 percent greater in the large unit case. The results of these calculations are given in Table 1. Note that total variance in the multi-unit case is n times the single unit variance, where n is the number of units.

Table 1
Power Availability Variance* as a
Function of Unit Size and FOR

1000 MW, FOR = .15	$\sigma_s^2 = .1275$
1000 MW, FOR = .10	$\sigma_s^2 = .0900$
10 units, 100 MW, FOR = .10	$\sigma_s^2 = .0090$

*Variance is in units of 1000 MW²

The interpretation of these variances will be discussed more fully in Section 3. They are related to backup requirements. What is of interest for now is the comparison of relative effects. Small units contribute disproportionately smaller variance. There is an economy of back-up or reserve margin. ⁽¹²⁾

Within the category of generators that are in principle controllable and can plausibly be modelled on the unit level as independent random variables, scale is the main distinction between distributed and conventional. Nuclear plants and conventional coal fired generators typically come in large sizes, i.e., 800-1000 MW. ⁽³⁶⁾ On the other hand, such distributed generators as fuel cells or geothermal plants are expected in small sizes, i.e., 25-50 MW. The FOR values listed in Table 1 are also consistent with EPRI estimates. ⁽³⁶⁾

2.2 Correlated Output Technologies: Consequences for Variance

Wind turbines and solar electric generating devices cannot be modeled as independent random variables for capacity availability purposes. Even over hundreds of miles separation, there is usually a correlation between insolation at pairs of sites. The same is true for wind power. This property has been modeled explicitly in the wind resource studies.⁽¹⁰⁾ It is implicit in simulation studies of solar electric technology such as reference 1.

A subtle but significant problem involved in modeling such correlated resources is choosing the geographical region over which correlations should be estimated. This involves both resource and institutional issues because the optimal geographical region from the physical viewpoint is unlikely to coincide with smaller utility company service area boundaries. The optimality of a region over which to integrate power system planning depends upon the total mix of resources. In reference (11) Kahn has found smaller capacity credit for a region with "better" wind availability compared to one with somewhat poorer wind availability. Such results show the influence of the other elements in a power system.

The most useful simplification of the problem is the development of some measure of the relation between correlation, geographical diversity and power availability variance. We address this by looking at the average variance of a correlated array of sites compared to the variance of an individual site, as follows:

$$\sigma_A^2 = \sigma_i^2 \left[\frac{1 + (n-1)\bar{\rho}}{n} \right] \quad (3)$$

where

- σ_A^2 = average array output variance
- σ_i^2 = individual site output variance
- n = number of sites in array
- $\bar{\rho}$ = average array correlation coefficient

A transformed version of Eq. (3) is graphed in Figure 1, where the level

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Figure 1. Array to single site standard deviation ratio
 σ_A/σ_i for array with n sites and cross correlation $\bar{\rho}$.

curves show the ratio of array standard deviation σ_A to individual site standard deviation σ_i .

Figure 1 shows that the ratio, which expresses the effect of geographical diversity on the square root of output variance, is insensitive to n when it is greater than 10. Therefore once we know something about the average correlation coefficient $\bar{\rho}$ for either wind or solar and something about σ_i^2 , the typical variance at a site, we can estimate the behavior of arrays. For large arrays, Fig. 1 suggests we can neglect details of the siting configuration and just parameterize by $\bar{\rho}$. Although the array problem can be posed for joint systems of both dispersed wind and solar, data is insufficient even for our limited purposes. Instead we will assess wind and solar arrays separately. Even here the data is of poor quality, but we can derive some first approximations from it.

Wind speed correlations have been studied for California (10,11). Qualitatively, the behavior of individual sites shows a lot of variance compared to the mean. For example, Sacramento data yields a coefficient of variation of .52, which is the ratio of the standard deviation of wind speed to $\bar{V}$, the mean wind speed (11). This gets considerably reduced in arrays since the average correlation coefficient is low, roughly 0.3. We can convert this data to power using a linear approximation introduced by Justus (10). In section 3 we will develop such data for use in the normal model of LOLP. The technical details of this adaptation are discussed in Appendix 1.

Insolation correlations have not gotten a very extensive treatment. For data relevant to California we rely on one limited simulation study done for EPRI by Aerospace Corporation (1). This data shows that the individual sites have much less relative variance compared to wind sites. The coefficient of variation is typically .17. A rough estimate of correlation shows much greater values than in the case of wind. In Appendix 2 we develop an estimate of $\bar{\rho} = .8$ and σ/μ the coefficient of variation. The product of these effects shows that solar arrays have much less variance per unit than wind turbine arrays. These results are suggestive, but uncertain even in comparison with the wind statistics. In Section 3 below we will neglect these uncertainties and deal only with solar cogeneration units that are

assumed to have enough storage to be modelled as independent random variables.

3.0 THE COMPARATIVE STORAGE PROBLEM FOR CALIFORNIA: SCENARIOS AND ANALYSIS

The comparative storage problem for conventional and intermittent resources was formulated in a particularly clear and simple form by Martin Ryle.⁽²⁴⁾ Ryle postulates a conclusion to the problem by simply asserting that storage requirements for a power system dominated by nuclear generators would be the same as that for one dominated by intermittents. The qualitative argument supporting this conclusion rests on what might be called a "baseload excess." The nuclear units would produce excess energy at seasonal or daily low points in demand and would not be sized to meet peak demand. Thus storage would be needed to track demand fluctuations. Otherwise a huge amount of excess baseload capacity would be necessary. Even today large generators are being required to cycle in response to supply and demand patterns.⁽³¹⁾ If Ryle's hypothesis were correct, then economic evaluation could be performed by comparing average unit capacity factors. If the nuclear plants operated at an average rate equivalent to 60 percent full utilization and the wind generator at 30 percent average, then twice the capacity of wind machines is required to displace a given nuclear capacity. The comparative economics are only this simple if Ryle's equal storage hypothesis holds. We will analyze this thesis by specifying a context in which the relative storage needs can be calculated.

It is convenient to rely upon the scenarios developed by the UC-LBL Distributed Technology Study for a hypothesized 2025 California energy system.⁽⁵⁾ In this study a demand level for electric energy is specified by end-use methods and matched to specified energy resources. A comparison is sketched between two supply mixes, called Centralized and Distributed. Both share a more or less common base of low variance technologies (hydro, geothermal, cogeneration). These provide just under half the energy. The remainder of the energy is supplied by roughly 40 large central station power plants (coal or nuclear) in one case, and by wind generators in the other. No attempt was made in that study to look at reliability and systems integration questions. In this section we will ask Ryle's question of the Centra-

lized and Distributed Supply systems (5). To do this we must first introduce and discuss the main methodological tool of the analysis, the normal model of LOLP.

3.1 Normal Model of LOLP

In section 2.1 we introduced expressions for the mean ($\bar{S}$) and variance (σ_S^2) of available capacity for individual conventional generators. Let us suppose we know these parameters for a given power system, that is, the sum of contributions from many generators. The Loss of Load Probability (LOLP) is the measure of reliability which tells what the chances are of failing to meet additional load when the system is already responding to some load W . We can write this as follows:

$$\text{LOLP} = \text{Prob} [S - W = 0]$$

where S is the (random variable) available supply and W is the load.

