The Physics & Design Issues of a High Gain FEL

Donald Prosnitz

Presented at the LLNL Free Electron Laser
Design Workshop

October 2, 1985

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed or represents that its use would not infringe privately owned right. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed or represents that its use would not infringe privately owned right. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed or represents that its use would not infringe privately owned right. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

MASTER

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED ²⁶

The free-electron laser

- Directly converts the kinetic energy of a relativistic electron beam into light
- Has no solids, liquids, or gases that can ionize or be damaged inside the laser
- Can be very efficient

- The coupling between electrons and light
 - The wiggler and wiggle motion
 - The laser electric field
 - $\vec{v} \cdot \vec{E}$ coupling
 - Resonance

- Electron beam properties that affect the FEL
 - Emittance
 - Energy spread
 - Energy loss

- Tapering the wiggler

Overview (cont)

- Focusing the electron beam
- Guiding the light by the electron beam
- Scaling

[⊙ High-gain, single-pass FEL amplifiers only will be discussed here—not FEL oscillators or RF linac driven FEL's]

ENERGY TRANSFER IN THE FREE ELECTRON LASER (FEL)

FEL intensity gain:

$$\frac{dI_s}{dz} \propto [\text{Electron energy loss}] \propto \vec{V} \cdot \vec{E}_s$$

$$\propto \sqrt{I_s}$$

The laser electric field

- Horizontally polarized:

- $E_x = E_s \cos(kz - \omega t + \phi)$

- with $k = 2\pi/\lambda$, $\omega = 2\pi \nu$

ELF: $\lambda = 8.6 \text{ mm}$ (34.6 GHz)

PALADIN: $\lambda = 10.6 \mu\text{m} = 10,600 \text{ \AA}$

- $E_s = E_s(x, y)$, $\phi = \phi(x, y) = \text{phase}$

The wiggler—a periodic, vertical magnetic field

- Electron beam
- Magnetic field lines

On midplane $\vec{B} = B_{y0} \cos k_w z \quad k_w = 2\pi/\lambda_w$

The wiggler — examples

- ELF - the microwave FEL next door

$$\lambda_w = 9.8 \text{ cm}$$

$$B_{y0} \cong 3.8 \text{ kG} \text{ (0 - 5 kG)}$$

~ 3 cm gap between magnets

$$L_w = 3 \text{ m}$$

- PALADIN - the infrared FEL to be built at ATA

$$\lambda_w = 8 \text{ cm}$$

$$B_{y0} \cong 2.5 \text{ kG}$$

~ 3 cm gap

$$L_w \cong 25 \text{ m}$$

○ In general $2 \text{ cm} \lesssim \lambda_w \lesssim 10 \text{ cm}$

$$1 \text{ kG} \lesssim B_{y0} \lesssim 5 \text{ kG}$$

$$0.5 \text{ m} \lesssim L_w \lesssim 2 \text{ m}$$

Wiggle motion — examples

$$\left| \frac{v_x}{c} \right|_{\max} \approx \frac{e B_{y0}}{\gamma m c k_w}$$

an angle at which
electrons cross the
 $x = 0$ plane

$$|x|_{\max} \approx \frac{e B_{y0}}{\gamma m c k_w^2}$$

maximum displacement
from straight-line orbits

-6-

	ELF	PALADIN
γ	7	100
Angle	0.5 rad	0.02 rad
Displacement	8 mm	200 μm = 0.2 mm

Electron-laser coupling

- Moving with the average motion of an electron

- $\vec{V} \cdot \vec{E}$ always greater than zero $\Rightarrow$ steady energy loss of electron $\Rightarrow$ steady energy gain of laser

Electron-laser coupling (2)

$$\begin{aligned} \frac{d\gamma}{dt} &= -\frac{e}{mc^2} \vec{v} \cdot \vec{E} = -\frac{e}{mc^2} v_x E_x \\ &= -\frac{e}{mc^2} \frac{e B_{y0}}{\gamma m k_w} E_s \sin k_w z \cos (kz - \omega t + \phi) \end{aligned}$$

$$\propto -\sin [(k + k_w) z - \omega t + \phi]$$

$$+ \sin [(k - k_w) z - \omega t + \phi]$$

- If $(k + k_w) z - \omega t + \phi > 0$ and ~ constant, then

$$\frac{d\gamma}{dt} < 0$$

Electron-laser coupling (3)

If, for a single e^- , $(k + k_w) z - \omega t + \phi \cong \text{const}$ then

- $(k - k_w) z - \omega t + \phi \cong -2 k_w z + \text{const} \Rightarrow$
rapidly varying and zero average for second term

- $\underbrace{\frac{dz}{dt} \cong \frac{\omega}{k + k_w}}_{\text{the FEL resonance condition}} < c, \text{ since } \frac{\omega}{k} = \lambda \nu = c$

the FEL resonance condition

Electron-laser coupling (4)

