A major purpose of the Technical Information Center is to provide the broadest dissemination possible of information contained in DOE’s Research and Development Reports to business, industry, the academic community, and federal, state and local governments.

Although a small portion of this report is not reproducible, it is being made available to expedite the availability of information on the research discussed herein.
LA-uR--88-4197
DE89 005465

TITLE Some Recent Results in Hadronic Physics with Pions

AUTHORS W. R. Gibbs, T-S

SUBMITTED TO Proceedings of LAMPF Users Group 1988

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States
Government. Neither the United States Government nor any agency thereof, nor any of
their employees, makes any warranty, express or implied, of assumes any legal liability or responsi-
bility for the accuracy, completeness, or usefulness of any information, apparatus, product, or
process disclosed, or represents that its use would not infringe privately owned rights. Refer-
ence herein to any specific commercial product, process, or service by trade name, trademark,
manufacturer or otherwise does not necessarily constitute or imply its endorsement, recommenda-
tion or favoring by the United States Government or any agency thereof. The views and
opinions of authors expressed herein do not necessarily state or reflect those of the
United States Government or any agency thereof.

Los Alamos National Laboratory
Los Alamos, New Mexico 87545

MASTER
Some Recent Results in Hadronic Physics with Pions

W. R. Gibbs

Theoretical Division, Los Alamos National Laboratory
Los Alamos, New Mexico 87545

Submitted to: Proceedings of LAMPF Users Group 1988
SOME RECENT RESULTS IN HADRONIC PHYSICS WITH PIONS

W. R. Gibbs

Theoretical Division, Los Alamos National Laboratory
Los Alamos NM 87545

ABSTRACT

Three topics in modern hadronic physics are developed with regard to their fundamental importance to our understanding of the strong interaction in general and nuclei in particular. These three subjects are: low energy pion nucleon scattering and charge exchange, the study of the three nucleon system with elastic scattering of pions, and double charge exchange of pions on nuclei. In each case the studies are presented in terms of the fundamental motivations underlying them and the spectacular new data which is bringing new insight into these areas.

Introduction

I wish to discuss three topics in the field of strong interaction physics. These research areas are on the very forefront of our understanding of hadronic interactions. I will talk about them in intuitive terms and emphasize motivations rather than presenting technical details. One of the things that I hope you will appreciate is the spectacular quality of the data which is being taken to address these questions.

Low Energy Pion-nucleon Scattering

While this subject is an old one, the questions it poses to modern hadronic physics are no less demanding. In fact, it has recently taken on additional interest for several reasons. First of all it is the data on this process which lies at the basis of the determination of the "hidden" sigma term, the measure of baryon symmetry breaking in the strong
interaction. This quantity represents the amount that the mass of the nucleon is altered by the fact that we live in a world in which chiral symmetry is not perfect. The numerical value of the sigma term is based on principle from the extrapolation of a combination of the s-wave proton-nucleon scattering amplitudes to a negative energy point. While there has been a great debate over the years on just how this should be done or if it still rages, the most common value obtained for this number is around 10 MeV. From theories based directly on quark models one calculates a value around 30 MeV. This discrepancy has been known for a long time. This was pointed out a few years ago by Donoghue and Nappi[1] that if one assumes that there is a sea of quark-antiquark pairs with about one quarter of them being strange quarks, then these two numbers could be reconciled. Recent experiments has been a measurement of the hydrogen atomic level shift[2] which, if correct would change the low energy s-wave nucleon parameters enough to move the experimental determination of the sigma term to 30 MeV, thus obviating the need for any strange quarks in the proton sea. There are also some pion-nucleon phase shift analyses which give smaller numbers for the sigma term so that the question of the experimental value of the sigma term cannot be considered as closed.

Let us examine the general situation for the low energy s-wave phase shifts. There are two of them, of course, the isospin 1/2 and 3/2 waves. Figure 1 shows these phase shifts (from an analysis[3] of low-energy charge exchange data[4]) plotted as a function of the center-of-mass momenta up to a kinetic energy of 100 MeV. They are plotted vs. \( k \) because that is the variable used in an effective range expansion. The slope of these curves at zero energy is the scattering length. The isospin 1/2 phase shift is essentially linear below 1 MeV, while the isospin 3/2 line has curvature even down to 5 MeV. The qualitative conclusion that one draws is that the interactions for the two isospins have very different ranges. The curves shown were calculated in terms of separable potentials to data. The range for the derived potential for the 1/2 case is slightly greater than 1 fm while the 3/2 case has a range of less than 1.3 of that. In we understand physically any of these differences, consider the simple view of the nucleon consisting of a quark core

\[ \text{core} \oplus \text{neutron} \]

This model is thought of as being a dense cloud. If we take this picture, then a single nucleon can interact with either the core or the cloud

\[ \text{core} \oplus \text{quark} \]

\[ \text{cloud} \oplus \text{quark} \]
Note that in this picture, the isospin selection in the π-nucleon system makes an important selection in the hadronic interaction.

