THE PROPAGATION OF PHASE-CHANGE FRONTS IN MOVING FLUIDS

PETER G. KROEGER

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The Propagation of Phase-Change Fronts in Moving Fluids

Peter G. Kroeger

ABSTRACT

The transient propagation of a phase-change front during the depressurization of a subcooled liquid has been analysed for the case of thermal equilibrium without friction or heat addition, using wave diagram solutions.

Various modes of phase-change front propagations have been identified and it appears that certain ranges of propagation speeds are impossible while others require the simultaneous formation of shocks.

Qualitative and quantitative results are given for two simple geometries.

TABLE OF CONTENTS

1. Introduction	1
2. Problem Formulation	3
3. Phase-Change Front Propagations	5
4. Applications	8
4.1 Qualitative Demonstration of Phase-Change Front Solutions	8
4.2 Application to Real Non-Ideal Fluids	15
5. Conclusions and Recommendations	19
References	22
Notation	24
Appendix A: Review of Important Features of Wave Diagrams	25
Appendix B: State Equation for Assumed Hypothetical Gas	27
Appendix C: Summary of Conservation Equations Across a Normal Moving Shock	29

List of Illustrations

<u>Figure</u>		<u>Page</u>
1	Schematic of wave diagram solutions in the presence of phase-change fronts	31
	(a) Phase-change front location being determined by Region 1 alone	
	(b) Phase-change front location being determined by both regions	
2	Phase-change front propagation speeds ($v > 0$; $a_1 > a_2$)	32
3	Schematic of wave diagram solutions in the presence of phase-change fronts	32
	(a) Conditions for Sector 2 of phase-change front propagation cases	
	(b) Conditions for Sector 4 of phase-change front propagation cases	
4	Wave diagram for a hypothetical gas with phase change, discharging from dead-ended pipe ($P_o = 4$ bar, $h_o = .625$ MJ/kg; $\lambda = 2$; $\kappa = 1.4$)	33
5	Schematic of phase-change front location during depressurization in a dead-ended pipe	33
6	Schematic of short-time wave diagram at origin of expansion fan	34
7	Wave diagram for a hypothetical gas with phase change, discharging through a pipe connected to a reservoir ($P_o = 2.1$ bar; $p_{sat} = 1.75$ bar; $h_o = .625$ MJ/kg; $\lambda = 2.0$; $\kappa = 1.4$) ^o	34
8	Schematic of phase-change front propagation beyond point A	35
9	Schematic of phase-change front	35
	(a) Without shock formation	
	(b) With shock formation	
10	Properties of water along an isentrope from $p_o = 70$ bar, $h_o = 1$ MJ/kg to atmospheric pressure ($p_{sat} = 28.81$ bar)	36

List of Illustrations

<u>Figure</u>		<u>Page</u>
11	Wave diagram for high pressure subcooled water discharging from a dead-ended pipe	36
12	Expanded view of first wave reflection of wave diagram for high pressure subcooled water discharging from a dead-ended pipe	37
13	Wave diagram for high pressure subcooled water discharging through a pipe connected to a reservoir	37
14	Expanded view of shock wave formation during discharge of subcooled water through a pipe connected to a reservoir	38
C-1	Schematic presentation of right moving shock	38

INTRODUCTION

During the transient depressurization of a subcooled liquid flashing can occur as the saturation pressure of the liquid is reached. A phase-change front, or boiling boundary, separating the single-phase and the two-phase region will then propagate through the fluid. One very important example of such a process is the blowdown in a pressurized water reactor during a hypothetical loss of coolant accident. Other examples are cavitation in pipe lines and normal transients in conventional power plants as well as in steam generators.

The computation of the phase-change front progression is severely complicated by the fact that for fluids of interest the density changes continuously across the phase-change front, however, its derivatives and thus the sonic velocity change discontinuously. A rigorous treatment of this discontinuity requires separate wave diagram solutions for both sides of the front, with proper matching at the front, as in the treatment of shocks (Rudinger, 1969).

The phase-change front propagation differs from the propagation of contact surfaces separating different fluids, as for instance treated by Rudinger, 1969, since the phase-change front is not a material surface, but the locus of all points in the time-space plane where the saturation pressure has been reached. It also is different from the combustion or chemical reaction shock (Chapman-Jouget shock): The phase-change front here is the onset of the region in which change of phase is occurring, and the pressures, densities, and velocities remain in general constant across the front, while in Chapman-Jouget shocks the whole reaction is lumped into the shock itself, and the pressures, densities, and velocities are discontinuous across this boundary.

Large codes analyzing reactor transients cannot deal with the details at the phase-change front if computing efficiency is to be maintained. In many

cases, in particular for rapid transients, approximate algorithms which interpolate across the phase-change front have been quite successful (for instance Hancox et al., 1975, or Kroeger 1976). Problems have been encountered in some cases, in particular for slower transients (for instance Mathers et al., 1976, or Kroeger 1976). The difficulty is similar to the inaccuracies incurred in solidification front propagations during the solidification of liquids unless special attention is given to the freezing front as shown by Murray and Landis, 1959.