We can also consider W a random variable. This will be useful in some applications below. In any event, $S - W$ is a random variable which we can call the margin. Let us standardize this variable and re-write the definition,

$$\text{LOLP} = \text{Prob} \left[\frac{S - \bar{S}}{\sigma_S} = \frac{W - \bar{S}}{\sigma_S} \right].$$

LOLP will vary continuously with load. For a fixed power system we can look at just the right hand side of the equality and then

$$\text{LOLP} = \text{Prob} \left[\frac{W - \bar{S}}{\sigma_S} \right]$$

At this point we invoke the normal approximation technique. This amounts to assuming that the distribution of the variable of interest has the following form:

$$\text{LOLP} = F_N \left(\frac{W - \bar{S}}{\sigma_S} \right), \text{ where}$$

$$F_N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-t^2/2} dt \text{ is the cumulative}$$

normal distribution function.

A little algebra allows us to re-express this relation in terms of the complementary error function,

$$\text{LOLP} = \frac{1}{2} \text{erfc}(b), \text{ where} \quad (4)$$

$$b = \frac{\bar{S}-W}{\sqrt{2}\sigma_S} \text{ and} \quad (5)$$

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2} dt. \quad (6)$$

this form (4)-(6) allows easy numerical calculations with the standard error function tables.

The justification of the normal approximation is usually made by appeal to the Central Limit Theorem. Sums of large numbers of random variables approach the normal distribution. In the case of our particular application, however, there is also a numerical study which compares (4)-(6) to the standard discrete model of LOLP (2). These authors found that the accuracy of the approximation improved with increasing system size, average forced outage rate and number of generators. Typical minimum values for these variables in the cases where the approximation is reasonable are 20,000 MW and above for system size, 10% and above for FOR and 150 and above for the number of units. Such results can be unified under the heading of large variance. The normal approximation yields good results in systems with large variance. This will justify our calculations below.

3.2 Gross Storage Capacity Requirements

We now apply equations (4)-(6) to the alternate supply scenarios. The data and calculations are given in Table 2 for the central station case and Table 3 for the distributed/intermittent supply case. In both cases the assumed demand is 1400×10^{12} BTU or roughly 410×10^9 kWh. This is the level postulated in ref. (5). For our purposes it is an essentially arbitrary but convenient level from which to work. The fraction of energy supplied by central stations in one case and wind arrays in the other is just over half the total. In both cases the remaining energy comes from small variance distributed technologies. Although the mix of technologies differs in the two cases, results are

not sensitive to the differences. The reason for this will become apparent after some further argumentation.

3.2.1 Specification of System Parameters

We start with an estimate of energy from each resource. Next, a capacity factor is assumed for all technologies. This will tell us what the total rated capacity must be for each resource. For conventional technologies we then assume a forced outage rate and use equations (1) and (2) to calculate normal parameters. We assume that our solar cogeneration installations have sufficient on-site storage for us to model them simply as conventional cogeneration units. This means we treat them as independent random variables and ignore solar correlations and partial cloudiness. For technical reasons solar cogeneration requires on-site storage (1, 26). Where possible, capacity factor and FOR estimates are consistent with engineering experience.

Having specified supply mixes we must now choose an LOLP criterion. To begin with, we use the conventional 1 day in 10 years rule of thumb. Since this rule has many interpretations, we rely on the variation used by the Pacific Gas and Electric Company. This version specifies an upper limit on LOLP of 8.5×10^{-3} . For simplicity, we assume at first that all the risk is concentrated at the peak, i.e., we need only consider the LOLP for our maximum demand. This assumption is relaxed below. Equation (4) and the error function tables translate this requirement into the value $b = 1.69$. That is, $\frac{1}{2} \text{erfc}(1.69) = 8.5 \times 10^{-3}$.

We cannot use our model without some estimate of the peak power and load distribution expected by the systems. A first approximation involves at least a load factor which converts assumed energy into peak demand by specifying the ratio of average demand to the peak. Power systems today which must meet significant summer air conditioning peaks typically experience load factors of 50-55%. Although models adequate to predict changes in basic parameters of the load forecast distribution are still in the development stage (17), some techniques have been developed to forecast load factors. We adapt an approach used by the California Energy

Commission (19). The details of our application are contained in Appendix 3. Since such exercises are inherently uncertain, it is best to try to bound an expected forecast result by the application of an additional constraint.

High load factor alone is not an absolute benefit in system planning. At some point, as load factor increases, risk will emerge in traditionally off-peak periods. Perhaps there won't be enough seasonal slack to do maintenance. A recent EEI study documents the increased reserve margin requirements associated with high load factor (8). Although results varied across systems, the most reasonable load shifting pattern showed that a 5% shift of the peak required roughly 20% extra in reserve margin. The next 5% shift required proportionately larger back-up. Qualitatively the increasing nature of the problem with increasing load factor is clear.

For our purposes, it seems unreasonable to rely solely on the statistical regression technique of Appendix 3. One of the major limitations of that approach is its failure to include load management explicitly. In light of the second-order effect on reserve margin just discussed, we limit the peak shaving to 5% of that forecast by the Appendix 3 method. This will result in a final forecast peak of 70.0 GW and a load factor of 67%.

3.2.2 Results, Interpretation and Major Technical Uncertainties

We re-write equation (5) solving for $\bar{S}$, the mean available capacity,

$$\bar{S} = W + b \sqrt{2\sigma_S^2} \quad (7)$$

Our problem reduces to the following calculation. We have specified $W = 70.0$, $b = 1.69$ for both systems. We have estimated variance σ_S^2 and mean available capacity $\bar{S}$ in Tables 2 and 3. We find in both cases that by solving (7) for $\bar{S}$ with the estimated σ_S^2 we get a value greater than that which is available. This difference is just the storage capacity requirement, which we will call $\Delta\bar{S}$. This is summarized in (8)

$$\Delta\bar{S} = \bar{S}_{(7)} - \bar{S}_A \quad (8)$$

where

$\Delta \bar{S}$ = Storage Requirement

$\bar{S}$ = Available Capacity Needed for
(7) Reliability estimated from eq. (7)

$\bar{S}$ = mean available capacity from energy
(7) conversion units.

The calculation using equation (8) tells us only part of the story about storage. Namely how much capacity we need to meet peak demand. It says nothing about the amount of energy to be stored. Answering such questions requires greater knowledge of load distribution than we have assumed. Indeed, the accounting for storage losses would require make-up energy on the supply side, increasing energy production requirements. We ignore such complications to concentrate on the more limited results at hand.

Table 2 shows a storage requirement of 12.4 GW for the central station case compared to 15.3 GW in the distributed intermittent case of Table 3. For comparison, the hydroelectric system in California as projected to 1995 would have 9.2 GW of capacity. This suggests a major storage requirement for either outcome, the distributed case requiring roughly 25% more capacity than the central station use. This result represents an estimate within a range of outcomes since there are major uncertainties involving the average capacity factor of both central station and wind turbine technologies. To assess the impact of these uncertainties we have examined the sensitivity of results to capacity factor estimates.

Capacity factor is an important parameter because it determines capacity requirements for a fixed energy level. In the Central Station Scenario we assume capacity factors of .65 for such units. Historical performance of nuclear plants, for example, is closer to .55. Our results in Table 4 show the trade-off between capacity factor, storage and capacity requirements. When we reduce estimated capacity factor to 60%, we must add 3.0 GW of extra capacity and we remove 2.35 GW of storage. For capacity factor = .54, we must add 7.0 GW of capacity and we remove 3.70 GW of storage. These results show that we pay a high price to reduce storage with poorer unit performance.

