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Towards Spectroscopy of Partially Coherent Sources

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Abstract

Traditional spectroscopy is concerned with spatially incoherent sources. In relatively recent times, after the invention of the lasers in 1960, spectroscopy of coherent sources has also been gradually developed. In the last few years it was found that spectroscopy of partially coherent sources i.e., those which are neither completely uncorrelated nor fully correlated, encounters certain problems. They arise from the fact predicted theoretically in 1986 and confirmed experimentally soon afterwards that, in general, the spectrum of light generated by a partially coherent source changes on propagation, even in free space. Such changes may take many different forms. For example a spectral line may be shifted or distorted or it may be split into several lines. In this talk the basic physical principles underlying this phenomenon will be described and some recent developments in this field will be reviewed.

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Spectroscopy is based on the implicit assumption that the (relative) energy distribution in the spectrum of the electromagnetic radiation emitted by a source is entirely determined by the chemical composition of the source and that it is independent of the location, where the spectrum of the emitted radiation is measured. In a paper published in 1986 it was shown theoretically^[1] that this assumption is incorrect and that, in general, the spectrum of the field depends not only on the spectrum of the source but also on some of its statistical properties, characterized by the so-called second-order degree of spectral coherence. It was also shown, that only for radiation generated by the most commonly occurring sources, for example blackbody sources and lasers, this effect may be absent under usual experimental conditions. This prediction has been verified experimentally soon afterwards^[2] and has been demonstrated since then in many different laboratories.^{[3]–[8]}

Implications of this discovery for several areas of optical physics, optical engineering and astronomy are currently being explored. These investigations have already led to some interesting developments and are beginning to lay the foundations to a new discipline, namely spectroscopy of partially coherent sources. In this talk I will review very briefly the basic principles underlying these developments and I will describe some recent contributions to this subject.

Let me begin by stressing that any realistic optical source and, consequently, also the field which the source generates will in the course of time undergo random fluctuations. The fluctuations have to be characterized statistically, either in terms of appropriate probability distributions or in terms of various correlation functions. Suppose that $Q(\mathbf{r}, t)$ represents a fluctuating source variable, say a Cartesian component of the current density, at a point whose position is specified by a vector $\mathbf{r}$, at time t , and that $V(\mathbf{r}, t)$ represents the field which the source generates.^{#1} Assuming that the fluctuations are statistically stationary, i.e. that the underlying probabilities are invariant with respect to time translation, the simplest of the source correlation functions characterizes the "statistical similarity" of the source fluctuations at two space-time points, say $(\mathbf{r}_1, t)$ and $(\mathbf{r}_2, t + \tau)$. It is defined by the formula

$$\Gamma_Q(\mathbf{r}_1, \mathbf{r}_2, \tau) = \langle Q^*(\mathbf{r}_1, t) Q(\mathbf{r}_2, t + \tau) \rangle, \quad (1)$$

where the angular brackets denote a statistical average. In a similar way we may define the correlation function of the field at two space–time points by the formula

$$\Gamma_V(r_1, r_2, \tau) = \langle V^*(r_1, t) V(r_2, t + \tau) \rangle. \quad (2)$$

The functions Γ_Q and Γ_V characterize correlations in the *space–time domain*. For many purposes, particularly for the analysis of spectral properties of the field, it is more appropriate to employ functions which characterize correlations in the *space–frequency domain*. These correlation functions are just the Fourier transform of Γ_Q and Γ_V , viz.,

$$W_Q(r_1, r_2, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Gamma_Q(r_1, r_2, \tau) e^{i\omega\tau} d\tau, \quad (3)$$

$$W_V(r_1, r_2, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Gamma_V(r_1, r_2, \tau) e^{i\omega\tau} d\tau, \quad (4)$$

known as the *cross–spectral densities* of the source variable and of the field variable respectively. Actually the formulas (3) and (4) do not show that the cross–spectral densities themselves are correlation functions. This was demonstrated only relatively recently,^[10] when it was shown that one may always introduce ensembles of random, frequency–dependent variables $\{U_Q(r, \omega)\}$ and $\{U_V(r, \omega)\}$ such that

$$W_Q(r_1, r_2, \omega) = \langle U_Q^*(r_1, \omega) U_Q(r_2, \omega) \rangle_\omega \quad (5)$$

and

$$W_V(r_1, r_2, \omega) = \langle U_V^*(r_1, \omega) U_V(r_2, \omega) \rangle_\omega. \quad (6)$$

In these two formulas the angular brackets, with the subscript ω , indicate averaging over these ensembles of frequency–dependent functions. The functions $U_Q(r, \omega)$ and $U_V(r, \omega)$ are *not* the

Fourier transforms of the variables $Q(r, t)$ and $V(r, t)$ because, as is well-known, the sample functions of stationary random processes do not possess Fourier representation within the framework of ordinary function theory^{#2}. The functions U_Q and U_V have to be introduced in terms of the eigenfunctions and the eigenvalues of a Fredholm integral equation, whose kernel is the cross-spectral density W_Q .

