A COLLECTION OF SCHOTTKY-SCAN NOTES *

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Introduction

This paper is a republication of ISR-RF notes and performance reports on work done in 1974-75. The original notes have been edited, corrected and, in most cases, shortened.

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NOISE, ERRORS AND THE SCHOTTKY SCAN*

Summary: An elementary signal-processing analysis of the Schottky scan technique is presented. The following points are considered:

- 1) proper treatment of the signals to give a linear representation of current density vs. momentum;
- 2) instrumental noise and zero-baseline;
- 3) measurement errors due to additive noise and finite processing time.

Introduction

The existing Schottky scan processing system is shown schematically in Fig. 1.

We discuss three problems:

- 1) how to process the signals so that at the output of the system we have a linear representation of current density in the stack versus momentum;
- 2) how to establish a good baseline;
- 3) how to estimate some measurement errors.

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Following the recent Schottky scan paper¹, the line charge density, λ , at a pick-up station (point (a), Fig. 1) is, for the n th harmonic of the revolution frequency,

$$\sum_i \lambda_i n(t) = \frac{2e}{2\pi R} \sum_i \cos n(2\pi f_i t + \theta_i) \quad (1)$$

The summation over i refers to the i 'th particle, as f_i refers to the revolution frequency of the i 'th particle

- e = electron charge
- $2\pi R$ = circumference of the storage ring
- θ_i = initial phase of the i 'th particle

The pick-up station has a sensitivity $\frac{V}{\lambda}$.

The signal is matched to a cable leading to the SRC by an amplifier with voltage gain G , then enters the spectrum analyzer. The filtering action of the analyzer is assumed to be ideal, the filter having a near-perfect rectangular bandpass with width δf .

The function which relates the density of particles contributing to our signal in the neighbourhood of f , within the "house" or spectral band belonging to revolution harmonic n is $N\left(\frac{f}{n}\right)$. This function is itself directly proportional to the distribution of particles versus momentum, $N(p)$.

We define a new sensitivity constant for the pick-up station

$$K = \frac{V \sqrt{2} e G}{\lambda 2\pi R} \quad (\text{the unit of } K \text{ is volts})$$

to simplify things, and write for the voltage at point (b), Fig. 1

$$V_{b \text{ rms}} = K \sqrt{2} \left\langle \left[\sum_{\delta f} \cos n(2\pi f_i t + \theta_i) \right]^2 \right\rangle^{\frac{1}{2}} \quad (2)$$

The phases θ_i are assumed to be random, and the average is taken over time. The summation is over all particles i whose revolution frequencies, multiplied by n , place them within the bandwidth δf centered on f .

The result of the summation is, at point (b), Fig. 2,

$$V_{b \text{ rms}} = K \left[N \left(\frac{f}{n} \right) \frac{\delta f}{n} \right]^{1/2} \quad (3)$$

Thus the rms signal voltage and the power at the output of the filter are proportional to the square root of particle density. We let $Z(\text{system}) = 1$ so that $P = V_{\text{rms}}^2$.

The extra factor of $1/2$ which appeared between equation (2) and equation (3) is due to the rms value of the cosine function in the sum².

Detection and Filtering

The signal is now detected. We shall consider two types of detectors, the full-wave linear detector and the full-wave square-law detector³.

Linear Detector

The full-wave "linear" detector has the non-linear transfer characteristic

$$V_{\text{out}} = b |V_{\text{in}}|$$

In Fig. 2(a), we see the band-limited noise with power spectral density = A at the input to the detector, centred on the spectrum analyzer IF frequency. After detection and filtering, we have (Fig. 2b):

- 1) a delta-function at zero frequency with area

$$(DC) = 2 \frac{b^2 A \cdot \delta f}{\pi}$$

This area has units of volts²·s⁻¹.

- 2) a triangular spectral density function symmetric about zero frequency. This spectrum is just the auto-correlation of each rectangular spectral slice in Fig. 2(a).

If the spectral slices in Fig. 2(a) are not rectangular, it is obvious that the area of the central delta-function will be proportional to the spectral area:

$$P = \int_{f_1}^{f_2} pdf$$

the power in the slice.

