NEAR SPHERICAL ILLUMINATION OF ION-BEAM AND LASER TARGETS

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ABSTRACT

A procedure is developed for reducing energy-deposition asymmetry in spherical targets driven directly by ion or laser beams. This work is part of a strategy for achieving illumination symmetry in such targets, which is proposed as an alternative to those in the literature.¹⁻⁶ This strategy allows an axially symmetric placement of beamlets, which would be convenient for some driver or reactor scenarios. It also allows the use of beam currents or energy fluxes and beam transverse profiles to help reduce deposition asymmetry with fewer beamlets. In the ideal limit of thin deposition layers and controlled beam profiles, at most six beamlets are needed for target symmetry.

1. Introduction

Many inertial fusion schemes allow a near-spherical target illumination but leave residual asymmetries centered around an axis that is determined by the driver or reactor scenario. For example, this occurs in the PBFA pulse-power facilities at Sandia, Albuquerque (and in related facilities), and in heavy ion reactor concepts such as the German HIBALL. In both these cases, driver and reactor considerations discourage the use of beam ports directly above or below the fusion chamber. In other such chambers, particularly those with vertically flowing liquid metals, it might simplify design to have fewer vertical planes occupied by beam ports. Our strategy provides just such flexibility.

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Even if no preferred axis arises in a particular illumination scheme, beam placement and beam current can be used to eliminate long-wavelength (low spherical-harmonic) asymmetries in the deposition profile. Removal of higher harmonics can depend on beam transverse profiles. Under conditions of thin deposition layers and ideal beam profiles, at most six beamlets are needed for target symmetry. For more general situations, a systematic strategy is outlined.

Typically, our strategy first eliminates explicitly long-wavelength asymmetries in the form of coefficients of spherical harmonics rather than just reducing these coefficients as in many symmetric (Platonic-solids) beamlet placement schemes. Low-spherical-harmonic asymmetries are eliminated, independent of deposition depth and up to values of l and $|m|$ that depend on the number of beamlets and the number of different beamlet strengths used (Section III). Many harmonics of higher l and $|m|$ are also eliminated, depending on what particular symmetries arise from the choices of beamlet orientations and strengths (Section II). The convergence ratio of the target determines how much smoothing of higher-order terms is required. Our procedure shows what penalties must be paid, in terms of higher harmonics not removed, if the driver or reactor system makes it desirable to eliminate certain symmetries in placement of beamlets.

In general (even with symmetric beam placement), higher harmonics can be removed using physical effects such as transverse beam profiles (see Section IV, also Refs. 1,6), dispersion in beamlet angles, particle (or photon) energy, thermal effects of target plasmas, or focal shift relative to the target center^{1,2,4}; for laser drivers, additional techniques include refraction smoothing⁴ and smoothing due to decoupling of absorption and ablation layers. Because our strategy can eliminate more long-wavelength (low-harmonic) asymmetry than can symmetric beam placement, we can use the free parameters in the other physical effects exclusively against shorter-wavelength asymmetries.

Here are two examples of the smoothing of lower harmonics. We could eliminate all harmonics with $1 \leq l \leq 5$ and $|m| \leq 5$ by appropriate choice of beamlet angles with 18 beamlets of identical strength. With 32 beamlets, of two different strengths (in the ratio $\sim 15/8$), we could eliminate all harmonics with $1 \leq l \leq 7$, and $|m| \leq 7$. Convenient arrangements for several ion-beam and laser driven illumination schemes are described later in Section V.

11. Use of General Symmetries of Beam-Placement in Axial Geometry

Our procedure consists of expanding the deposition profile of each beamlet in spherical harmonics and choosing the beamlet orientations (θ_n, ϕ_n) and amplitudes ϵ_n so that harmonics of specific (l, m) vanish in the net deposition profile. We assume that all beamlets are directed towards the center of the target. The same shape is assumed for each beamlet; only the beamlet strengths vary, so that we let (θ_n, ϕ_n) represent the orientation of the central axis of beamlet n . With the geometry of Fig. 1, (r, θ, ϕ) designates an arbitrary point on the target surface and γ_n represents the angle between the directions (θ_n, ϕ_n) and (θ, ϕ) . Assuming that the deposition profile $E_n(r, \theta, \phi, t)$ of beamlet n depends only on (r, γ_n, t) , we expand E_n as

$$E_n(r, \theta, \phi, t) = \epsilon_n \sum_{l=0}^{\infty} E_l(r, t) P_l(\cos \gamma_n) \quad , \quad (1)$$

