6-12-81
2400p to ATIS

M.E. ①

R. 4982

UCID- 19055

MASTER

MATHEMATICAL MODELS FOR THE EPIC CODE

H.L. Buchanan

June 3, 1981

Lawrence
Livermore
Laboratory

This is an informal report intended primarily for internal or limited external distribution. The opinions and conclusions stated are those of the author and may or may not be those of the Laboratory.

REPRODUCTION OF THIS DOCUMENT IS UNLIMITED

MATHEMATICAL MODELS FOR THE EPIC CODE*

H.L. Buchanan

Abstract

EPIC is a fluid/envelope type computer code designed to study the energetics and dynamics of a high energy, high current electron beam passing through a gas. The code is essentially two dimensional (x, r, t) and assumes an axisymmetric beam whose r.m.s. radius is governed by an envelope model. Electromagnetic fields, background gas chemistry, and gas hydrodynamics (density channel evolution) are all calculated self-consistently as functions of r, x , and t . The code is a collection of five major subroutines, each of which is described in some detail in this report.

DISCLAIMER

This document contains information which is classified "Secret" under Executive Order 11652, dated 5-22-64, and is exempt from automatic downgrading and declassification. It is the property of the Department of Defense and is loaned to your agency. It and its contents are not to be distributed outside your agency without the express written approval of the Department of Defense. This document is to be stored in a secure location and is to be destroyed when it is no longer needed for official use. It is to be destroyed in accordance with the instructions of the Department of Defense. It is to be destroyed in accordance with the instructions of the Department of Defense.

*The Lawrence Livermore National Laboratory is operated by the University of California for the Department of Energy under Contract No. W-7405-Eng-48.

This work is performed by LLNL for the Department of Defence under DARPA (DOD) ARPA Order 3718, Amendment #33 and 34, monitored by NSWC under Contract No. N60921-81-LT-W0009, Amendment No. 1 and N60921-81-LT-W0026.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

The computer code EPIC (Electron Pulse Interactions with a Channel) was designed as a tool to be used to study the axisymmetric energetic interaction of a series of arbitrarily spaced electron pulses (referred to here as a bolt) with a dynamic background gas. A desire to treat a wide range of pulse separation times forces the inclusion of a large number of physical phenomena, not all of which will apply to any single situation. For example, a high current, high energy electron pulse will ionize the background gas at a very rapid rate. This ionization will certainly constrain the evolution of the electromagnetic self-fields of the pulse and so must be followed on a sub-picosecond time scale. On the other hand, the decay of this ionization and the attendant conductivity is essentially a chemical kinetic process and takes place over a much longer time scale, perhaps over microseconds. Longer still are the time scales over which the background gas can respond hydrodynamically, tens of microseconds, and yet this hydrodynamical density development significantly affects the chemistry of the gas which in turn couples into the beam self-field. In the past a particular time scale regime has been selected for examination and processes taking place much faster are regarded as instantaneous while slower processes are regarded as static. EPIC represents an attempt at a more self consistent treatment of the whole beam-gas interaction problem.

The formulation is axisymmetric and as such it can shed no light on transverse phenomena (tracking, hose instability, etc.). Rather it is hoped that the energetics of successive pulses in an evolving gas channel may be examined in some detail. In particular, we would hope to explore macroscopic phenomena such as beam pulse erosion, pulse to pulse head expansion, joule heating of the background gas, intra-pulse conductivity build up and decay,

and overall energy loss scaling. This list of objectives, although partial, is already ambitious and calls for a slight departure from traditional computational technique. The code EPIC should not be regarded as a user oriented production code, but rather a loosely coupled set of mutually compatible subroutines which can be combined like building blocks to apply to the particular situation at hand. There are, in fact, five major subroutine packages, each of which advance one or more related quantities a single step in time at each call. The design of the main routine depends on the situation under examination. While this procedure will seldom optimize the computation of any one problem it has the advantage of great flexibility with only a small investment of labor.