To see this, let us decompose the I=1/2 nucleon into its I=1/2 core and the I=1 pion in the cloud. It is clear, with a little recoupling, that the π-π interaction in the I=1/2 pion-nucleon system must be only in the I=0 state and by the same token, it must be in the I=2 state for the I=1 pion-nucleon system (there is no I=1 π-π state in the s-wave). To see this directly note that if the two pions are coupled to I=0 then the I=1-2 core can only stretch to a total isospin of 1/2 and if the two pions are in an I=2 state we can only reach I=3/2. Now the π-π interaction in the isospin zero state is about an order of magnitude larger than that in I=2. This tells us that there should be a strong contribution from the pion cloud in the I=1/2 state, but not for the I=3/2 wave. The pion cloud represents the largest extension in the system so it would lead to a long range potential, so the picture is consistent with the ranges found. If we try to calculate the magnitude of the I=1/2 π-nucleon scattering from the π-π I=0 scattering we don't do too badly. The π-π scattering amplitude is not very well known and we don't know how many pions to put into the cloud, so getting a precise number with this simple model is not easy. Putting one pion into the cloud, one gets to within a factor of two of the right answer.

Since the π-π interaction in the I=3/2 π-nucleon scattering is very small we expect that what remains must be dominant, i.e. the interaction with the quark core. This also makes sense if we compare with specific calculations as we will now see.

It was pointed out many years ago [5] that, if one uses any model which involves quark exchange, there should be relationships between different reactions with the same number of "active" quarks to be exchanged. A good candidate for this comparison is π⁺ and K⁺-proton scattering. If we neglect the antiquark in each case (the antiquark for the kaon is strange and hence is inactive while for the pion its annihilation with the down quark in the proton would lead to an I=1/2 core resonance which is very high in energy) then the number of up quarks to be exchanged (along with one or more gluons, for example) is two. And therefore the two systems evolve under the action of the same potential. Because the two mesons don't have the same mass, the scattering problem will be different, but assuming a reason for the potential one is able to use one of the scattering amplitudes to predict the other with this in mind or at least with a fair amount of agreement.

Therefore, let us now compare the π⁺-n and K⁺-n scattering amplitudes. When comparing the scattering amplitude of the two reactions, it is easy to see why the K⁺-n scattering amplitude is smaller than that of the π⁺-n in the I=1/2 state. To do this, let us decompose the K⁺ as an I=1/2 core and the I=3 pion in the cloud. As we noted before, the I=1/2 nucleon can only stretch to a total isospin of 1/2, so the I=0 state is not possible. Therefore, the π⁺-n scattering amplitude is larger than that of the K⁺-t in the I=1/2 state. This is also consistent with the ranges found. If we try to calculate the magnitude of the K⁺-n scattering from the π⁺-n I=0 scattering we don't do too badly. However, the K⁺-n scattering amplitude is not very well known and we don't know how many pions to put into the cloud, so get the precise number with this simple model is not easy. Putting one pion into the cloud, one gets to within a factor of two of the right answer.

Since the K⁺-n interaction in the I=3/2 π-nucleon scattering is very small we expect that what remains must be dominant, i.e. the interaction with the quark core. This also makes sense if we compare with specific calculations as we will now see.
that, in fact, the cloudy bag model is able to get the correct value for the \( \pi^+ \)-proton and \( K^+ \)-proton scattering lengths. We note that for the isospin 1/2 case the cloudy bag model is a disaster, that is until meson exchange contributions (sigma etc.) are included. This is simply another indication of what I said before; the meson cloud dominates the \( I=1/2 \) and the quark core dominates the \( I=3/2 \) \( \pi^- \)-proton s-wave scattering. Thus nature has provided us with a laboratory for separating quark and meson degrees of freedom.

We can even obtain an estimate for the purity of the separation from the following arguments. For the \( K^+ \)-nucleon scattering case the fact that the neutron has half as many up quarks as the proton means that the quark prediction of the \( K^+ \)-neutron scattering amplitude is only half that of the \( K^+ \)-proton amplitude. In terms of isospin this means that the \( I=0 \) amplitude is zero. Experimentally it is found to be very small. Applying the same arguments to the pion case we find that the quark prediction for the \( I=1/2 \) amplitude is 1/4 of the \( I=3/2 \) amplitude. Since \( I=3/2 \) scattering length is only (in magnitude) 1/2 that of the \( I=1/2 \) scattering length we may estimate the quark contamination of the \( I=1/2 \) scattering length to be only \(-1/8 \) or \(10^{-20}\). Above the very low energies we would need a more complete model to make such an estimate.

