CONF-810340--13

Lawrence Berkeley Laboratory

UNIVERSITY OF CALIFORNIA

Engineering & Technical Services Division

MASTER

Presented at the Seventh International Conference
on Magnet Technology, Karlsruhe, West Germany,
March 30-April 3, 1981

MODELING THE COOLDOWN OF FORCE-COOLED COILS

M.A. Green, S. Mitina, and G. Krafft

March 1981

Abstract - This paper describes a finite difference computer program which simulates the cooldown of force-cooled superconducting coils. The basic theory is discussed and the method of calculation used in the program is described. Some of the problems associated with computer modeling of a cooldown are discussed.

The program capability is demonstrated on a three dimensional model which represents the 1000 kg cryogenic model of the euratom LCT coil. From computer simulation using the program described here, a method for cooling down large forced cooled superconducting coils can be developed.

BASIC THEORY

The computer code described here calculates the temperature distribution in a variety of three dimensional shapes. The program establishes the problem in Cartesian coordinates. Other coordinates can be used by modifying the heat transfer functions within the code. The basic equation for heat transfer in three dimensions in Cartesian coordinates

$$c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left[k_x \frac{\partial T}{\partial x} \right] + \frac{\partial}{\partial y} \left[k_y \frac{\partial T}{\partial y} \right] + \frac{\partial}{\partial z} \left[k_z \frac{\partial T}{\partial z} \right] + Q_c \quad (1)$$

where

$$\begin{aligned} k_x &= k_x(x, y, z, T) \\ k_y &= k_y(x, y, z, T) \\ k_z &= k_z(x, y, z, T) \\ c &= c(x, y, z, T) \\ Q_c &= Q_c(x, y, z, T) \end{aligned}$$

T is temperature; t is time; k is thermal conductivity; c is specific heat per unit volume; Q_c is heat removed or added by convective heat transfer; and x, y, and z are coordinates in space. (Note: x, k, c and Q_c have functional relative to x, y, z, and T.)

The computer divides the problem into a three dimensional array of blocks which have mass and specific heat. These blocks may be made of any of the four materials or a mixture thereof. Heat is transferred through massless resistors which connect the faces of adjacent blocks. The specific heat and enthalpy of the block (the heat content) is a function of temperature. Heat transfer through the resistor between blocks 1 and 2 is calculated using the following relationship:

$$Q_T = R_T(T_1 - T_2) \quad (2)$$

where the value of the thermal resistor R_T

$$R_T = k(T_{ave}) \frac{A_T}{b} \quad (2a)$$

where

$$T_{ave} = \frac{T_1 + T_2}{2} \quad (2b)$$

and $k(ave)$ is the characteristic thermal conductivity of resistor as a function of temperature, T_{ave} ; A_T is the cross sectional area of the resistance element; b is the thickness of the resistance; T_1 is the temperature of block 1; and T_2 is the temperature of block 2.

Equation (1) can be restated in terms of an enthalpy change, Δh , and the volume of the block, V,

$$\Delta h = \Delta t \left[\sum_{N=1}^6 Q_T + Q_c \right] \quad (3)$$

the temperature change ΔT in the block can be characterized

$$\Delta T = \frac{\Delta h}{CV} \left[\sum_{N=1}^6 Q_T + Q_c \right] \quad (4)$$

where the summation is over the six faces of the block and Q_T is defined by (2). Q_c is the convective term, Δt is the time change, and C is the volume specific heat as a function of block temperature.

Q_c , the convective term, can be calculated using the following expression for convective heat transfer in a tube.

$$Q_c = h_c A_c (T_2 - T_f) \quad (5)$$

where T_f is the fluid temperature; T_2 , the block temperature; A_c is the inside surface area associated with block. The heat transfer coefficient h_c is defined for various Reynold number regions.

Laminar flow $Re < 2000$

$$h_c = \frac{4k_f}{D_H} \quad (5a)$$

turbulent flow $Re > 5000$

$$h_c = 0.023 Re^{0.8} Pr^{0.67} k_f / D_H \quad (5b)$$

the transition region between Reynolds of 2000 and 5000 is represented by a more complex merging equation.

k_f is the fluid [2] thermal conductivity; D_H is hydraulic diameter; Re is the Reynolds number; and Pr is the fluid Prandtl number (for helium over most of the temperature range, $Pr \approx 0.67$).

$$Re = \frac{v D_H \rho}{\mu_f} \quad (5c)$$

$$Pr = \frac{C_p \mu_f}{k_f} \quad (5d)$$

Manuscript received March 31, 1981.
M.A. Green and S. Mitina are with the Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720, USA. G. Krafft is with the Institute für Technische Physik at the Kernforschungszentrum Karlsruhe, W. Germany.

where v is the fluid velocity in the pipe; ρ is the fluid density; μ_f is the fluid viscosity; and C_p is the specific heat of the fluid with constant pressure. K_f and D_H have been previously defined.