This paper will concentrate on the details of the phase-change front propagation for the limiting case of thermal equilibrium in one dimensional flow without friction or heat addition, starting from a uniform state, i.e., for homentropic flow. In particular in very rapid transients thermal non-equilibrium can occur, and boiling may be delayed. However, this work will concentrate on the limiting case of thermal equilibrium and give some qualitative consideration to the required later extensions of this work to the important case of here is that the resulting wave diagram solutions are essentially exact solutions without any numerical diffusion, since the Rieman invariants remain truly constant (see Section 2). By applying computer codes using the full balance equations for the specific case of no friction, no heat addition, and thermal equilibrium, and comparing those results against these wave diagram solutions, one can obtain a good measure of the effect of numerical diffusion on the results of those codes.

The fluid of primary interest is water, and results for this fluid will be shown later. However, the discontinuity in density derivatives for water is

very severe within the pressure ranges of practical interest. Therefore, it was found that many of the important features of the phase-change front propagation can be demonstrated qualitatively in a more convenient scale, using a hypothetical fluid with only mild discontinuities in the density derivatives at saturation pressure.

The mathematical formulation of the problem will be given in Section 2. Different modes of the phase-change front motion will be classified in Section 3. Qualitative examples will be discussed in Section 4.1 and results for water will be presented in Section 4.2.

2. PROBLEM FORMULATION

The balance equations for one-dimensional transient flow of a compressible fluid without friction or heat addition are (see for instance Courant and Friedrichs, 1948):

Conservation of mass:

$$\frac{D\rho}{Dt} + \rho \frac{\partial v}{\partial z} = 0 \quad (1)$$

Conservation of momentum:

$$\frac{Dv}{Dt} + \frac{1}{\rho} \frac{\partial p}{\partial z} = 0 \quad (2)$$

Conservation of thermal energy:

$$\frac{Dh}{Dt} - \frac{1}{\rho} \frac{Dp}{Dt} = 0 \quad (3)$$

Equation (3) implies isentropic changes of state. With the state equation

$$\rho = \rho(h, p) \quad (4)$$

and using equation (3), equations (1) and (2) can be expressed as a set of two equations in v and p :

$$\frac{1}{a^2} \frac{Dp}{Dt} + \rho \frac{\partial v}{\partial z} = 0 \quad (5)$$

$$\frac{Dv}{Dt} + \frac{1}{\rho} \frac{\partial p}{\partial z} = 0 \quad (6)$$

These can be transformed into the ordinary differential equations in characteristic directions

$$\frac{1}{\rho a} \frac{dp}{dt} + \frac{dv}{dt} = 0 \quad \text{on} \quad \frac{dz}{dt} = v + a \quad (7)$$

and

$$\frac{1}{\rho a} \frac{dp}{dt} - \frac{dv}{dt} = 0 \quad \text{on} \quad \frac{dz}{dt} = v - a \quad (8)$$

Introducing the function (Sauer, 1966)

$$\pi(p) = \int_{p_1}^p \frac{dp}{a\rho} \quad (9)$$

Equations (7) and (8) can be expressed in terms of the Riemann invariants

$$P \equiv \pi + v = \text{const on } c_p = v + a \quad (10)$$

$$Q \equiv \pi - v = \text{const on } c_Q = v - a \quad (11)$$

For an ideal gas (10) and (11) of course reduce to the well known relationships

$$P = \frac{2}{k-1} a + v = \text{const on } c_p = v + a \quad (12)$$

$$Q = \frac{2}{k-1} a - v = \text{const on } c_Q = v - a \quad (13)$$

For general fluids the function π must be evaluated with the constraint of equation (3). The sonic velocity, a , can be expressed as

$$\frac{1}{a^2} = \left(\frac{\partial \rho}{\partial p} \right)_s = \left(\frac{\partial \rho}{\partial p} \right)_h + \frac{1}{\rho} \left(\frac{\partial \rho}{\partial h} \right)_p \quad (14)$$

where the density and its pressure and enthalpy derivatives are functions of p and h . Thus with $a = a(p, h)$ one can solve two simultaneous ordinary differential equations obtained from Equations 3 and 9 for h and π as functions of p along an isentrope beginning at an arbitrary initial point, p_0, h_0 :

$$dh = \frac{1}{\rho(h,p)} dp \quad (15)$$

$$d\pi = \frac{1}{\rho(h,p)a(h,p)} dp \quad (16)$$

obtaining h and π as functions of p and of h_0 and p_0 . For isentropic flow beginning from a given initial state one state variable alone, for instance the pressure, p , or the sonic velocity, a , fully describe the state, and π can be considered as a function of p or of a , whichever is more convenient

3. PHASE CHANGE FRONT PROPAGATIONS

For a given function $\pi(p)$ Equations 10 and 11 (or 12 and 13 for an ideal gas) can readily be solved as long as the coefficients of Equations (5) and (6) (i.e. ρ and a) and their derivatives remain continuous (Friedrichs, 1948).

However, at the phase-change front the sonic velocity, a , changes discontinuously and the solution cannot be continued across the front without special consideration.

