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Within Very Low Density Plastics and by Helium**

University of California

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THE COOLING OF DT AND T₂ CRYOGENIC PELLETS WITHIN VERY LOW DENSITY PLASTICS AND BY HELIUM

by

Joseph J. Devaney

ABSTRACT

We have established previously that typical DT pellets cannot be kept solid with external cooling through pure vacuum, and can only be maintained solid within DT gas-filled spaces over a limited range of cooling temperatures. This report examines whether low-density (0.01 g/cm³ or 0.05 g/cm³) polyethylene foam or helium gas have sufficient conductivity to allow inner tritium-containing shells to be kept solid. For both conductors the answer is affirmative, but some uncertainty exists concerning the conductivity of foams and the worst excursions might not provide enough conductivity. For helium much less than one millitorr pressure suffices for cooling typical pellets. For cooling the same nuclear fuel concentrated in the form of a central ball, the lightest foams are marginal but might be satisfactory at the highest excursion of our conductivity error. Required helium pressures for ball cooling are in the region of a millitorr. A simple model for the ratio of foam to solid conductivity of a substance is developed.

INTRODUCTION

Because radiative cooling is proportional to the fourth power of temperature, such cooling is inefficient at temperatures of solid hydrogen (the triple point of D₂ is 18.71 K and DT mixtures contain 46 to 31 at.% D₂). Consequently, typical DT shells for laser fusion cannot be maintained in the solid state even at an absolute zero vacuum.^{1,2} This report examines whether low-density plastic or low-density helium will provide sufficient thermal conduction to maintain typical DT shells in the solid state. The answer is generally in the affirmative for plastics whose voids are not too large and whose composition is not too pure, and for helium at even extremely low pressures (below a millitorr). However, hydrogen will maintain solid DT only over a limited range of coolant temperatures. See Refs. 1 and 2 for a discussion of such cooling.

We base our solid polyethylene conductivities on those reported by the National Bureau of Standards for 0.971 g/cm³ density.³ However, for a slightly lower density solid, 0.914 g/cm³, the conductivities at temperatures of 4 to 20 K are lower by factors of 2.3 to 4, respectively. At the moment, we do not have any directly measured polyethylene foam conductivities. Indeed, we do not have any foam conductivities at all at our required low temperatures of 4.2 to 19 K. Our lowest measured foam conductivities are those for polystyrene at 23 K (-400°F),⁴ and, because of the role of gas conductivity and condensation in the voids of the foam, extrapolation to lower temperatures is extremely uncertain. Consequently, our first task is to develop a theory of conductivity of a foam when its solid conductivity is known. We then compare the ratio of the two conductivities (foam to

solid) with some experimental ratios to determine the validity of our theory.

FOAM CONDUCTIVITY, THEORY

A simple model of foam will suffice for our accuracy. All voids in the foam are assumed spherical and of equal size, the centers of which are staggered in two dimensions equidistantly as shown in Fig. 1. We first calculate the conductivity across one cube, the conductivity of the solid being k_s , that of foam k_f . Let the side of a cube be $2X$ and the radius of the included spherical void be R , then if y is a vertical coordinate and x a horizontal coordinate measured from the center of the spherical void, the area of solid plastic across any cube at $y < R$ is

$$A = 4X^2 - \pi(R^2 - y^2) \quad (1)$$

The energy, $e \equiv dF/dt$, conducted across the cube is constant and equal to

$$e = k_s A \frac{dT}{dy}, \text{ or } dT = k_s A e^{-1} dy \quad (2)$$

The temperature change across the cube must equal

$$\Delta T = k_f 8X^3 e^{-1} = k_s e^{-1} \left\{ \int_{-R}^R dy [4X^2 - \pi R^2 + \pi y^2] + 8X^2(X - R) \right\} = k_s e^{-1} (8X^3 - 4\pi R^3/3) \quad (3)$$

whence

$$k_f = k_s (\text{Solid Volume/Total Volume}), \text{ or}$$

$$k_f = k_s (\rho_f / \rho_s) \quad (4)$$

where ρ_f is the foam density and ρ_s is the solid plastic density. This reduction in the solid conductivity does not include the tortuosity of the

