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Contaminant Transport from an Array of Sources

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Abstract

We show analytic solutions to the problem of contaminant dispersion from an array of point sources in a waste disposal site. These solutions are for waste sources in a fluid-saturated porous medium, and may be for isotropic or anisotropic dispersion. The solutions are illustrated through isopleths of contaminants for a planar array of point sources perpendicular to ground-water flow. The concentration fields several meters away from this plane can be approximated by equivalent plane sources.

Introduction

Nuclear waste in geologic repositories and hazardous materials in disposal sites will be emplaced in thousands of individual containers. In evaluating contaminant transport from such facilities, will it be necessary to consider each individual waste source? In this paper we compare the space-time-dependent concentration field predicted for an array of discrete sources with the concentration field predicted for an infinite-plane source. We will develop solutions for an array source by superpositioning solutions for individual point sources.

Analytic Solutions - Single Point Sources

For a single point source in an infinite porous medium, the governing equation for the dispersion of a contaminant is

$$K\frac{\partial c}{\partial t} + u_1\frac{\partial c}{\partial X_1} + u_2\frac{\partial c}{\partial X_2} + u_3\frac{\partial c}{\partial X_3} = \frac{\partial}{\partial X_1}\hat{D}_1(\frac{\partial c}{\partial X_1}) + \frac{\partial}{\partial X_2}\hat{D}_2(\frac{\partial c}{\partial X_2}) + \frac{\partial}{\partial X_3}\hat{D}_3(\frac{\partial c}{\partial X_3}) - \lambda K c,$$

$$X_i \in D_\infty, \quad i = 1,2,3, \qquad t > 0, \tag{1}$$

where the Cartesian coordinates were labeled X_1, X_2, X_3, c is the contaminant concentration, $\hat{D}_i$ the dispersion coefficient in the i^{th} direction, K is the retardation coefficient of the contaminant and D_∞ the unbounded space. Some solutions to this problem, without limit on the velocity field u_1, u_2, u_3, and for radioactive chains of arbitary length, were obtained by P. L. Chambré (1985).

The solution, for an uniform flow along the X_1 axis with a pore velocity of u is

$$c(X_1, X_2, X_3, t) = \int_0^t \frac{\dot{M}(\tau)e^{-\lambda(t-\tau)}}{\epsilon K(4\pi D(t-\tau))^{3/2}} exp\{\frac{-1}{4(t-\tau)}[\frac{x_1 - u(t-\tau)^2}{D_1} + \frac{x_2^2}{D_2} + \frac{x_3^2}{D_3}]\}d\tau (2)$$

where $\dot{M}(\tau)$= Contaminant input rate at time τ $[M/T]$
D_i = Normalized disperson coefficient in the X_i direction, $D_i = \hat{D}_i/K$, $[L^2/T]$
u = Normalized ground-water pore velocity in the X_1 direction, $u = u_1/K$, $[L/T]$
λ = Decay constant $[T^{-1}]$
ϵ = Porosity
$D = (D_1 D_2 D_3)^{1/3}$

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The dispersion coefficients and the ground-water velocity have been normalized by dividing them by the retardation coefficient of the contaminant.

If the rate of contaminant input is temporally constant, the solution in (2) can be simplified by the substitution of a constant input rate $\dot{M}$, and integrated to give

$$c(X_1,X_2,X_3,t) = \frac{\dot{M}e^{x_1u/2D_1}}{8\pi D^{3/2}[x_1^2/D_1+x_2^2/D_2+x_3^2/D_3]^{1/2}}$$

$$\Bigg[exp - \sqrt{(x_1^2/D_1+x_2^2/D_2+x_3^2/D_3)(\lambda+\frac{u^2}{4D_1})}\cdot$$

$$\{2-erfc\Big(\sqrt{(\lambda+\frac{u^2}{4D_1})t}-\sqrt{(x_1^2/D_1+x_2^2/D_2+x_3^2/D_3)(\frac{1}{4t})}\Big)\}$$

$$+exp\sqrt{(x_1^2/D_1+x_2^2/D_2+x_3^2/D_3)(\lambda+\frac{u^2}{4D_1})}\cdot$$

$$erfc\Big(\sqrt{(\lambda+\frac{u^2}{4D_1})t}+\sqrt{(x_1^2/D_1+x_2^2/D_2+x_3^2/D_3)(\frac{1}{4t})}\Big)\Bigg] \tag{3}$$

