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Choice of Momentum Compaction Factor
for the
APIARY Low-Energy Ring

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ABSTRACT

For the new low-energy ring of the APIARY B factory collider, there are several considerations that go into the choice of momentum compaction factor. In this note we enumerate these considerations and indicate the restrictions on momentum compaction factor that arise therefrom. Probably the most difficult condition to achieve is maintaining the same betatron tune modulation at the IP as occurs for the high-energy ring. Generally, however, we find that the constraints are rather loose, so the ring design is not heavily influenced.

INTRODUCTION

In designing a new storage ring to serve as the low-energy ring of the APIARY collider, one of the parameters that must be selected is the momentum compaction factor, α . We discuss here a number of beam properties that are influenced by this quantity, including:

- Natural bunch length
- Single-bunch instabilities (longitudinal microwave and transverse mode coupling)
- RF system momentum acceptance
- Tune modulation at the IP
- Natural emittance

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In this paper we will generally be concerned not with the momentum compaction factor itself but with a closely related parameter, η , the so-called phase-slip factor defined by

$$\eta = \alpha - 1/\gamma^2 . \quad (1)$$

For an electron ring the Lorentz factor γ is large, so the numerical difference between the two quantities α and η is negligible and they are often used interchangeably. Here we will explicitly use η in those formulae for which it is appropriate. Unless otherwise noted, the formulae and units used in this paper have been taken from Ref. 1.

NATURAL BUNCH LENGTH

The natural bunch length can be written, for short bunches, as

$$\sigma_z = \frac{\eta R}{v_s} \sigma_{p0} \quad (2)$$

where v_s is the synchrotron tune and $\sigma_{p0} \equiv (\delta p/p)_{\text{rms, natural}}$. In terms of the RF, lattice, and beam parameters, the synchrotron tune is given by

$$v_s = \frac{1}{\beta} \left[\frac{-V_{RF} h \eta \cos \phi_s}{2\pi (E/e)} \right]^{1/2} \quad (3)$$

where h is the harmonic number, and the other symbols have their usual interpretation. For the B factory parameter regime, where we use a high RF voltage to produce short bunches, the synchronous phase is always close to π . Thus, $\cos \phi_s \rightarrow -1$ and (taking $\beta = 1$) we can combine Eqs. (2) and (3) as

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$$\sigma_z = \left[\frac{2\pi (E/e) \eta}{V_{RF} h} \right]^{1/2} R \sigma_{p0} \quad (4)$$

Using appropriate parameters for the low-energy ring, $E/e = 3.1 \times 10^9$ V, $h = 3492$, $R = 350$ m, Eq. (4) can be written

$$\sigma_z = 8.267 \times 10^5 \sqrt{\frac{\eta}{V_{RF}}} \sigma_{p0} . \quad (4a)$$

At present, there are two extreme parameter sets that can be considered for the low-energy ring. We denote as "Case 1" the situation corresponding to the typical low-energy ring without any wigglers. Because the ring circumference is large for a 3.1-GeV storage ring ($C = 2200$ m), a modest bending magnet field that gives a bending radius, ρ , of 58 m leads to a natural momentum spread of about 3.5×10^{-4} . We refer to as "Case 2" the situation corresponding to the choice of equal damping decrements for the high- and low-energy APIARY rings (as part of the attempt to reach "energy transparency" conditions). In this case the synchrotron radiation energy loss per turn, U_0 , of the low-energy ring is adjusted by means of wigglers to give the same damping time as in the high-energy ring. The use of wigglers increases the natural momentum spread of the lattice to 1×10^{-3} .

Case 1

Without the energy transparency conditions, the natural momentum spread of a typical lattice is $\sigma_{p0} = 3.5 \times 10^{-4}$, and Eq. (4a) becomes

$$\sigma_z = 289.34 \sqrt{\frac{\eta}{V_{RF}}} . \quad (5)$$

Assuming a required bunch length of 0.01 m, we obtain

$$\frac{\eta}{V_{RF}} = 1.1945 \times 10^{-9} . \quad (6)$$

Table I
Typical Parameter Combinations for Case 1
(to produce a 1 cm bunch length)

V_{RF} (MV)	η
1	1.19×10^{-3}
3	3.58×10^{-3}
5	5.97×10^{-3}

Thus, we see that—for expected η values of a few times 10^{-3} —it is possible to produce the required 1-cm bunches with relatively modest voltage.