where the $P_l(x)$ are Legendre polynomials. In accordance with our assumptions, the $E_l(r, t)$ are common to all the beamlets, while the γ_n and ϵ_n may vary. To sum over beamlets we use a particular definition (see, e.g., Ref. 7) of spherical harmonics and the expansion of $P_l(\gamma_n)$ in the spherical harmonics $Y_{lm}(\theta_n, \phi_n)$ and $Y_{lm}(\theta, \phi)$. We thus obtain the net deposition profile

$$E(r, \theta, \phi, t) = \sum_{l=0}^{\infty} E_l(r, t) \sum_{m=-l}^l D_{lm} Y_{lm}(\theta, \phi), \quad (2a)$$

where

$$D_{lm} = \frac{4\pi}{2l+1} \sum_n \epsilon_n Y_{lm}^*(\theta_n, \phi_n) = A_{lm} \left[\frac{4\pi}{2l+1} \frac{(l+m)!}{(l-m)!} \right]^{1/2}. \quad (2b)$$

Retaining some symmetry in the beamlet distribution would by itself cause many of the A_{lm} or D_{lm} to vanish. Table 1 summarizes the results of symmetry about the origin, about the $z = 0$ plane, rotational symmetry about the z -axis (over specific angles that are rational submultiples of 2π), and a combination of these symmetries. Derivations are outlined in the Appendix.

III. The Benefits of Beam-Placement Using Legendre-Gauss Quadrature

To eliminate specific harmonics of low (l, m) , we assume that θ takes on the values θ_j , $j = 1, 2, \dots, L_0$, and that for each θ_j we have beamlets at angles $\phi = \phi_{jk} + \phi_{j0}$, $k = 1, 2, \dots, M_j$. The total number of beamlets is then $N_0 = \sum M_j$. Using properties of spherical harmonics,⁷ and doing some resuming, we reduce Eq. (2) to

$$E(r, \theta, \phi, t) = A_{00} E_0(r, t) + \sum_{l=2L_0}^{\infty} A_{l0} E_l(r, t) P_l(\cos \theta) + 2 \sum_{m=M_0}^{\infty} \sum_{l=m}^{\infty} \operatorname{Re}(A_{lm} e^{im\phi}) E_l(r, t) P_l^m(\cos \theta) \quad (3)$$

Here $P_l^m(\cos \theta)$ is an associated Legendre function, and

$$A_{lm} = \frac{(l-m)!}{(l+m)!} \sum_{j=1}^{L_0} \epsilon_j B_{mj} \exp(-im\phi_{j0}) P_l^m(\cos \theta_j), \quad (4)$$

$$B_{mj} = \sum_{k=1}^{M_j} \exp(-im\phi_{jk}) \quad (5)$$

We assume that the ϵ_n depend only on j and not on k , so that ϵ_j replaces ϵ_n .

In Eq. (3), we have chosen the (θ_j, ϕ_{jk}) and the ϵ_j so that the A_{lm} vanish for $m=0$ and $1 \leq l < 2L_0$ as well as for $1 \leq m < M_0$ and $l \geq m$. This is why the first sum (the $m = 0$ case) in Eq. (3) starts with $l = 2L_0$ rather than with $l = 0$, and why the second sum starts at $m = M_0$ rather than $m = 1$. That is, we have taken

$$\phi_{jk} = 2\pi k / M_j, \quad k = 1, 2, \dots, M_j, \quad (6)$$

and adopted

$$\cos \theta_j = x_j, \quad \epsilon_j = w_j / M_j \quad (7)$$

where x_j and w_j are the quadrature points and weights of Legendre-Gauss numerical quadrature (cf. Eq. 25.4.29 of Ref. 8).

The derivations are in the appendix. Figure 2 illustrates the advantage of Legendre-Gauss quadratures for an $L_0=4$ case. This illustration makes additional assumptions of thin deposition layer and axisymmetry over ϕ -angles which are not required in our general discussions of Sections II-III.

IV. Use of Beam Transverse Profile to Assist Reduction in Number of Beamlets

Our above methods can eliminate all low order harmonics up to certain (l,m) wave number as well as selected bands of higher harmonics. The reduction of all higher harmonics to a sufficiently small level must depend on other effects, some of which are listed in Section I. This section only shows that under ideal conditions of a thin deposition region, the single addition of control over beam profile should be sufficient to control all asymmetries.