Table 2
CENTRAL STATION SUPPLY

Resource	Energy (10^{12} Btu)	(10^9 kWh)	Capacity Factor	Rated Capacity (GW)
Cogeneration	330	96.7	.80	13.8 GW
Geothermal	200	58.6	.85	7.9 GW
Hydro	140	41.0	.50	9.2 GW
Central Station	730	213.9	.65	38.0 GW

Normal Parameters Resource	Unit Size (GW)	Number of Units	FOR	Mean	Variance
Cogeneration	.050	276	.10	12.42	.06
Geothermal	.100	79	.08	7.27	.06
Hydro	.050	184	.03	8.92	.01
Central Station	1.000	38	.15	<u>32.30</u>	<u>4.84</u>
		TOTALS		62.91	4.97

Storage Requirements

$$\text{Need } \bar{S}_n = 70.0 + 1.69 \sqrt{9.94}$$

$$= 75.33$$

for LOLP = 1 day in 10 years.

$$\text{Since } \bar{S}_A = 62.91, \text{ then } \Delta \bar{S} = 75.33 - 62.91$$

$$= 12.42 \text{ GW.}$$

$$\text{Storage Requirements} = \Delta \bar{S}$$

Data Sources

Capacity Factor:
and Forced Outage
Rates

Hydro and Geothermal represent
historical performance
Central Station from (36)
Cogeneration from (37) assuming
gas turbine

Unit Size:

Cogeneration, assuming gas turbine
Central station from (36)
Geothermal - current P.G.&E. size
Hydro - P.G.&E. dispatch
assumption

Table 3
Distributed/Intermittant Supply

Resource	Energy (10^{12} Btu)	(10^9 kWh)	Capacity Factor	Rated Capacity
Fuel Cogeneration	40	11.7	.80	1.70
Solar Cogeneration	130	38.1	.45	9.67
Geothermal	330	96.7	.85	13.00
Hydro	140	41.0	.50	9.20
Wind	760	222.7	.35	72.50

Normal Parameters Resource	Unit Size (GW)	Number of Units	FOR	MEAN	Variance
Fuel Cogeneration	.050	34	.10	1.53	.008
Solar Cogeneration	.050	193	.10	8.70	.043
Geothermal	.100	130	.08	11.96	.095
Hydro	.050	184	.03	8.92	.014
Wind				<u>38.06</u>	<u>36.25</u>
TOTALS				69.17	36.41

Storage requirements

$$\text{Need } \bar{S}_n = 70.0 + 1.69 \sqrt{72.83}$$

$$= 84.42$$

for LOLP = 1 day in 10 years.

$$\text{Since } \bar{S}_A = 69.17, \text{ then } \Delta \bar{S} = 84.42 - 69.17$$

$$= 15.25 \text{ GW.}$$

$$\text{Storage requirements} = \Delta \bar{S}$$

Table 4: Impact of Capacity Factor Uncertainty

Central Station Scenario: Sensitivity to Central Station Capacity Factor and FOR

	<u>Base Case</u>		
Capacity Factor	.65	.60	.54
Installed Capacity	38.0 GW	41.0 GW	45.0
$\bar{S}$	32.30	34.85	36.90
σ_S^2	4.84	5.22	6.64*
ΔS	12.42	10.07	8.71
Impact	Installed 3.0 GW Central station		Installed 7.0 GW Central station
	Remove 2.35 GW Bulk storage		Remove 3.70 GW Bulk storage

* FOR assumed .18

(Note: since capacity factor reflects both forced and planned outages, lower capacity factors require higher FOR's. The value chosen represents a modest adjustment for this effect.)

B. Distributed/Intermittent Scenario: Wind Array Performance

Varying capacity factor can be expressed as a variation on available capacity. Since the degree of these variations can only be determined by extensive simulation we estimate two changes for each capacity factor. The base case has $\bar{S} = (.525)$ Rated Capacity. For higher capacity factor this fraction should increase (we consider .55 and .57); conversely for lower capacity factor.

	<u>Base Case</u>		
Capacity Factor	.35	.40	.30
Installed Capacity	72.50GW	63.60GW	84.74GW
$\bar{S}$	38.06	(a)36.25 (b)34.96	(c)39.83 (d)42.87
$\Delta\bar{S}$	15.25	(a)17.05 (b)18.35	(c)13.49 (d)10.94
Impact		Remove 9.0GW wind turbine	Add 12.24GW wind turbine
		Add (a) 1.80GW (b) 3.10GW bulk storage	Remove(c)1.76GW (a)4.31GW bulk storage

Notes: (a) $\bar{S} = (.57)$ Rated Capacity

(b) = (.55) " "

(c) = (.47) " "

(d) = (.50) " "

Table 4 also shows the trade-off for wind generators. If average wind turbine capacity factor were 40%, then installed capacity requirements would drop by 9.0 GW and storage requirements go up by 1.80-3.10 GW. On the other hand, performance as poor as 30% capacity factor means an extra requirement of 12.24 GW of wind turbines with a storage savings of 1.76 to 4.31 GW. All these trade-offs are at equal system reliability. It would be useful to examine the sensitivity of our results to the somewhat arbitrary reliability criterion we have used. This subject requires a little deeper look at the nature of reliability and its relation to other notions. It is to this that we now turn.

4.0 PASSIVE RESILIENCE; OR RELIABILITY SENSITIVITY

All of our discussion so far has been in a static context. This is common in traditional generation reliability studies. The study of dynamic disturbances in power systems has been typically oriented toward the highly specified network analysis and the contingency enumeration approach (7, 28). There is a more abstract, generic approach to dynamic questions of reliability that has been postulated by Lovins and others. This is the study of system resilience.

Resilience is a difficult concept because it incorporates both a passive, behavioral notion and an active feedback control notion. A resilient system absorbs shock more easily than a "rigid" system; that is, it falls apart less easily. This is a passive characterization. The corrective response to disturbance is an active control notion. In the case of power systems, the corrective response ultimately involve the political economy in which the system is embedded. Regulatory agencies institute investigations of major disturbances and initiate action to reinforce perceived weaknesses. It is difficult to bring the entire social and political process implied by this reality into the framework of the probabilistic calculation techniques. Recently there have been efforts at partial analysis by bringing regulatory delay impacts into reserve margin and reliability analysis. These efforts are more suggestive than definitive (9).

Because the active control aspect of resilience is so difficult, we will concentrate attention on the passive aspects. From this perspective it is possible to compare the response of our two systems to changes in reliability criteria, to contrast the response to uncertainty and the impact of shocks. This discussion will proceed by examining the structure of the risk in our two supply systems.

4.1 Risk Structure and LOLP Criteria

In Figure 2 we graph LOLP as a function of load with storage capacity fixed from Tables 2 and 3. The calculation uses equations (4) and (5) where the parameter $b = b(W)$ is a function of W . The curves for the two systems are designed to intersect at our risk criterion of $\text{LOLP} = 8.5 \times 10^{-3}$ for the expected peak load $W = 70.0$ GW. What is of interest is the comparative shape and curvature.

Figure 2 shows that the central station scenario has a considerably steeper risk curve than the distributed/intermittent scenario. This geometry has several important implications. First, it says that if we were to set a more relaxed criterion for LOLP, the distributed system would carry more extra load than the central station system. We show one measure of this difference in Figure 2 with the line segment A. This is the difference between peak loads servable by the systems at $\text{LOLP} = .10$. This level is roughly speaking one day per year. That is only slightly higher than the level of 8 days in 10 years recently adopted by the New York Public Service Commission in an exploratory ruling (20). The line segment A shows the distributed system carrying roughly 4 GW of additional load beyond the extra load carried by the central station system.