Things get even more interesting when we realize that the chiral symmetry conserving combination of these two quantities that cancels at zero pion mass and at zero energy is exactly the same combination that occurs in the calculation of one part of the pion-nucleus optical potential for scattering from an isospin-zero nucleus. Of course, because of the difference in the energy dependence of the two isospin waves, the two contributions do not completely annihilate away from zero energy but the cancellation is still significant, as shown in figure 2. Here the separate contributions have been divided by \( k^2 \) because of a conventional factor included in that part of the optical potential which arises from the pion-nucleon s-wave. I have shown the \( b_0 \) obtained from the analysis quoted before \( \text{[3]} \), but also shown is the one which comes from Arndt's analysis \( \text{[4]} \). While in general, there is little difference between the two sets of phase shifts, the large cancellation accentuates this uncertainty. The values of \( b_0 \) needed to fit the pion-nucleus scattering data are well known to be more negative than those that I have shown here, predicted from the fundamental amplitudes. We also know that a part of this discrepancy comes about because some of the p-wave part of the \( \pi N \)
which is much stronger than the s-wave, gets mixed into $b_0$. However, it has always been difficult to find enough strength from this effect to agree with the experimentally determined $b_0$.

One explanation for the EMC effect has been that the bag-like core of the nucleon “swells” slightly thus partially deconfining the quarks of the increase in the size of the quark core is estimated to be of the order of 10-15%. If we assume that the pion cloud is unaffected by the immersion of the nucleon in the nucleus (obviously an oversimplification) and simply increase the core radius (and hence, in a hard-core model, the = 1 phase shift) by a factor of 1.1 the value of $b_0$ is made more negative. The curve $s$ labeled is also shown in figure 2. Because of the cancellation already noted, the 10% change in the bag size gives a 50% change in the value of $b_0$, at least at the lowest energies.

It is worthwhile pointing out that the pion wave lengths are, in fact, the right size to carry out this kind of investigation. In order to distinguish two different length scales the wave length should be in the range where the smaller size system has the appearance of a delta function (or at least a short range) in coordinate space and the other has a clear finite extension as evidenced by a momentum or energy dependence. That is to say, the wave length should be between the two scales. Since the pion wave length is typically of the order of 1 fm for low energy pion scattering this condition is satisfied.

I hope it is now clear from what I have just said that, from several points of view, the low energy pion-nucleon phase shifts constitute a crucial data set. How do we get an accurate measurement of them?

Note again that the isospin 3/2 phase shift is linear in $k$ below 50 MeV so that measurements below that energy are not essential for this isospin. Note also that $\pi^+$-proton scattering gives this number directly since it is purely isospin 3/2. Hence, good $\pi^+$-proton data down to 50 MeV are sufficient for the determination of this phase shift. Such data has recently been taken by Brack et al. [10] and should fill the bill.

The isospin 1/2 amplitude poses a different problem. First of all, there is no single experiment which directly measures this amplitude. There are two choices, extract it from either $\pi^-$-proton elastic scattering or from exchange data. Of course, one would like to have both sets of data to check that isospin violations due to the Coulomb potential and mass differences have been properly taken into account and that there are no nasty surprises from some other source. However, the $\pi^-$-proton data...
Fig. 3: Comparison of the new (preliminary) data of ref. 11 with the predictions of ref. 3. Note that the agreement at 40 MeV and 1 MeV is satisfactory while the comparison at 20 MeV indicates a readjustment of the parameters will be necessary.
is very difficult to do at the very low energies needed because the Rutherford amplitude is coherent with the strong scattering and tends to dominate. In this case one would have to measure the differential cross section very accurately in order to extract the strong component. Of course the pion beam must be transported to the target before scattering and from the target to the spectrometer after scattering in order to measure the absolute cross section with high precision and, at low energies, the pion decay makes this difficult. The charge exchange reaction is much more promising. First of all, there is no coherent Coulomb amplitude. That is not to say that there is no Coulomb effect, but only that it is much smaller. Secondly, the beam need only be transported to the target; the \( e^0 \) decays immediately and is detected by means of the two photons from the decay. This detection method is adequate since energy resolution is not an overwhelming consideration here.