In our case, the amplitude A (Fig. 2) is equal to $(V_{brms})^2$ (eq. (3)) and the DC voltage component at the output of the detector is:

$$V_c = K \left[N \left(\frac{f}{n} \right) \frac{\delta f}{n} \right]^{\frac{1}{2}} \quad \text{for} \quad \frac{2b^2}{\pi} = 1.$$

The rest of the signal power at the output of the detector (point (c), Fig. 1) appears as Gaussian noise superimposed on the DC level. (We call ISR noise Gaussian.)

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In order to measure the DC level exactly, we would have to average or filter the signal at point (c) for an infinite time. In filtering or averaging for a finite time T , we make an estimate whose standard deviation of error is

$$\epsilon = \frac{1}{\sqrt{T \cdot \delta f}} \quad (5)$$

This is a good approximation for the restricted class of noise and band-filtering outlined above. If one is solving equation (5) for T in a particular problem, one must remember that the equation applies to one measurement point only. A complete spectral scan consists of many points or channels; e.g. for the ISR transverse scan the scan width is 50 kHz and δf is 100 Hz. Thus we have a 500-point measurement, taken one point at a time, with our present measurement technique.

Thus, if we set our filter at many frequencies, we will construct a picture of $[N(P)]^{1/2}$. This is not quite what we need, and we can square each measured value of V_c by digital or other techniques to get $N(P)$.

Square-law Detector

A full-wave square-law detector has the transfer characteristic

$$V_{out} = b V_{in}^2.$$

The output of this detector is similar to the output of the "linear" detector (Fig. 2b) except that the area of the DC signal is

$$(DC) = 4a^2 A^2 (\delta f)^2.$$

This area has units of volts⁴·s⁻².

In terms of the beam noise, as before,

$$V_c = K \left[N \left(\frac{f}{n} \right) \frac{\delta f}{n} \right] \quad \text{for } 4a^2 = 1$$

The error in power estimation after averaging of the DC plus noise is the same as for the linear-detector case. We do not need to square the power measurements in this case, and thus the overall measurement is simpler.

Noise and Errors

Additive instrumental noise

Our measuring devices contribute an uncorrelated broadband noise "background" which adds to the Schottky noise. At some frequency f we have a noise power spectral density which gives a noise voltage $V_N(\text{rms})$ at the input to the first filter (Fig. 1). If f is also within a Schottky noise band, the total voltage is

$$\bar{V}_T = \left[\bar{V}_S^2 + \bar{V}_N^2 \right]^{1/2} \quad (6)$$

$\bar{V}_S$ is V_b of equation (3): all $\bar{V}$'s are rms.

After detection, averaging and squaring (this analysis is applicable to "linear" and square-law detectors), neglecting multiplicative constants,

$$\bar{V}_T^2 = \bar{V}_S^2 + \bar{V}_N^2$$

$\bar{V}_S^2$ is just V_c^2 of equation (4).

In the present Schottky-scan set-up at the ISR, the spectrum analyzer measures regions above and below the Schottky noise in frequency, thus establishing a baseline which is in fact the background noise level, $\bar{V}_N$. After squaring of each measured value of power density in the computer, the Schottky-noise level is defined as amplitude above the background noise level. This process can be expressed as

$$\bar{V}_S^2 = \bar{V}_T^2 - \bar{V}_N^2.$$

Since $\bar{V}_T^2 = \bar{V}_S^2 + \bar{V}_N^2$, the value of $\bar{V}_N^2$ is immaterial.

Errors in the estimate

If every voltage, $\bar{V}_T$ and $\bar{V}_N$, is measured separately for each f , the ϵ for the desired quantity $\bar{V}_S$ is

$$\epsilon_{\text{tot.}} = \frac{[(\bar{V}_S \epsilon_S)^2 + (\bar{V}_N \epsilon_N)^2]^{\frac{1}{2}}}{\bar{V}_S}$$

Since ϵ_S is usually equal to ϵ_N , in this case

$$\epsilon_{\text{tot.}} = \epsilon \frac{[\bar{V}_S^2 + \bar{V}_N^2]^{\frac{1}{2}}}{\bar{V}_S}$$

or, for $R = \frac{\bar{V}_S}{\bar{V}_N}$

$$\epsilon_{\text{tot.}} = \epsilon \left[\frac{2 + R^2}{R^2} \right]^{\frac{1}{2}}$$

Since $\epsilon = \frac{1}{\sqrt{T}}$, the required averaging time τ for a desired $\epsilon_{\text{tot.}}$ has the proportion,

$$\tau \propto \frac{2 + R^2}{R^2}$$

Thus it is not worth struggling for $R > 2$.