This report describes each of the five sections in terms of the overall model used with the caveat that even the routines themselves may change in response to unique circumstances. The sections are described loosely in the order of increasing time scale beginning with the electromagnetic field solver, ASOLVE, which must obviously treat quantities changing at light speed; then SIGSOLVE, which treats conductivity, electron density, and electron temperature; RSOLVE, which calculates beam radius, energy, and emittance as a function of distance, z ; TGSOLVE, which uses the thermodynamics of the background to calculate the gas temperature; and finally FLUID, which follows the hydrodynamics of the background gas to calculate a gas density as a function of radius. The functional relationship of these quantities is depicted in fig. 1.

Actual numerical detail is omitted except where a particular technique may be significant in the interpretation of a result. In particular, we may describe the overall code as two dimensional and Eulerian (fixed radial

grid) but with two distinct radial grids in use. In routines ASOLVE, SIGSOLVE, and TGSOLVE an expanded center approach is used in which the calculational grid is uniform in the variable $u = \ln(1+r)$. This allows fine gridding near the beam where quantities are required with accuracy and fewer grid points near the maximum radius where the electrical boundary condition is applied. The FLUID routine uses uniform gridding allowing for the fact that hydrodynamic development far from the beam may be important at the beam at some later time, hence no region of space deserves particular emphasis. The routine RSOLVE deals in beam averaged quantities and requires no radial resolution.

ASOLVE

As in many recent beam propagation codes, the electromagnetic field in EPIC is described by a single variable $A = A_z + \phi$ where A_z and ϕ are components of the traditional vector and scalar potentials. As described in reference (1), this quantity gives, to an excellent approximation in relativistic beam geometry, the electric fields, E_z and E_r , (at fixed spatial coordinate, x), as

$$E_z = - \frac{\partial A}{\partial x}; \quad E_r = - \frac{c}{4\pi\sigma} \frac{\partial^2 A}{\partial r \partial x} \quad (A.1)$$

$$(x = ct - z)$$

and the transverse force (due to electric and magnetic fields) as

$$F_r = q \frac{\partial A}{\partial r} \quad (\text{A.2})$$

This new potential evolves in x (a doppler shifted time) according to

$$\frac{1}{r} \frac{\partial}{\partial r} r \left(1 + \frac{c}{4\pi\sigma} \frac{\partial}{\partial x} \right) \frac{\partial}{\partial r} A = \frac{4\pi}{c} \left(\sigma \frac{\partial A}{\partial x} - J_b \right) \quad (\text{A.3})$$

where we have assumed that Ohm's law is valid for the plasma return current $J_p = \sigma E_z$ and that an omitted term in $\frac{\partial^2 A}{\partial x^2}$, which is part of the displacement current, is negligible. This equation is third order (two orders in r and one in x) and non-linear since σ will depend in a complicated way on the fields. Further, well-posedness requires only the single boundary condition

$$A(r = b) = 0 \quad (\text{A.4})$$

That this is true can be seen by transforming the problem to x, y space and noting that the above boundary condition gives A at both x and y boundaries.

Differencing of the field equation in a time (x) centered implicit way yields a simple tri-diagonal inversion matrix which is easily solved by conventional methods. Stability of the numerical scheme is assured regardless of step size but step size is constrained by accuracy considerations. The expanded center feature is accomplished by changing variables to

$$u = \ln(1+r) \longrightarrow r = (e^u - 1); \quad \frac{\partial}{\partial r} \rightarrow e^{-u} \frac{\partial}{\partial u}$$

This introduces variable coefficients to each derivative term but these coefficients need to be calculated only once since the u-grid is fixed. In final form

$$\frac{\partial}{\partial u} S \left(\frac{\partial}{\partial u} A + \frac{c}{4\pi\sigma} \frac{\partial}{\partial u} \frac{\partial A}{\partial x} \right) = - \frac{4\pi}{c} T \left(J_b - \sigma \frac{\partial A}{\partial x} \right) \quad (\text{A.5})$$

where $S = 1 - e^{-u}$

and $T = e^u(e^u - 1)$

SIGSOLVE

The subroutine SIGSOLVE advances the electrical conductivity, $\sigma(r)$, one time step. The conductivity is a function of both the new electron temperature and the new electron density so these quantities must be advanced as well. To determine electron temperature we take the energy equation to be

$$\begin{aligned} \frac{3}{2} n_e k \frac{dT_e}{dt} = & \sigma E_z^2 + J_b n_g \kappa(7\text{ev}) - n_e v_i (15\text{ev}) \\ & - \frac{3}{2} n_e k (T_e - T_g) v_u n_g \end{aligned} \quad (\text{B.1})$$