Case 2

If we require the energy transparency conditions, the natural momentum spread of the low-energy ring increases to $\sigma_{p0} = 1 \times 10^{-3}$ and Eq. (4a) becomes

$$\sigma_z = 826.70 \sqrt{\frac{\eta}{V_{RF}}} . \quad (7)$$

A 1-cm bunch then requires

$$\frac{\eta}{V_{RF}} = 1.4632 \times 10^{-10} . \quad (8)$$

Corresponding parameters are summarized in Table II below.

Table II
Typical Parameter Combinations for Case 2
(to produce a 1 cm bunch length)

V_{RF} (MV)	η
5	7.30×10^{-4}
10	1.46×10^{-3}
15	2.19×10^{-3}
20	2.93×10^{-3}

For values of the momentum compaction factor comparable to those for Case 1 (see Table I), an eight times higher voltage is required to produce the 1-cm bunch.

BUNCH LENGTHENING

In the previous section, we examined the constraints on momentum compaction factor associated with the requirement to maintain a natural bunch length of 1 cm. Here we consider the possibility of bunch lengthening (and widening) driven by the longitudinal broadband impedance of the ring, i.e., we consider the effects of the longitudinal microwave instability. This instability will cause an increase in the bunch length and momentum spread of a beam bunch if the single-bunch current exceeds a threshold value given by:

$$I_b = \frac{\sqrt{2\pi} |\eta| (E/e) \sigma_{p0}^2 \sigma_z}{R \left| \frac{Z}{n} \right|_{eff}} \quad (9)$$

where the effective broadband impedance is defined as

$$\frac{|Z|}{n_{eff}} \equiv \frac{|Z|}{n_0} \left(\frac{\sigma_l}{b} \right)^{1.68} \quad (10)$$

We take the low-frequency broadband impedance, $|Z|/n_0$, to be 1.5Ω and, for a beam pipe radius of $b = 5$ cm, a 1-cm bunch sees an effective impedance of about 0.1Ω . Substituting numerical values into Eq. (9) we obtain

$$I_b = 2.22 \times 10^6 \eta \sigma_{p0}^2 \quad (11)$$

Case 1

Here we take $\sigma_{p0} = 3.5 \times 10^{-4}$ so, from Eq. (11), $I_b = 0.272\eta$. Thus, to reach the required single-bunch current of 1.28 mA (2.23 A in 1746 bunches) requires $\eta \geq 4.71 \times 10^{-3}$, i.e., about double the momentum compaction factor of the present PEP ring. As can be seen from Table I, however, this value of η still leads to a reasonable value (4 MV) for the RF voltage needed to maintain a 1-cm bunch at the longitudinal microwave instability threshold. If it were necessary or desirable to run the low-energy ring at its maximum design current of 3 A (still with 1746 bunches), the required single-bunch current would increase to 1.7 mA, corresponding to $\eta \geq 6.32 \times 10^{-3}$. In this case, a higher voltage of 5.3 MV would be needed to keep the bunch length to 1 cm.

Case 2

If the energy transparency guidelines are followed, the natural momentum spread increases to 1×10^{-3} , in which case Eq. (11) gives $I_b = 2.22\eta$. The nominal single-bunch current of 1.28 mA means that $\eta \geq 5.8 \times 10^{-4}$ is needed. As can be seen from Table II, this

is not a significant constraint in terms of RF parameters, again corresponding to $V_{RF} = 4$ MV. At the maximum design current of 3 A, the single-bunch current increases to 1.7 mA, and the momentum compaction factor must increase to $\eta \geq 7.7 \times 10^{-4}$, or $V_{RF} = 5.3$ MV for a 1-cm bunch.

The fact that the same RF voltage is needed for both Cases 1 and 2 is a result of Eq. (4a). For a given bunch length, the ratio V_{RF}/η is proportional to the square of σ_{p0} . Thus, in Eq. (11) the product $\eta\sigma_{p0}^2$ depends only on V_{RF} .

TRANSVERSE MODE COUPLING

The transverse mode-coupling instability limits the maximum single-bunch current in the present PEP ring, and is likely to be the limiting factor for the B factory low-energy ring as well. The scaling of the threshold current for this instability goes as

$$I_b \propto \frac{(E/e) v_s}{\langle \text{Im}(Z_{\perp}) \beta_{\perp} \rangle} \quad (12)$$

At PEP, a threshold current of 8.4 mA was obtained under conditions where $E/e = 14$ GV, $v_s = 0.046$, $Z_{\perp} = 0.8$ M Ω /m, and $\beta_{\perp} = 87$ m. For the low-energy ring we have $E/e = 3.1$ GV, $Z_{\perp} = 0.4$ M Ω /m, and $\beta_{\perp} = 20$ m. Thus, we expect the threshold current to scale as

$$\begin{aligned} I_b &= \frac{3.1 v_s}{(0.4)(20)} \cdot \frac{(0.8)(87)}{(14.5)(0.046)} 8.4 \text{ mA} \\ &= 339.65 v_s \text{ mA} \end{aligned} \quad (13)$$

To reach the required single-bunch current of 1.28 mA, a synchrotron tune of $v_s = 3.8 \times 10^{-3}$ is needed. To reach the maximum design current of 1.7 mA per bunch, $v_s = 5.0 \times 10^{-3}$ is needed.