Tails of stacks

One important use of the Schottky scan is the determination of the size and extent of low-intensity "tails" on the ends of stacks. The structure of the stack may be quite clear at central momenta, but errors due to additive noise in the measurement system are much more serious for tails.

Let us say $R = 10$ for the major part of the stack, and there is a tail of 5% (tail amplitude/central amplitude). If $\epsilon = 0.05$ for the central amplitude, $\epsilon_{\text{tot.}} = 0.0502$. However, for the tail, $\epsilon_{\text{tot.}} = 0.11$. One must fight signal-to-noise much harder in order to see clearly low-intensity regions of a stack.

A final note

All of the preceding analysis is applicable to the transverse Schottky-scan sidebands due to the betatron amplitudes of the protons as well as the longitudinal scans. In the case of the transverse scans, the function $N\left(\frac{f}{n}\right)$ is proportional to the product of $N(P)$ and $A[Q(P)]$, a function which gives betatron oscillation amplitudes as a function of Q : Q itself is a function of P .

Acknowledgement

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References

1. "Non-destructive diagnostics of coasting beams with Schottky noise", J. Borer *et al.*, CERN/ISR-DI/RF/74-23.
2. ISR Perf. Rep., 4.5.72, H.G. Hereward.
3. Details of the detection process are given in "Random Signals and Noise" by W.B. Davenport and W.L. Root, McGraw-Hill, 1958, p.250 *et seq.*

Fig. 1. For a,b,c etc., see text.

(a) Input power spectral density

(b) Output power spectral density about DC

Fig. 2. Input and output of linear detector $V_{\text{out}} = b|V_{\text{in}}|$.

SPEEDING UP THE SCHOTTKY SCAN*

Introduction

One of the main factors limiting the usefulness of the Schottky scan technique is the time required to obtain a scan of useful accuracy. Currently, one scans the spectrum many times with a high-frequency spectrum analyzer, averaging the output of many scans. With "real-time" spectrum analysis techniques, it is possible to increase the rate at which scans can be obtained by a factor equal to the relative resolution of the scan, $\Delta f/f$, for a fixed level of accuracy. In the ISR system as now set up, this factor is 500.

Outline of the problem

The Schottky signal is Gaussian noise¹. Although the voltage from a pick-up is a random function of time, the signal has a definite spectral shape. "White" noise has a spectrum which is flat for all frequencies (in practice, flat for all frequencies of interest) and "pink" noise has a spectrum which is a non-constant function of frequency. The ISR Schottky noise is an example of pink noise.

When one measures the spectrum of a signal which is made up of coherent spectral lines, e.g. a frequency-modulated carrier, the accuracy and speed of the measurement is limited only by such factors as (1) extraneous noise introduced with the signal, (2) noise produced by the receiver/analyzer, (3) the measuring time needed to satisfy the $\Delta f \Delta t \approx 1$ relationship for the filters in the receiver.

* Published as ISR performance report, 23 May 1975

In measurements of noise power spectra, the power density in a frequency interval is only defined statistically: the signal must be measured over a finite time to obtain a power estimate of a given accuracy. For a filter bandwidth B and a measurement time T , the standard deviation of error is

$$\epsilon = \frac{1}{\sqrt{BT}} \quad \text{for } \epsilon < 0.1$$

As an example: for the transverse Schottky scan, $B = 100$ Hz, and since the full width of the scan is usually 50 kHz, T in the relation above is (averaging time/500). For averaging time = 10 min, $\epsilon = 0.091$.

There is no way of improving the data collection speed by using better amplifiers, larger pick-ups etc. The statistical nature of the signal itself sets the necessary times.

Possible improvements

The signal processing is slow because we have only one filter looking at the entire signal. If we had 500 fixed frequency receivers, or perhaps some smaller number of filters being swept, we could reduce our processing time by a factor equal to the number of receivers. Fortunately, such measures are unnecessary. There exist measurement problems in radio astronomy², acoustics³, mechanical vibration analysis and radar which are very similar, and the techniques developed in these fields are directly applicable.