The line segment A is probably an over-estimate of the impact--due to a change in LOLP criterion. The reason is the problem of accumulation of risk. Let us digress a bit into this subject. We have assumed up to now that the total risk could be represented by the risk at the time of the peak load. However, we pointed out in sec. 3.2.1 that, with the increasing load factor, risk may emerge at other times. One measure of this problem can be found using the ratio of the second highest load to the peak load

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Figure 2

LOLP as a Function of Load

and looking at the risk impact of this next biggest load. Pacific Gas and Electric Company estimates that this ratio is between .963 and .979. Using an intermediate value of .97 we get a second highest load of 67.9 GW. Figure 2 shows risk in the central station decreases by a factor of over 15, while in the distributed case the decrease is less than a factor of 3.

There are several ways in which we can correct for the accumulation of risk in systems such as our distributed scenario, provided we accept the measure associated with the second highest load. We can ask how much load must be dropped to stay at a 70.0 GW limit if we add the risk of the second highest load to the risk at peak. Let us express this as follows:

$$\begin{aligned} \text{LOLP } (W = 70.0) + \text{LOLP } (W = 67.9) &= \text{LOLP } (W = x) \\ \text{Drop load} &= 70.0 - x. \end{aligned} \quad (9)$$

Equation (9) says simply that the sum of two risks is equivalent to the risk at some load $W = x$ greater than 70.0. If we take that difference and reduce the allowable peak by it (interruptible load) we remain at our specified risk level. Doing the calculation thus sketched for the distributed case we find $x = 70.80$ GW at $\text{LOLP} = 1.2 \times 10^{-2}$. This means we must curtail 800 MW of load at peak.

A more stringent accounting for risk accumulation would begin by looking at the slope of the two curves and use the inverse of ratio of slopes to scale the ratio of allowed LOLP at peak. We express this as follows:

$$\left(\frac{\partial \text{LOLP}_{\text{CS}}}{\partial W} / \frac{\partial \text{LOLP}_{\text{DIST}}}{\partial W} \right)^{-1} = \frac{1}{2} \text{erfc}(b_{\text{CS}}) / \frac{1}{2} \text{erfc}(b_{\text{DIST}}) \quad (10)$$

where the subscripts CS and DIST stand for central station and distributed, respectively. We fix $\frac{1}{2} \text{erfc}(b_{\text{CS}}) = 8.5 \times 10^{-3}$, and approximate the derivatives by a straight line. The ratio on the left of (10) turns out to be 2/5. This means we find b_{DIST} such that $\frac{1}{2} \text{erfc}(b_{\text{DIST}}) = (.4)(8.5) \times 10^{-3}$ which is 3.4×10^{-3} . The appropriate $b_{\text{DIST}} = 1.91$. Using this value in our original storage requirement calculation of Table 3 increases storage capacity by 1.90 GW. This measure is more than twice as stringent as the previous calculation of interruption requirement. It should be noticed that none of these corrections will affect the sensitivity analysis of Table 4.

Now we return to the question of altering the LOLP criterion. We

will use equation (10), this time estimating the derivatives of LOLP = .10. The ratio changes to .50. The appropriate value of b for LOLP = .10 is .905. Using eq. (8) we get b = 1.165 for the distributed case. We re-estimate storage requirements $\Delta \bar{S}$ from equation (8);

$$\Delta \bar{S} = \bar{S}_{(7)} - \bar{S}_A.$$

We present the results in Table 5.

Table 5: Impact of LOLP Criterion

	$\bar{S}_{(7)}$	S_A	$\Delta \bar{S}$
Central Station	72.85	62.91	9.94
Distributed	79.94	69.17	10.77

The gap in required storage between the two systems has narrowed from the base case difference of 2.83 GW (= 15.25 - 12.42) to .81 GW. In Figure 2 we represent the impact of a relaxed LOLP criterion with risk accumulation by the line segment B, which is about 2 GW, as expected (2.83 - .81 ~ 2.0).

These calculations have shown that the distributed system can carry more extra load than the central station system with less risk, and that this effect grows with relaxed LOLP criterion. This is one measure of static or passive resilience. Another measure is the impact of exogenous uncertainties. It is to this subject that we now turn.

4.2 The Impact of Exogenous Uncertainty

To proceed with any analysis we have had to make assumptions about demand and supply parameters on the basis of relatively little information. Although limited information is typical in planning, in our case the uncertainties are greater than what we normally expect. In some sense, uncertainty is inherent in long range analysis. Therefore some explicit account of it should be made. We can treat uncertainties explicitly and parametrically in the context of the normal model of LOLP

by returning for a moment to its initial formulation.

The discussion of section 3.1 began by defining the margin between randomly available supply and load, S-W. We then standardized this variable and applied the normal theory. We pointed out that the load W can also be considered a random variable. Now we take up this interpretation explicitly. If we assume that demand is distributed normally, then we can re-write our definition of the parameter b to include the mean value of load and its variance as follows:

$$b = \frac{\bar{S} - \bar{W}}{\sqrt{2(\sigma_S^2 + \sigma_W^2)}} \quad , \quad (11)$$

where $\bar{W}$ = expected load, σ_W^2 = variance of load.

The assumption of normally distributed loads has been used in the planning literature (27). Typical value for σ_W^2 in reliability studies are around 3% of peak load (26).

We are interested in uncertainties of greater magnitude. The source of these uncertainties lies in both supply technology and demand behavior. Let us consider one example involving the interaction of supply and demand in distributed systems. Suppose that the occurrence of weather sensitive cooling demand were correlated with lulls in the wind. Such a relationship is plausible intuitively, but we don't have any data to test it one way or another. We should also include in our inventory of uncertainty such events as droughts which reduce hydroelectric energy availability, coal strikes or other labor disturbances which interrupt fuel transport, and major interregional supply deficiencies. It doesn't really matter for our analysis whether we call these supply or demand uncertainties, since they are equivalent in equation (11). The disruptive uncertainties we are interested in are the kind of phenomena which can destroy the integrity of a system. We want to see how the two systems respond to large perturbations of expected values.

In Figure 3 we exhibit curves which plot LOLP for the two systems fixed by the storage requirements calculated in Tables 2 and 3 as a function

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Figure 3.

LOLP as a Function of Exogenous Uncertainty

of uncertainty σ_W^2 . These curves show more dramatically than Figure 2 the slower growth of LOLP in the distributed case compared to the central station case. We should expect that risk as a function of uncertainty would grow slower than risk as a function of load for a given system. This follows from analysis of equations (4)-(6). Roughly speaking, the derivative of LOLP with respect to σ^2 goes as $1/\sigma^2$, whereas the derivative of LOLP with respect to W varies as $e^{-(\ell(W))^2}$ for the linear function $\ell(W)$. These are discussed more fully in Appendix 4.

The basic phenomenon in Figure 3 is the difference between supply systems with regard to given levels of uncertainty. It matters less to the risk of the distributed system that variance goes up some amount than to the central station system. The reason for this is evident when we consider how $\partial \text{LOLP} / \partial \sigma^2$ behaves. In Appendix 4 we derive the following expression for this partial derivative

$$\frac{\partial \text{LOLP}}{\partial \sigma^2} = \frac{be^{-b^2}}{\sigma^2 2\sqrt{\pi}} \quad (12)$$

We put the appropriate numbers into equation (12) and draw the graph of this function for our two systems. The data are given in Table 6, the graph is Figure 4. We also include for comparison the function $1/\sigma^2$. For uncertainties on the order of 10% or greater, the derivative is reasonably well approximated by $1/\sigma^2$ times some constant.