What is really sensational is that such data have just recently been taken down to 10 MeV, and preliminary results reported by Isenhower et al. [11.] Figure 3 shows this data compared with the predictions of the potential analysis mentioned earlier. It is interesting that the agreement is very good around 40 MeV (it should be, since the fits were made to the previous charge-exchange data [4] in this energy region) and at the lowest energy, but that there is a noticeable difference around 20 MeV. This means that, in a reanalysis, the curvature is going to be somewhat different than that obtained before. It will very interesting to see what effect the results of an analysis of the final data will have on the value of the sigma term (and the number of strange quarks in the nucleon?) Remember that the sigma term comes from an extrapolation of the data below threshold so that a knowledge of the effective ranges is as important as that of the scattering lengths. The accurate determination of this curvature is significant.

\[ ^{\ast} \text{and } ^{\ast}\text{ Scattering on the \(^1\text{He}/T\text{ Systems}} \]

The n-p force is slightly stronger than the n-n p-p force. The neutron is bound and the n-p spin-singlet scattering length is larger in magnitude than the n-n scattering length \( = 1.1 \). Therefore one expects the radius of the odd nucleon in the trinucleon system to be smaller than radius of the like pair. That is, the proton radius should be smaller than the neutron radius. State-of the art follows.
calculations employing contemporary nucleon-nucleon force models yield a difference of about 0.16 fm.

In the absence of the Coulomb interaction between the two protons in $^3$He, the $^3$H and $^3$He systems would be identical. Including the Coulomb interaction in the Faddeev calculations leads one to an increase in the proton radius of 0.03 - 0.04 fm. The repulsive Coulomb interaction also affects the neutron radius. The increased separation of the two protons means that the neutron is less bound. That is, the neutron distribution is also expanded, and the neutron radius is increased by 0.02 - 0.03 fm.

The proton radii of $^3$H and $^3$He are known experimentally from elastic electron scattering:

$$r_p(^{3}\text{He}) = 1.76 \pm 0.04 \text{ fm}$$

$$r_p(^3\text{H}) = 1.57 \pm 0.04 \text{ fm}$$

The difference of 0.19 fm is consistent with the results of the Faddeev calculations: 0.16 + (0.03 - 0.04) fm.

What can be said about the neutron radii? It is difficult to extract a neutron radius for $^3$He from magnetic electron scattering, because meson exchange current corrections are sizeable. It is impossible to extract a neutron radius for the $^3$H because the odd nucleon, which carries most of the spin, is the proton.

Thus, one is led to pursue pion scattering to determine the relative radii in the A=3 systems. Meson exchange current contamination is minimal. Near resonance, the $\pi^-p$ interaction dominates the $\pi^+ scattering and the $\pi^-n$ interaction dominates the $\pi^-$ scattering. Assuming that multiple-scattering effects can be properly accounted for, ratio measurements should be very sensitive to differences in the odd nucleon and like nucleon matter distributions.

One might ask about the effect of three-nucleon forces on these systems. Contemporary two-pion-exchange three-nucleon force models were included in the above mentioned Faddeev calculations. These proposed three-body force models are isoscalar in nature. Thus, they tend to decrease the difference between the proton and neutron radii. One can see in Fig. 4 from ref. 12 that, while the introduction of a three-body force can improve the binding energy (and low-energy properties such as radii, three-body forces do not resolve the discrepancy between theory and
Fig. 4. \(^3\)He and \(^3\)H form factors from the Faddeev calculations in Ref. 1 with and without three-body forces.
experiment for the higher momentum transfer region of the charge form
factors.

Let us consider three ratios of pion-trinucleon cross sections.
First, the ratio

\[ r_1 = \frac{\sigma(\pi^+^3H)}{\sigma(\pi^-^3He)} \]

involves primarily the pion strong interaction with the odd nucleon in each case. That is, in the region of the (3,3) resonance, \( \pi^+p \) and \( \pi^-n \) scattering dominate over \( \pi^-p \) and \( \pi^+n \). Clearly the coherent Coulomb scattering does not cancel from the ratio, but the strong interaction should be much more important. Thus, \( r_1 \) should be sensitive to the ratio of the odd-nucleon form factors -- in the single scattering (impulse) approximation, this is what one would calculate keeping only the dominant \( \pi^+p \) and \( \pi^-n \) interactions. Both spin-flip and non-spin-flip scattering from the odd nucleon are important.

Second, the ratio

\[ r_2 = \frac{\sigma(\pi^-^3H)}{\sigma(\pi^-^3He)} \]

involves primarily the pion strong interaction with the like nucleons in each case. Again the Coulomb effects do not cancel in the ratio. However, because the like nucleons are essentially paired in spin (to spin 0), spin-flip scattering is minimal. Thus, \( r_2 \) is sensitive to the ratio of the like-nucleon form factors.