Analytic Solutions - Array Sources

For an array of $W \times Y \times Z$ point sources, the contaminant concentration field resulting from the array is given by

$$c^a(X_1,X_2,X_3,t) = \frac{\dot{M}}{8\pi\epsilon K D^{3/2}}\sum_{w=1}^{W}\sum_{y=1}^{Y}\sum_{z=1}^{Z}\frac{e^{(x_1-x_1^w)u/2D_1}}{[\frac{(x_1-x_1^w)^2}{D_1}+\frac{(x_2-x_2^y)^2}{D_2}+\frac{(x_3-x_3^z)^2}{D_3}]^{1/2}}\cdot$$

$$\Bigg[exp-\sqrt{(\frac{(x_1-x_1^w)^2}{D_1}+\frac{(x_2-x_2^y)^2}{D_2}+\frac{(x_3-x_3^z)^2}{D_3})(\lambda+\frac{u^2}{4D_1})}\cdot$$

$$\{2-erfc\Big(\sqrt{(\lambda+\frac{u^2}{4D_1})t}-\sqrt{(\frac{(x_1-x_1^w)^2}{D_1}+\frac{(x_2-x_2^y)^2}{D_2}+\frac{(x_3-x_3^z)^2}{D_3})(\frac{1}{4t})}\Big)\}$$

$$+exp\sqrt{(\frac{(x_1-x_1^w)^2}{D_1}+\frac{(x_2-x_2^y)^2}{D_2}+\frac{(x_3-x_3^z)^2}{D_3})(\lambda+\frac{u^2}{4D_1})}\cdot$$

$$erfc\Big(\sqrt{(\lambda+\frac{u^2}{4D_1})t}+\sqrt{(\frac{(x_1-x_1^w)^2}{D_1}+\frac{(x_2-x_2^y)^2}{D_2}+\frac{(x_3-x_3^z)^2}{D_3})(\frac{1}{4t})}\Big)\Bigg] \tag{4}$$

where c^a = concentration from the array source.
The location of the individual point sources in the array is given by

$$x_1^w = d_1(w-\frac{W+1}{2}), \qquad w=1,\ldots,W$$
$$x_2^y = d_2(y-\frac{Y+1}{2}), \qquad y=1,\ldots,Y$$
$$x_3^z = d_3(z-\frac{Z+1}{2}), \qquad z=1,\ldots,Z$$

where d_i are the separation or pitches along the i^{th} coordinate.

Numerical Illustrations

We present numerical illustrations from a 3x3 planar array of point sources, perpendicular to the flow of ground water, as shown in Figure 1.

In these calculations we assumed

- Each point source is of equal and time-invariant strength.
- The center of the array is at the origin of the axes.
- The constant source strength was derived from a separate model based on solubility-limited dissolution of the contaminant from the source. The details are given in [P. L. Chambré, *et al.*, 1987].
- Ground-water velocity is along X_1 only and is constant.
- The dispersion coefficients are constant in space and time. This is not a requirement of the solutions. Actually, in the solutions as stated above, the dispersion coefficients can be functions of velocity, although not of position.

The separations between sources have been chosen arbitarily as

$$d_2 = 3.7\,\text{meters}$$
$$d_3 = 36.7\,\text{meters}$$

The other parameter values used in the calculation are

$$D_i = 50\,\text{m}^2/\text{yr} \text{ or } 5\,\text{m}^2/\text{yr}, \text{ as stated}$$

$$u = 1\,\text{m/yr}$$
$$\dot{M} = 0.48\,\text{g/yr}$$
$$\lambda = 0$$
$$K = 1$$
$$\epsilon = 0.05$$

Figure 2 shows a contaminant plume from the array source of Figure 1, after *local steady-state* has been reached. This is of the order of a thousand years after the start of contaminant dispersion. In this case the transverse dispersion coefficients are one-tenth the longitudinal dispersion coefficient. It can be seen that the plumes from all nine point sources have merged into an overall plume, and that this plume has moved downstream.