Fig. 1. Spherical voids in cubes.

path that the displaced voids, as in Fig. 1., provide. The two-dimensional path increase per Fig. 1 due to inclination of the path from layer to layer is roughly $\sqrt{X^2 + R^2}$ versus X . The actual path goes from $2X$ per cube for an $R = 0$ void to about a half circumference, $\pi R = \pi X$, per cube for a maximum spherical void, where $R = X$; in three dimensions X for the latter case becomes $\sqrt{2}\pi X$. Combining these considerations, the effective increase in path length for heat flow is the factor,

$$f = \sqrt{1 + \frac{R^2}{X^2} \left(-1 + \frac{\pi^2}{2}\right)} \quad (5)$$

which has the requisite property of being zero for $R = 0$ and being $\pi/\sqrt{2}$ for $R = X$, together with a behaviour of $\sqrt{1 + \text{const. } R^2}$ in between, as we would wish. Since

$$(4\pi R^3/3)/8X^3 = (\text{Void Volume})/(\text{Total Volume}), \quad (6)$$

or

$$\pi R^3/6X^3 = 1 - (\text{Solid Volume})/(\text{Total Volume}),$$

whence

$$R^2/X^2 = (6/\pi)^{2/3} \left[1 - (\rho_f/\rho_s) \right]^{2/3},$$

so that insertion of this relation into Eq. (5) and division of Eq. (4) by f gives our theoretical foam conductivity as a function of the solid conductivity and the relative densities,

$$k_f = k_s (\rho_f/\rho_s) \sqrt[3]{1 + 6.057 \left[1 - (\rho_f/\rho_s) \right]^{2/3}}. \quad (7)$$

In making comparisons of Eq. (7) with measured conductivities⁵ of polystyrene foam and polyvinylchloride foam divided by their respective solid conductivities all at 100 K, we find Formula (7) is 4.1 times lower than the former and 7.6 times lower than the latter. We suggest that the discrepancy is somewhat in the gas conductivity in the voids at least and that formula (7) is more likely right at lower temperatures than is 4 to 8 times Eq. (7). Again we have conductivities of polystyrene foam⁴ at 23.15 K (0.32 g/cm³ is 0.004 W/m K, 0.099 g/cm³ is 0.01 W/m K) leading to conductivity ratios of 5.4 and 4.3 times that of Eq. (7). Although air is no longer present as gas in the voids (N₂ condenses at 77.4 K and O₂ at 90.2 K at SP), some hydrogen gas is possible (H₂ condenses at 20.3 K at SP) and a little hydrogen makes a good conductor. However, for a conservative lower solid conductivity as noted above,³ we shall use the values of Eq. (7) multiplied by a factor of 1/3.2 and for a conservative higher conductivity, we use the average of the values immediately above to multiply Eq. (7) by a factor of 5. These factors should denote the extent of our experimental error. The solid conductivity of polyethylene k_s, is then given by

$$k_{so} = 303.21 \bar{T}^{1.6826} \text{ ergs/s} \cdot \text{cm} \cdot \text{K}, \quad (8)$$

where $\bar{T}$ is the average temperature, and as noted above, we will also use the following limiting values when substituting into Eq. (7) for foam.

$$k_{s-} = k_{so}/3.2, \text{ and} \quad (9)$$

$$k_{s+} = 5k_{so}. \quad (10)$$

STEADY-STATE TEMPERATURES OF FOAM FILLED CRYOGENIC PELLETS

We choose for our pellet example one derived from that proposed by W. S. Varum,¹ which we modify to omit details extraneous to our present purposes. This pellet was designed for use with 1 MJ incident laser light so that it is larger than expected for use in Antares or Helios, but the principles and the results scale to be (roughly) the same. Also with this choice, comparison may also be directly made with the calculations of Ref. 1. The pellet structure then consists of concentric spherical shells as given in Table I. For this pellet the shells are so thin that each can be considered as one dimensional. In addition, the conductivity of the DT or (T₂) shell is so high that temperature differences across such shells are at most the order of 30 μK (at 10 K),¹ so that we can take the conductivity of the DT or T₂ to be infinite. For each shell, i , we determine a mean radius $\bar{r}_i$, a conductivity k_i , and a thickness Δr_i so that the energy, E , produced in the tritium of the shell 1-2 is related to the temperature change across the shell, ΔT_i , by

$$e \equiv (dE/dt) = k_i 4\pi \bar{r}_i^2 \Delta r_i^{-1} \Delta T_i. \quad (11)$$