Examination of Figure 4 shows that there is a kind of inertia to risk. For systems with relatively little variance the impact of uncertainties is large and even grows larger at first. Then after enough variance has been absorbed or accumulated, the marginal impact of variance gets smaller. The theme of this story might be called "biting the bullet". A system such as our distributed supply scenario has already absorbed statistical fluctuation and is relatively less affected by more of it than the central station system. The bullet of uncertainty has been bitten in the one case, but not in the other. From this perspective the central station plants have neither the reliability benefit of the small variance technologies, nor the passive resilience of large variance technologies.

Figure 4. Partial Derivative of LOLP with respect to Variance

Table 6: Partial Derivative of LOLP with Respect to Variance

		b	σ^2	$\partial \text{LOLP} / \partial \sigma^2$	$1/\sigma^2$
Base Case	DIS	1.69	36	7.6×10^{-4}	2.8×10^{-2}
	CS	1.69	5	5.5×10^{-3}	2.0×10^{-1}
$\sigma_W^2 = 7.0$					
	DIS	1.55	43	9.2×10^{-4}	2.3×10^{-2}
	CS	1.09	12	7.8×10^{-3}	8.3×10^{-2}
$\sigma_W^2 = 17.5$					
	DIS	1.39	53.5	1.1×10^{-3}	1.8×10^{-2}
	CS	.795	22.5	5.3×10^{-3}	4.4×10^{-2}
$\sigma_W^2 = 35.0$					
	DIS	1.21	71	1.1×10^{-3}	1.4×10^{-2}
	CS	.596	40	2.9×10^{-3}	2.5×10^{-2}
$\sigma_W^2 = 49.0$					
	DIS	1.10	85	1.1×10^{-3}	1.2×10^{-2}
	CS	.513	54	2.1×10^{-3}	1.8×10^{-2}
$\sigma^2 = \sigma_S^2 + \sigma_W^2$					

5.0 ECONOMIES OF SCALE IN STORAGE

The discussion of storage needs has focused on capacity requirements, i.e., GW, rather than energy requirements, GWh the amount of energy to be stored. This limitation results from our incomplete knowledge about the future time structure of demand. Other than annual load factor, we have specified nothing about diurnal and seasonal load variations. Indeed our estimates on the resource side are limited to summer afternoons only, since this is the forecasted peak period. Therefore, we have neither the detailed demand nor supply variations necessary to model the typical cycles of a storage system. A recent study of the marginal problem for a single intermittent resource shows the detailed data requirements and the varying integration assumptions which characterize a fully adequate storage assessment (14). Since this level of detail has been suppressed from our analysis for lack of data, what then is the appropriate use and interpretation of our results? The answer to this question will come after a brief digression on the physical nature of the main bulk storage technology available in California, hydrologic reservoirs.

There are two kinds of hydro electric resources, the run-of-river type which essentially cannot be controlled, and storage or reservoir hydro. Although this distinction is not absolute, it is useful (30). The integration of hydro resources with thermal resources is done in the most sophisticated cases by a constrained optimization technique which minimizes expected system production cost, subject to engineering and water resource constraints (30). This amounts to a scheduling problem. The calculations include the reliability aspect of cost minimization in a limited and static framework. Storage hydro is represented by its turbine capacity and energy stored. When the thermal system experiences a significant forced outage, hydro capacity can fill in the gap. An outage duration is necessary to evaluate how much energy must be supplied by the hydro back-up. The optimal design of a hydro system is a function of all these parameters.

Let us consider the following examples. The long duration outages (on the order of weeks) of central station units are replaced by short duration outages from intermittent resources (on the order of days). Then the hydro system might need re-optimization. We will argue that this is exactly what is likely to happen and that the consequences of this change

argue for a change in the existing hydro system toward bigger units at given sites. In essence there is a relationship between risk and storage capacity that suggests there is an economy of scale in storage capacity. Since these economies of scale will affect the number of storage reservoirs needed for a fixed capacity requirement, we may find a significant opportunity to reduce storage costs and actually limit the impact of hydro-electric facility requirements on the land resources of a region. To proceed with the argument we must abstract the problem.

5.1 Sharkey's Theorem

The hydro design optimization problem we have sketched is more complex than the scheduling problem. In fact it is a special case of the more general question of production economics with uncertain demand. In our case the demand for storage hydro is the demand to back up either the central station or the intermittent output technologies. There is a formulation of the general problem and a stylized solution to it that is particularly applicable to our question. The results we will quote are due to Sharkey (25).

Let us introduce a little notation. We are interested in the situation where productive capacity is fixed ex ante and there is random demand. For a plant of size q the operating cost is assumed to be linear, varying h units per unit output. We can call h the avoidable cost. It is the cost we avoid by shutting down the plant in question. We will denote by b the benefit, per unit production. One way to think of b in our setting is that it represents the marginal fossil fuel cost that would be incurred if there were no hydro production. That is, b is a cost which is deferred or displaced. We let g represent the fixed costs per unit product (assumed the same for all units). Finally let q_i be the size of the i^{th} plant, n the number of plants, and R a measure of demand uncertainty. Sharkey proves the following distribution of plant size maximizes net expected consumer surplus considering both fixed and avoidable costs.

Theorem : If $b > h+g$, then a unique maximum of net expected consumer surplus exists for the following distribution of plant size:

$$q_1 = \frac{b(b-h-g)R}{(b-h) [2n-1)b+h]}, \quad (13)$$

$$q_j = 2q_{j-1} = 2^{j-1}q_1, \quad j = 2, \dots, n. \quad (14)$$

where

b = benefit, per unit, of production
h = avoidable cost
 q_i = capacity of i^{th} plant
g = fixed costs per unit product
R = measure of demand uncertainty
n = number of plants

This interesting result requires discussion. First, it is assumed that n, the number of plants, is fixed. Although Sharkey proves a further result on the optimal number n, the more limited result has interesting implications. Hydro resources are limited; the constraint is geologic. Indeed for our application we might want to consider our "plants", q_i , to be hydrologic basins, since storage is really a function of the entire basin. Second, equations (13) and (14) imply that the largest plant, q_n , will have capacity equal to one-half the capacity of the entire system. This can easily be seen by the following calculation:

$$\frac{q_n}{\sum q_i} = \frac{(q_1)2^{n-1}}{q_1(2^n-1)} = \frac{2^{n-1}}{2^n-1} \approx \frac{1}{2}.$$

A quick look at California's hydroelectric system, with total capacity of around 9 GW shows that its largest dam, at Lake Oroville, has less than 1 GW of capacity. This suggests that some re-optimization will be desirable in any future evolution of the electricity system.

From our perspective it is important to understand the dependence of total capacity on the risk measure $\$$. It is to this subject that we now turn.

5.2 Anatomy of Risk

Sharkey's model postulates demand behaving as if it were a real number drawn at random from the interval $[0, R]$. By this he means that each demand level z has probability $1/R$ of occurring. This is the uniform distribution. It is not a good model for our problem where we have argued for a normally distributed available supply margin. Our model incorporates a normally distributed demand. It also says that risk is normally distributed. To apply Sharkey's Theorem in our setting we must see how to replace the

measure R by parameters of our model.

Essentially R in equation (13) is just the inverse of the probability density function of demand. If we assume that (13) and (14) generalize to the case of normally distributed risk, then R would be replaced by the inverse of normal density function, $f(x)$; where

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right), \quad (15)$$

for μ = the expectation of x

σ = the standard deviation of x.

In the base case of $LOLP = 8.5 \times 10^{-3}$, the exponential term will be the same for both supply systems. The constant $1/\sigma$ will differ between systems and its inverse, σ , will take the place of R in (13). As we have argued previously, σ in the distributed system will always be larger than σ in the central station system. This implies that hydro capacity optimized for the former will be larger and have larger units than for the latter.