Several possible configurations can arise. Without loss of generality we assume the existence of a single-phase region, Region 1, and a two-phase region, Region 2. Region 1 is at the left and has a higher sonic velocity than Region 2, i.e.,

$$a_1 > a_2 \quad (17)$$

If the phase-change front travels towards the left into Region 1, at a speed equal to or higher than $|c_{Q1}|$, one can advance the solution through Region 1 until the saturation pressure p_{sat} is reached. This is shown schematically in Part a of Figure 1. The resulting line $p = p_{sat}$ in the $z-t$ plane thus established from the solution in Region 1 will in general not be a characteristic

of the second region, and, therefore, constitutes an initial value curve from which the solution can be continued into Region 2 as indicated in Part a of Figure 1. (For actual applications see also Figures 4 and 12). The same construction applies if the phase-change front moves towards the right with a speed equal to or higher than to c_{p2} . In these two cases the phase-change front location is completely determined by the flow field of one of the regions alone.

As a steady state is approached, for instance, during the discharge of a subcooled liquid from a large or heated reservoir into a region of lower pressure, the phase-change front velocity must ultimately approach zero. In such cases the solution for the locus $p = p_{sat}$ has to be obtained by interaction of characteristics from both regions. This is shown schematically in Part b of Figure 1. The locus of all points $p = p_{sat}$ is determined by the solution at the intersection of P_1 waves and Q_2 waves which have P and Q values such that the resulting pressure at the intersection is just equal to the saturation pressure, p_{sat} . In these cases the phase-change front location is affected simultaneously by the flow fields of both regions. This condition prevails when the phase-change front travels left or right with a velocity c_f

$$c_{Q_2} < c_f < c_{P_2}.$$

At any point A on the phase-change front in the z - t plane one can draw the P and Q characteristics through that point separating the domain of possible phase-change front velocities into five sectors, as shown in Figure 2. Sectors 1 and 5 correspond to Part a of Figure 1, where the flow field in the original phase alone determines the phase-change front location. Sector 3 corresponds to Part b of Figure 1, where the phase-change front location is affected by the flow fields of both regions.

Sectors 2 and 4 remain to be investigated: Part a of Figure 3 shows the conditions for Sector 2. Only P_1 characteristics can reach the front which results in insufficient information to construct solutions. For all flow fields and all fluids investigated to date no case has ever arisen where the phase-change front speed would fall into this range. Without being able to prove it at this time, however, based on the fact that sufficient characteristics and thus sufficient information for a solution can never reach a phase-change front moving at such a speed, as demonstrated in Part a of Figure 3, it appears that the phase-change front cannot propagate at the velocities of Sector 2, i.e.,

$$c_{Q_1} < c_f < c_{Q_2} \text{ appears to be impossible.}$$

The case of Sector 4 is shown in Part b of Figure 3. Here the P and Q waves of Region 2 as well as the P waves of Region 1 can reach the front. However, since this does require that waves of the same kind intersect (here P_1 and P_2 waves) a shock must be formed simultaneously. Such cases will be shown in Section 4.1 and 4.2.

In summary, the phase-change front propagation is limited to certain velocity ranges. In some cases the flow field of the original phase alone will determine the solution, resulting in very high front propagation speeds (Sectors 1 and 5). In other cases both flow field regions interact and affect the front position, resulting generally in slower front propagation speeds (Sector 3). The range of impossible front propagation velocities (Sector 2) can be very wide for instance for water where the sound speed changes by almost two orders at the phase-change front. Similarly the range of front propagation velocities which requires the simultaneous formation of shocks (Sector 4) can be very wide.

4. APPLICATIONS

Several solutions to the equations of Section 2 will now be presented. At first the results for a hypothetical "semi-ideal gas", with a mild discontinuity in sound speed will be given to present qualitatively some of the important features of the phase-change process. Thereafter, results for water will be presented with a change in sound speed from 1300 m/sec to 20 m/sec at the phase-change front.

In the subsequent discussion of wave diagram solutions it will be assumed that the reader is familiar with these techniques as presented in many text books on gas dynamics (for instance Courant and Friedrichs, 1948, Owczarek, 1968, Shapiro, 1953, or Rudinger, 1969.) Some of the important features of wave diagrams are also recalled in Appendix A of this paper.

4.1 Qualitative Demonstration of Phase-Change Front Solutions

The hypothetical "semi-ideal gas" considered in this subsection is an ideal gas for $p \geq p_{\text{sat}}$, and has a continuous density, but a discontinuous density derivative and, thus, a discontinuous sound speed at $p = p_{\text{sat}}$. Its assumed state equation and several resulting relationships are given in Appendix B.

A solution for the discharge of such a hypothetical gas from a dead ended pipe, initially pressurized, and with a membrane bursting at $\tau=0$ is shown in Figure 4. At the open end an expansion fan is formed at $\tau=0$ with Q waves of successively decreasing strength. As long as these waves intersect with P waves originating from the undisturbed region ($P=P_0$) a simple wave region is formed and the state and velocity will remain constant along each Q wave, and along one of them, say $Q=Q_f$, the value of Q is such that $a=a_{\text{sat}}$ or $p=p_{\text{sat}}$. Thus the original propagation of the phase-change front from the open end will occur along a Q wave, $Q=Q_f$, traveling with velocity c_0 into the single-phase region,

corresponding to the upper bound of Sector 1 in Figure 2. At point A in Figure 4 the phase-change front Q wave, Q_f , intersects with the last P wave from the undisturbed region ($P=P_0$). The continuation of the front beyond this point is indicated schematically in Figure 5. Beyond Point A the Q_f wave will interact with waves of decreasing P, thus a and p drop below their saturation values along the further path of the Q_f wave, and this area must belong into the two-phase region. Consider a wave shortly prior to the Q_f wave in the expansion fan, $Q=Q_f+\epsilon$. This wave has pressure and sonic speed slightly above saturation values up to point B. Beyond that point the pressure will drop and will just reach the saturation value at intersections with the wave $P=P_0-\epsilon$ at point C.