According to an obvious generalization of the Wiener-Khintchine theorem^[11] the spectral density $S_Q(r, \omega)$ of the source distribution and the spectral density $S_V(r, \omega)$ of the field distribution are just the "diagonal elements" of the cross-spectral densities $W_Q(r_1, r_2, \omega)$ and $W_V(r_1, r_2, \omega)$, i.e.

$$S_Q(r, \omega) = W_Q(r, r, \omega), \quad (7)$$

and

$$S_V(r, \omega) = W_V(r, r, \omega). \quad (8)$$

In view of the formulas (5) and (6) the spectral densities may be expressed in intuitively more appealing forms as

$$S_Q(r, \omega) = \langle U_Q^*(r, \omega) U_Q(r, \omega) \rangle_\omega \quad (9)$$

and

$$S_V(r, \omega) = \langle U_V^*(r, \omega) U_V(r, \omega) \rangle_\omega. \quad (10)$$

Let us now consider light propagation in free space, from a localized source. Because the field variable and the source variable are then related by the inhomogeneous wave equations, one can show that the cross-spectral densities are rigorously related by the equation^[12]

$$(\nabla_1^2 + k^2)(\nabla_2^2 + k^2) W_V(r_1, r_2, \omega) = (4\pi)^2 W_Q(r_1, r_2, \omega). \quad (11)$$

Here ∇_j^2 , ($j = 1, 2$), is the Laplacian operator, acting with respect to the coordinates of the point r_j and

$$k = \omega/c, \quad (12)$$

c being the speed of light in vacuo. The solution of Eq. (11) for the cross-spectral density function of the field radiated by the source may readily be shown to be given in the formula

$$W_V(r_1, r_2, \omega) = \int_D \int_D W_Q(r'_1, r'_2, \omega) \frac{e^{ik(R_{22} - R_{11})}}{R_{11} R_{22}} d^3 r'_1 d^3 r'_2, \quad (13)$$

where

$$R_{11} = |r_1 - r'_1|, \quad R_{22} = |r_2 - r'_2| \quad (14)$$

and the integrations extend twice independently over the source domain D , assumed to be finite.

Suppose now that the field points are situated in the far zone of the source, at the same distance r from an origin O in the source region, in directions specified by unit vectors u_1 and u_2 respectively (Fig. 1). Then

$$R_{11} \sim r - u_1 \cdot r', \quad R_{22} \sim r - u_2 \cdot r' \quad (15)$$

($r = |r|$) and the formula (13) becomes (with superscript " ∞ " indicating the far-zone value)

$$W_V^{(\infty)}(ru_1, ru_2, \omega) = \frac{1}{r^2} \bar{W}_Q(-ku_1, ku_2, \omega), \quad (16)$$

where

$$\bar{W}_Q(K_1, K_2, \omega) = \int \int W_Q(r'_1, r'_2, \omega) e^{-i(K_1 \cdot r'_1 + K_2 \cdot r'_2)} d^3 r'_1 d^3 r'_2. \quad (17)$$

Because $W_Q(r'_1, r'_2, \omega) = 0$ when either r'_1 or r'_2 are position vectors of points located outside the source region D , the integrals on the right-hand side of Eq. (17) may be taken formally to extend over the whole space. We see that $\bar{W}_Q(K_1, K_2, \omega)$ is just the six-dimensional spatial Fourier transform of the cross-spectral density function of the source distribution. From Eqs. (8) and (16) it follows at once that the spectral density of the field in the far zone, at a point specified by the position vector $r \equiv ru$ ($u^2 = 1$) is given by the formula

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$$S_V^{(\infty)}(\mathbf{r}\mathbf{u}, \omega) = \frac{1}{r^2} \tilde{W}_Q(-\mathbf{k}\mathbf{u}, \mathbf{k}\mathbf{u}, \omega). \quad (18)$$

