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LASER OSCILLATOR AT THE ATF(BNL)

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Juan C. Gallardo

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PRELIMINARY STUDIES OF AN INFRARED FREE-ELECTRON LASER OSCILLATOR AT THE ATF(BNL).

Juan C. Gallardo
Brookhaven National Laboratory
Upton, New York 11973

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ABSTRACT

In this report I present results of a theoretical 1-D model discussed by G. Dattoli, A. Renieri and myself¹. The validity of a steady-state analysis is discussed and some estimates are given of the effects introduced by the "lethargy" of the laser pulse due to the finite length of the e^- pulse. Also, I present analytical expressions for the laser pulse in terms of supermodes (wave-packets of cavity modes), their evolution, physical width as well as the associated frequency spectrum.

Next, I present results obtained with a 3-D code² for the single pass, small-signal gain. In the appendix, I summarize the symbols used in this report.

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Introduction

A schematic of the proposed oscillator infrared Free-Electron Laser (IRFEL) is shown in Fig. 1.

In Table 1 and Table 2 we summarize the parameters of the electron beam, the laser pulse and the near concentric (stable) optical resonator for the IRFEL in the wavelength range $2\mu\text{m} \leq \lambda \leq 50\mu\text{m}$ for peak electron currents $15\text{ A} \leq I_p \leq 100\text{ A}$.

Analysis

The laser evolution equation in the long pulse limit $\mu_c = \frac{N\lambda}{\sigma_e} \ll 1$ where $s = N\lambda$ is the slippage distance and σ_e is the length of the electron pulse, is given by

$$\begin{aligned} \frac{\partial}{\partial t} A(\zeta, t) = & \left(G_1(\nu_0, \sigma_E, \sigma_\perp) - \frac{2Q}{j_0} \right) A(\zeta, t) + \mu_c (G_2(\nu_0, \sigma_E, \sigma_\perp) - \Theta) \frac{\partial}{\partial \zeta} A(\zeta, t) \\ & + \frac{1}{2} G_3(\nu_0, \sigma_E, \sigma_\perp) \frac{\partial^2}{\partial \zeta^2} A(\zeta, t) - \frac{1}{2} \zeta^2 G_1(\nu_0, \sigma_E, \sigma_\perp) A(\zeta, t) \end{aligned} \quad (1)$$

ζ is the spatial variable normalize to σ_e . The coefficients in Eq. (1) are

$$G_1(\nu_0, \sigma_E, \sigma_\perp) = 2i \int_0^1 \int_0^\tau d\tau' (\tau - \tau') \frac{e^{-i\nu_0(\tau-\tau')} e^{-\frac{1}{2}\sigma_E^2(\tau-\tau')}}{[1 - i\sigma_\perp(\tau - \tau')]} \quad (2a)$$

$$G_2 = i \frac{\partial}{\partial \nu_0} G_1, \quad G_3 = -\frac{\partial^2}{\partial \nu_0^2} G_1 \quad (2b)$$

where j_0 is the current coefficient, Q is the loss coefficient at the mirrors and $\Theta = \frac{4\delta\mathcal{L}}{4j_0}$. $\delta\mathcal{L}$ is the detuning cavity length; it represents the shortening of the optical resonator, due to lethargy, with respect to the nominal synchronous length $\frac{cT}{2} = L_c$, where T is the separation time (or multiples) between micropulses.

The eigenfunctions of Eq. (1) $A_n(\zeta)$ are a collection of interacting “cold” longitudinal modes of the optical cavity or wave packets, with the property of being self-reproducing after each round trip in the resonator up to a complex number, the eigenvalue λ_n . These $A_n(\zeta)$ are known as SUPERMODES of the FEL oscillator. $\text{Re } \lambda_n > 0$ is the gain and $\text{Re } \lambda_n < 0$ corresponds to absorption.

In summary, the most important results of this model of an FEL in the small-signal and low gain regime, for a long electron pulse are:

• Eigenvalues

$$\lambda_n(\nu_0, \mu_c, Q, j_0, \sigma_E, \sigma_\perp) = -(n + \frac{1}{2})\omega_0 + \Gamma \quad (3)$$

with

$$\omega_0 = \mu_c \sqrt{G_1 G_3} \quad \text{and} \quad \Gamma = G_1 - \frac{2Q}{j_0} - \frac{1}{2} \frac{(G_2 - \Theta)^2}{G_3} \quad (4)$$

The fundamental supermode has the highest gain $\text{Re } \lambda_0 = -\frac{1}{2}\omega_0 + \Gamma$.