For any point on the phase-change front we have with Equation A-4

$$P + Q = 2 \pi(p_{sat}) \quad (18)$$

Thus for Q_f intersecting with P_0 waves

$$Q_f = 2 \pi(p_{sat}) - P_0 \quad (19)$$

Equation 18 implies that in regions of decreasing P the locus of points $p = p_{sat}$ will be found on waves of higher Q. With the dead end conditions $P = Q$ (equation A-9) it follows from Equation 18 that the front will reach the dead end as the wave $Q=\pi_{sat}$ reaches it.

The interesting feature here is that the phase-change front, the locus of points $p = p_{sat}$, can propagate faster than the Q waves which travel with sonic speed relative to the fluid. While pressure disturbances cannot propagate faster than the sonic speed the condition $p = p_{sat}$ can be reached on two neighboring waves even at the same time, thus implying an "infinite" speed of the phase-change front. It is also important to note, that this solution for the phase-

change front did not at all require the use of the hypothetical gas. Up to this front the solution for an ideal gas during the depressurization up to and including the isobar $p = p_{\text{sat}}$ has been determined by well accepted conventional techniques, using only the ideal gas state equation.

It is assumed now that the gas begins a change of phase at $p = p_{\text{sat}}$ as described in Appendix B, i.e., with continuous density but discontinuous sound speed. The isobar $p = p_{\text{sat}}$, established above, can now be considered as the initial value line for the solution for Region 2, with the constraint that this solution is only a true solution if

$$p \leq p_{\text{sat}}$$

To develop further detail in the exit region, it is assumed that the exit membrane is broken and that the exit plane pressure decreases linearly over a very short time $t_{\text{rupture}} \ll L/a$. The t - z plane for the exit region during this period is presented schematically in Figure 6. The exit flow is not choked at this time. The Q waves traveling from the open end into the pipe follow from the exit plane pressure prescribed as function of time and from the strength of the arriving P waves ($P=p_0$) as

$$Q = 2\pi [\pi(p(t)) - \pi(p_0)] \quad (20)$$

and travel left with the wave velocity

$$c_Q = \pi(p_0) - \pi[p(t)] - a[p(t)] \quad (21)$$

Noting that $\pi(p)$ decreases monotonically with p (see Sauer, 1966) Q is decreasing and c_Q is increasing with time, resulting in the expansion fan of Figure 4.

Noting that the function $\pi(p)$ is continuous across the phase-change front, while the sonic velocity is discontinuous, one can see that at the time where

$p = p_{\text{sat}}$ is reached in the exit plane two Q waves leave, one at $c_Q = v - a_1$ and the other at $c_Q = v - a_2$, where a_1 and a_2 are the sonic speeds for regions $p > p_{\text{sat}}$ and $p < p_{\text{sat}}$ respectively. Both waves are of equal strength, $Q = Q_f$. The triangle between these two Q waves of Figure 4 or Figure 6 is a region in which the solution is determined by Q and P waves of Region 2, emanating from the phase-change front prior to Point A. With Q and P being the same for any point in this triangle it will be a region of constant state and velocity, $p = p_{\text{sat}}$, $a = a_2(p)$, i.e., it is a region of constant state, being at saturation pressure but belonging to the two-phase region. If one were to try to extend the solution of Region 1 into this domain one would find the pressure dropping below p_{sat} , which is not possible in the case of thermal equilibrium i.e., for a fluid strictly following its state equations.

It will be seen later that these triangles next to the phase-change front, belonging to the two-phase region but being of constant state at $p = p_{\text{sat}}$ and $h = h_{\text{sat}}$ are very important for phase change with water, since they occupy a very large portion of the z - t plane due to the large change of the sound speed across the phase-change front.

The remainder of the wave diagram construction for the two-phase region is very straight forward, following standard procedures. The change in the slope of the Q waves across the phase-change front left from Point A in Figure 4 is clearly seen.

The next case to be considered is that of a pipe connected to a large reservoir of constant state. The conditions will be chosen so that the phase-change front remains inside the pipe, at least for the period of interest. The boundary conditions between the reservoir and the pipe will be those commonly used for such compressible flow problems (Shapiro, 1953, Owczarek, 1964), namely

isentropic expansion in the reservoir with quasi steady flow from the reservoir to the inlet, i.e.

$$h_{in} + \frac{1}{2} v_{in}^2 = h_o \quad (22)$$

From the arriving Q wave and the above boundary condition the inlet state and the strength of the leaving P wave can then be computed; considering a , h , and π as functions of p :

$$\pi(p_{in}) - v_{in} = Q_{arr} \quad (23)$$

$$h(p_{in}) + \frac{1}{2} v_{in}^2 = h_o \quad (24)$$

and

$$P_{leaving} = \pi(p_{in}) + v_{in} \quad (25)$$

$$c_p = v_{in} + a(p_{in}) \quad (26)$$

For a general fluid Equations 23 and 24 must be solved as a set of simultaneous non-linear equations. For an ideal gas they have an analytical solution using a_o and a_{in} as state variables for the stagnation and inlet state:

$$a_{in} = \frac{k-1}{k+1} \left[Q_{arr} + \sqrt{\frac{k+1}{k-1} a_o^2 - \frac{k-1}{2} Q_{arr}^2} \right] \quad (27)$$

and

$$v_{in} = \frac{k-1}{k+1} \left[\sqrt{\frac{2}{k-1} \frac{2}{k-1} \frac{k+1}{k-1} a_o^2 - Q_{arr}^2} \right] \quad (28)$$

For the present demonstration cases a constant pressure boundary condition could have been applied, resulting in several simplifications. However, this boundary condition for a pipe coupled to a reservoir is more general.