We will now show that this simple-looking formula has important physical implications regarding the spectra of fields produced by sources of different states of coherence. For this purpose we first introduce that degree of spectral coherence at frequency ω of the source distribution by the formula^[13]

$$\mu_Q(\mathbf{r}'_1, \mathbf{r}'_2, \omega) = \frac{W_Q(\mathbf{r}'_1, \mathbf{r}'_2, \omega)}{\sqrt{S_Q(\mathbf{r}'_1, \omega)} \sqrt{S_Q(\mathbf{r}'_2, \omega)}}. \quad (19)$$

It can be shown that for all values of its variables,

$$|\mu_Q(\mathbf{r}'_1, \mathbf{r}'_2, \omega)| \leq 1. \quad (20)$$

The extreme value $|\mu_Q| = 1$ characterizes complete correlation (complete spatial coherence) and the other extreme, $\mu_Q = 0$, characterizes complete absence of correlation (complete spatial incoherence) of the source fluctuations at the points $\mathbf{r}'_1$ and $\mathbf{r}'_2$, at frequency ω .

Suppose now that the spectrum of the source is the same at every source point. We may then write $S_Q(\omega)$ in place of $S_Q(\mathbf{r}, \omega)$ and Eq. (19) shows that in this case

$$W_Q(\mathbf{r}'_1, \mathbf{r}'_2, \omega) = S_Q(\omega) \mu_Q(\mathbf{r}'_1, \mathbf{r}'_2, \omega). \quad (21)$$

It will be useful to introduce the normalized far-zone spectrum

$$s_V^{(\infty)}(\mathbf{r}\mathbf{u}, \omega) = \frac{S_V^{(\infty)}(\mathbf{r}\mathbf{u}, \omega)}{\int_0^\infty S_V^{(\infty)}(\mathbf{r}\mathbf{u}, \omega) d\omega}, \quad (22)$$

whose integral, over all frequencies has evidently the value unity. On substituting from Eqs. (18) into Eq. (22) and making use of the formula (21) we finally obtain the following expression for the normalized far-zone spectrum:

$$s_V^{(\infty)}(\mathbf{ru}, \omega) = \frac{1}{N(\mathbf{u})} S_Q(\omega) \tilde{\mu}_Q(-\mathbf{ku}, \mathbf{ku}, \omega), \quad (23)$$

where

$$\tilde{\mu}_Q(\mathbf{K}_1, \mathbf{K}_2, \omega) = \iint \mu_Q(\mathbf{r}'_1, \mathbf{r}'_2, \omega) e^{-i(\mathbf{K}_1 \cdot \mathbf{r}'_1 + \mathbf{K}_2 \cdot \mathbf{r}'_2)} d^3 r'_1 d^3 r'_2 \quad (24)$$

is the six-dimensional spatial Fourier transform of the degree of spectral coherence of the source and

$$N(\mathbf{u}) = \int_0^\infty S_Q(\omega) \tilde{\mu}_Q(-\mathbf{ku}, \mathbf{ku}, \omega) d\omega \quad (25)$$

is a normalization factor.

The formula (23) is one of the main results relating to spectra of radiation fields produced by sources of any state of coherence. It shows that *the normalized far-zone spectrum depends not only on the source spectrum but also on the degree of spectral coherence of the source*. Moreover, as the formula also shows, the far-zone spectrum depends, in general, also on the direction $\mathbf{u}$ of observation.

It is clear from Eq. (23) that the coherence properties of the source may drastically affect the form of the spectrum of the radiated field. In particular, it is evident that if the source spectrum $S_Q(\omega)$ is a single spectral line centered on frequency ω_0 and if for some direction $\mathbf{u}$, $\tilde{\mu}_Q(-\mathbf{ku}, \mathbf{ku}, \omega)$ is sharply peaked around some frequency $\omega_1 \neq \omega_0$, the normalized far-zone spectrum $s_V^{(\infty)}(\mathbf{ru}, \omega)$ will no longer be centered on ω_0 but rather on a frequency that is closer to ω_1 . In particular if $\omega_1 < \omega_0$ the spectrum $s_V^{(\infty)}$ will be centered at a frequency lower than ω_0 , i.e. it will exhibit a *redshift*, whereas when $\omega_1 > \omega_0$ it will be centered on a frequency higher than ω_0 i.e. it will exhibit a *blueshift*. Such spectral changes^{#3} were discussed in refs. [14] and [15] and are illustrated by an example in Fig. 2.