Finally, the "super ratio"

\[ R = r_1 r_2 \]

\[ = \frac{\sigma(\pi^+^3H) \sigma(\pi^-^3H)}{\sigma(\pi^-^3He) \sigma(\pi^-^3He)} \]

should be least sensitive to model uncertainties in the treatment of the pion-nucleus scattering theory (as well as experimental normalizations). While the Coulomb interaction does not cancel, the calculation of \( R \) should be less sensitive to any model dependence on those effects than the individual ratios \( r_1 \) and \( r_2 \).
Because $^3$He is expected to be larger than $^3$H, such that its form factor falls faster, we anticipate (in general) that $R > 1$. Similar conclusions can be reached for $r_1$ and $r_{1n}$, although they are subject to greater uncertainty due to Coulomb interference effects.

Looking at Fig. 5, we see the relevant form factor (impulse approximation retaining only the strongest interaction) ratios plotted as dashed lines. The solid lines represent pion-trinucleon scattering calculation results in which variations among the strong interaction model parameters ($<n$ s-wave off-shell range, $<n$ p-wave off-shell range, $<n$ spin-flip off-shell range, and energy shift) were made. It is clear that the model dependence in terms of the $<n$ interaction is minimal between 0$^\circ$ and 80$^\circ$. Also, the multiple scattering results do follow the general trend of the form factor ratios.

In Fig. 6 we display the same set of curves BUT for trinucleon matter densities which have been modified to account more reasonably for the existing data. We have assumed that the shape of the trinucleon distributions are adequately defined by the Faddeev calculations. Thus, the difference in the $^3$He/$^3$H structure between the calculations presented in Figs. 5 and 6 is given entirely in terms of the rms radii of the odd nucleon and like nucleon pair, for each nucleus (i.e. the radius variable in each density was rescaled so that the cited rms radius was obtained, the normalization being corrected as well). We have furthermore assumed that the radii determined by elastic electron scattering from $^3$He (the radius of the like protons) and from $^3$H (the radius of the odd proton) are fixed by those measurements. Therefore, the odd-nucleon radius of $^3$He was decreased (1.61 - 1.57 fm) to improve the theoretical ratio $r_1$. Similarly, the like-nucleon radius in $^3$H was decreased (1.71 - 1.67 fm) to improve the fit to $r_2$ and $R$. The changes made (0.04 fm) are no larger than the absolute uncertainties in the measured values of $r_1(^3$He) and $r_1(^3$H). However, the relative sizes of the resulting radii for $^3$He and for $^3$H disagree completely with the predictions for the odd-nucleon and like-nucleon radii given by the Faddeev calculations. Thus we see that the measurement of the relative radii provides a much more stringent test than the comparison of the two proton radii alone.

It is clear from looking at the model differences reflected in the plots shown in Figs. 5 and 6 that the ratios are much more sensitive to the relative sizes of the matter distributions of the trinucleons than to the proton-nucleus scattering model uncertainties. It is also evident that the
relative sizes of the odd-nucleon and like-nucleon matter distributions in
the trinucleons can be more precisely determined from the proposed r.
measurements than they are now known from the absolute measurements made
via elastic electron scattering. Recent preliminary data taken at 2.56 MeV
and presented at Santa Fe [14] shows that the super ratio is less than one.
This not only violates our naive expectation expressed above but also
disagrees with the extrapolations from both calculations just discussed.
The individual ratios $r_1$ and $r_2$ are not yet available and we await the
final data reduction before attempting even a speculative explanation.

Pion-Nucleus Double Charge Exchange

The DCX reaction has been considered for many years as one of the best
hopes for probing the correlation structure of the nucleus. This is due to
the fact that (at least) two nucleons must be affected by this reaction;
there is no first order (or single scattering) term. How to actually
extract information on the nucleon-nucleon correlations from this reaction
has not been clear. The problems are the usual ones, i.e. the nucleus is a
many-body problem and scattering is, at least, a many-plus-one-body
problem. Clearly approximations and insight are needed to develop a
technique for extracting information.

To do this we begin with the shell model, starting with the simplest
form and gradually adding increasing complexity as warranted by the data
and our ability to deal with the scattering aspects. From this point of
view we start with the simplest, non-trivial, case we can find. The system
chosen, for both experimental and theoretical reasons, is that of the
calcium isotopes and more generally, the $f_{7/2}$ shell. We assume, to begin
with, that all active particles are in $f_{7/2}$ orbitals. The case of the
transitions to the double analog from calcium isotope targets is the most
straightforward. From the nuclear structure point of view we note that
for the case of only neutrons in a single shell, $j=7/2$ the seniority
model [15] is exact in the sense that it gives the same answer as a full
diagonalization of the type of, say, MBZ [16]. In the seniority model in
general, one finds for DCX, as in the case of the original formula for
energy levels, that there are only two amplitudes that contribute and that
one of the amplitudes is long range, being sensitive to the entire nuclear
volume, while the other depends only on the components of the wave
of the two active nucleons representing the situation when they are in...
to each other. It was not obvious that such a formula is valid for $\text{DCX}$ since its derivation for the nuclear energy levels (for which it was originally created) depends on the assumption of a scalar interaction between the two nucleons and the DCX scattering operator is by no means a spatial scalar. This same simplification comes about in this case because the transition proceeds from a $0^+$ initial state to a $0^+$ final state so that only the scalar part of the DCX operator is sampled.