We will now consider the depressurization of this pipe, coupled to the reservoir, as a membrane at its other end is ruptured. At least initially as the fluid at the inlet is accelerated $\pi(p)$ will decrease less rapidly than v

increases, and after the last undisturbed Q wave arrives at the inlet P will be increasing, as can easily be established by linearizing Equations 23 and 24 for small changes from the undisturbed condition and considering arriving Q waves at $Q = Q_0 - \epsilon$.

This case differs, therefore, from the previous case by the fact that at least initially P will increase at $z = 0$ while it decreased at the dead end of the previous pipe.

The initial propagation of the phase-change front will be identical to that of the previous case, up to the intersection with the first reflected P wave at Point A in Figure 7.

As the $Q = Q_f$ wave continues beyond Point A it intersects with P waves of increasing strength. Thus, the pressure increases and this domain, therefore, belongs to the single-phase region. On the other hand, the continuation of the P_0 wave beyond Point A will intersect with $Q = Q_f$ waves emanating from the phase-change front $Q = Q_f$ and propagating with the two-phase region wave speeds, as indicated schematically in Figure 8. Thus, the continuation of the P_0 wave beyond Point A is part of the phase-change front. P waves from the two-phase region will intersect with this wave, however, since they are of equal strength no shock will be formed. (This result can also be obtained more rigorously by applying the shock relationships of Appendix C to the intersection of single-phase P waves originating just left from Point A, $P_1 = P_0 + \epsilon$, with two-phase P waves, $P_2 = P_0$, originating just right from Point A.)

The phase-change front moved at first along a Q wave of the single-phase region corresponding to the upper bound of Sector 1, in Figure 2. It shifted at Point A, to move backward from A to C (Figure 7) along a single-phase P wave corresponding to the upper bound of Sector 5.

As the state along the line AC is constant the Q waves entering the single-phase region above this line are of equal strength, resulting in a simple wave region. As these waves of uniform strength reach the inlet it can be seen from Equations 23 to 26 that the P waves leaving will be uniform, thus, resulting in the constant state triangle DEF and a subsequent simple wave region EFGH.

Beyond Point C the phase-change front will begin to intersect with two-phase Q waves of decreasing strength, coming from the open end. Two possibilities exist for the further motion of the phase-change front, depending on the magnitude of the discontinuity in sound speed at the phase-change front. Both possibilities are shown schematically in Figure 9.

Beyond Point C the Q waves will decrease in strength, and the interface relationship, Equation 18, will be satisfied at the intersection of these waves with P waves of higher strength. If the discontinuity in sound speed is mild the development will be as shown in Part a of Figure 9, resulting in a motion of the phase-change front, slower than the P waves of Region 2, i.e., in terms of Figure 2 the interface now moves with speeds of Sector 3 and the front position is determined by interaction between P waves of Region 1 and Q waves of Region 2.

If the discontinuity in the speed of sound is sufficiently strong, so that the single-phase region P wave of strength $P_0 + \epsilon$ intersects with the two-phase region P wave $P_2 = P_0$ before it reaches the $Q_2 = Q_f - \epsilon$ wave, then P waves of differing strength intersect resulting in shocks. This is shown in Part b of Figure 9. The state in Region 2 can be determined from its P and Q waves up to and including the two-phase side of the shock. The shock relationships of Appendix C must then be used to obtain the state of the corresponding point on the single-phase side of the shock and the shock speed. This mode of phase-

change front propagation corresponds to Sector 4 of Figure 2. The pressure is in general below saturation on the two-phase side of the shock, and above saturation on the single-phase side of the shock i.e., the phase-change front falls inside the shock.

Finally after a sufficient decrease in the strength of P_1 or Q_2 waves the Point G will be reached where the saturation pressure is reached on the single-phase side. The phase-change front will then leave the shock and move towards the single-phase region, while the shock continues to travel towards the exit.

A specific example of this case will be shown for water in Section 4.2.

In the discharge shown in Figure 7, with a mild discontinuity in sound speed, such shocks did not arise, and the discharge proceeded as shown in Part a of Figure 9 until Point G was reached. Beyond Point G in Figure 7 the P waves remain at uniform value and the state along the Region 1 Q wave from G to H will be constant, equal to the saturation state. Just like at the origin, a new constant state and velocity triangle GHJ belonging to Region 2 is formed. The process from Point A is repeated at H and the process of Point C is repeated at J.

In this exceedingly complex manner the phase-change front moved with speeds corresponding to Sector 1 and 5 as well as Sector 3 of Figure 2. Any motion corresponding to Sector 2 appears to be impossible and Sector 4 motion, with shocks, did not occur in this case, however, it will be shown to exist in a case in Section 4.2.