Fig 2

The preceding analysis was concerned with influence of correlations involving three-dimensional primary sources. However, the phenomenon we are discussing is much more general and is manifested also in situations which involve secondary sources of various kinds, such as illuminated apertures^{[1], [5], [6], [7], [16]} or illuminated scattering media whose macroscopic parameters vary randomly in space^[17] and possibly also in time.^{[18] - [23]} In some cases the spectral changes produced by scattering from suitably correlated fluctuating random media may imitate the Doppler effect, even though the source, the scattering medium and the observer are all at rest with respect to each other. An example is given in Fig. 3.

Perhaps the simplest situation which illustrates correlation-induced spectral changes involves two small apertures illuminated by partially coherent light^{#4}. An example of this kind, due to Gori, Palma and Padovani^[25], is illustrated in Fig. 4. It makes use of two, mutually independent beams B_α and B_β , with nearly the same spectrum, illuminating pinholes P_1 and P_2 . A phase difference is introduced between the beams by means of a beam splitter BS. By superposing the two beams the light at the two pinholes becomes partially coherent and its degree of coherence can be varied by changing the orientation of the beam splitter. With suitable orientations redshifts as well as blueshifts of spectral lines can be produced.

Another conceptually simple demonstration of the effects of source correlations on the spectra of radiation is by means of the Young interference experiment with partially coherent light, provided the bandwidth of the light incident on the pinholes is sufficiently broad or its degree of spectral coherence varies rapidly enough with frequency over the bandwidth.^{[26], [27]} For example when an uniform, circular, quasi-homogeneous,^[28] secondary source illuminates two symmetrically placed pinholes P_1 and P_2 as shown in Fig. 5a, the light at the two pinholes will be partially coherent and its degree of coherence will depend on the pinhole separation^{#5}. In consequence the spectrum of the light in the region of superposition will differ from the spectrum of the light incident on the two pinholes. In Fig. 5b spectra are shown which would be observed

at an axial point Q_0 , if the pinholes were illuminated with thermal radiation whose spectrum is given by Planck's law^{#6}.

So far we described correlation-induced spectral changes entirely on the basis of classical, statistical wave theory, but this effect is not restricted to the domain of classical physics. First quantum analysis of a phenomenon of this kind was developed by Varada and Agarwal^[29] who studied frequency shifts due to cooperative effects at the microscopic level. They examined the spectrum of radiation emitted by an atomic system consisting of two identical two-level atoms which interact with a thermal field and which are coupled by the dipole-dipole interaction. They found that for directions of observation perpendicular to the line joining the two atoms, the steady-state spectrum consists of a single line which is, however, shifted with respect to the line that would be generated by each atom separately. The direction of the shift was found to depend on the sign of the dipole-dipole interaction term, which in turn is determined by the interatomic distance. With a mean number of thermal photons $\bar{n} = 0.1$ they found that when the distance between the atoms is greater than about half a wavelength, the spectral line was blueshifted, whereas for somewhat smaller separations it is redshifted (Fig. 6). For much smaller interatomic separation the spectrum breaks up into two distinct resonances.

Some experiments which illustrate correlation-induced spectral changes and which are most naturally analyzed quantum mechanically were performed by Drabe *et al.*^[30] They found that the fluorescence spectra of radiating molecules change drastically when a mirror is placed in their vicinity and that the spectra are different at different points in space (Fig. 7). These spectral changes may be regarded as manifestation of correlations which exist between the molecules and their mirror image.

In this short review we were only able to present an account of the basic principles underlying the rapidly developing field of spectroscopy of partially coherent sources and to note some of the recent contributions to this field. Although the first papers on this subject appeared only six years ago, there are by now about sixty publications dealing with this topic and some applications of the underlying phenomenon of correlation-induced spectral changes have been

considered. Some of these developments are briefly described in two recent review articles.^{[31], [32]}

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Footnotes

- #1 Throughout this paper we use the complex analytic signal representation ([9], Sec. 10.2) of the source variable and of the field variable. Also in order to bring out the essential features of correlation-induced spectral changes we ignore vector features of the source and of the field, i.e. we take $Q(r, t)$ and $V(r, t)$ to be scalar quantities.
- #2 For discussion of this point see, for example, ref. [10a], Sec.2.
- #3 First experimental demonstration of shifts of spectral lines due to source correlations was made with acoustical sources by M. F. Bocko, D. H. Douglass and R. S. Knox, *Phys. Rev. Lett.* **58**, 2649-2651 (1987).
- #4 The possibility that when two beams with identical normalized spectra are superposed, the spectrum of light in the region of superposition may differ from that of the beams was first pointed out by L. Mandel [24] in his research on cross-spectral purity.
- #5 This fact follows from one of the reciprocity relations for fields generated by quasi-homogeneous sources [28] which is analogous to the well-known van Cittert-Zernike theorem ([9], Sec. 10.4.2).
- #6 Since this talk was given, spectral changes produced in Young's interference experiment were observed in beautiful experiments performed by M. Santarsiero and F. Gori, (to be published).