The gain is a quadratic function of $\delta\mathcal{L}$.

• Eigenfunction $A_0(\zeta)$

$$A_0(\zeta) = \frac{1}{\sqrt{\sigma_L} \sqrt{\pi}} e^{-\frac{\zeta_0^2}{4\mu_c} e^{-\frac{1}{2\sigma_L^2}} (\zeta - \zeta_0)^2} \quad (5)$$

gaussian function of dispersion $\sigma_L^2 = \mu_c \sqrt{\frac{G_3}{G_1}}$ and maximum amplitude at $\zeta_0 = -\frac{(G_2 - \Theta)}{G_1 G_3} = 0$; i.e. $\text{Re } G_2(\nu_0, \sigma_E, \sigma_\perp) = \Theta$ and $\text{Im } G_2 = 0$. This is the synchronism condition to achieve optimal overlap of both pulses and consequently, maximum laser power. This condition insures the timing of arrival of the laser pulse bouncing off the mirrors of the cavity and the electron pulse coming from the R.F. Linac but also, insures that both pulses stay overlapped during the trip through the undulator.

The spectrum is also gaussian with dispersion

$$\sigma_\lambda = \frac{\lambda^2}{2\pi\sqrt{s\sigma_e}}$$

The typical width of the spontaneous emission curve is $\frac{\Delta\lambda}{\lambda} \sim \frac{1}{N}$, hence the spectrum width of the fundamental supermode is $\frac{1}{2\pi} \sqrt{\frac{s}{\sigma_e}}$ narrower than the spontaneous incoherent spectrum.

The dimensionless current parameter used in the theory is

$$\left(\frac{16e\sqrt{\pi}}{mc^2c} \right) \frac{N(KL)^2 I_p}{\gamma^3 \sigma_e^2} j_1(x_\perp) = j_0 j_1(x_\perp) [J_0(\xi) - J_1(\xi)]^2 \quad (6)$$

where $\xi = \frac{K^2}{2(1+K^2)}$. The examples shown in Table 1 and Table 2 correspond to the values of j_0 presented in Table 3. The slippage parameter $\mu_c = \frac{s}{\sigma_e} = \frac{N\lambda}{\sigma_e}$ is the relevant quantity

that quantifies the short-pulse effects. σ_e is the length of the electron pulse. **Example** $\sigma_e = cT_e = 0.15$ cm for $T_e = 5$ ps.

• Undulator I

$N = 27$ and $2 \mu m \leq \lambda \leq 53 \mu m$ we obtain

$$0.04 \leq \mu_c \leq 0.954.$$

Short pulse effects become progressively more important as we increase the wavelength.

• Undulator II

$N = 31$ and $2.0 \mu m \leq \lambda \leq 50 \mu m$ corresponding to

$$0.04 \leq \mu_c \leq 1.03.$$

For $\mu_c \geq 1$ the short pulse effects are very important. The length of the optical cavity $L_c = 367.5$ cm is chosen to obtain synchronization between the round trip of the laser pulse in the resonator and the next e^- micropulse, see Fig. 2

$$T_c = \frac{2L_c}{c} = 24.5 \text{ ns} \approx 2T = 2/81.61 \text{ MHz}$$

The cavity length L_c correspond to a situation where there are two laser pulses in the cavity simultaneously.

The spectrum of the fundamental supermode has an rms width σ_λ . The ratio of

$$\frac{\sigma_\lambda}{\sigma_{SE}} = \frac{1}{2\pi} \sqrt{\frac{s}{\sigma_e}} = \frac{1}{2\pi} \mu_c^{\frac{1}{2}}$$

where σ_{SE} is the rms width of the spontaneous emission curve $\approx 4\%$.

$\sigma_\lambda = \frac{\Delta\lambda}{\lambda}$ the rms width of laser is,

$$0.12\% \lesssim \frac{\Delta\lambda}{\lambda} 100 \lesssim 0.6\%.$$

The inhomogeneities of the electron beam degrade the gain if the rms deviation in resonance parameter ν_0 introduced by energy spread and finite e^- beam radius are larger or equal to the width of the gain curve

$$\sigma_e, \sigma_\perp \leq \sigma_\nu$$

with $\sigma_\nu \sim 1.76j_0^{1/6}$ in the high-gain and $\sigma_\nu \sim \pi$ in the low-gain regime.