The important question is when can simpler mathematical models of array sources give equally valid predictions? Figure 3 shows the comparison between two sets of predictions. The ordinate is the steady-state contaminant concentration along the X_1 axis predicted for the discrete array sources, normalized to the concentration predicted for an infinite-plane source of the same areal dissolution rate, plotted as a function of a distance parameter $\theta = \sqrt{zD_T/v}$. When the ratio is unity, the two models give identical predictions. For a value of the distance parameter of approximately ten meters, the ratio is above unity and the detailed array-source model should be used. Beyond this region, the infinite-plane source model either gives identical predictions, or it overestimates, conservatively. Where the concentration ratio becomes less than unity with increasing θ, the concentration field can be accurately predicted by replacing the discrete-source array by an equivalent finite plane source.

We next compare the predictions of the detailed array source model with those of a single point source at the origin. This single point source has the strength of all the point sources in the array combined. The results are shown in Figure 4. Close to the center of the array plane and thus near the single equivalent point source, the strength of the single point source overwhelms the predictions for the array of sources. At a distance parameter of about 100 meters and greater, transverse dispersion has reduced the prediction for the single equivalent point source to that for the array of sources. For the values of velocity and dispersion coefficients used, a distance parameter of 100 corresponds to a downstream distance of 10,000 meters. Thus, for predicting contaminant concentrations at large distances, a single point source can replace the array of sources.

Conclusion

Analytic solutions for contaminant dispersion in ground water from an array of point sources in a porous medium are given and illustrated. The numerical illustrations indicate that for distances tens of meters downstream from the sources simpler plane-source models might give equally satisfactory predictions.

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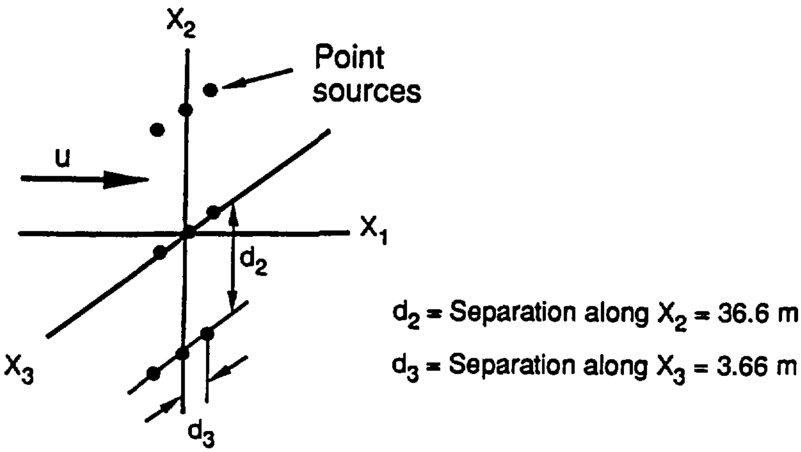