The story gets more subtle when we consider other loci on the risk curves. Then Figure 2 tells us that the exponential term in (15) will differ between the two supply systems. Roughly speaking we may think of the equivalent to R in the normal version of Sharkey's Theorem as σe^{b^2} . Then looking at Figure 2 we conclude that above $W = 70.0$, σe^{b^2} will always be greater for the distributed system than for the central station system. This follows, since both σ and b are greater for the former system in this region. The situation will reverse between $W = 68.0$ and $W = 69.0$. For loads less than $W = 68.0$ the exponential term will dominate for the central station system. Only in this region would the central station system require a larger hydro capacity than the distributed system.

We might finally ask what the situation would be if we choose a less stringent LOLP criterion, say, $LOLP = 0.10$. As argued in section 4.1, we would remove roughly 2.0 GW of storage from the central system and 4.5 GW from the distributed system. The curves in Figure 2 would then

intersect at $W = 70.0$, $LOLP = 0.10$. Their relative positions wouldn't change. What would change is the cross-over point discussed in the previous paragraph.

Qualitatively we may conclude from this discussion that the kind of risk seen in a distributed/intermittent resource system requires a bigger back-up capacity than the central station system. The wind lulls are likely to be considerably shorter than central station forced outages. However, the magnitude of lulls is likely to be greater than outages of a central station system. This result accounts for correlations. For example, Justus and Hargraves found that a wind lull on their Pacific Coast array which reduced power to about $1/3$ the summer mean would last for 15 hours with 95% probability (10). For 99% availability of only $1/6$ the summer mean, the lull would last about 10 hours. For comparison, however, major outages of LWR generators have an average duration of 300 hours (38). The LWR outages can be expected to be independent of one another. Hence the expected capacity deficit of any one time is likely to be less than for the wind system.

If we can assume that back-up energy requirements are the same in the two systems, then we should have more hydro capacity run at lower capacity factor in the distributed case compared to the Central station case. This kind of hydro optimization has been called "over-machining" since it is based on the installation of turbines designed to run at lower than historical capacity factors. This makes the hydro system more of a peaking system. Where base load excess is expected, such hydro re-optimization is required. This strategy is currently being pursued by the Bonneville Power Administration.

It is still premature to assess adequately the role of hydro storage in the integration of future power systems. A parametric representation of systematic load variation is necessary to do the job properly. Better data on the correlation of risk and resource is also needed. The tools for analysis of these factors can be developed. They must be tempered in their application by the other constraints on the hydro system. Such a research agenda should be cast in the generic framework we have tried to adopt here. Only then will the long run choices available for planning become sufficiently clear for informed choice.

6.0 CONCLUSIONS

We have found significant differences in reliability planning requirements (and therefore costs) for systems dominated by central station plants as opposed to those dominated by intermittent resource technologies. Although storage requirements in the base case are greater for the intermittents than the central station system, the uncertainties in capacity factor make this conclusion sensitive to actual performance.

More robust differences emerge in other areas. The shape of the risk curves differ significantly between the two systems. The central station system has a steeper curve than the intermittent system. Several consequences follow from this. First, risk is more concentrated in the central station system. This means that there is no diffusion of risk which would make for a resilient response to exogenous uncertainties or disturbances. The flatter risk curve for the intermittent system also means that less storage is needed compared to the central station case if the LOLP criterion is relaxed. This benefit survives even when risk accumulation in the intermittent system is accounted for.

Looking more carefully at the impact of uncertainties we also find that central station plants occupy an intermediate position between the small variance technologies and the large variance intermittents. Central station plants have neither the reliability benefit of the former, nor the resilience benefit of the latter.

Finally we argue that existing hydro-electric facilities need re-optimization. These plants provide the only currently existing bulk power storage in electric energy systems. Using this storage as back-up to either central station or intermittent generations requires sizing that considers both magnitude and duration of outages. Preliminary data suggests that intermittents would require greater back-up capacity for shorter duration than central station plants. This suggests that more of the intermittent back-up requirement could come from re-optimized existing hydro compared to central station back-up demand.

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Appendix 1--Normal Parameters for California Wind Arrays

Data on the mean power and standard deviation for California wind arrays is developed in (11) based on analysis in (10). The probability distribution used in these models is the Weibull. The Weibull distribution is characterized by two parameters conventionally denoted by k for the shape parameter and c for the scale parameter. The mean and variance are expressed in terms of c and k . For our purposes we need to know if the normal distribution is a reasonable approximation to the Weibull for our given parameters c and k .

The first question to address is which of the many distributions developed in (11) should form the data for assessing the accuracy of the normal approximation. Since our purpose is to provide input to LOLP analysis we should concentrate on those distributions which are co-incident with peak demand. Our assumptions about the structure of future load shape and its determinants are contained in Appendix 3. The main conclusion of interest is that summer afternoons remain the peak period. The argument of (11) also shows how wind power output distribution varies over geographical area. For simplicity we will choose an array representing the summer afternoon wind power distribution for the whole state of California. This array has the least relative variance of the various configurations within the state, but more than a Pacific Coast regional array.

The wind speed distribution parameters characterizing the California array are listed in Table 1-1 below.

Table 1-1: California Wind Array: Summer Afternoon

Weibull parameters	Mean Wind Speed, $\bar{V} = 9.17$ m/s
$k = 3.45$	Standard Deviation = 2.93 m/s
$c = 10.40$	

Bury⁽³⁾ shows that the normal model is a reasonable approximation to the Weibull distribution when k is between 2 and 6. The symmetry property of the normal distribution holds for the Weibull when $k = 3.6$. Thus, although

wind speed distribution at individual sites is not well modelled by the normal (13), when the large array data is analyzed it does resemble the normal model reasonably well.

The final step in developing normal parameters for our hypothetical wind turbine array involves the transformation from wind speed distribution to power output. Although theoretically the power law for this relation is cubic, wind turbine designs with low rated speeds narrow the effective range of this effect. The DOE wind energy program is based on such a design philosophy and the modelling underlying our data assumes rated wind speeds of under 11 m/s (10). Therefore Justus, for example, has found that a linear approximation to the power curve is reasonable for such machines. We write down Justus' linearization below:

$$P_j = (a + b V_j/V_r) P_r \text{ where} \quad (1)$$

P_j = power at velocity V_j

V_j = velocity of interest

V_r = rated wind speed of generator

P_r = rated power of generator.

For our data the rated (i.e. maximum) wind speed is 10.6 m/s; at this speed $P_r = 2000$ kW/machine. The constants a and b are estimated by Justus to be $a = -0.42$, $b = 1.14$ for our data (10).

To calculate the variance on power availability we square the standard deviation of wind speed $\sigma = 2.93$ and use the squared velocity ($= 8.58$) as V_j in (1). This yields a variance on power of 1000 per 2000 kW machine. Using the mean wind speed from Table 1-1 in equation (1) is likely to give an overly optimistic mean power. The reason is that there is one particularly good wind site out of six which is considerably above the average of the remaining five sites. To scale this average including the good site may be unrealistic for a large array of many machines which must necessarily include many poor sites. Therefore we will use a lower mean wind speed, one which characterizes the Pacific Coast regional array at this time, namely

8.8 m/s. This corresponds to a mean power of 1050 kW per 2000 kW machine.

Our technique finally reduces to assuming that these ratios, $\sigma_S^2 = .50$ rated power, $\bar{S} = .525$ rated power yield the normal parameters for our wind array. Once we have a total rated capacity, the ratios give normal parameters for the LOLP model.