Assuming that the deposition profile is spatially highly peaked (or thin) in depth, we can immediately show that control over the beam transverse profile allows reduction of higher (l, m) asymmetries to arbitrarily small levels. Under the above conditions, the deposition profile E_n of beamlet n (Eq. 1) can be written as

$$E_n = F_n(\nu_n) A(\nu_n) \left| \nu_n \right|, \quad (8a)$$

where

$$\nu_n = \cos \gamma_n. \quad (8b)$$

$F_n(\nu_n)$ and $A(\nu_n)$ represent the distribution of beam energy flux and fraction of energy absorbed. Under ideal conditions, the product $F_n(\nu_n) A(\nu_n)$ vanishes at $\nu_n = 0$ and reaches a peak at $\nu_n = 1$, so that we can expand it as an odd-order Legendre series in ν_n

$$F_n(\nu_n) A(\nu_n) = \sum_{k=0}^{\infty} a_{2k+1} P_{2k+1}(\nu_n) \quad (9)$$

Clearly from Eqs. (8) and (9) we can identify the coefficients E_k of our earlier formal expansion (1). Namely, for $k = 0, 1, 2, \dots$

$$E_k = \begin{cases} \frac{1}{3} a_1 & , \text{ for } k = 0 \\ \frac{1}{2k-1} a_{k-1} + \frac{k+1}{2k+3} a_{k+1} & , \text{ for } k = \text{even integer} \neq 0 \\ 0 & , \text{ for } k = \text{odd integer} \end{cases} \quad (10)$$

We may note that our above beam placement scheme of Section III eliminates the effects of E_k for small k . At the same time the higher order E_k ($k \geq 2L_0$ or $k \geq M_0$) can be controlled by choosing the beam profile according to Eq. (9) so that a_{2L_0-1} and higher coefficients are sufficiently small.

If beam profile can be controlled arbitrarily, we can even have $(F_n A_n) = \nu_n$ so that only six beamlets are needed for symmetry ($L_0 = 2, M_0 = 3$, cf. also result of Schmitt, Ref. 6). This ideal profile $(F_n A_n = \nu_n)$ is illustrated in Fig. 3. It is not so far from most typical beam profiles. (Here ξ_n is radial distance from axis of beamlet in units of the deposition layer radius $R, \xi_n = R \sin \gamma_n$.) Nevertheless, our present strategy in assuming the more general Eq. (9) and the results of Sections II-III already assume deviations from such ideal control over beam profiles and deposition layer thickness. Additional investigations are needed to address other effects of realistic target driving conditions.

V. Some Illustrative Examples of Axisymmetric Beamlet Placement

We should consider both even and odd values of L_0 , since both are convenient for driver concepts that allow beamlets to impinge from all lateral directions around the reactor chamber. In the simplest case of interest, for which $L_0 = 2$ with $M_1 = M_2 \geq 4$, deposition harmonics with $1 \leq k \leq 3$ and $|m| \leq 3$ will vanish, provided that $\theta_1 \sim 54.74^\circ$ and $\theta_2 \sim 125.26^\circ$. Since $w_1 = w_2$ for $M_0 = 2$, the beamlets can all have the same c_n ; this is useful for the PBFA or HIBALL (if accelerator) geometries. It relies

on beam profiles (Section IV) and other effects (Section I) to eliminate higher harmonic asymmetries.

Figure 4(a) illustrates the beamlet placement for $L_0 = 2$, $M_1 = M_2 = 10$ (the HIBALL-like case). Note that having $M_{1,2} > 2L_0 = 4$ does not help remove even-integer $l \geq 4$, but $m = 0$ deposition asymmetries of the form $P_l(\cos \theta)$. This is true despite the fact that many higher harmonics with $m = 0$ can be eliminated (we could take $M_1 = M_2$ to be an even integer and choose phases ϕ_{j0} so that the first three symmetries of Table 1 hold: A_{lm} would then vanish unless l, m are even and m is an integer multiple of M_1). Thus this illumination generates more asymmetry in the θ direction than in the ϕ direction. Of course, if θ_1 and θ_2 were not at the Legendre-Gauss quadrature points, it could not even be guaranteed that the $l = 2$ harmonics would be minimized in this case.