Carrying out the calculation assuming such wave functions \( \psi \) yields the following table for double analog transitions in the calcium isotopes. The nuclei listed at the right of the table are the particle-hole conjugates of those on the left and completely equivalent insofar as the shell structure is concerned.

\[
\begin{align*}
^{42}\text{Ca} & \quad |A+B|^2 & \quad ^{54}\text{Fe} \\
^{44}\text{Ca} & \quad 6|A+\frac{1}{2}B|^2 & \quad ^{52}\text{Cr} \\
^{44}\text{Ca} & \quad 15|A-\frac{1}{2}B|^2 & \quad ^{50}\text{Ti} \\
^{48}\text{Ca} & \quad 28|A-\frac{1}{2}B|^2 \\
\end{align*}
\]

Table I. Expressions for the analog cross section for double charge exchange in terms of the two amplitudes "A" and "B".

The amplitude "A" corresponds to the long range (uncorrelated) part of the total amplitude and, if it were the only contributor, the cross section would be proportional to the "pairs factor" appearing in the front of the expression, so called because it is simply the number of excess neutron pairs. We see that a violation of this pairs factor rule is a sign that, either the assumptions made in deriving these formulae are wrong, or that the "B" term, representing correlations, is present.

It has been known for some time that the pairs factor rule is broken by a considerable amount, especially at low energies where, e.g., the \(^{44}\text{Ca}\) cross section was measured to be only 1/2 of that of \(^{44}\text{Ca}\) instead of 6 times greater as predicted by this simple rule. Thus it seemed likely that the "B" term, arising from correlations, was playing a significant role.

How do we prove to ourselves that the understanding of DCX truly lies in the existence of the correlation term "B"? We can use measurements of several of these isotopes to perform a test. Notice that "A" and "B" are
two complex amplitudes and, since the overall phase is irrelevant, there are only 3 independent numbers which must describe all of the cross sections at each energy and angle (at least in the pure seniority model). Thus the measurement of 3 isotopes determines these numbers and permits the prediction of additional cross sections based on these formulae. The following table presents a series of measurements made at 35 MeV in the summer of 1987 to check these relationships. This kind of analysis was made by Z. Weinfeld but the cross sections given here are actually due to Mike Leitch [18].

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Double Analog Transition</th>
<th>Ground State Transition</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experiment</td>
<td>Prediction</td>
</tr>
<tr>
<td>$^{42}$Ca</td>
<td>2.27 ± 0.29</td>
<td>&lt;2.27&gt;</td>
</tr>
<tr>
<td>$^{44}$Ca</td>
<td>1.09 ± 0.16</td>
<td>&lt;1.09&gt;</td>
</tr>
<tr>
<td>$^{50}$Ti</td>
<td>1.55 ± 0.27</td>
<td>1.47</td>
</tr>
<tr>
<td>$^{52}$Ca</td>
<td>2.70 ± 0.90</td>
<td>&lt;2.70&gt;</td>
</tr>
<tr>
<td>$^{52}$Ti</td>
<td>2.53 ± 0.35</td>
<td>4.52</td>
</tr>
<tr>
<td>$^{54}$Fe</td>
<td>1.30 ± 0.40</td>
<td>2.27</td>
</tr>
<tr>
<td>$^{42}$Ca</td>
<td>1.90 ± 0.30</td>
<td>&lt;1.90&gt;</td>
</tr>
<tr>
<td>$^{44}$Ca</td>
<td>1.10 ± 0.15</td>
<td>&lt;1.10&gt;</td>
</tr>
<tr>
<td>$^{50}$Ti</td>
<td>1.47 ± 0.18</td>
<td>1.45</td>
</tr>
<tr>
<td>$^{52}$Ca</td>
<td>2.40 ± 0.60</td>
<td>&lt;2.40&gt;</td>
</tr>
<tr>
<td>$^{52}$Ti</td>
<td>2.11 ± 0.30</td>
<td>3.69</td>
</tr>
<tr>
<td>$^{54}$Fe</td>
<td>0.90 ± 0.20</td>
<td>1.90</td>
</tr>
<tr>
<td>$^{42}$Ca</td>
<td>0.40 ± 0.08</td>
<td>&lt;0.40&gt;</td>
</tr>
<tr>
<td>$^{44}$Ca</td>
<td>0.16 ± 0.04</td>
<td>&lt;0.16&gt;</td>
</tr>
<tr>
<td>$^{50}$Ti</td>
<td>0.71 ± 0.13</td>
<td>0.83</td>
</tr>
<tr>
<td>$^{52}$Ca</td>
<td>2.20 ± 0.50</td>
<td>&lt;2.20&gt;</td>
</tr>
<tr>
<td>$^{52}$Ti</td>
<td>0.47 ± 0.12</td>
<td>0.95</td>
</tr>
<tr>
<td>$^{54}$Fe</td>
<td>0.04 ± 0.03</td>
<td>0.40</td>
</tr>
</tbody>
</table>