Given the "fatal" nature of this instability, it is prudent here to apply a safety margin, say a factor of 2–3. Thus, we should choose parameters that will give a synchrotron tune of at least $\nu_s = 0.01$. In terms of the ring and RF parameters, the synchrotron tune is given by Eq. (3). Making the same approximations and choosing the same parameters as were used to obtain Eq. (4), we can rewrite Eq. (3) for a synchrotron tune of $\nu_s \geq 0.01$ as

$$\sqrt{V_{RF}\eta} \geq 23.6 . \quad (14)$$

Even at the lowest acceptable voltage of 4 MV considered above for Case 2 (which has the lowest momentum compaction factor), the left hand side of Eq. (14) does not drop below 48. Thus, essentially any RF voltage capable of producing a 1-cm bunch gives a synchrotron tune sufficient to avoid a limitation from the transverse mode-coupling instability.

MOMENTUM ACCEPTANCE

An acceptable RF bucket height is required to ensure adequate beam lifetime. In principle, there are two loss processes to consider—quantum excitation and Touschek scattering. In practice, the Touschek lifetime is not of concern for an RF bucket height beyond about 0.5%, as indicated in Fig. 1. For adequate quantum lifetime, it is prudent to maintain a bucket height of $10\sigma_{p0}$ or more. For Case 1 ($\sigma_{p0} = 3.5 \times 10^{-4}$), this means $\Delta p_{RF} = 0.0035$, whereas Case 2 ($\sigma_{p0} = 1.0 \times 10^{-3}$) requires $\Delta p_{RF} = 0.01$. Again taking the limit $\cos \phi_s \rightarrow -1$, we can write the RF bucket height as

$$\Delta p_{RF} \approx \left[\frac{2V_{RF}}{\pi h \eta (E/e)} \right]^{1/2} . \quad (15)$$

Substituting the appropriate ring parameters, we can rewrite Eq. (15) as

$$\Delta p_{RF} = 2.43 \times 10^{-7} \sqrt{\frac{V_{RF}}{\eta}} \quad (15a)$$

Case 1

From Eq. (5) we obtain immediately (for a 1-cm bunch) that

$$\sqrt{\frac{V_{RF}}{\eta}} = 2.8934 \times 10^4 \quad (16)$$

so that $\Delta p_{RF} = 0.7\%$ always. This will give a Touschek lifetime (see Fig. 1 below) of about 20 hours, which is not of consequence because the beam lifetime from gas scattering is only about 4 hours. In terms of quantum lifetime, this voltage corresponds to a bucket height of $20\sigma_{p0}$, which is more than adequate. We conclude, therefore, that the momentum acceptance under these conditions is of little concern.

Case 2

From Eq. (7), a 1-cm bunch has

$$\sqrt{\frac{V_{RF}}{\eta}} = 8.267 \times 10^4 \quad (17)$$

giving $\Delta p_{RF} = 2.0\%$ always. The Touschek lifetime corresponding to this acceptance (see Fig. 1) is about 200 hours, and the quantum lifetime for a bucket height of $20\sigma_{p0}$ is, as mentioned, more than sufficient. Thus, the momentum acceptance is not of concern.

TUNE MODULATION

It has been discussed by Myers,² and confirmed by Chin³ for the B factory parameter regime, that it is of benefit for the beam-beam interaction if the tune modulation at the interaction point (IP) is equal in the two rings, that is, we want

$$\left(\frac{\nu_s \sigma_t}{\beta^*} \right)_+ = \left(\frac{\nu_s \sigma_t}{\beta^*} \right)_- \quad (18)$$

where the subscripts + and – refer to the positron and electron ring, respectively. From Eq. (2), however, we see that the two factors in the numerators of Eq. (18) are not independent parameters but are constrained to satisfy the relationship

$$\nu_s \sigma_t = \eta R \sigma_{p0} . \quad (19)$$

Case 1

For Case 1, where we have not tried to satisfy the other energy transparency matching conditions, it is illogical to subsequently attempt to match the tune modulations, but we will nevertheless consider this possibility here. Substituting Eq. (19) into Eq. (18), and

eliminating the common radius value, we can rewrite the tune modulation condition as

$$\left(\frac{\eta \sigma_{p0}}{\beta^*} \right)_+ = \left(\frac{\eta \sigma_{p0}}{\beta^*} \right)_- \quad (20)$$