The characteristic rms deviation due to

- energy spread

$$\sigma_E = \frac{4\pi N}{(1+K^2)} \left(\frac{\Delta\gamma}{\gamma} \right)$$

- finite electron beam radius

$$\sigma_\perp = \frac{4\pi N}{(1+K^2)} \frac{K^2 k_0^2 \epsilon_n \beta^*}{2 \gamma}$$

where β^* is the undulator natural focusing function $\frac{1}{\beta^*} = \left(\frac{K k_0}{\gamma \sqrt{2}} \right)$.

For e^- beam with:

- Parameters in Table 1

$$\pi \leq \sigma_\nu \leq 4.9$$

- Parameters in Table 2

$$\pi \leq \sigma_\nu \leq 5.3$$

- Energy spread

$$\sigma_E = 1.7$$

- Finite e^- beam radius

$$\sigma_\perp = 0.18 (K=1) \quad , \quad 0.32 (K=3).$$

The number of passes required to reach saturation

$$\frac{P_{sat}}{P_{SE}} \sim 10^7 \equiv \frac{P^{(n)}}{P^{(0)}} = \frac{P^{(n)}}{P^{(n-1)}} \times \frac{P^{(n-1)}}{P^{(n-2)}} \cdots \frac{P^{(1)}}{P^{(0)}}$$

where $P^{(j)}$ is the power at end of the j -th pass. The small-signal gain definition is $G = \frac{P^{(n)}}{P^{(n-1)}} - 1$ therefore

$$\frac{P_{sat}}{P_{SE}} \sim 10^7 = (G + 1)^{N_{pass}}$$

Accordingly, for a gain $G \approx 100\%$, the net single pass gain and the number of passes to reach saturation is $N_{pass} \sim 25$.

The 3-D single pass code shows that the laser field is fairly well described by a gaussian function all along the undulator as shown in the phase (Fig. 3) and amplitude (Fig. 4) plots. The $1/e$ radius of the laser field is shown in Fig. 5; at the end of the undulator the laser beam waist $\omega_0 (\tau = 1)$ is equal to the initial electron beam radius. Finally, in Fig. 6 we plot the gain coefficient as a function of the normalized undulator length.

Appendix

Summary of Symbols

Laser wavelength	$\lambda = \frac{2\pi}{k}$
Slippage distance	$s = N\lambda$
Slippage parameter	$\mu_c = \frac{s}{\sigma_e}$
Dimensionless laser field	$A(\zeta, t)$
electron beam density	$\hat{n} = \frac{eN}{\sigma_e r_e^2}$
electron beam radius	r_e
normalize electron energy	γ
peak electron current	I_p
length electron beam	σ_e
dimensionless current parameter	$j_0 = 8N\hat{n} \frac{(e\pi KL)^2}{\gamma^3 mc^2}$
electron phase	$\xi = (k + k_0)z - \omega t$
electron phase velocity	$\nu_0 = \frac{d\xi}{d\tau}$
normalize time variable	$\tau = \frac{ct}{L}$
Number of undulator periods	N
undulator wavelength	$k_0 = \frac{2\pi}{\lambda_0}$
undulator length	$L = N\lambda_0$
undulator parameter	$K = \frac{eB_0}{mc^2 k_0}$
Peak undulator magnetic field (linear)	$B_0 \left(\frac{B_0}{\sqrt{2}} \right)$

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Figure Captions

Fig. 1: Schematic of an IRFEL oscillator. L_c length of the optical cavity; L is the undulator length and λ_0 is the period.

Fig. 2: Electron beam micropulse structure. T_e micropulse time duration; T pulse to pulse time separation; T_m macropulse duration; f accelerator repetition rate.

Fig. 3: Phase of the laser field minus the quadratic gaussian phase at several position along the undulator corresponding to the parameters shown in Table 1 and Table 2.

Fig. 4: Amplitude of the laser field at several position along the undulator corresponding to the parameters shown in Table 1 and Table 2.

Fig. 5: Evolution of the $1/e$ width of the laser amplitude along the undulator.

Fig. 6: Evolution of the gain length coefficient L_G along the undulator.

Table 1: Electron beam and undulator parameters used in the first set of examples.