Table II. DCX cross sections (in µb/sr) at 35 MeV. Angle brackets "<" indicate values used for the fit. Parentheses indicate predictions beyond the seniority model.

The cross sections in the angle brackets are the values used to fix the amplitudes A and B and the rest are predictions of the theory. First let us look at the predicted cross sections for $^{50}$Ti which is the
particle-hole conjugate of $^{40}$Ca, the later being a rather expensive "error" for pions. One sees that the predictions are equal to the experimental cross sections, within errors, at each of the three angles which means that the expressions involving the correlation term work very well for this simple case where the seniority model is exact. This is a test of the assumption of the pure $f_{7/2}$ model for the calcium isotopes, or at least for the constancy of the correction to this model across the shell. Note that this is not a trivial result: to be able to predict three cross sections within 15% is significant.

Next, let us examine the cross sections for $^{44}$Ti. For this case the seniority model is not equivalent to the shell model and we must go beyond the two-amplitude expression [19]. In order to calculate a cross section from the amplitude already determined experimentally we use a correction (the numbers given in parentheses below the numbers given for the seniority model) which has some model dependence. We see that the seniority model does not work, as was expected, but that the full (single orbital $f_{7/2}$) shell model does predict the cross section within the 15% errors.

We now proceed to the case of $^{56}$Fe which is the p-h conjugate of $^{40}$Ca so may be expected to have the same cross section. However that expectation assumes that, among other things, the nuclei are of the same size. But these two nuclei are at opposite ends of the shell as implied by the conjugate relationship. The orbitals should have rather different spatial extensions so we should not be surprised to find a difference in the form of a more rapid fall-off of the iron differential cross section. This is, in fact, what is observed. Our microscopic calculations indicate that the difference seen is about the right size. Of course it is also possible that the structure of $^{56}$Fe contains different components from that of $^{40}$Ca as well.

It is possible to predict, not only the analog transitions, but the transitions to the final ground state both in the seniority scheme and the more general model [19]. Table II shows the cross section for the one ground state that has been measured at 35 MeV for $^{40}$Ca. We note that the agreement with the extended prediction is marginally satisfactory. This same data is shown plotted in figure 7 to demonstrate the evolution of the angular distribution from $^{40}$Ca to $^{40}$Ca with the later being much more nearly isotropic, a feature which can also be understood from the equations given above. Since it is beyond the scope of the talk here I refer you to reference 19.
This past summer additional data were taken to measure the energy dependence of the cross sections. There were several surprises. First of all, enough data could be assembled at 65 MeV to start a similar analysis to that at 35 MeV. Here we note that the ground state of $^{44}$Ca (as corrected for the full $f_{7/2}$ shell model) was needed to help fix the parameters. It is clear that there is a problem with the ground state of $^{44}$Ca; actually the problem is with the ratio $^{44}$Ca/$^{40}$Ca. It is much too small to fit into the scheme. It is possible that the problem is related to the rapid energy dependence to be discussed below.

![Graph showing energy dependence of cross sections](image)

**Table III**  
DCC cross sections (in $\mu$b/sr) at 15° and 65 MeV

<table>
<thead>
<tr>
<th>Double Analog Transition</th>
<th>Ground State Transition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment</td>
<td>Prediction</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>$^{44}$Ca</td>
<td>138+0.60</td>
</tr>
<tr>
<td>$^{40}$Ca</td>
<td>&lt;0.6</td>
</tr>
<tr>
<td>$^{42}$Ca</td>
<td>0.4+0.11</td>
</tr>
</tbody>
</table>

Note: The energy dependence is not explicitly shown in the table but is implied by the data shown in the graph.
Before going on to present the results achieved so far for the nuclear dependence let me explain why it is so interesting.