4.2 Application to Real Non-Ideal Fluids

In this subsection the phase-change front propagation in water will be shown for the same configurations that were presented in Subsection 4.1, first for the discharge of subcooled high pressure water from a dead-ended pipe, and

then for the flow of water through a pipe from a large reservoir of high pressure subcooled water to atmospheric conditions.

For the homentropic flows considered here the state along an isentrope was computed, integrating Equations 15 and 16 from an initial subcooled state of $p_0=70$ bar, $h_0=1$ MJ/kg to atmospheric pressure, using available material property routines for the density and its derivatives (Wulff, et al., 1976). The results were supplied to a cubic spline fit routine for continuous computation of $h(p)$ and $\pi(p)$ along this isentrope. The sonic velocity and the function $\pi(p)$ are shown in Figure 10 for this isentrope.

The discharge of high pressure subcooled water from a dead ended pipe is presented in Figures 11 and 12. In principle these wave diagrams have the same regions as Figure 4 for the hypothetical gas. However, the discontinuous change of the sound speed at the phase-change front is very drastic for water, from 1300 m/sec to 20 m/sec for the case shown here. Correspondingly the phase-change front moves essentially completely along a Q wave satisfying Equation 21 and only in a very small region of Figure 11 does it move to earlier Q waves as shown in the expanded view of Figure 12.

The constant region of two-phase mixture of "zero quality" between the two Q waves of strength $Q=Q_f$ is now very large. It is separated by only a very narrow simple wave region from another large region of constant state. However, in this second region of constant state the pressure has dropped below the saturation value corresponding to a void fraction $\alpha=0.16$. As a consequence of this constant state and velocity triangle a large simple wave region follows it on its right. Only thereafter will the outflow be described wholly by non-simple wave regions. The regions of constant state shown here were also observed in other analyses of the discharge of subcooled water from a dead ended pipe

(Kroeger, 1976). However, for actual flows with shear, and also due to the numerical diffusion present in all three-and four-equation numerical solutions, these regions are not as distinct as here, for the idealized isentropic case.

The wave diagram for high pressure subcooled water discharging through a pipe connected to an infinitely large reservoir of constant stagnation pressure and enthalpy is shown in Figure 13. The case is in principle similar to the case of Figure 7, except that due to the very strong discontinuity in sound speed the simple and non-simple wave regions have been condensed almost into lines, and the constant state triangles dominate the t - z plane. The phase-change front moves back and forth within the lines shown traversing almost the whole pipe, except for a small part at the exit and a very small part at the inlet. At any point in the interior of the pipe very strong, almost square wave shaped pressure oscillations occur. The two-phase triangles have a two-phase mixture of "zero void fraction" and only in the relatively small region close to the exit does actual boiling begin.

The propagation of the phase-change front corresponding to the line CG of Figure 7 will now occur with shock formation, as it was described in part (b) of Figure 9. Details from the outer tip at $\tau \approx 2$ and $\zeta \approx .975$ are, therefore, shown in Figure 14. As the phase-change front reaches Point A a shock forms due to the intersection of single-phase P waves with the two-phase P wave emanating from A. Beyond Point C the single-phase waves remain of equal strength, coming from the constant state and velocity triangle ranging from $\tau \approx 1$ to $\tau \approx 3$ at the inlet. When the Q waves from the exit have reached sufficiently low strength the pressure at the single phase side of the shock reaches p_{sat} . Saturation conditions are thus established on the single-phase side of the shock at this point (G). Since all single-phase waves are of equal strength in this

part of the wave diagram, the state will remain constant along the single-phase Q wave leaving from point G. Thus, this line becomes part of the phase-change front. The shock itself will travel towards the exit, with two-phase fluid on both sides now.

The computations were executed here using the well established weak shock relations (Equations C-14 to C-15). However, for the shock region between A and G the pressure ratios get relatively high, reaching a peak value of 1.98 at Point C. For ideal gases the weak shock relations still give good results at these pressure ratios, but for water this remains to be established. The case should be recalculated using the full shock relations (Equations C-4 to C-8) if more accurate results in this region are required. However, it would only affect the results in the very small region, shown in expanded form in Figure 14. Once the phase-change front separates from the shock, the remaining shock is very weak ($p_1/p_2 \leq 1.02$) and the weak shock relationships are well justified. An identical shock can be seen to leave the tip at $\tau \approx 4$ in Figure 13.

With the present equilibrium model the enthalpy of the fluid does not drop below saturation value in the two-phase constant state triangles of Figure 13. In real flows thermal non-equilibrium could occur resulting in pressure and enthalpy undershoots with subsequent return to the saturation values indicated in Figure 13.

One should also note that the boundary condition of constant stagnation pressure and enthalpy may require extremely large reservoirs for a fluid of very low compressibility. For instance, for the case of Figure 13, considering a pipe of 1 m length and 13 cm diameter the vessel would have to be 6 m^3 large to keep the vessel pressure within a 1% band for the extent of the transient shown in Figure 13. Thus, for many actual transients the change of state in the vessel should be modelled simultaneously with the wave diagram solution.

5.0 CONCLUSIONS AND RECOMMENDATIONS

It has been established that there are several different modes of phase-change front propagations. The phase-change front propagation may be determined by the flow field of either the single-phase region or the two-phase region only, or by interaction between the flow fields in both regions.