On the basis of target-deposition symmetry alone (the point of view taken here), it is preferable to have $M_0 \geq 2L_0$ but to have the two not too different from one another. Doing this as an alternative to the case of Fig. 4(a), we could obtain somewhat better deposition symmetry, i.e., the elimination of harmonics for $1 \leq l \leq 5$, all odd l , and $|m| \leq 5$, with only 18 beamlets. Such an illumination scheme could be achieved with $L_0 = 3$; $\theta_1 \sim 39.23^\circ$, $\theta_2 = 90.00^\circ$, and $\theta_3 \sim 140.77^\circ$; $M_1 = M_3 = 5$, $M_2 = 8$; symmetry about the origin; and $\epsilon_1 = \epsilon_2 = \epsilon_3$. The Legendre-Gauss weights $w_j = \epsilon_j J_j$ in this case are in the exact ratio $w_2/w_1 = w_2/w_3 = 8/5 = 1.6$. With the same basic conditions (18 beamlets, $L_0 = 3$, and the same values of θ), we could eliminate all harmonics for $1 \leq l \leq 5$, $|m| \leq 5$, all odd l and m , and $m/6 = \text{integer}$ with $M_1 = M_2 = M_3 = 6$ and $\epsilon_2/\epsilon_1 = \epsilon_2/\epsilon_3 = 8/5 = 1.6$. This is illustrated in Fig. 4(b). Clearly other such combinations are possible.

For some driver scenarios, it might be convenient to arrange the beamlets into several wide bunches, directed at the target from two sides. Take the case $L_0 = 4$ and $M_0 = 8$, which eliminates deposition harmonics with $1 \leq l \leq 7$ and $|m| \leq 7$, as well as those shown for the first three symmetries in Table 1. The beamlets are arranged in four cone-like bunches with cone angles $\theta_1 \sim 30.56^\circ$, $\theta_2 \sim 70.12^\circ$, $\theta_3 \sim 109.88^\circ$, and $\theta_4 \sim 149.44^\circ$. The Legendre-Gauss weights satisfy $w_1 = w_4$, $w_2 = w_3$, and $w_2/w_1 = 1.8748$. With $M_1 = M_4 = 8$ and $M_2 =$

$M_3 = 15$ (for a total of 46 beamlets), we note the apparent coincidence that $M_2/M_1 = 1.8750 = w_2/w_1$: thus for this case the $\epsilon_j M_j = w_j$ are the Legendre-Gauss weights without our even having to choose different ϵ_j . Alternatively, one could also accomplish the same result with only 32 beamlets by choosing $M_1 = M_2 = M_3 = M_4 = 8$ and $\epsilon_2/\epsilon_1 = w_2/w_1 \sim 15/8$; Fig. 4(c) illustrates this latter case. This comparison shows again the advantage (e.g., 32 versus 46 beamlets) for judicious use of different beamlet currents whenever convenient for the driver.

In the longer term we feel confident that further flexibility could be provided to the reactor designer. In particular the two wide bunches of beams in this last example should be replaceable by two narrow bunches (if considered useful) with say up to 30° beamlet inclination from a chosen axis.

Further studies are also needed to confirm that higher-order asymmetries can be kept to tolerable levels by the use of the various physical effects mentioned earlier in this article.

Appendix

Here we give a few details of derivations leading to Table 1 and Eq. (3) of Sections II-III. Symmetry about the origin and about the $z=0$ plane involve effects of pairs of beamlets while rotational symmetries involve more beamlets. For example, symmetry about the origin implies that for every beamlet n , there exists a complementary one n' such that $\epsilon_{n'} = \epsilon_n$, $\theta_{n'} = \pi - \theta_n$, $\phi_{n'} = \pi + \phi_n$ so that

$$\begin{aligned} \sum_n \epsilon_n Y_{lm}^*(\theta_n, \phi_n) &= \sum_n \epsilon_n Y_{lm}^*(\pi - \theta_n, \phi_n + \pi) \\ &= (-1)^l \sum_n \epsilon_n Y_{lm}^*(\theta_n, \phi_n) \end{aligned} \quad (A1)$$

which is possible only if l =even integer as in Table 1. On the other hand, the last row of Table 1 assumes

$$\begin{aligned} \sum_n \epsilon_n Y_{lm}^*(\theta_n, \phi_n) &= \sum_n \epsilon_n Y_{lm}^*(\pi - \theta_n, \phi_n + \frac{2\pi}{N_2} N_1) \\ &= \exp \left[-i \frac{2\pi}{N_2} N_1 m - i 2\pi \frac{(l-m)}{2} \right] \sum_n \epsilon_n Y_{lm}^*(\theta_n, \phi_n) \end{aligned} \quad (A2)$$

which can only be true if $[(l-m)/2 + mN_1/N_2]$ is an integer.