Using Eq. (11) and the various conductivities of Eqs. (7) through (10) as well as the heat productions of Table I, we calculate the temperature of the DT or T₂, i.e., the temperature T_1 or T_2 when T_5 is maintained at 10 or 4.22 K. The results are given in Table II.

Note that we exceed the triple point of D₂, a constituent of DT, only for the higher quantity of tritium and for the lowest conductivity of the least dense foam. Consequently, it is concluded that it is possible to keep internal DT shells in the solid state within a tenuous foam in typical pellet designs except for the most extreme circumstances. It is not possible to keep such shells in vacuum.^{1,2}

TABLE I
PELLET DESIGN

<u>Number</u>	<u>Radius (cm)</u>	<u>Material</u>	<u>Mass (mg)</u>	<u>Heat Production (erg/s)</u>
1	0.165	DT (or T_2)	0.61 (or 0.999)	1190 (or 3247)
2	0.173	Plastic	3.46	0
3	0.182	Foam, Gas, or Void	Various	0
4	0.3016	Plastic	17.5	0
5	0.3166			

TABLE II
TEMPERATURE OF INNER FUEL FOR A COOLANT TEMPERATURE OF 10 or 4.22 K

<u>Source (r_1 to r_2) and Insulation (r_3 to r_4)</u>	<u>Temperature, T_2 (K) at Coolant Temperature, T_5</u>	
	<u>$T_5 = 10$ K</u>	<u>$T_5 = 4.22$ K</u>
0.61 mg DT, 0.01 g/cm ³ polyethylene	12.7 (10.6 to 16.7) ^a	9.9 (6.3 to 15.4)
0.999 mg T_2 , 0.01 g/cm ³ polyethylene	16.0 (11.6 to 22.9)	14.6 (8.3 to 22.5)
0.61 mg DT, 0.05 g/cm ³ polyethylene	10.6 (10.1 to 11.9)	6.2 (4.8 to 8.7)
0.999 mg T_2 , 0.05 g/cm ³ polyethylene	11.8 (10.4 to 14.3)	8.3 (5.5 to 12.3)

^aThe parentheses indicate the range of temperatures. High temperature is for a poorly conducting solid polyethylene base, Eq. (9), and low temperature is for a (fictitious) highly conducting solid polyethylene base, Eq. (10).

STEADY-STATE TEMPERATURE OF HELIUM-FILLED CRYOGENIC PELLETS

The formula

$$k = 305.42 T^{0.7217} \text{ ergs/s.cm.k} , \quad (12)$$

fits the helium thermal conductivity⁶ within $\pm 2\%$ over the range 5 to 18 K and is suitable when the dimensions of a chamber exceed the mean free path, independent of the pressure.¹ The diameter, d , of the helium atom is $2.576 \text{ \AA}$,⁷ the mean free path is given by,⁷

$$\ell = (\pi n d^2 \sqrt{2})^{-1} , \quad (13)$$

where n is the molecular density. If we set ℓ equal to $r_4 - r_3 = 0.1196 \text{ cm}$ (Table I), we find a critical atomic density of

$$n_{\text{crit}} = 2.8 \times 10^{15} / \text{cm}^3 , \quad (14)$$

above which Formula (12) holds. Using Formulas (8), (11), and (12), the pellet of Table I containing 0.61 mg DT rises to a maximum temperature of only 10.12 K if r_6 is kept at 10 K and to 4.45 K if r_6 is kept at 4.22 K whenever the helium atom density exceeds n_{crit} of Formula (14).