Certain propagation speeds appear to be impossible, while others will occur only with the simultaneous formation of shocks.

In particular for the discharge of subcooled water from a large reservoir through a pipe the phase-change front will oscillate rapidly with simultaneous pressure oscillations. The shocks formed in the exit region can have quite significant strength. Pressure ratios of up to 2.0 have been observed in the examples given here.

The results shown were developed for isentropic flow and for constant upstream reservoir conditions. As exact wave diagram solutions they do not include any numerical diffusion.

The following extensions of this work are suggested:

1. Replace the upstream boundary conditions of constant stagnation conditions by ordinary differential equations for finite sized reservoirs, which may be more realistic for many applications of interest.
2. Extend the discharge of water through a pipe coupled to a reservoir to larger times to follow the further growth of the exit region where net vapor generation occurs.

This extension should also focus on temporarily supersonic flow which has been observed in some such preliminary runs, close to the exit, due to compression waves coming from the inlet.

3. Make corresponding analyses with a three equation homogeneous equilibrium model, including shear and heat addition to determine how much these results would differ for real, non-homentropic fluid flow in pipes. By running that model with zero friction, and comparing it to the results shown here, the effect of numerical diffusion on that model can be assessed separately from the effects of friction and heat addition. Inclusion of the proper phase-change front propagation model and shock treatment might also permit the investigation of such effects as double choking in a pipe. The effects of cross sectional area changes should also be investigated with such a model.
4. The model presented here can also serve to assess the effect of numerical diffusion in any more advanced models by running these models without friction and heat addition and for the case of thermal equilibrium, and comparing the results to those presented here.
5. More advanced codes cannot deal with the details of the phase-change front propagation. By interpolating in the vicinity of the phase-change front between neighboring nodes and/ or time steps one violates the rigorous solution at the phase-change front. One can then, for instance, obtain propagation speeds which fall into the sector which appears to be impossible. The insight gained here can be utilized in the more advanced codes, for instance by prohibiting a phase-change front propagation in the impossible speed range, which is a very large sector for water.

6. Extensions to thermal non-equilibrium and vapor slip should be considered, possibly extending the model suggested in Point 3.

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Notation

a	sonic speed (Eqs. 14)
c	wave propagation speed
h	enthalpy
k	specific heat ratio (c_p/c_v)
p	pressure
P	Rieman invariant on right traveling waves
Q	Rieman invariant on left traveling waves
t	time
v	velocity
z	length coordinate
ρ	density
ζ	dimensionless length coordinate = $\frac{z}{L}$
τ	dimensionless time coordinate = $\frac{at}{L}$
π	function of state (Eqn. 9)

Subscripts

o	initial homentropic state
sat	state of saturation pressure
f	phase-change front

APPENDIX A

Appendix A

Review of Important Features of Wave Diagrams

P and Q waves in general:

$$P = \pi(p) + v = \text{const} \quad \text{on } c_p = v + a \quad (\text{A-1})$$

$$Q = \pi(p) - v = \text{const} \quad \text{on } c_Q = v - a \quad (\text{A-2})$$

For ideal gas:

$$\pi(p) = \frac{2}{k-1} a(p) \quad (\text{A-3})$$

At intersection of a P wave with a Q wave

$$\pi(p) = \frac{1}{2} (P+Q) \quad (\text{A-4})$$

$$p = \pi^{-1} \left[\frac{1}{2} (P+Q) \right] \quad (\text{A-5})$$

For ideal gas

$$a = \frac{k-1}{4} (P+Q) \quad (\text{A-6})$$

For any fluid

$$v = \frac{1}{2} (P-Q) \quad (\text{A-7})$$

At dead end:

$$v = 0 \quad (\text{A-8})$$

Thus

$$P = Q \quad (\text{A-9})$$

In regions where P and Q do not vary from wave to wave, the state and velocity remain constant. (Marked as C in wave diagrams of this paper).

In regions where all waves of one kind have the same P or Q value the state and velocity do not vary along waves of the other kind which are, therefore, straight lines. (Simple wave regions, marked as S in wave diagrams of this paper).

Regions in which both P and Q vary are called "non-simple" and are marked NS in the wave diagrams of this paper. Both waves are generally curved in such regions.

APPENDIX B

State Equation for Assumed Hypothetical Gas

The gas is ideal at $p \geq p_{sat}$, with densities being continuous but density derivatives and sonic velocities being discontinuous at $p = p_{sat}$.

The assumed state equation is

Region 1

$$p \geq p_{sat}$$

$$\rho = \frac{k}{k-1} \frac{p}{h}$$

Region 2

$$p \leq p_{sat}$$

$$\rho = \frac{k}{k-1} \frac{p}{h} \left[\lambda - (\lambda-1) \frac{p_{sat}}{p} \right] \quad (B-1)$$

where λ is a parameter used to select the desired magnitude of the sound speed discontinuity. From equation (B-1) follows:

Region 1

$$p \geq p_{sat}$$

$$a^2 = (k-1) h$$

Region 2

$$p < p_{sat}$$

$$a^2 = (k-1) \frac{h}{1 + (\lambda-1)k} \quad (B-2)$$

i.e., at the phase-change front

$$a_{sat+}^2 = (k-1) h_{sat} \quad (B-3)$$

$$a_{sat-}^2 = \frac{(k-1)}{1 + (\lambda-1)k} h_{sat} \quad (B-4)$$

and for the function $\pi(a)$ defined in Equation 9

$$p \geq p_{sat}$$

$$p < p_{sat}$$

$$\pi = \frac{2}{k-1} a$$

$$\pi = \pi_{sat} + \frac{2}{k-1} \left[1 + (\lambda-1)k \right] (a - a_{sat-}) \quad (B-5)$$

Note that π is continuous at $p = p_{\text{sat}}$

This state equation can only be used in a limited range of pressures depending on the value of λ and is only valid for demonstration purposes, it does not represent any real fluid.