The first term on the right hand side is simply the joule heating of the electrons by the induced electric field. This energy is simultaneously added to the gas under the assumption that the electrons cool against the gas in a time short compared to a hydro-time. The second term models the fact that knock-on electrons resulting from collisions of beam particles with gas

molecules are generated with an average energy of 7ev. The constant κ , gives the number of electrons generated in gas of density n_g according to the measured stopping power $dE/dx = 2.52 \text{ Mev/g/cm}^2$. It is calculated to be

$$\kappa = \frac{dE/dx}{\Delta E_e} = 1.554 \times 10^{14} \left[\text{cm}^2/\text{g esu} \right]$$

where we have assumed the 33.75 eV is required for each ion pair. The third term accounts for the fact that electrons created by avalanche break-down acquire an average of 15ev of potential energy at the expense of the available kinetic energy. The last term models the exchange of energy from hot electrons to gas molecules by collisions. The energy exchange frequency, ν_u , is taken to be a simple straight line approximation to experimental data⁴ shown in fig. 3. of the form

$$\nu_u = \begin{cases} 1.731 \times 10^{-11} \text{ sec}^{-1} \text{cm}^3 & T_e < 1200^\circ\text{K} \\ 1.203 \times 10^{-17} T_e^2 \left[\text{eV} \right] \text{ sec}^{-1} \text{cm}^3 & T_e > 1200^\circ\text{K} \end{cases}$$

Electron density is a balance between production by collisional and avalanche ionization and depletion by ionic recombination. We model the gas chemistry by following only two species. For air at moderately high temperatures we describe the chemistry in terms of two reactions

The rate equations are then written

$$\frac{dn_e}{dt} = S_e - \alpha n_e (n_e + n_{O_2}^-) - \beta n_e + \beta' n_{O_2}^-$$

(B.2)

$$\frac{dn_{O_2}^-}{dt} = S_{O_2}^- - \alpha n_{O_2}^- (n_e + n_{O_2}^-) + \beta n_e - \beta' n_{O_2}^-$$

where the reaction rates are taken to be⁴

$$\alpha = 2 \times 10^{-7} \left(\frac{300}{T_e} \right)^{1/2} \left[\frac{cm^3}{sec} \right]$$

$$\beta = 4.04 \times 10^8 \left(\frac{300}{T_e} \right) e^{-600/T_e} \left(\frac{\rho_g}{\rho_o} \right)^2 \left[\frac{1}{sec} \right]$$

$$\beta' = 1.61 \times 10^9 \left(\frac{T_e}{300} \right)^{1/2} e^{-5590/T_e} \left(\frac{\rho_g}{\rho_o} \right) \left[\frac{1}{sec} \right]$$

S_e and $S_{O_2}^-$ represent source terms due to ionization by the beam, hydrodynamic convection of the gas, etc. We have then three coupled, non-linear equations to solve for T_e , n_e , $n_{O_2}^-$. Numerically, the problem is even more complicated since the coupled set is stiff, that is it contains several widely differing time scales which must be treated simultaneously. Rather than address the numerical difficulties, we obtain approximate analytic solutions of these three equations in EPIC.

We begin with eq. (B.1) and lump all source terms together to be held constant throughout one time step. For temperatures greater than 1200°K we may write

$$\frac{dT_e}{dt} = -a (T_e - T_g) T_e^2 + b \quad (\text{B.3})$$

$$\text{where } b = \frac{\sigma E_z^2 + J n_g \kappa (7\text{ev}) - n_e v_i (15\text{ev})}{\frac{3}{2} n_e k}$$

$$a = 1.203 \times 10^{-17} n_g$$

if we now let $T = \frac{T_e}{T_g}$, $x = \frac{bt}{T_g}$, and $\xi = \frac{a T_g^3}{b}$ our equation becomes

$$\frac{dT}{dx} = -\xi T^2 (T-1) + 1$$

we see immediately that the steady state temperature T^* satisfies

$$\xi T^{*2} (T^*-1) = 1$$

if we now redefine $T \rightarrow \frac{T}{T^*}$ we obtain

$$\frac{dT}{dx} = -\frac{1}{(T^*-1)} T^2 \left(T - \frac{1}{T^*}\right) + \frac{1}{T^*} \quad (\text{B.4})$$