Appendix 2--Insolation Statistics

1. Coefficient of Variation

We first develop estimates of the coefficient of variation, σ/μ , at individual sites. We do this in two independent ways. First we tabulate data on Santa Maria and Inyokern California that is presented in (1). From this we will derive the mean and standard deviation by the following relations:

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i, \quad (1)$$

and

$$S_x^2 = \frac{1}{n-1} \sum f_i (x_i - \bar{x})^2, \quad (2)$$

where

- $\bar{x}$ = sample mean
- x_i = i^{th} sample value
- n = number of values
- S_x^2 = sample variance
- f_i = frequency of x_i .

Table 2-1: Percent of Time that Insolation is Greater than x_i
(Normalized to Peak = 100)

Cumulative Frequency (5)	Santa Maria	Inyokern
0	90	90
5	80	82
10	78	80
15	75	80
20	70	78
25	67	73
30	58	68
35	46	62
40	27	55
45	10	40
50	0	0

Normalizing the data in Table 1 so that the cumulative frequency is 100% and assigning frequencies to each insolation value, we find that all states except 90 have frequency = 0.1. From this we calculate $\bar{x}$ and $\sqrt{S_x^2}$ for each site. These are $\bar{x} = 54.6$ for Santa Maria and $\bar{x} = 64.4$ for Inyokern, and $\sqrt{S_x^2} = 8.84$ for Santa Maria and $\sqrt{S_x^2} = 7.68$, therefore the coefficient of variation is .16 for Santa Maria and .12 for Inyokern.

An alternate estimate can be developed from attempts to collapse frequency data into the representation of forced outage rates (FOR). One such effort is a study done by Southern California Edison (26) which also looks at Inyokern. Using equations (1) and (2) from section 2.1, we can write the coefficient of variation as follows:

$$\begin{aligned} \frac{\sigma}{\mu} &= \frac{\sqrt{\alpha_i^2 (1-L_i) L_i}}{\alpha_i (1-L_i)} \\ &= \sqrt{\frac{L_i}{1-L_i}} \end{aligned} \quad (3)$$

At the time of peak demand, the total forced outage rate is estimated to be 2.3%, adding partial outages would bring this to 2.7%. For these values, eq. (3) gives $\sigma/\mu = .15$ and $.17$.

2. Estimation of Solar Output Correlations

The following discussion is designed to provide support for the proposition that the correlation of solar output across regions in California is high. We will perform a stylized calculation to support an estimate of roughly $\rho = .8$. The argument is indirect. We will use the normal model of LOLP to fit results calculated by the Aerospace EPRI study (1). In Table 1 we list the parameters of the system studied.

Table 2-2

Base System: 18 units, 1.0 GW/unit, FOR = .07
 20 units, .5 GW/unit, FOR = .05
 42 units, .2 GW/unit, FOR = .03
 LOLP, peak month = .456 hrs/mo.

$$\approx 8.3 \times 10^{-4}$$

Geographical Dispersal Results:

3000 MW of Solar displaces 6 units of .5 GW each

Back-Up Capacity Required

Santa Maria 950 MW

Inyokern 550 MW

1/3 Santa Maria,

2/3 Inyokern 390 MW

Abstracting from several complications, we can use the normal model of LOLP to account for the changes in back-up requirements as we go from one site to another and finally to the 2-site pattern. Attributing the changes in back-up to changes in output variance reduces the problem to calculating changes in the term $b\sqrt{2\sigma_S^2}$. To make such calculations we must estimate the power availability variance due to the base system, and the solar siting pattern. We also need the value of b corresponding to $LOLP = 8.3 \times 10^{-4}$. The last task is simple. Since $LOLP = \frac{1}{2} \text{erfc}(b)$, we just search the normal probability table for the appropriate value. It is $b = 2.2$. Equation (2) from section 2.1 applied to the base system of Table 2-2 (with 6 units of .5 GW each displaced) yields the following variances:

number of units	units of size (GW)	FOR	σ_S^2
18	1.0	.07	1.172
14	.5	.05	.166
42	.2	.03	.050
			<u>1.388</u>

Finally, the sample standard deviations calculated in Part 1 above allows us to estimate power variances for the individual sites as follows:

$$\begin{aligned}
 \text{Santa Maria} &= 30 (8.8)^2 / 1000 \\
 &= 2.323 \\
 \text{Inyokern} &= 30 (7.7)^2 / 1000 \\
 &= 1.778.
 \end{aligned}$$

The difference between Santa Maria and Inyokern can now be calculated.

Santa Maria

$$\begin{aligned} b\sqrt{2\sigma_S^2} &= 2.2\sqrt{2(1.388 + 2.323)} \\ &= 5.99 \end{aligned}$$

Inyokern

$$\begin{aligned} b\sqrt{2\sigma_S^2} &= 2.2\sqrt{2(1.388 + 1.788)} \\ &= 5.54 \end{aligned}$$

$$\begin{aligned} \Delta &= 5.99 - 5.54 \\ &= .45 \text{ or } 450 \text{ MW.} \end{aligned}$$

This is close to the simulation result of $\Delta = 400$ MW.

Next we scale the Inyokern standard deviation down using equation (3) of section 2.2. Here we are really looking for the correlation coefficient ρ . It is easiest to assume a value, then check if that reduces back-up by the required amount, namely 160 MW (= 550-390). We show that $\rho = .8$ does this. From eq. (3) section 2.2,

$$\begin{aligned} \sigma_A / \sigma_{\text{Inyokern}} &= \frac{\sqrt{1 + .8}}{2} \\ &= .949, \end{aligned}$$

where σ_A is the standard deviation of the array consisting of Inyokern and Santa Maria.

Since $\sigma_{\text{Inyokern}} = 7.7$, $\sigma_A = 7.3$; therefore the array variance = $30/(7.3)^2/1000$ = 1.60.

$$\begin{aligned} \text{Finally, we compute } b\sqrt{2\sigma_S^2} &\text{ for the array configuration. This is just} \\ b\sqrt{2\sigma_S^2} &= 2.2\sqrt{2(1.60 + 1.39)} \\ &= 5.38 \\ \Delta &= 5.54 - 5.38 \\ &= .16 \text{ or } 160 \text{ MW as required.} \end{aligned}$$

This calculation neglects several factors that could be important. The back-up capacity itself contributes variance. This has been neglected on the grounds that it is small (back-up is typically peaking units which come in small sizes and are reliable). More significantly perhaps we have not used data on mean outputs from sites. The data in Table 2-1 is averaged over all times of the day. For LOLP purposes we want mean power co-incident

with peak. A priori the Table 2-1 data says nothing about this. By assuming that all dispersed benefits is due to changes in array variance we are really estimating a lower bound for ρ , the correlation coefficient. If dispersal also increased mean array power co-incident with the peak, then our estimate of $\rho = 0.8$ would have to be higher. For the purpose of our argument a lower bound on $\bar{\rho}$ is sufficient.