For the high-energy ring, the parameters are $\eta_- = 0.00245$, $\sigma_{p0} = 6.1 \times 10^{-4}$, and $\beta^* = 3$ cm. The low-energy ring has $\sigma_{p0} = 3.5 \times 10^{-4}$, and $\beta^* = 1.5$ cm. To have equal tune modulation, then, requires $\eta_+ = 0.87\eta_- = 2.13 \times 10^{-3}$. As noted earlier, for the Case 1 parameters the minimum η value that will avoid bunch lengthening (at 1.28 mA) is 4.71×10^{-3} , so some bunch lengthening would be expected under the conditions corresponding to matching the tune modulations.

Case 2

In this case, the low-energy ring has $\sigma_{p0} = 1 \times 10^{-3}$, and equal tune modulation requires $\eta_+ = 0.31\eta_- = 7.47 \times 10^{-4}$. In contrast to the situation for the Case 1 parameters, this η value is compatible with the bunch lengthening constraints (see earlier discussion). In a previous design of the low-energy ring for the APIARY-IV parameters (see "New Red Book"⁴), a value of $\eta_+ = 1.15 \times 10^{-3}$ was used, so the tune modulation was mismatched by about 50%. The calculated luminosity for a beam-beam parameter of $\xi = 0.03$ did not significantly degrade, which gives a measure of the sensitivity to this effect. Nonetheless, the ability to achieve this matching condition is a potential benefit of the Case 2 scenario.

EMITTANCE ADJUSTMENT

From Sands,⁵ we can write the natural emittance of a separated-function lattice as

$$\epsilon_0 \approx \frac{C_q \alpha R \gamma_0^2}{J_x \rho v_x} \quad (21)$$

Thus, the emittance will scale as $\epsilon_0 \propto \alpha/v_x$. For a typical FODO lattice, the momentum compaction factor scales roughly as

$$\alpha \propto \frac{1}{v_x^2} \quad (22)$$

so the emittance varies inversely as the cube of the horizontal betatron tune. This scaling however, assumes that the lattice structure is left untouched. In reality, it is possible to adjust the value of the momentum compaction factor independently of the emittance. For example, a region of negative dispersion can be added to the ring to reduce the momentum compaction factor, or the emittance can be adjusted up or down by means of suitably chosen wigglers. Thus, the lattice emittance does not greatly constrain the value of α , *provided that this independence is designed into the lattice from the beginning.*

SUMMARY

In this paper we have explored those issues that potentially reflect upon the choice of momentum compaction factor for the APIARY low-energy ring. Two scenarios were considered. In Case 1, we used a low-energy ring lattice with $\rho = 58$ m and no wigglers; in this circumstance the natural momentum spread of the beam will be small. Case 2 corresponds to a similar lattice but with wigglers whose strength has been chosen to give damping times in the low-energy ring the same as those in the high-energy ring, in an attempt to achieve energy transparent conditions; here the natural momentum spread of the low-energy ring increases roughly threefold compared with that in Case 1.

To store the nominal single-bunch current of 1.28 mA without bunch lengthening, a voltage of 4 MV is needed in either case, i.e., the voltage requirement is essentially independent of the natural momentum spread of the ring. Reaching the maximum single-bunch current of 1.7 mA (corresponding to 3 A in 1746 bunches) requires a voltage

of 5.3 MV in either case. At these voltages, the synchrotron tune is high enough to avoid any problems from the transverse mode-coupling instability.

Due to the enhanced longitudinal stability arising from an increased momentum spread, a lower value for the momentum compaction factor is sufficient to ensure stability in Case 2. This is of benefit in matching the tune modulation in the low-energy ring to that in the high-energy ring. If the momentum spread of the lattice is low, as in Case 1 (where no wigglers are used), it would not be possible to match the tune modulation without suffering some bunch lengthening. In this case, however, the damping decrements are very different, so the rationale for doing the matching is probably lost anyway.

Touschek lifetime considerations favor the Case 2 scenario, but this is not a dominant concern in making a parameter choice.

It presently seems best to aim for a low-energy ring with a large momentum spread ($\sigma_{p0} \approx 1 \times 10^{-3}$) and a low momentum compaction factor ($\alpha \approx 1 \times 10^{-3}$). This choice appears to permit the best flexibility in matching parameters of the two rings. When the range of parameter variation in the high-energy ring is finalized, the issues here should be revisited.

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