Depending on the coefficients a and b , T_e is either increasing or decreasing towards T^* . For a decaying solution let $T = 1 + y$ to obtain

$$\frac{dy}{dx} = -y \left[\frac{3T^*-2}{T^*(T^*-1)} \right] - y^2 \left[\frac{3T^*-1}{T^*(T^*-1)} \right] - y^3 \left[\frac{1}{T^*-1} \right] \quad (B.5)$$

for a growing solution we let $T = 1-y$ and obtain the same equation except for the sign of the middle term. For decaying solutions $0 < y < \infty$ and we make the approximation that when y is large its slope is governed by the last term and when y is small its slope is determined by the first. The middle term is dropped and we obtain

$$\frac{dy}{dx} = -\frac{y}{(T^*-1)} \left[\frac{3T^*-2}{T^*} + y^2 \right] \quad (B.6)$$

This may be integrated over one Δx

$$y_{n+1}^2 = \frac{C \psi \alpha^2}{(1-C \psi)} \quad \psi = \exp \left[-\frac{2 \alpha^2 \Delta x}{(T^*-1)} \right] \quad (B.7)$$

$$\text{where } \alpha^2 = \frac{(3T^*-2)}{T^*} \quad \text{and} \quad C = \frac{y_n^2}{\alpha^2 + y_n^2}$$

A slightly better approximation to the original equation is obtained by changing the factor of 2 in ψ to 3. This partially compensates for the missing middle term.

For growing solutions y is bounded as $0 < y < 1$ and a different procedure is used. Returning to the original equation (changing the sign of the middle

term) we replace $y^3 \rightarrow y^2 y_n$ where y_n is, again, y at the beginning of the Δx step. We then obtain an equation that we can integrate and the solution is

$$\frac{y_{n+1}}{\alpha} = C \exp \left[\frac{(3T^* - 2) \Delta x}{(T^* - 1) T^*} \right] + 1$$

where $C = 1 + \frac{\alpha}{y_n}$ and $\alpha = \frac{3T^* - 2}{(3 - y_n) T^* - 1}$

Hence through our definition of $T \approx 1 + y$ we can calculate T_e^{n+1} from T_e^n .

To solve the rate equations we first set the source terms to zero. In our scheme, we will apply our sources to n_e and $n_{O_2^-}$, separately, half at the beginning and half at the end of each time step thus simplifying our problem a bit. The homogeneous equations are still coupled and still non-linear but an exact solution can be found as

$$n_e^{n+1} = \Delta \left[\frac{\beta' (n_e^n + n_{O_2^-}^n)}{(\beta + \beta')} (1 - \phi) + n_e^n \phi \right] \quad (B.9)$$

$$n_{O_2^-}^{n+1} = \Delta \left(n_e^n + n_{O_2^-}^n \right) - n_e^{n+1}$$

$$\Delta = \frac{1}{1 + \alpha \left(n_e^n + n_{O_2}^n \right) \Delta t} \quad \phi = \exp \left[-(\beta + \beta') \Delta t \right]$$

Note that in order to solve the equation for T_e we had to assume n_e was constant throughout the step and to solve for n_e and n_{O_2} we had to assume T_e was constant throughout the step. Actually, in regions of no beam and no electric field $T_e \approx T_{gas}$ and T_e changes very little with time or n_e so there is no difficulty. In regions of high field or large current the final values of n_e and T_e at $t + \Delta t$ is obtained by iterating on the analytic solutions.