Realization

All of the analysis techniques to be considered are digital, thus one can only process signal within the frequency range of digital-to-analog converters.

One need not work directly at the original (baseband) frequencies of the Schottky noise, which may range into the 300 MHz region for longitudinal scans. The signals themselves are narrowband, and no information is lost if they are heterodyned down to low frequencies. We have performed simple experiments in the RF group to realize this technique with a spectrum analyzer as a receiver and a frequency synthesizer as a heterodyne oscillator.

One may consider the transverse Schottky, the broadest signal that we now look at, as having an effective upper frequency cut-off of 50 kHz, since we may place zero frequency wherever we wish by tuning the receiver or the heterodyne oscillator: thus we need a signal processor that functions up to 50 kHz.

A straightforward approach to the problem is:

- (1) digitize the information at a suitable rate for a suitable period
- (2) transfer the data to a Fourier transform processor of some sort
- (3) store the spectrum
- (4) average the successive spectra
- (5) repeat from (1)

These operations are repeated until the averaged spectrum has the requisite accuracy.

We must define "suitable rate" and "suitable period". If a signal of bandwidth B is sampled at a rate of $2B$, it is completely characterized by the samples, and no information is lost. This is the well-known Nyquist sampling criterion. For a frequency resolution Δf , one must sample for a period $T = 1/\Delta f$. The information content of one sample is so small that the difference in information collection rate between one-bit digitization and ideal digitization (infinite bits) is 2 (ref. 2).

If one can digitize fresh data and process already digitized data simultaneously without any breaks and without falling behind in the analysis, the spectrum analysis is said to be "real time". Processing speed is always the practical limit, rather than digitization speed, so one often reaches a point at which there are breaks in the data collection during which the processor finishes transforming and averaging the last block of data. The upper limit of real time processing speed is the real-time bandwidth.

A data analysis system of this type, operating beyond its real-time bandwidth, is still very useful, and its speed gain over a single channel frequency analyzer is

$$G = p \cdot E \quad E \leq 1$$

where p is the number of frequency resolution channels and E is the efficiency, $E = (\text{real-time bandwidth}/\text{operating frequency})$. E cannot exceed 1.

Conclusion

There exist several commercially available instruments which perform real-time spectral analysis and averaging real-time bandwidths from 5 kHz to 50 kHz. A single instrument should suffice for all Schottky scans, since with $E = 1$ at 50 kHz, a scan which now takes 10 min could be completed in 1.2 sec. It seems likely that beam setup and machine development work would benefit from such rapid feedback of stack conditions.

References

1. For a good definition of Gaussian noise, see "Random signals and noise" by W.B. Davenport and W.L. Root, McGraw Hill, 1958, p. 154.
2. "The Jodrell Bank Radio Frequency Digital Autocorrelator", D. Davies, E.B. Ponsonby, L. Pointon, J. De Jaeger, Nature, Vol. 222, 7th June 1969, p. 933.
3. "Modern Techniques of Power Spectrum Estimation", IEEE Trans., Audio and Electroacoustics, Vol. AU 15, pp. 56-66, June 1967.

SCHOTTKY MARKERS AND FAST SCHOTTKY SCANS*

Introduction

During this run, we used a prototype fast Schottky scan¹⁾ set-up to look at transverse Schottky signals. This allowed us to look in greater detail at the markers, and served as a test of the usefulness and practicality of the fast Schottky technique.

Fast Schottky prototype

We used the same raw signals which the standard ISR transverse Schottky system uses. A spectrum analyzer and a special heterodyne system were used to bandlimit the signal and bring it down in frequency close to DC (Fig. 1). A digital fast Fourier transform processor^{**} with built-in averager computed the power spectral density of the signal.

There were no especial problems in using or tuning the system, and it worked exactly as one would predict from theory.

Markers

The main effort in this run was to see how narrow the markers appeared with the new higher-resolution system, with the new ability to measure them within 30 ns after they were made.

