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MAGNETOHYDRODYNAMIC BALLOONING INSTABILITIES EXCITED
BY ENERGETIC TRAPPED PARTICLES

By

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ABSTRACT

A new branch of magnetohydrodynamic ballooning modes is shown to be destabilized by energetic trapped particles. Both the real frequencies and growth rates of the instabilities are comparable to the trapped-particle precession frequencies. The theoretical results are also shown to be consistent with the high-frequency (~ 100 kHz) oscillations observed during the high-power beam-injection experiments in PDX.

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I. INTRODUCTION

The influence of energetic particles or particle beams on magnetohydrodynamic (MHD) ballooning and interchange modes has now been an intensive field of research for several years.¹⁻⁸ Although the first expectation was that the energetic particles should be destabilizing, the results turned out to be a very weak influence or a significant stabilizing effect.¹⁻⁸ The main reasons for the stabilizing effect in the regime $\omega \ll \omega_{dh}$, where ω_{dh} is the energetic particle curvature drift frequency, are a stabilizing finite Larmor radius (FLR) type term⁵ that enters as $k_{\perp}^{-1}$ and stabilizing drift resonance effects.⁶ The injection of energetic trapped particles has actually been suggested as a way of reaching the second stability region for ballooning modes in tokamaks.⁸ The first indication of a destabilizing effect of energetic particles was in connection with high-power perpendicular neutral injection on the PDX device.⁹ In these experiments, "fishbone" oscillations, rotating toroidally with a frequency comparable to the precession frequency of the energetic trapped beam ions, were observed. These oscillations were later explained¹⁰ as a trapped particle-induced new branch of the internal kink mode with a frequency of the order of the trapped particle precession frequency. Thus, although the energetic particles may have a stabilizing influence on the usual magnetohydrodynamic (MHD) mode branch with $\omega \ll \omega_{dh}$, they may introduce a new unstable branch at $\omega \sim \omega_{dh}$ and a similar situation for ballooning modes could be expected.

In the present work we study the influence of energetic trapped particles on ballooning modes in the frequency range $\omega \sim \bar{\omega}_{dh}$ where $\bar{\omega}_{dh}$ is the precession frequency, i.e., the bounce averaged curvature drift frequency, using the same formalism as in Ref. 10 but assuming now radially localized modes with $n \gg 1$. Here, n is the toroidal mode number. We arrive at a dispersion

relation of the similar type as in Ref. 10. The excitation of this kinetic instability is strongest near MHD marginally stable situations in either the first or the second stability region. Finally, we discuss qualitatively how this trapped particle-induced MHD ballooning instability could explain the high-frequency (~100 kHz) MHD-type oscillations also observed in the beam-heated PDX.¹¹

II. ANALYTICAL FORMULATION

We start by considering the energetic particle dynamics. Describing the perturbed magnetic field by two scalars, $\delta A_{\parallel}$ and $\delta B_{\parallel}$, where $\delta B_{\parallel}$ is due to compression and assuming the unperturbed distribution function to depend on the particle constants of motion $E = (1/2)v^2$ and $\mu = (1/2)(v_{\perp}^2/B)$ where v is particle velocity and $\perp$ denotes perpendicular to the unperturbed magnetic field B , we may write the gyrokinetic equation for perturbations as^{12,13}

$$\delta F = \frac{1}{m} \left(\delta \phi \frac{\partial}{\partial E} - \frac{\nu}{n} \frac{\delta B_{\parallel}}{c} \frac{\partial}{\partial \mu} \right) F_0 + \delta H e^{iL} \quad (1)$$

where

$$[\nu_{\parallel} \hat{e}_{\parallel} - \hat{v} - i(\omega - \omega_d)] \delta H = i \frac{q}{m} Q \delta \psi$$

$$Q = \left(\omega \frac{\partial}{\partial E} + \hat{\omega}_* \right) F_0$$

and

$$\delta \psi = \left(\delta \phi - \frac{v_{\parallel}}{c} \delta A_{\parallel} \right) + \frac{v_{\perp}^2}{2n} \frac{\delta B_{\parallel}}{c}.$$

Here, we have taken the small Larmor radius limit, ϕ is the electrostatic potential, q and m are particle charge and mass, Ω_c is the cyclotron frequency, F_0 and δF are the unperturbed and perturbed parts of the distribution function, $L = (\vec{v} \times \hat{e}_\parallel) \cdot \vec{k} \Omega_c$, $\vec{k}$ is the wave vector, $\hat{e}_\parallel = (\vec{B}/B)$, $\hat{\omega}_* = (-i/\Omega_c)(\hat{e}_\parallel \times \vec{\nabla} \ln F_0) \cdot \vec{\nabla}$, and $\vec{\nabla}$ operates only on the perturbation. In order to solve Eq. (1) we must introduce suitable ordering relations. For the derivation of an eigenvalue equation we will assume that the inverse aspect ratio is small $\epsilon = a/R < 1$ and the core β , β_c , is of order ϵ . Here, $\beta \equiv$ plasma pressure/magnetic pressure and the core plasma is taken to be Maxwellian. For the hot energetic particle β , β_h , we assume the formal ordering $\beta_h \sim O(\epsilon^2)$. We furthermore assume that $\omega \sim \bar{\omega}_{dh} \gg \omega_{*c}$ where ω_{*c} is the diamagnetic drift of the core plasma and that $\bar{\omega}_{dh} < \omega_{th}$, ω_{bh} where ω_{th} and ω_{bh} are the transit and bounce frequencies of hot particles, respectively. Note that the condition $\bar{\omega}_{dh} < \omega_{bh}$ is equivalent to the condition $k_\perp \rho_h < \epsilon^{1/2}$. We further assume $T_c/T_h \sim \epsilon^2$ leading to $\omega_{*c}/\bar{\omega}_{dh} \sim \epsilon$ and $n_h/n_c \sim \epsilon^3$. Since for electromagnetic modes typically $\delta A_\parallel \sim \phi k_\perp c/\omega$, we find that this ordering means that the terms $v_\parallel \hat{e}_\parallel \cdot \vec{\nabla} \delta H$ and $v_\parallel \delta A_\parallel / c$ dominate in Eq. (1). It is then convenient to write