The fact that the lowest order A_{lm} coefficients of Eq. (4) vanish for $l \leq m < M_0$ and $l \geq m$ follows immediately from the choice (6) for the angles ϕ_{jk} because then if $(m/M_j) \neq \text{integer}$,

$$B_{mj} = \sum_{k=1}^{M_j} \exp \left[-i \frac{2\pi m}{M_j} k \right] \quad (A3)$$

$$= \exp \left(-i \frac{2\pi m}{M_j} \right) \left\{ \frac{1 - \exp[-i2\pi m]}{1 - \exp[-i \frac{2\pi m}{M_j}]} \right\} ,$$

so that,

$$B_{mj} = \begin{cases} 0, & \text{if } (m/M_j) \neq \text{integer} \\ M_j, & \text{if } (m/M_j) = \text{integer} \end{cases} \quad (A4)$$

Then if $M_0 = \min(M_j)$, the A_{lm} vanish for $1 \leq m < M_0$. If all the M_j were equal, this same result would follow more succinctly from rotational symmetry, as can be seen from Table 1. Our derivation of Eq. (6) is an alternate to that of Schmitt.⁶

Our choice for the θ_j is so that $A_{l0} = 0$ for $1 \leq l < 2L_0$. For $m = 0$, we have $B_{0j} = M_j$ and

$$A_{l0} = \sum_{j=1}^{L_0} w_j P_l(\cos \theta_j) , \quad (A5)$$

where $w_j = c_j M_j$. This sum vanishes for $1 \leq l \leq (2L_0 - 1)$ if we choose the $x_j = \cos \theta_j$ and w_j to be the positions and weights of a Legendre-Gauss numerical quadrature formula of order L_0 . [This quadrature is exact for polynomials of order $l \leq 2L_0 - 1$; this is implied by Eq. 25.4.29 of Ref. 8 and by the orthogonality relations of the Legendre functions (p. 774 of Ref. 8).]

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Table 1. Symmetries for which particular A_{lm} coefficients vanish
(we assume $\epsilon_{n'} = \epsilon_n$).

Symmetry	Condition: for every n , there exists an n' such that the relations below are valid	Result: $A_{lm} = 0$ unless quantity below is an integer
About origin	$\theta_{n'} = \pi - \theta_n$ $\phi_{n'} = \pi + \phi_n$	$l/2$
About $z = 0$ plane	$\theta_{n'} = \pi - \theta_n$ $\phi_{n'} = \phi_n$	$(l \pm m)/2$
Of rotation about z -axis ^a	$\theta_{n'} = \theta_n$ $\phi_{n'} = \phi_n + (2\pi/N_2)N_1$	$(m/N_2)N_1$
About $z = 0$ plane with rotation ^a	$\theta_{n'} = \pi - \theta_n$ $\phi_{n'} = \phi_n + (2\pi/N_2)N_1$	$(l \pm m)/2 + (m/N_2)N_1$

^aHere N_1 and N_2 are integers.

Figure Captions

Fig. 1. Geometry used in deriving the results given in this article. The angle γ_n is measured between the axis of beamlet n and an arbitrary point on the target (r, θ, ϕ).

Fig. 2. This illustrates the advantages of using gaussian quadrature angles for beam placement. It assumes in addition a model of thin deposition layer (circle in Figure) that is also symmetrized over ϕ -angles (the exact methods of Section II-III does not depend on this model). Each pair of dashed lines with same inclination to z axis gives the beamlet outer envelope for a case where beamlets are located on four θ -angles only ($L_0=4$); the reader could fill in by reflection the rays originating from $z < 0$. Concentrating attention on the ray with a 30.56° angle from the z -axis, the center of this ray is clearly also at the intersection of several other beamlets, thus it is not surprising a local maximum of deposition occurs near that position. Also, the upper edge of this 30.56° beamlet is also near the center and intersection of other beamlets, so that a local maximum should occur there also. Altogether, this physical picture illustrates the mathematically exact result that the final deposition profile should be a higher order Legendre function.

Fig. 3. Ideal beamlet profile that allows use of very few beamlets for illumination symmetry in a thin deposition layer model. Plotted is the beamlet flux times fraction absorbed $F_n A_n$, as function of ξ_n , the radial distance from beamlet axis. Although this profile seems easy to achieve, our paper outlines a variety of methods to achieve symmetry and does not depend on strict application of one concept alone which would require highly ideal conditions.

Fig. 4. Three beamlet-placement schemes based on the strategy outlined in this article: (a) $L_0 = 2$, $M_1 = M_2 = 10 \geq 2L_0$ (similar to the HIBALL reactor scenario); (b) fewer beamlets than in (a), but greater deposition symmetry; (c) two open bunches of beamlets from two sides, giving still greater symmetry.