Knowing $n_e(t + \Delta t)$ and $T_e(t + \Delta t)$ we may at last calculate the conductivity according to

$$\sigma(t + \Delta t) = \frac{2.54 \times 10^8 n_e(t + \Delta t)}{v_m + \frac{37.5 n_e(t + \Delta t)}{T_e^{3/2}}} \quad (8.10)$$

where the collision frequency is given by another straight line fit to experimental data⁵ shown in fig. 4

$$v_m = \begin{cases} 2.318 \times 10^8 \left(\frac{\rho}{\rho_0} \right) T_e & T_e < 19726^\circ K \\ 4.573 \times 10^{12} \left(\frac{\rho}{\rho_0} \right) & T_e > 19726^\circ K \end{cases}$$

We have thus far neglected any mention of any dependence on r . In fact, dependence on r arises predominately through the heating profiles of σE_z^2

and $\kappa n_g J_b$. In addition we must account for the fact that in order to maintain charge neutrality, plasma electrons must convect radially outward with the expanding gas. This will take place on a time scale slow compared to electron heating and is therefore ignored in the temperature equation. This time scale for the gas chemistry is much longer, however and the source terms become

$$S_e = \kappa n_g J_b + n_e v_i \left(\frac{E_z}{P} \right) + \frac{n_e}{n_g} \frac{\partial n_g}{\partial \tau}$$

$$S_{O_2} = \frac{n_e}{n_g} \frac{\partial n_g}{\partial t}$$

As previously described, the first electron source term represents knock-ons (beam gas collisions). The second term describes the generation of electrons due to the avalanche ionization. This process is modeled as a rate v_i which scales with E_z/P and is taken to be⁶

$$v_i = \frac{AS^3}{1 + BS + CS + DS^3} P$$

where $S = \frac{E_z^2}{P^2}$ (E_z in statvolts and P in atmospheres)

$$A = 1.423 \times 10^{-4}$$

$$B = 9.179 \times 10^{-6}$$

$$C = 2.656 \times 10^{-10}$$

$$D = 2.820 \times 10^{-17}$$

The terms proportional to $\frac{\partial n_g}{\partial x}$, a quantity returned from the hydrodynamic algorithm, models the depletion of electrons and ions that are carried away with expanding gas.

TGSOLVE

The subroutine TGSOLVE advances the gas temperature, $T_g(r)$, one time step by integrating an energy equation of the form

$$\rho T_g \frac{dS}{dt} = \rho T_g \left(\frac{\partial S}{\partial t} + v \frac{\partial S}{\partial r} \right) \quad (C.1)$$

$$= \sigma E_z^2 + \kappa n_g J_b (33 \text{ eV}) - \frac{4 \sigma_{SB} T_g^4}{r} + \frac{1}{r} \frac{\partial}{\partial r} r K_{th} \frac{\partial}{\partial r} T_g$$

Here T_g is the gas temperature and S is the entropy per gram of the gas. The first term on the right hand side results from the joule heating of the gas by the induced plasma current, the second term describes the direct deposition of energy in the gas by beam particles, and the final term represents thermal conduction of energy in the radial direction. The direct deposition coefficient κ has been described previously and the thermal conduction coefficient, K_{th} , is obtained from tables⁴ as a function of gas temperature and pressure. It is shown in fig. 6. The third term models energy loss due to radiation in the approximation that the gas channel is at all points optically thin. A Planck mean length is found from a look-up table as a function of gas temperature and density² and is, like K_{th} , implicitly a function of radius. The Planck mean length is graphed in fig. 5.

In order to obtain an equation for the gas temperature, T_g , we need an expression for the entropy. Here the code offers two options; the first, the long time option, assumes that the gas is always at chemical equilibrium, i.e. that at any particular temperature and density the gas instantly takes on its equilibrium fractional ionization. This is achieved by analytically fitting a table of entropy values measured at equilibrium to obtain the dimensionless form

$$\frac{S}{R_a} = 0.289 \left(\frac{T_g}{0_K} \right)^{0.5} \left(\frac{\rho}{\rho_0} \right)^{-0.12} \quad (C.2)$$

R_a is the universal gas constant times the molecular weight. When this expression is substituted into the gas temperature equation a second order PDE is obtained in terms of known quantities.