Let us consider two of the ways that the double charge exchange reaction might take place. The first, and most often computed, is the sequential process shown in figure 8a. There the reaction proceeds through two independent single charge exchanges, (although not necessarily with the intermediate nucleus in the single analog state). The part of the

\[
\begin{array}{c}
\text{n} \\
\downarrow \\
\pi^+ \\
\downarrow \\
\text{p} \\
\uparrow \\
\pi^0 \\
\downarrow \\
\text{n} \\
\uparrow \\
\pi^- \\
\downarrow \\
\text{p}
\end{array}
\]

\[
\begin{array}{c}
\text{n} \\
\downarrow \\
\pi^- \\
\downarrow \\
\text{p} \\
\uparrow \\
\pi^+ \\
\downarrow \\
\text{n} \\
\uparrow \\
\pi^0 \\
\downarrow \\
\text{p}
\end{array}
\]

(a) \hspace{2cm} (b)

\textbf{Figure 8.} Diagrams for the a) sequential and b) "meson exchange current" process.

amplitude which arises from the intermediate analog route in the sequential model is to be identified with the amplitude "A". The rest of the possible intermediate states contribute to "B". Figure 9 shows a plot of \(|A|\) and \(|B|\) (computed with the sequential mechanism and without distortion) as a function of the internucleon distance. That is to say, what is plotted is the value that these two quantities would have if there were no contribution from inside the corresponding internucleon range which labels the abscissa. As we see, the quantity "A" has contributions from the entire nucleus while "B" only receives strength from short internucleon spacings. We may well believe that this sequential model is suitable for the calculation of "A" since reactions occurring far apart are likely to be independent. However, for the "B" amplitude the sequential model is questionable since it assumes independence even when the nucleonic constituents are overlapping.

The pion clouds associated with the nucleons should sometimes overlap and in this case the double charge exchange reaction can take place in a
single step as shown in figure 8b. This process has been considered for a number of years [20] and calculations of it have always claimed to give substantial cross sections. An interesting feature of this mechanism is that the cross section does not depend on energy (in plane-wave calculations with a constant $\pi - \pi$ vertex) but only on momentum transfer. Therefore at $0^\circ$ the DCX cross section would be independent of energy. Of course the energy dependence arising from the variation of the distortion of the initial and final waves is present in any realistic calculation.

On the other hand for the sequential process, aside from this same energy variation arising from the distorted waves, there are two additional sources of energy dependence coming from the transition amplitude itself -- the two delta resonances (one at each charge exchange) and the s-p interference at 50 MeV. The idea presents itself that perhaps we can separate the contributions of these two mechanisms by examining the energy variation of the DCX cross sections.

Figure 9 shows a distorted wave calculation of the $^{40}Ca$ double analog cross section with the meson exchange mechanism only. We see that the nuclear transparency around 50 MeV causes a large structure in the cross section. Of course the sequential calculation will show similar, but more complicated structure.
Fig. 10: Distorted wave calculation of the MEC contribution at 0° as a function of pion energy

Fig. 11: Experimental measurements of the analog and ground state cross sections (Ref. 20)
Figure 11 shows the measured energy dependence of the analog and ground states as presented by Mike Leitch (preliminary data) at the Santa Fe DNP [20] meeting. We see several interesting features. For one thing, the rapid energy variation around 50 MeV causes us to question assumptions such as: the corrections due to the difference in Q-values are small. If the outgoing pion energy differs by 10 MeV between two different cases, that can make a significant difference to the cross section and might explain the difficulties mentioned above for the ratio of the $^{48}$Ca/$^{44}$Ca ground states.

One of the most striking features is the structure in the $^{42}$Ca analog cross section. It very much resembles that shown in figure 10 for the meson exchange current. It would be premature, however, to conclude that we have seen evidence for such an effect since the sequential process can produce similar structure. Additional information is available from the fact that the meson exchange graph contributes to A and B in a well defined manner. We note that the ability to separate the reaction into two parts, (plus the sharp energy dependence) is apparently providing us with a microscopic view into the nucleus, permitting us to investigate the very basis of the structure of the hadronic interaction.

Acknowledgements

Since this talk consisted primarily of a review and discussion of recent experimental data, and its interpretation, I have a great number of people to thank. First of all I must thank Donald Isenhower and Mike Sadler (and the rest of the team) for allowing me to quote their striking data on low energy pion-nucleon charge exchange before publication. That experiment is a real tour de force.

Next I thank Barry Berman, Ben Nefkens, Bill Briscoe, Kalvir Dhuga (and the rest of the group) for letting me use their exciting data on the ratio measurements on the three nucleon system. I especially thank Ben Gibson for helping me to think through the relevant physics. The expertise on the three body system is totally his.

For the low energy DCX work I thank to the whole team, but especially Mike Leitch who gave me the data, provided the fits I have shown and even allowed me to steal some of his figures. I also wish to thank Bill Kaufmann, a collaborator in the DCX work, especially for the calculation of the meson exchange cross section.
This work was supported by the U.S. Department of Energy.

References

8. R. Arndt, SAID online program.