Appendix 3

Long-Range Electric Utility Peak Load Estimation

In this appendix we discuss procedures for estimating peak loads of electric utilities. The effort is simplified for California utilities because the California Energy Commission has done work in this area.⁽¹⁹⁾ Their forecast is based on regressions which separate peak demand into weather-sensitive and non-weather-sensitive components. The form of the relations is as follows (we consider the case of the Pacific Gas and Electric Company):

$$\begin{aligned}
 \text{PEAK} &= \text{NWD} + \theta \text{TOTTAU} & (1) \\
 \text{NWD} &= \text{non-weather-sensitive demand} \\
 &3 \\
 \text{TOTTAU} &= \sum_{J=1}^3 [(\text{TEMP}_J > 75^\circ) \times \text{AC}_J] \\
 \theta &= \text{weather-sensitive coefficient} \\
 J &= \text{each weather station} \\
 \text{TEMP} &= \text{dry-bulb, maximum temperature} \\
 \text{AC} &= \text{number of air conditioning units}
 \end{aligned}$$

A separate estimate is made for NWD which correlates this part of the load with projected energy (kWh) consumption. The results of the ERCDC regression for this part of the PG&E peak are:

$$\begin{aligned}
 \text{NWD} &= 1120 + (.1316) \text{EST} & (2) \\
 \text{EST} &= \text{total sales in millions of kWh}
 \end{aligned}$$

The ERCDC estimates of the PG&E peak are developed for the period 1975-1995. We tabulate the results below.

Year	NWD (MW)	Weather- Sensitive Demand (WSD)	Total Demand (MW)
1975	9,245	3615 MW	12,860
1980	11,111	4215 MW	15,326
1985	12,868	4853 MW	17,721
1990	14,699	5353 MW	20,052
1995	16,657	5774 MW	22,431

To examine a 2025 PG&E area peak we will use (1) and (2) with suitable modifications and the scenario developed by the Distributed Energy Systems Study Group.⁽⁵⁾ The data required to make this calculation are the number of air conditioning units (AC), millions of kilowatt-hours used (EST) and a projected value for θ , the weather-sensitive coefficient. For EST we use the values given in (5) allocated from a California basis down to the PG&E area. We use the scenario from (5) which assumes 410×10^9 kWh for California. This corresponds to an average annual growth rate of 2.0% from 1975. PG&E constitutes about 45% of the California total electricity consumption today. Projecting this fraction to 2025, we allocate 184×10^9 kWh to PG&E for this scenario. Using eq. (2), this yields 2025 NWD of 25,340 MW.

A little more ingenuity is required to estimate the number of air conditioners in 2025. Using ERCDC projections out to 1995, we can fit them to a logistic curve and extend out to 2025. Recall the equations of a logistic growth curve.

$$\frac{dx}{dt} = rx(1 - \frac{x}{K}) \quad , \quad (3)$$

where

- x = the growing variable
- r = rate of growth
- K = resource or saturation limit.

The solution to (1) is given by

$$x(t) = \frac{K}{1 - ce^{-rt}} \quad , \quad c = \frac{K-x_0}{x_0} \quad (4)$$

For our purposes we interpret x as the saturation of air conditioners, that is, the ratio of air conditioners to households. Therefore, K = .95 in equations (3) and (4) is a reasonable limit based on (4). We take the following data from the PG&E CFM submissions.⁽²¹⁾ This shows a 1977 saturation of 27.7% and a projected 1998 saturation of 32.5%. We let $x = s(t_0) = .277$ to determine c in equation (4). This gives c = 2.43. Then the 2025

saturation, which is 48 years after 1977, can be found once we know r , the growth rate, just by solving for $x(48)$. We determine r by calculating the compound growth rate between 1977 and 1998; we get $r = 0.76\%$. The unit growth rate is faster than this. So

$$\begin{aligned} \text{2025 saturation} &= x(48) \\ &= .95/1 + 2.43e^{-.367} \\ &= .95/2.68 = .354. \end{aligned}$$

From (5) we have an estimate of total California households in 2025, namely, 16 million. This estimate is based on the same population projections as the ERCDC residential forecasting model. Allocating 45% of these households to PG&E area we get 7.2 million. Multiplying by the estimated .354 saturation gives 2,548,000 air conditioning units in 2025.

It remains to make adjustments for temperature diversity and increased efficiency of air-conditioners. In eq. (1) the variable TOTTAU is summed over three weather stations. For practical purposes the average maximum temperature is 105°F . We can use a diversity factor to avoid disaggregating air conditioners by weather station. Therefore the ratio of WSD in 1995 to WSD in 1975 should be equal to the ratio of the number of 1995 air conditioners to the number of 1975 air conditioners multiplied by a diversity factor. The ratio of AC units is $1380/735 = 1.88$, whereas the ratio of WSD_{95} to WSD_{75} is $5774/3615 = 1.60$. Therefore the diversity factor is $1.60/1.88 = .85$.

The final adjustment is for improved end-use efficiency. We measure this by EER. The average EER for an air conditioner has been estimated at 6.5.⁽¹⁸⁾ An energy intensity (the reciprocal of efficiency) of .55 for new air conditioners in 2000 is estimated in (5). This would correspond to an average EER of 11.8. This value is considerably higher than the minimum EER of 8.7 that will soon be required in California. Since air conditioners have a lifetime much less than 25 years, we can use the energy intensity from (5) to correct the weather-sensitive coefficient θ in eq. (1). From (19) the value of $\theta = .2175 \text{ GW}$. We calculate WSD as follows:

$$\begin{aligned}\text{WSD} &= \theta \text{ (efficiency factor) (2025 \# of air conditioners)} \\ &\quad (30) \text{ (diversity factor)} \\ &= .2175(.55) (2.55 \times 10^6) (30) (.85) \\ &= 7.78 \text{ GW}\end{aligned}$$

Therefore, peak = 25.34 + 7.78 = 33.12 GW.

Because it is difficult to make intuitive sense of any long-range projection, it is useful to translate these calculations into a more common index of load shape, the load factor. Load factor is defined as the ratio of average to peak demand. Our estimate of annual electric energy is 184×10^9 kWh or 21.00 GW on the average ($= 184 \times 10^9$ kWh/8760 hrs.). For a peak demand of 33.12 GW, the load factor is 63.4%. For comparison, the 1975 PG&E load factor was 54% (62.6×10^9 kWh and peak = 13.3 GW). It is plausible to find load factor increasing in this scenario because baseload and off-peak uses grow faster in the scenarios from (5) than residential and commercial demands.

Appendix 4 Derivatives of LOLP

Equations (4)-(6), the normal model of LOLP, can be differentiated explicitly to examine the sensitivity of risk to the various parameters. The partial derivative of LOLP with respect to some variable x can be written as follows:

$$\begin{aligned}\frac{\partial \text{LOLP}}{\partial x} &= \frac{\partial}{\partial x} \left(\frac{1}{2} \text{erfc}(b) \right) \\ &= \frac{d}{db} \left(\frac{1}{2} \text{erfc}(b) \right) \frac{\partial b}{\partial x} \\ &= \frac{-e^{-b^2}}{\sqrt{\pi}} \frac{\partial b}{\partial x}\end{aligned}$$

We are interested in the cases where the variable x is the load W and the variance σ^2 . We compute $\partial b / \partial W$ and $\partial b / \partial \sigma^2$ below:

$$\frac{\partial b}{\partial W} = \frac{\partial}{\partial W} \left(\frac{\bar{S} - W}{\sqrt{2}\sigma} \right)$$

$$= -1/\sqrt{2}\sigma, \text{ and}$$

$$\frac{\partial b}{\partial \sigma^2} = \frac{\partial}{\partial \sigma^2} \left(\frac{\bar{S} - W}{\sqrt{2}\sigma} \right)$$

$$= -b/2\sigma^2.$$

The resulting partials are:

$$\frac{\partial \text{LOLP}}{\partial W} = \frac{e^{-b^2}}{\sigma\sqrt{2\pi}}$$

and

$$\frac{\partial \text{LOLP}}{\partial \sigma^2} = \frac{be^{-b^2}}{\sigma^2 2\sqrt{\pi}}.$$

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