The second thermodynamic option involves the assumption that the gas had had insufficient time to change its composition (in particular, particle number) and is called the short time option. Entropy is now given by the Sacur-Tetrode expression for diatomic gases and is written

$$\frac{S}{R_a} = \ln \left[1.330 \times 10^4 \left(\frac{\rho}{\rho_0} \right) T_g^{3/2} \right] + \ln \frac{T_g}{\theta_r} + \frac{\alpha_v/T_g}{\left(e^{\alpha_v/T_g} - 1 \right)} - \ln \left(1 - e^{-\alpha_v/T_g} \right) \quad (C.4)$$

The first term corresponds to the translational contribution, the second is due to rotations where θ_r is the rotational temperature (for N_2 , $\theta_r = 2.88^\circ K$), and the last two terms describe the vibrational contribution (for N_2 , $\theta_v = 3374^\circ K$). Using this expression we find that

$$\frac{\partial}{\partial t} \left(\frac{S}{R_a} \right) = \frac{C_v}{R_a} \left[\frac{1}{T} \frac{\partial T}{\partial t} - \left(\frac{\rho}{\rho_o} \right) \frac{\partial}{\partial t} \left(\frac{\rho}{\rho_o} \right) \right] \quad (C.5)$$

and a similar expression for $\frac{\partial}{\partial r} \left(\frac{S}{R_a} \right)$

$$\text{where the heat capacity } \frac{C_v}{R_a} = \frac{5}{2} + \left(\frac{\theta_v}{T} \right)^2 \frac{\frac{\theta_v}{T_g}}{\left(e^{\frac{\theta_v}{T_g}} - 1 \right)^2}$$

is the normal value for ideal diatomic molecules, a function of temperature only. If instead we had used the long time option the result would have been

$$\frac{\partial}{\partial t} \left(\frac{S}{R_a} \right) = \frac{C_v}{R_a} \frac{1}{T} \frac{\partial T}{\partial t} + 0.240 \frac{C_v}{R_a} \left(\frac{\rho}{\rho_o} \right) \frac{\partial}{\partial t} \left(\frac{\rho}{\rho_o} \right) \quad (C.6)$$

$$\text{where } \frac{C_v}{R_a} = .1445 \left(\frac{\rho}{\rho_o} \right)^{-0.12} \left(\frac{T}{T_g} \right)^{0.5} \quad \text{which is now a weak function of}$$

density as well as temperature. Comparison of the two values of C_v at room temperature and ambient density shows (fig. 2) that the two agree well at room temperature and ambient density but differ by an order of magnitude at high temperatures. A final change of variable from r to $u = \ln(1+r)$ yields the gas temperature equation in the short time option as

$$\begin{aligned} C_v \rho \frac{dT}{dt} - R_a T \frac{d\rho}{dt} = \sigma E_z^2 + \kappa n_b J_b - \frac{4\sigma S_B T_g^4}{\ell'} \\ + \frac{1}{e^u (e^u - 1)} \frac{\partial}{\partial u} (1 - e^{-u}) K_{th} \frac{\partial}{\partial u} T_g \end{aligned} \quad (C.7)$$

where $\frac{d}{dt} \rightarrow \frac{\partial}{\partial t} + v(1-e^{-v}) \frac{\partial}{\partial u}$. The numerical differencing of this equation is accomplished according to the explicit method of Dufort-Frankel to insure stability independent of step size.

FLUID

The subroutine FLUID advances the gas density $\rho(r)$ and the radial gas velocity $v(r)$ one time step according to a conventional fluid model consisting of an equation of motion (Eulerian)

$$\rho \left(\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial r} \right) = - \frac{\partial}{\partial r} P \quad (D.1)$$

and an equation of continuity

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \rho v) = 0 \quad (D.2)$$

where ρ is the gas mass density, v the radial velocity, and P the radial pressure. In order to close the set we require an equation of state which, of course, must be consistent with the gas thermodynamic assumptions outlined during the discussion of TGSOLVE. For the two options we have the pressure (in dynes/cm²) given as

$$P = \begin{cases} 2.618 \times 10^6 T_g \rho & \text{short time} \\ 3.848 \times 10^4 T_g^{1.463} \rho^{0.899} & \text{long time} \end{cases} \quad (D.3)$$

where T_g [°K] and ρ [g/cm³]. It will be noticed that at standard temperature and density the long time equation of state gives a pressure that is about one third normal. This is a result of the fact that the

parameterization of eq. C.6 was fit to data for temperatures greater than 10^4 K and gives error at lower temperatures.