For the sake of completeness the basic equation for pressure drop in the pipe is given as follows

$$\Delta P = \rho \frac{v^2}{2} f \frac{\Delta L}{D_H} \quad (6)$$

where ΔP is the pressure drop in a pipe segment with a length, ΔL ; and f is the Fanning friction factor [3] which is four times the drag-friction coefficient.

THE COMPUTER PROGRAM

The computer program consists of a main program and a number of subroutines. The calculation of fluid properties, pipe pressure drop, pipe heat transfer coefficient, and solid material properties is done in subroutines outside of the main program.

The main program sets up a problem. In general, nearly any pipe configuration can be handled by the code. The solid part of the problem can be divided into as many as 3000 blocks arranged in a three dimensional array. The pipe which carries the helium can run anywhere in or on the array. The properties of the different blocks are read in by various name list statements. The pipe properties are read in the same way. The thermal resistor boundaries are also defined by a series name list statement. By defining the properties using name list statements, one can calculate the problem using Cartesian coordinates. Conversion to other coordinate systems is accomplished by how the blocks and resistors are defined using the input data.

The program starts the problem by defining a preliminary pressure distribution, based on the fluid temperature in the pipe being the same as the initial temperature of the blocks adjacent to the pipe. The temperature distribution along the pipe is calculated using the following approximate expression

$$\Delta T_f = \frac{h_c A_c}{m C_p} (T_2 - T_f) \quad (7)$$

where ΔT_f is the temperature change of the fluid flowing along a piece of pipe with a convective heat transfer area, A_c ; m is the fluid mass flow; C_p is the fluid specific heat; and h_c the heat transfer coefficient. Equation 7 applies if $\gamma C_p / m C_p < 0.1$. In most cases, it becomes necessary to divide the pipe into many pieces in order to calculate the temperature distribution with any accuracy.

Once the temperature distribution in the pipe has been found, the temperature distribution in the solid can be calculated using (4). One starts by dividing the time into increments of Δt . The number of time increments must be at least five times the sum of the number of blocks in each of the three directions i.e., if the problem is 20 blocks long, 5 blocks wide, and 2 blocks thick, the minimum number of time steps is 175. The program does not iterate to find an exact solution at any given time step. The temperature at the farthest block does not start to change until enough time steps have progressed [4]. Figure 1 illustrates this concept in a two dimensional matrix. A typical problem has about 1000 time steps.

Fig. 1. Progression of the temperature distribution in a two dimension slab with cooling at the origin. The number of nodal points which become involved in the problem increases as the more time steps are taken.

Since the program has many time steps and many blocks, it is important to eliminate any iterative calculation which would make individual heat transfer calculation time consuming (a typical problem may have between 10^6 and 10^7 individual heat transfer calculations). The calculation of solid and fluid properties does not have to be made at each time step. This further reduces the computer time needed to solve the problem.

A typical time step for a problem, which has a mass of 1000 kg, might be 100 seconds. As the temperature drops to 50 K or lower, the specific heat of the solid material becomes very low, and the term $\Delta t/CV$ in (4) becomes large. When Δt becomes larger than, one-half the temperature difference between the solid and the fluid, the time step should be reduced.

FLUID AND SOLID MATERIAL PROPERTIES

The computer code has a subroutine which calculates the properties of helium at temperature from 10 to 400 K and pressures from 0 to 20 bar. There are existing helium property codes, but these are too slow. Thousands of fluid property calculations are done during the process of simulating the cooldown of a superconducting magnet. As a result, the subroutine is programmed to calculate fluid properties very quickly.

The helium equation of state is a first order connection from the ideal gas equation of state. When pressure and temperature are given, the subroutine calculates enthalpy, entropy, specific heat, density, thermal conductivity, viscosity, and Prandtl number. Within the range of the temperatures and pressure given, the fluid properties calculations are accurate to a few percent [5]. A thousand helium property calculations can be done in less than 500 ms on an IBM 370 computer.

The program has subroutines which calculate the thermal conductivity, specific heat and enthalpy for copper, aluminum, stainless steel, and epoxy. Most superconducting magnets contain a combination of several of these materials.

The specific heat for these materials is curve fit using a cubic spline and its first and second derivative. The curve fit matches the specific heat and its first and second derivatives (the derivatives must be continuous) at 13 different points in the range of temperatures from 1 to 400 K. An interpolation routine calculates the specific heat between these 13 points. The solid material enthalpy is found by integration of the spline fit. For copper, stainless steel, and aluminum, the specific heat and enthalpies agree within a percent or two over the full range of temperatures from 1 to 400 K [6,7]. Epoxy properties are fitted using the specific heat of Plexiglass as a guide.