The minimum density of helium required to keep the internal DT from melting is considerably lower than critical. The flow across a spherical shell filled with gas below critical density is given by⁸

$$d^2 E / dt dA = (3P/8) \sqrt{3R/M\bar{T}} \cdot (T_3 - T_4) , \quad (15)$$

where P is pressure, R the gas constant, M the molecular weight, and $\bar{T}$ the mean temperature $(T_3 + T_4)/2$. Applying Eq. (15) to our pellet with 0.61 mg DT, the minimum pressure of He needed is 1.8×10^{-4} torr at a mean temperature of 14.4 K

if T_2 is 18.71 K and T_5 is 10 K, which is a particle density of $1.2 \times 10^{14} / \text{cm}^3$. The minimum pressure of He needed is 9.6×10^{-5} torr at a mean temperature of 11.5 K if T_6 is 4.22 K, which is an atomic density of $8.1 \times 10^{13} / \text{cm}^3$. Both values are less than n_{crit} of Eq. (14) justifying the use of Eq. (15).

If now our pellet contains 0.999 mg of T_2 we will need 5.1×10^{-4} torr helium at a mean temperature of 14.4 K or an atomic density of $3.4 \times 10^{14} / \text{cm}^3$ if T_6 is kept at 10 K, and we will need 2.7×10^{-4} torr helium at a mean temperature of 11.5 K or an atomic density of $2.3 \times 10^{14} / \text{cm}^3$ if we cool r_5 to 4.22 K. Again both values are less than n_{crit} of Eq. (14). Thus, even a high vacuum of helium is highly effective in cooling an inner shell. These values are summarized in Table III. Hydrogen is also effective, but its temperature must not fall below about 13.3 K mean for this pellet, because otherwise the atom density will then be too low to conduct enough heat from the pellet to forestall melting.^{1,2}

STEADY-STATE COOLING OF CENTRAL BALLS WITH FOAM CONDUCTORS

As noted above, even a low-density plastic or a low-density gas (helium or hydrogen in a restricted temperature range), can keep a cryogenic multishell pellet cool enough to prevent melting. Because of the smaller surface area, cooling is much more difficult if the fuel is in the form of a central ball. In that case, one cannot use slab-geometry heat transfer, but must go to spherical geometry. By using Eq. (2) with $A = 4\pi r^2$ and d replaced by dr we integrate from a radius r_a to r_b to find

$$\begin{aligned} T_a - T_b &= (e/4\pi k) \int_{r_a}^{r_b} r^{-2} dr \\ &= (e/4\pi k) (r_a^{-1} - r_b^{-1}), \end{aligned} \quad (16)$$

if the thermal conductivity k is independent of temperature. On the other hand, our thermal conductivities obey well the dependence

TABLE III

MINIMUM HELIUM PRESSURES REQUIRED TO KEEP
HYDROGEN FUEL SHELLS IN THE SOLID STATE
(i.e., DT or T₂ kept at 18.71 K)^a
(Helium pressure in mtorr at a mean gas temperature in Kelvin)

Fuel	<u>T₅ Temperature</u>	
	<u>10 K</u>	<u>4.22 K</u>
0.61 mg DT	0.18 at 14.4 K	0.096 at 11.5 K
0.999 mg T ₂	0.51 at 14.4 K	0.27 at 11.5 K

^aWe have used the triple point of D₂, 18.71 K⁹ as the temperature of onset of softenings for both DT and T₂ for comparison. This value is appropriate for DT because, as noted, DT contains considerable D₂. However, the pure T₂ triple point is 20.62 K⁹ and consequently lower helium pressures than listed would suffice for T₂.

$$k = CT^C ; \quad (17)$$

thus, integrating we find in this case

$$T_a^{C+1} = T_b^{C+1} + [e(c+1)/4\pi C] (r_a^{-1} - r_b^{-1}) . \quad (18)$$

At a density of 0.215 g/cm³,¹⁰ a 0.61-mg DT ball has a radius r_a of 0.08817 cm. For polyethylene foam of 0.01 g/cm³, $k = 1.179T^{1.6826}$, out to $r_b = 0.3016$ cm which is at a temperature of 10 K (T_b). Then the DT temperature $T_a = 17.65$ K. If $T_b = 4.22$ K, then the DT temperature $T_a = 16.27$ K. For the same foam (0.01 g/cm³) with 0.999-mg T₂ ball, of 0.255 g/cm³ density and 0.09782 cm radius, we get a ball temperature of 23.09 K when cooling the foam to 10 K at $r_b = 0.3016$ cm, and obtain 22.24 K if the foam is cooled to 4.22 K at 0.3016 cm.