* ISR Performance Report, 2 October 1975

** Nicolet Scientific Corp., on loan from the manufacturer.

Fig. 2 shows a good collection of markers. There is a cluster of four markers closely spaced at the left, measured 12 minutes after they were made, while the most recent marker, the second spike from the right, is only 1 minute old. The first spike to the right is a real resonance, presumably 7th order, at the bottom of the stack.

All of these markers were made with a waveform from the HP frequency synthesizer, 10 steps up, 10 ms/step, 5 Mh/step. This waveform has a bandwidth of approx. 175 Hz. Details of the expected apparent width of the markers are given in a performance report on run 586. The measured width of the markers as they appear in Fig. 2 and other photos is approx. $1\frac{1}{2}$ channels wide FWHM, each resolution channel being 62.5 Hz wide. The measured markers are thus narrower than one would expect, and since we are looking at the slow-wave sideband, the frequency spread in revolution frequency should also widen rather than narrow the markers.

The small negative-going dips in signal associated with each marker are real, and not instrumental artifacts. In other measurements, these dips were always absent on real resonance signals. It seems plausible that these dips are at the original frequency of excitation, and that the larger-amplitude particles have shifted up in betatron frequency due to nonlinearities of the lattice and space-charge effects.

Reference

- 1) "Speeding up the Schottky Scan", ISR Perf. Rep. 23rd May 1975, A. Sabersky.

Figure 1

Figure 2

SOME ENGINEERING ASPECTS OF THE FAST SCHOTTKY SCAN*

We have demonstrated the usefulness of the fast Schottky scan technique in recent MD runs, and it seems certain that we will buy a fast processor in the near future. One characteristic of any type of parallel processor that we must contend with is that its bandwidth is always equal to its frequency range, that is, it works from DC to f , its upper frequency.

In order to bring a Schottky noise band within its range, we must heterodyne the band of interest down so that its lower edge is near zero frequency. This type of heterodyning leads to special problems, as does the use of a digital processor. The low-frequency edge of the signal must be very sharply attenuated, otherwise adjacent signals will fold over about DC and give spurious signals.

Fig. 1

The upper edge should also be sharply attenuated, otherwise signals with frequencies about $\frac{1}{2}$ sampling rate of the processor will be digized, giving rise to aliasing, another type of foldover. The upper edge filtering will be aided by the built-in filters in the processor, which fall at about 70 dB/octave.

There exist standard flat-topped filters of the correct bandwidth (50 kHz) with reasonable bandpass ripple (1 dB) and sharp edges (70 dB/octave) at around

* Published as ISR Perf. Report 10 October 1975

10-30 MHz. These filters are made for the radio communications industry.

The use of such filters immediately leads us to think of a double-conversion superheterodyne.

Note: No detector!

Fig. 2

The first L.O. (Fig. 2(c)) and first mixer (Fig. 2(b)) bring the appropriate sideband into the range of the high-quality filter (Fig. 2(d)).

This L.O. should probably be a computer-controllable frequency synthesizer for stability and flexibility.

The second L.O. and mixer (Fig. 2, (e) and (f)) shift the band down near DC. Let us call the centre frequency of the filter (d), Fig. 2) f , its bandwidth B , and the frequency distance required to attenuate foldover ΔB . Then the frequency of the second L.O. should be:

$$\omega_2 = f \pm \left(\frac{B}{2} + \Delta B \right)$$

If we use the π case, the Schottky band will be inverted in frequency with respect to its orientation as it leaves the first mixer (Fig. 2(b)). Similar considerations apply to the frequency of the first L.O.

The second L.O. can be a fixed crystal oscillator. For added flexibility, one might wish to have the capability of switching in sharp-edged filters (Fig. 2(d)), of different bandwidth, some wider and some narrower than 50 kHz. Since the digital processor has a resolution which is a fixed fraction of its bandwidth, one needs a narrower-band filter and a different second L.O. frequency shift to do high-resolution measurements. This could easily be accomplished by having several filters to switch in, simultaneously switching in appropriate second L.O. crystals.

Fig. 2 omits necessary amplifiers: we already know that such a receiver yields excellent results from the experiments done by Thorndahl and Vaughan.¹

One receiver of the type described should be sufficient for all the scans, if one uses a synthesizer for the first local oscillator, and coaxial switches to select the proper signal.

Reference

1. ISK Perf. Rep., run 593, 25.4.75

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