For cases of our interest, we expect the creation and propagation of shock-like structure to be endemic, hence careful numerical treatment is required. We have chosen to adopt a two-step Lax-Wendroff procedure with the incorporation of flux correction to the transport of both density and velocity. This procedure has been described in detail elsewhere by its inventors Book, Boris, and Hain⁹ under the name of Phenical Lax-Wendroff and so will only briefly outlined here. Basically, the procedure is designed to propagate a quantity characterized by steep gradients along a fixed Eulerian grid, explicitly and stably, without introduction of spurious, numerically generated, structure on the quality. This is accomplished by adding an artificial diffusion to the quantity to insure, prior to each time step, smooth transport and then removing the diffusion at the end of the step to recover the original character of the quantity. The key to the process lies in removing the diffusion (anti-diffuse) in a way that will introduce no new maxima or minima in the transported quantity. As adopted in the present code, this procedure allows us to follow the highly non-linear evolution of the gas channel in the presence of intense heating by the beam and its associated fields with a modest investment in time and space. While flux corrected transport (FCT) does tend to square off rounded edges, precise shock profiles are not of interest here.

Unlike other sections of the code, TGSOLVE works in the variable r rather than u since fluid behavior at radii large compared to the beam radius may have a significant effect on the gas density and temperature on the axis of

the beam. In addition, available experimental data of gas behavior likely comes from acoustic sensors placed away from the beam center so that comparison would require accurate numerical treatment there.

RSOLVE

Propagation of beam pulses in z is accomplished within the framework of an envelope model⁸ in which the beam profile changes self-similarly in r.m.s. radius, R , according to the equation

$$\frac{d^2 R}{dz^2} + \frac{qE_z}{\gamma mc^2} \frac{dR}{dz} = -\frac{U^2}{R} + \frac{E^2}{R^3} \quad (E.1)$$

E is defined to be a beam emittance, a quantity indicative in some respects of transverse beam temperature. $\gamma^2 E^2$ can be thought of as a transverse phase area of beam particles in a particular beam segment and can be expected to nearly constants in z , increasing only by scattering and non-linear effects according to

$$\frac{dE^2}{dz} + \frac{2qE_z^2}{\gamma mc^2} E^2 = -2\alpha U^2 E \frac{(E^2 - U^2 R^2)}{(E^2 + U^2 R^2)} + \left(\frac{dE^2}{dz} \right)_{\text{scatt}} \quad (E.2)$$

The first term on the right hand side is taken as a model of emittance increase due in the main to anharmonic betatron oscillations of the beam particles. The result of this particular ansatz is to damp out radial beam oscillations due to this anharmonic "phase mixing" of the beam particles. The constant α (of order 0.5), is chosen to correspond to damping as observed in

beam particle simulations and is a function of beam profile. The second term is the rate at which emittance increases due to the many small angle scatterings of beam particles against gas molecules. It is taken to be

$$\left(\frac{\partial E^2}{\partial z}\right)_{\text{scatt}} = \frac{R^2}{\gamma^2} \frac{r_e}{mc^3} \frac{4 C_N}{\lambda_R} \quad (\text{E.3})$$

C_N - Nordsieck constant = 3.7×10^{12} watts

λ_R - Bremsstrahlung radiation length in the background gas

$$= (3 \times 10^4 \text{ cm}) \left(\frac{\rho}{\rho_0}\right)$$

r_e - classical electron radius = e^2/mc^2

m - electron mass

The quantity U^2 is the mean of the betatron wave number times r^2 averaged over the beam profile. To be more precise, it is

$$U^2 = \langle k_\beta^2 r^2 \rangle = \int_0^{\alpha} dr \left(\frac{2 \pi r J_b}{l_b} \right) \left(- \frac{e}{\gamma mc^2 r} - \frac{\partial A}{\partial r} \right) r^2 \quad (\text{E.4})$$

and represents the average magnetic pinch force that is felt by the beam particles due to the beam's own self field. In fluid terms, Eq. (E.1) expresses the competition between the magnetic pinch pressure tending to radially compress the beam and the transverse thermal pressure tending to expand the beam. Electrostatic repulsion of beam particles is not included explicitly as the effect of space charge has already been included in the field equation A.3.