- $\psi \equiv (k + k_w) z - \omega t + \phi$
 - Slowly varying phase in a potential well of width $\cong \lambda$
- $\frac{d\gamma_i}{dt} \propto -\sin \psi_i$ for i^{th} electron
- $\psi_i > 0$ means wiggle motion in phase with $\vec{E}$ of laser $\Rightarrow$ energy loss
- $\psi_i < 0$ means wiggle motion out of phase with $\vec{E}$ of laser $\Rightarrow$ energy gain
- Sign of ψ_i is random for initial e^- beam

The Free Electron Laser is tuned by changing the electron beam's parallel energy

Resonant electron rest frame

therefore $\lambda^s = \frac{\lambda_w}{2\gamma^2}$

Gain mechanism

- Electrons with $\psi_i < 0$ gain energy, move faster, move toward $\psi_i \cong 0$
- Electrons with $\psi_i > 0$ lose energy, slow down, move toward $\psi_i \cong 0$

⇒ Spontaneous bunching at $\psi_i \cong 0$, where electrons neither gain nor lose energy,

but they shift the phase ϕ of the electric field, $\frac{d\phi}{dz} > 0$

- Emittance – a measure of the “temperature” of the transverse (x, y) electron velocities $\Rightarrow$ electrons with same γ but different random v_x, v_y (superimposed on wiggle motion) will have different $\beta_{||}$'s scattered around resonance
- Energy spread

Electron beam emittance characterizes the electron's average trajectories

Trajectories

Phase space

- The shape in phase space may change, but for constant energy, the area is preserved
- For accelerated or decelerated beams the normalized emittance, $\gamma\epsilon$, is constant
- The spread in parallel energy ($\Delta\gamma_{||}$) can be related to the phase space area

$$\mathcal{J} = \frac{2I\pi^2}{\gamma^2 d^4 V} \approx \frac{I\pi^2}{(\gamma\epsilon)^2} = \text{beam brightness}$$

- **Energy loss – essential to gain mechanism and FEL operation**
 - As e^- lose energy, they drop out of resonance, or (more precisely) they fall back to $\psi < 0$ and start taking energy back from the light

Hence TAPERING

Tapering the wiggler

$$\lambda = \frac{\lambda_w}{2\gamma_{\parallel}^2} = \lambda_w \frac{(1 + a_w^2)}{2\gamma^2} \quad \text{resonance condition}$$

λ fixed , laser wavelength

As γ decreases (energy loss) , resonance can be maintained if

- λ_w decreases down the wiggler , or
- $a_w \propto B_{y0}$ decreases down the wiggler , or
- some combination

LLNL approach is to decrease B_{y0} with increasing z - experimental flexibility

Taper design (in theory)

- Set $a_w(z) = \frac{e}{\sqrt{2 m c k_w}} B_{y0}(z)$ so that one electron stays at constant $\psi = \psi_r$ as it decelerates
- As a_w and γ both decrease, $\gamma_{||}$ (and $\beta_{||}$) remain constant for this design electron
- All other e^- decelerate differently (or not at all!)

Wiggler field

90-A-1085-179

Wiggler field

Trapped electrons

- Electrons near the design electron that stay within $-\pi < \psi < \pi$, and decelerate with the design electron are "trapped" in the potential well
- Trapped electrons undergo "synchrotron" oscillations around the design electron

Trapped electrons (2)

- Only trapped electrons are continuously decelerated in a tapered wiggler — so

$$\begin{aligned} \text{Power extracted from } e^- \text{ beam} &\cong \\ \text{Initial } e^- \text{ beam power} &\times \\ \text{Trapped fraction} &\times \\ \text{Fractional deceleration in } \gamma & \end{aligned}$$

- Simplest design tradeoff — too rapid deceleration $\Rightarrow$ less trapping

Transport of an optically bunched beam

Transport of an optically bunched beam (2)

- Electron beam focusing must preserve $10\ \mu$ bunches inside 1 cm e^- beam
- Focusing with curved pole faces can (in theory) accomplish this feat

External focusing in linear wigglers can lead to extensive detrapping

- Our tapered wiggler amplifier designs lower “buckets” of trapped particles many bucket heights over the full wiggler length
 - Detrapping can strongly reduce gain
- Long linear wigglers require focusing in the wiggle plane
 - External focusing (e.g. quadrupoles) is usual choice
 - $(a_w^2 + \gamma^2 \beta_\perp^2)$ varies over an individual particle's betatron orbit
 - $\gamma_\parallel$ variation destroying FEL resonance → detrapping
- In worst case, resonance between betatron and synchrotron motion can lead to resonant detrapping exceeding that from curved optical phase fronts
- Simulation runs confirmed problems with quadrupole focusing