Again these parameters have a range of values, see Table IV. Although the lightest foams probably can be used with small solid DT balls, it seems that the higher heat sources in the form of balls are unlikely to be kept at 18.71 K or below with the lightest foams. Heavier foams or He-filled foams appear to be generally satisfactory even for larger ball sources.

STEADY-STATE COOLING OF CENTRAL BALLS, HELIUM CONDUCTORS

If now we replace the foam by the minimum density of helium required to keep the hydrogen fuel solid, we must first calculate the critical density for collisionless cooling from Eq. (13). That number for the DT radius is

$$n_{crit} = 1.59 \times 10^{15} \text{ He/cm}^3 ; \quad (19)$$

for a T₂ ball it is,

$$n_{crit} = 1.66 \times 10^{15} \text{ He/cm}^3 . \quad (20)$$

We then apply Eq. (15) where the cooling atoms that hit the ball come directly from the outer walls. Thus the appropriate area to use with Eq. (15) is that of the ball.

The average temperature, $\hat{T}$, of Eq. (15) is nearly T_4 because most atoms are at the wall temperature, the ratio of atom temperatures being proportional to the areas. Calculating the values we find that $\hat{T} = 1 + T_4$ can be used for both $T_4 = 10$ K and $T_4 = 4.22$ K for simplicity, the differences are small. Use of Eq. (15) with a 0.999-mg

T_2 ball leads to minimum helium densities of $2.29 \times 10^{14}/\text{cm}^3$ for T_4 temperatures of 10 K. This particle density exceeds the critical density, Eq. (20) of $1.66 \times 10^{15} \text{ He}/\text{cm}^3$, implying that one should rather use the collisional formula of Eq. (16) with conductivity given by Eq. (12). Use of these latter equations leads to ball temperatures,

T_a , of 11.07 K for cooling temperatures T_b of 10 K and 6.02 K for cooling to 4.22 K implying that near the critical density the values of Eq. (15) are conservative, so we remain with the latter equation even though we do slightly exceed critical densities for the T_2 (however we do not so exceed for DT). Our results are presented in Table V.

TABLE IV

TEMPERATURE OF A SOLID DT OR T_2 BALL FOR COOLANT TEMPERATURES OF 10 OR 4.22 K
(Outer polyethylene foam radius, 0.3016 cm;
foam density, $0.01 \text{ g}/\text{cm}^3$)

Ball Source	Ball Temperature, T_a (K), at Coolant	
	Temperature, T_b	
	$T_b = 10 \text{ K}$	$T_b = 4.22 \text{ K}$
0.61 mg DT	17.6 (12.2 to 25.6) ^a	16.3 (9.3 to 24.9)
0.999 mg T_2	23.1 (15.0 to 36.6)	22.2 (13.1 to 36.2)

^aThe parentheses indicate the range of temperatures. High temperature is for a poorly conducting solid polyethylene base, Eq. (9), and low temperature is for a (fictitious) highly conducting solid polyethylene base, Eq. (10).

TABLE V

MINIMUM HELIUM PRESSURES REQUIRED TO KEEP
HYDROGEN FUEL BALLS IN THE SOLID STATE
(i.e., DT or T_2 kept at 18.71 K)^a
(In mtorr at a mean gas temperature in Kelvin)

Fuel, Ball Radius	Coolant Temperature, T_4	
	10 K	4.22 K
0.61 mg DT, 0.088 cm	1.17 at c11 K	0.49 at c5.2 K
0.999 mg T_2 , 0.098 cm	2.6. at c11 K	1.08 at c5.2 K
(critical densities are slightly exceeded, see text)		

^aWe have used the triple point of D_2 , 18.17 K⁹ as the temperature of onset of softening for both DT and T_2 for comparison. This value is appropriate for DT because, as we've noted, it contains considerable D_2 . However, the pure T_2 triple point is 20.62 K⁹ and consequently lower helium pressures than listed in the table would suffice for T_2 .

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