Finally we must model the loss of energy by beam particles themselves. We assume that all particles within a segment of the beam (region in the neighborhood of a fixed x) may be characterized by a single value of total energy γmc^2 . We may then account for changes in energy by writing for a beam of negatively charged particles

$$\frac{\partial}{\partial z} \gamma mc^2 = -\frac{\gamma mc^2}{\lambda_R} - \left(\frac{\partial \gamma mc^2}{\partial z} \right)_{\text{scatt}} - eE_z \quad (\text{E.5})$$

the three terms on the right hand side represent losses due to Bremsstrahlung, deposition of energy by the beam into the background gas, and Ohmic heating of the gas. The depositional term represents a sum of the effects of many processes and may be approximated by the use of a constant stopping power formula

$$\left(\frac{\partial \gamma mc^2}{\partial z} \right)_{\text{scatt}} = 3.25 \times 10^{-3} \left(\frac{\rho}{\rho_0} \right) \frac{\text{MeV}}{\text{cm}} \quad (\text{E.6})$$

The subroutine RSOLVE uses the above set of equations to advance R , γ , and E one step in z at a fixed x . It is called at the completion of each x -step (for which $R \neq 0$) and the R , γ , and E for that segment is immediately replaced with the advanced value. This substantially reduces the required amount of storage space.

REFERENCES

1. E.P. Lee, "The New Field Equations", LLNL, UCID 17286, 4 Oct. 76.
2. R.R. Johnston, O.R. Platas, and L.M. Tannenwald, "Atomic Line Transitions and Radiation from High Temperature Air", LMSC Technical Report N-3L-70-1, 15 July 70.
3. J.M. Yos, AVCO Report, RAD-TM-63-7 (196) (revised 1967).
4. M.H. Bortner and T. Baurer Editors-in-Chief, Defense Nuclear Agency Reaction Rate Handbook, DNA 1948H, General Electric Co., Space Division, Space Sciences Laboratory, P.O. Box 8555, Philadelphia, Penn. 19101.
5. A.G. Engelhardt, A.V. Phelps, and C.G. Risk, "Determination of Momentum Transfer and Inelastic Collision Cross Sections for Electrons in Nitrogen Using Transport Coefficients", Phys. Rev. 135, A1566 (1964).
6. P. Felsenthal and J.M. Proud, Physical Review 139 (6A) 1796 (1965).
7. Ya. B. Zel'dovich and Yu. P. Raizer, "Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena Academic Press", New York and London, 1966.
8. E.P. Lee and R.K. Cooper, Particle Accelerators, Vol. 7, pp. 83-95 (1976).
9. D.L. Book, J.P. Boris, and K. Hain, "Flux-Corrected Transport II: Generalizations of the Method", J. Comput. Phys. 18 (1975), 248-283.

Fig. 1. Schematic depiction of the working relationship of the subroutines in the code EPIC. The left hand loop is advanced on a fast time scale τ_f , while the right hand loop is advanced on a slow time scale τ_s .

Fig. 2. Comparison of the specific heat obtained from the two thermodynamic models described in the text.

Fig. 3. The energy exchange frequency, ν_u , as a function of energy and the straight line approximation used in EPIC.

Fig. 4. The momentum exchange frequency, ν_m , and the straight line approximation used in EPIC.

Fig. 5. The Planck mean length for air as a function of temperature and density as used in EPIC.

Fig. 6. The thermal diffusion coefficient K_{th} as a function of temperature and density as used in EPIC. K_{th} is given in units of 10^{-4} ($\text{erg cm}^{-1} \text{sec}^{-1} \text{°K}^{-1}$).