Thermal conductivity is fitted using simple exponential equations which are accurate over various ranges of temperature. The fit is within 5 percent over most of the range for copper, stainless steel, and aluminum. The epoxy thermal conductivity was calculated by fitting the Plexiglass curve.

A TEST CASE

The computer code has been used to model the cooldown of the cryogenic model of the Euratom LCT coil. The test coil has a mass of 1000 kg. It can be cooled by flowing helium through the forced cooled conductor or it can be cooled down using a tube attached to the stainless steel case. The latter case model is presented here.

The cryogenic model is simulated using a composite slab which consists of two types of blocks. The slab is 20 blocks long (the x direction), 5 blocks wide (the y direction), and 2 blocks thick (the z direction). The cooling tube is attached to the type 2 blocks at the center (see Fig. 2). The two types of blocks (100 of each type) contain all of the mass. The lower blocks (type 1), consist

Fig. 2. The computer simulation model used to model the cryogenic model for the Euratom LCT coil.

predominantly of copper. They are assumed to have a mass of 2.9 kg each. The upper blocks (type 2) consist of stainless steel and have a mass of 7.1 kg each.

The resistance elements between blocks vary depending on location and direction of heat flow. This model has five types of resistors which are characterized as follows:

- Resistor type 1: the resistor between the type 2 blocks in the z direction; consists of stainless steel with $R_T = 50 \text{ k}_{SS}\text{WK}^{-1}$ (k_{SS} is the thermal conductivity of stainless steel in cgs units).
- Resistor type 2: the resistor between the type 1 and type 2 blocks in the y direction; consists of epoxy with $R_T = 2000 \text{ k}_E\text{WK}^{-1}$ (k_E is the thermal conductivity of epoxy in cgs units).
- Resistor type 3: the resistor between the type 1 blocks in the x direction; consists of epoxy with $R_T = 2000 \text{ k}_E\text{WK}^{-1}$.
- Resistor type 4: the resistor between type 2 blocks in the x direction; consists of stainless steel with $R_T = 0.32 \text{ k}_{SS}\text{WK}^{-1}$.
- Resistor type 5: the resistor between type 1 blocks in the x direction; consists of copper with $R_T = 0.32 \text{ k}_{Cu}$ (k_{Cu} is the thermal conductivity of copper in cgs units).

The cooling tube is 993 cm long. It has a hydraulic diameter of 0.66 cm and a cross-sectional area of 1.5 cm^2 . The heat transfer area is the pipe is about 9.1 cm^2 per centimeter of length. As an initial condition, the helium flows along the pipe at the rate of 4.5 gs^{-1} . The entering temperature and pressure are 50 K and 10 atm. The initial temperature of all 200 blocks is 300 K.

Figure 3 shows a plot of temperature vs distance in the x direction in the type one block 2 farthest from the cooling pipe (at the corner). The various

The author acknowledges the help given to us by the computer center at the Kernforschungszentrum Karlsruhe and the Lawrence Berkeley Laboratory. The work was supported in part by the Institute für Technische Physik, Kernforschungszentrum Karlsruhe. It was also supported in part by the Director of the Office of Energy Research, Office of High Energy Physics of the U. S. Department of Energy under contract No. W-7405-ENG-48 with the Lawrence Berkeley Laboratory.

REFERENCES

- [1] M. S. Carslaw and J. C. Jaeger, Conduction of Heat in Solids, Oxford Press, London, 1959.
- [2] K. Kreith, Principles of Heat Transfer, International Textbook Co., Scranton, 1961.
- [3] John K. Vennard, Elementary Fluids Mechanics, John Wiley and Sons, New York, 1954.
- [4] The Schmidt method for transient heat transfer in composite solid as discussed in Kreiths Book [2] and Carslaw and Jaeger [1].
- [5] "Thermophysical Properties of Helium 4 from 2 to 1500 K with Pressures to 1000 atm," NBS Technical Note 631 (Nov. 1972).
- [6] "A Compendium on the Properties of Materials at Low Temperatures," NBS Cryogenics Laboratory, Boulder Colorado, WAOD Tech. Report 60-56 (December 1961).
- [7] Handbook of Materials for Superconducting Machinery, Metals and Ceramics Information Center, Columbus Laboratories, 505 King Avenue, Columbus, Ohio 43201, MC1C-HB-04 (Nov. 1974).

Fig. 3. Temperature vs x direction distance on the type 1 block farthest from the cooling pipe for various times in the cooldown.
 Initial Temperature 300 K
 Entering Helium Temperature 50 K
 System Mass 1000 Kg

curves in Fig. 3 show the progression of the cooldown at various times from 0 to 10 hours.