APPENDIX C

Summary of Conservation Equations Across a Normal Moving Shock

The general conservation equations across a shock in terms of velocities u_i relative to the shock are (Courant and Friedrichs, 1948):

$$\rho_1 u_1 = \rho_2 u_2 \quad (C-1)$$

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2 \quad (C-2)$$

$$h_1 + \frac{1}{2} u_1^2 = h_2 + \frac{1}{2} u_2^2 \quad (C-3)$$

In terms of absolute velocities v , and the shock velocity U these relationships can be expressed as

$$(\rho_1 - \rho_2) U = \rho_1 v_1 - \rho_2 v_2 \quad (C-4)$$

$$p_2 - p_1 = \rho_2 (v_2 - U)^2 \left[\frac{\rho_2}{\rho_1} - 1 \right] \quad (C-5)$$

and the Hugoniot relationship

$$h_1 - h_2 = \frac{1}{2} \left(\frac{1}{\rho_1} + \frac{1}{\rho_2} \right) (p_1 - p_2) \quad (C-6)$$

For the shock schematically shown in Figure C-1 the state and velocity on Side 2 are obtained by standard methods from characteristics P_2 and Q_2 . Equations C-4 to C-6 with the state equations for Side 1

$$\rho_1 = \rho(h_1, p_1) \quad (C-7)$$

and the Side 1 P wave

$$\pi(p_1) + v_1 = P_1 \quad (C-8)$$

present five nonlinear equations for

$$\rho_1, p_1, v_1, h_1, \text{ and } U \quad (C-8)$$

which can be solved at any particular point.

Expressing the Side 1 variables as

$$y_1 = y_2 + \Delta y \quad (C-9)$$

with

$$\Delta U \equiv U_1 - U_2 \quad (C-10)$$

Equations C-4 to C-6 can be written as

$$\frac{\Delta p}{\rho_2} \Delta U - \left(1 + \frac{\Delta p}{\rho_2}\right) \Delta v = 0 \quad (C-11)$$

$$\left(1 + \frac{\Delta p}{\rho_2}\right) \Delta p - \Delta \rho \Delta U^2 = 0 \quad (C-12)$$

$$\Delta h - \left(\frac{1 + \frac{\Delta p}{\rho_2}}{1 + \frac{\Delta \rho}{\rho_2}} \right) \frac{1}{\rho_2} \Delta p = 0 \quad (C-13)$$

At modest to small shock pressure ratios ($\Delta p/p \rightarrow 0$), as the shock Mach number $M_s = U/a_2$ approaches unity, the entropy increase across the shock goes to zero with the third power of $(M_s - 1)$. For such weak shocks the entropy increases across the shocks can, therefore, be neglected. In this case the computations are greatly simplified, since then (Rudinger, 1969):

$$Q_1 = Q_2 \quad (C-14)$$

i.e., the value of Q remains constant across the shock and Side 1 values are now fully given by P_1 and Q_1 (see Figure C1). However, the shock speed even of a weak shock is not equal to that of the characteristics. It is (Owczarek, 1964) for a weak shock:

$$U = \frac{1}{2} (c_{p1} + c_{p2}) \quad (C-15)$$

Figure 1. Schematic of wave diagram solutions in the presence of phase-change fronts. (a) Phase-change front location being determined by Region 1 alone. (b) Phase-change front location being determined by both regions.

Figure 2. Phase-change front propagation speeds
($v > 0$; $a_1 > a_2$).

Figure 3. Schematic of wave diagram solutions in the presence of phase-change fronts. (a) Conditions for Sector 2 of phase-change front propagation cases. (b) Conditions for Sector 4 of phase-change front propagation cases.

Figure 4. Wave diagram for a hypothetical gas with phase change, discharging from dead-ended pipe ($P_0 = 4$ bar, $h_0 = .625$ MJ/kg; $\lambda = 2$; $\kappa = 1.4$).

Figure 5. Schematic of phase-change front location during depressurization in a dead-ended pipe.

Figure 8. Schematic of phase-change front propagation beyond point A.

Figure 9. Schematic of phase-change front. (a) Without shock formation. (b) With shock formation.

Figure 10. Properties of water along an isentrope from $p_0 = 70 \text{ bar}$, $h_0 = 1 \text{ MJ/kg}$ to atmospheric pressure ($p_{sat} = 28.81 \text{ bar}$).

Figure 11. Wave diagram for high pressure subcooled water discharging from a dead-ended pipe.

Figure 12. Expanded view of first wave reflection of wave diagram for high pressure subcooled water discharging from a dead-ended pipe.

Figure 13. Wave diagram for high pressure subcooled water discharging through a pipe connected to a reservoir.