Electron Beam

Energy, γ	119.2 , 50.8 , 34.4
Peak Current, I_p	15 – 100 A
Energy Spread [%]	0.5
Normalize emittance, ϵ_n [m-rad]	31.4×10^{-6}
Electron Beam Radius, r_e [cm]	0.03

Undulator I

Period λ_0 [cm]	4.0
Length, $L = N\lambda_0$ [cm]	108
Undulator parameter, K	0.92 – 3.69

Optical Resonator

Laser wavelength, λ [μm]	2 – 53
Cavity length, L_c [cm]	367.5
Mirror losses, Q [%]	1.5
Mirror Radius of Curvature, R [cm]	189.0
Optical waist size, ω_0 [cm]	
at 2 μm	0.045
at 11 μm	0.104
at 24 μm	0.155
at 53 μm	0.229
Outcoupling fraction, [%]	2

Table 2: Electron beam and undulator parameters used in the second set of examples.

Electron Beam

Energy, γ	152.7 , 68.3
Peak Current, I_p	15 – 100 A
Energy Spread [%]	0.5
Normalize emittance, ϵ_n [m-rad]	31.4×10^{-6}
Electron Beam Radius, r_e [cm]	0.03

Undulator II

Period λ_0 [cm]	6.45
Length, $L = N\lambda_0$ [cm]	200
Undulator parameter, K	0.92 – 3.53

Optical Resonator

Laser wavelength, λ [μm]	2 – 50
Cavity length, L_c [cm]	367.5
Mirror losses, Q [%]	1.5
Mirror Radius of Curvature, R [cm]	201.89
Optical waist size, ω_0 [cm]	
at 2 μm	0.061
at 10 μm	0.136
at 50 μm	0.303
Outcoupling fraction, [%]	2 – 4

Table 3: Dimensionless current parameter (j_0) for various electron energy (Γ), peak current (I_p) and undulator parameter (K).

Undulator I, Peak current $I_p = 15$ A

Electron Energy, γ		j_0	
119.2	3.31	11.93	36.44
50.8	42.75	157.62	481.0
34.4	137.7	496.6	1515.5

Undulator I, Peak current $I_p = 100$ A

Electron Energy, γ		j_0	
119.2	22.1	79.56	246.1
50.8	285.0	1051.0	3207.0
34.4	917.9	3310.5	10103.2

Undulator II, Peak current $I_p = 15$ A

Electron Energy, γ		j_0	
152.7	8.51	38.54	120.1
68.3	95.14	430.7	1341.7

Undulator II, Peak current $I_p = 100$ A

Electron Energy, γ		j_0	
152.7	56.7	257.0	800.4
68.3	634.3	2871.3	8944.8

Fig. 1

Fig. 2

Linear undulator

$$\lambda_0 = 4 \text{ cm}$$

$$K = 2$$

$$I_p = 172.4, \gamma = 119.$$

$$\lambda = 7.04 \mu\text{m}$$

FIG. 3

PHASE DIFFERENCE

time step= 0.60

time step= 0.80

FIG. 3

FIG. 3

FIG. 4

TRANSVERSE PROFILE OF LASER FIELD

$w_0[\text{cm}] = 3.48 \times 10^{-1}$
 $r_b[\text{cm}] = 1.39 \times 10^{-1}$
 $\text{energy_sp} = 1.70 \times 10^0$
 $\text{ang_sp} = 1.86 \times 10^0$

time step= 0.20
 current= 79.56
 $\text{nu}_0 = 2.60$
 $\ln(1+\text{gain}) = 0.002$

$w_0[\text{cm}] = 3.48 \times 10^{-1}$
 $r_b[\text{cm}] = 1.39 \times 10^{-1}$
 $\text{energy_sp} = 1.70 \times 10^0$
 $\text{ang_sp} = 1.86 \times 10^0$

time step= 0.40
 current= 79.56
 $\text{nu}_0 = 2.60$
 $\ln(1+\text{gain}) = 0.043$

FIG. 4

TRANSVERSE PROFILE OF LASER FIELD

$w_0[\text{cm}] = 3.48 \times 10^{-1}$
 $r_b[\text{cm}] = 1.39 \times 10^{-1}$
 $\text{energy_sp} = 1.70 \times 10^0$
 $\text{ang_sp} = 1.86 \times 10^0$

time step= 0.60
 current= 79.56
 $\nu_0 = 2.60$
 $\ln(1+\text{gain}) = 0.181$

$w_0[\text{cm}] = 3.48 \times 10^{-1}$
 $r_b[\text{cm}] = 1.39 \times 10^{-1}$
 $\text{energy_sp} = 1.70 \times 10^0$
 $\text{ang_sp} = 1.86 \times 10^0$

time step= 0.80
 current= 79.56
 $\nu_0 = 2.60$
 $\ln(1+\text{gain}) = 0.546$

FIG. 4

FIG. 4

FIG. 5

FIG. 6