Curved pole faces provide focusing solution

- Curved, parabolically-shaped pole faces give focusing in both planes
 - Invariance of $(a_w^2 + \gamma^2 \beta_\perp^2)$ retained over a betatron orbit
- Focusing in wiggler plane (**x**) due to non-zero average of B_y over a wiggler period
- Betatron frequencies in wiggler and non-wiggler planes are given by

$$k_{\beta,y} = k_y a_w / \gamma, \quad k_{\beta,x} = k_x a_w / \gamma \quad \text{where}$$

$$k_x^2 + k_y^2 = k_w^2$$

- If $k_x = k_y = k_w / \sqrt{2}$
 - an injected axisymmetric beam remains so through the wiggler
 - linear wiggler performs identically to helical wiggler of same rms magnetic field strength except for Bessel function coupling factor (= 0.9) between particles and field

Matched electron orbit

Laser optics in the wiggler

- Diffraction – the laser light would, in vacuum, spread out

$$E_s(r) \propto e^{-r^2/w^2}$$

$$\text{with } w = w_0 \left[1 + \left(\frac{z}{z_R} \right)^2 \right]^{1/2} \cong w_0 z/z_R$$

laser beam
radius at z

smallest beam
radius ("waist")

$$z_R \equiv \frac{\pi w_0^2}{\lambda} \text{ is "Rayleigh range"}$$

= characteristic expansion distance

Mismatched electron orbit

Mismatched electron orbit

- For long wigglers, diffraction could be a problem: the laser light can diffract away from the electron beam, reducing (or eliminating) the electron-laser coupling

But

- Optically bunched electron beam has an effective index of refraction $n > 1$, and can guide the light like an optical fiber

The index of refraction of the e^- beam

- $n > 1$ means $d\phi/dz > 0$; ie., phase velocity of light reduced
- Optical fiber acts as a distributed lens, trying to focus the light toward the axis
- Light propagates with constant profile along a fiber, as the focusing precisely balances diffraction
- Electron bunches with $\psi \cong 0$ produce no gain, but phase shift $d\phi/dz > 0 \Rightarrow n > 1$

A bunched electron beam behaves as an optical fiber and guides the optical beam in a high gain FEL

- 36 -

Simple scaling consideration

Why do we need high energy?

<u>Relation</u>	<u>Consequence</u>	<u>"Issue"</u>
$\lambda_s = \frac{\lambda_w}{2\gamma^2} (1 + a_w^2)$	High γ , small λ_w	\$/MeV
$\frac{d\gamma}{dZ} = \frac{e E_s}{\sqrt{2} mc^2} \frac{a_w}{\gamma} \sin \psi$	Large E_s , large a_w , small γ	Long wigglers may be diffraction limited

- Small λ_w limits maximum acceptable emittance and hence current
- Small λ_w limits maximum a_w

A shorter wiggler period requires a reduced electron beam emittance

$$\frac{\Delta\gamma_{\parallel}}{\gamma_{\parallel}} \approx \frac{(2\pi a_w)^2}{2} \frac{1}{1+a_w^2} \left(\frac{r_b}{\lambda_w}\right)^2 F(d^4V)$$

- r_b/λ_w should be held constant

$$\left(\frac{r_b}{\lambda_w}\right)^2 = \frac{1}{2\pi a_w} \left(\frac{\gamma\epsilon}{\lambda_w}\right)$$

- Beam emittance must be reduced linearly with λ_w
- Beam current is consequently reduced as λ_w^2
(fixed brightness)

A shorter wiggler period reduces the maximum a_w

- $a_{w \text{ (rms)}} \approx 2.2 \lambda_w \exp \left(\frac{-g}{\lambda_w} (5.47 - 1.8 g/\lambda_w) \right)$

$g = \text{full magnet gap} \approx \lambda_w/3$

There are several pathways to a .5 micron FEL

Constant gain, 0 is 10.6 microns, * is .5 microns

Why are high peak powers required

- High electric fields enhance trapping

$$\left(\frac{\Delta\gamma_{\parallel}}{\gamma} \right)_{\text{trapped}} \propto E_s^{1/2}$$

- High electric fields are required to decelerate the electron beam

$$\frac{d\gamma}{dZ} \propto -E_s \approx \left(\frac{2.9 \text{ MeV}}{m} \left(\frac{I}{100 \text{ GW/cm}^2} \right)^{1/2} \right)$$

- High currents are required for high gains

$$\frac{dE_s}{dZ} \propto J_{\text{trapped}}$$

Simple scaling is complicated by several issues

- Optical guiding — how long a wiggler is acceptable
- Exponential gain — how does the FEL saturate and what instabilities occur in very high gain systems
- Beam (both laser & electron) profiles

Summary

- The FEL is a "simple" enough system to be effectively modeled
- Scaling must consider for 3D effects
- Scaling must consider real hardware
 - Drive laser limits
 - Magnetic field limit
 - Mechanical tolerances
 - Optical beam train limits
- The FEL is new enough that all designs have a safety factor built in

"No cliff designs"