Figure 1 Array of Point Sources in Porous Media

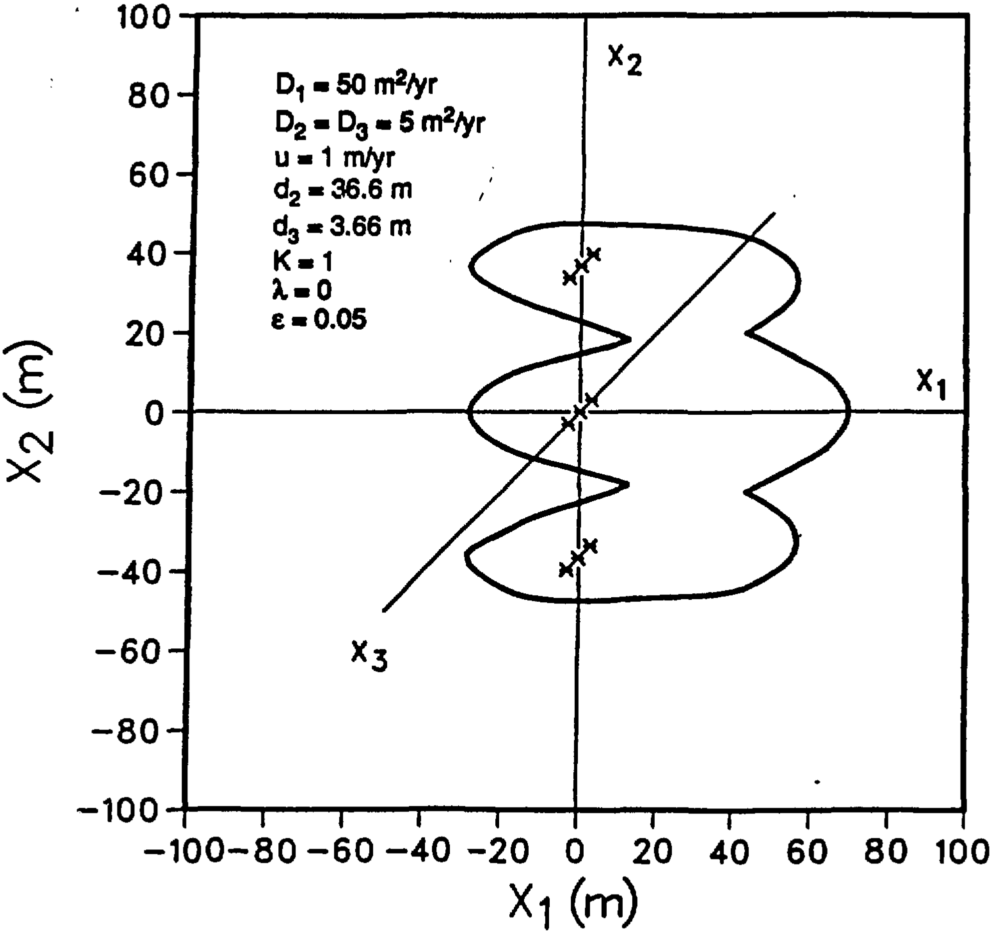

Figure 2 Isopleth of a contaminant at concentration 0.01 g/m^3 for steady-state, anisotropic dispersion for the array of point sources in Figure 1

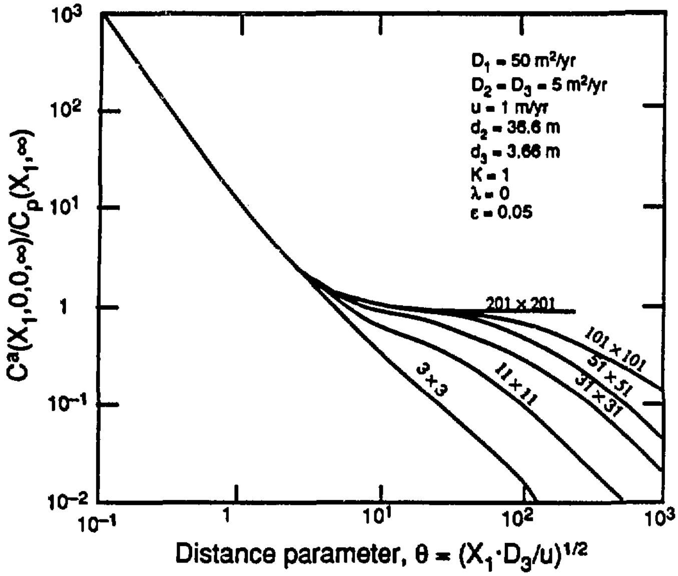

Figure 3 Comparison of concentration from array model with concentration from infinite-plane model (at steady state, along X_1 axis, anisotropic dispersion)

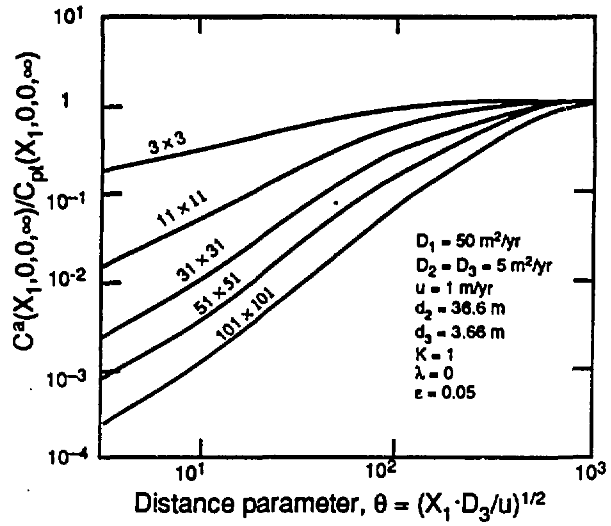

Figure 4 Comparison of concentration from array model with concentration from a single equivalent point source (at steady state, along X_1 axis)