Figure Captions

Fig. 1: Illustrating the notation relating to the calculation of the cross-spectral density and of the spectrum of the field at point P_1 and P_2 in the far zone of a three-dimensional source.

Fig. 2: Normalized spectra $s_V^{(\infty)}(\omega)$, in units of $(1/\delta\sqrt{2\pi})$, of the far field generated by sources with the same normalized source spectrum $s_Q(\omega) = (\delta\sqrt{2\pi})^{-1} \exp[-(\omega - \omega_0)^2/2\delta^2]$ and degree of spectral coherence $\mu_Q(r', \omega) = \exp[-r'^2/2\zeta^2]$, with $\omega_0 = 3.887 \times 10^{15} \text{ s}^{-1}$ ($\lambda_0 = 4.861 \text{ \AA}$) and $\delta = 9.57 \times 10^{13} \text{ s}^{-1}$, for several values of the effective source correlation length ζ . The solid curve ($\zeta \rightarrow 0$) also represents the source spectrum [adapted from E. Wolf [14]].

Fig. 3: Two O III lines ($\lambda = 4959 \text{ \AA}$ and $5007 \text{ \AA}$) as seen at rest (*solid line*), Doppler shifted (dotted line) and shifted by dynamic scattering on a suitably correlated random medium (dashed line). For details see the paper by D. F. V. James, M. Savedoff and E. Wolf [20], from which this figure is reproduced.

Fig. 4: Experimental demonstration of shifts of spectral lines due to correlation between two secondary sources. (A) Layout of the experiment: B_α, B_β are mutually uncorrelated beams of essentially the same spectrum, BS is a beam splitter, P_1, P_2 are pinholes (secondary sources), SA is a spectral analyzer. (B) Results of observations shown by heavy dots. Redshifted spectrum (a) and blueshifted spectrum (b). The curve (c) shows the spectrum, multiplied by a factor 2, produced at the point of observation by light passing through only one of the pinholes. For the sake of clarity, the experimental points

have been connected by lines. {After F. Gori, G. Guattari, C. Palma and C. Padovani [25]}.

Fig. 5: Spectral changes produced in Young's interference experiment. (a) Illustrating the geometry and the notation. (The distances and angles are exaggerated to show detail). (b) The changes in the Planck spectrum, produced at an axial point Q_0 by interference, for different pinhole separations d . The circular, quasi-homogeneous source was assumed to be at temperature $T = 3000^\circ\text{K}$ and to subtend an angular semi-diameter $\alpha = 2.96 \times 10^{-5}$ radians at O . The units on the vertical axis are arbitrary, but are the same for all values of d . {After D. F. V. James and E. Wolf [26]}.

Fig. 6: The spectra $S(\omega)$ of radiation emitted by a pair of identical two-level atoms, which are interacting with a thermal field at a fixed temperature and which are coupled by the dipole-dipole interaction. The direction of observation was chosen to be perpendicular to the line joining the two atoms and the mean number of thermal photons was taken as $\bar{n} = 0.1$. The different curves correspond to different values of the (dimensionless) interatomic distance $k_0 r_{12}$. The curves (a) and (b) correspond to $k_0 r_{12} = 3$ and 2 respectively (blueshifted lines), whereas (c), (d) and (e) correspond to $k_0 r_{12} = 1, 0.7$ and 0.5 respectively (redshifted lines). The normalization factor γ which appears in the variable that indicates the relative frequency shift (horizontal axis) is one-half of the Einstein A-coefficient. {After Varada and Agarwal [29]}.

Fig. 7: Fluorescence spectra observed at different points of radiating molecules placed in the neighborhood of a mirror. Both spectra were obtained from the same sample. {After H. K. E. Drabe, C. Cnossen, D. A. Wiersma, H. A. Ferwerda and B. J. Hoenders [30]}.

Fig. 1

Fig. 2

(A)

(B)

Fig. 4

Fig. 5a

Fig. 5b

Fig. 6

Fig. 7

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