$$\delta H_h = \hat{\delta H}_h + \delta G_h, \quad (2)$$

where

$$\hat{\delta H}_h = -\frac{e}{m\omega} Q_h \hat{\psi}$$

and $\hat{\psi}$ is defined as

$$\hat{e}_{\parallel} \cdot \hat{\nabla} \hat{\psi} = i \frac{\omega}{c} \delta A_{\parallel} \quad .$$

For the circulating hot particles, the dominance of $\hat{\delta H}$ will lead to an adiabatic response without drift resonances. Since $\beta_h/\beta_c \sim \epsilon$, the influence of circulating hot particles is thus negligible. For the trapped hot particles, on the other hand, the bounce average will average out the bounce motion $v_{\parallel}$ and the curvature resonance $(\omega - \omega_d)$ term will become important terms. Performing bounce averaging, we obtain

$$\delta G_{h,t} = \frac{1}{\omega - \bar{\omega}_{dh}} \frac{e}{m} Q_h \left[\left(1 - \frac{\omega_{dh}}{\omega}\right) \hat{\psi} - \delta \phi - \frac{v_{\perp}^2}{2\Omega_c} \frac{\delta B_{\parallel}}{c} \right] \quad (3)$$

where the bounce average $\bar{\cdot}$ is defined as

$$\bar{A} = \frac{[\oint (A/|v_{\parallel}|) d\ell]}{[\oint (d\ell/|v_{\parallel}|)]} \quad .$$

Since $E_{\parallel} = -\hat{e}_{\parallel} \cdot \hat{\nabla} \delta \phi + i(\omega/c) \delta A_{\parallel}$, we find that in the MHD limit when $\delta E_{\parallel} = 0$, we have $\hat{\psi} = \delta \phi$. Since in our present ordering $k^2 \rho^2 \ll 1$ and $\omega \ll \omega_{ci}$ for the core plasma and since the trapped particles do not generate a parallel electric field, we may use this approximation here (see Appendix). Using the fact that the leading order drift velocity is the $\delta \vec{E} \times \vec{B}$ drift, we have from Ohm's law

$$\delta \vec{E} + \frac{1}{c} (\vec{u} \times \vec{B}) = 0 \quad (4)$$

and

$$\frac{\partial \delta B_{\parallel}}{\partial t} = \hat{e}_{\parallel} \cdot [\hat{\nabla} \times (\vec{u} \times \vec{B})] = -\vec{B} \cdot \hat{\nabla} \cdot \vec{u}_{\perp} - \vec{B} \cdot \vec{\kappa} \cdot \vec{u}_{\perp} - (\vec{u}_{\perp} \cdot \hat{\nabla}) B \quad (5)$$

where

$$\vec{k} = (\hat{e}_{\parallel} \cdot \vec{\nabla}) \hat{e}_{\parallel}.$$

Introducing now

$$\vec{u}_{\perp} = \frac{c}{B} (\hat{e}_{\parallel} \times \vec{\nabla} \delta \phi)$$

and

$$\omega_d = \frac{1}{i} \vec{v}_d \cdot \vec{\nabla}$$

where

$$\vec{v}_d = \frac{1}{\Omega_c} \hat{e}_{\parallel} \times \left(\frac{1}{2} v_{\perp}^2 \vec{\nabla} \ln B + v_{\parallel}^2 \vec{k} \right),$$

we obtain

$$\delta \bar{G}_{h,t} = \frac{i}{\omega(\omega - \bar{\omega}_{dh})} \frac{e}{m \Omega_c} Q_h \overline{[(v_{\parallel}^2 + (1/2)v_{\perp}^2)(\hat{e}_{\parallel} \times \vec{k}) \cdot \vec{\nabla} \delta \phi]} \quad (6)$$

For $\beta_h \sim 0(\epsilon^2)$ it will furthermore be possible to neglect the $\delta B_{\parallel}$ term for hot particles in Eq. (1). We then find that the first part of $\delta \bar{B}$ cancels the first part of Eq. (1) and we obtain

$$\delta F_{h,t} = -i \frac{c}{\omega B} (\hat{e}_{\parallel} \times \vec{\nabla} \delta \phi) \cdot \vec{\nabla} F_0 + \delta \bar{G}_{h,t} \quad (7)$$

where the first part is due to $\vec{E} \times \vec{B}$ convection. We now multiply Eq. (7) by

v^2 and integrate over velocity space to obtain the pressure perturbation. We then introduce the variables $E = v^2/2$ and $\alpha = u/E$ and obtain the pressure tensor

$$\begin{aligned} \hat{\delta p}_{h,t} = & -\frac{ic}{\omega B} (\hat{e}_\parallel \times \nabla \delta \phi) \cdot \nabla [p_\parallel \mathbf{I} + (p_\parallel - p_\perp) \hat{e}_\parallel \hat{e}_\parallel]_h \\ & + \hat{\delta p}_\perp \mathbf{I} + (\hat{\delta p}_\parallel - \hat{\delta p}_\perp) \hat{e}_\parallel \hat{e}_\parallel \end{aligned} \quad (8a)$$

where

$$\left\{ \begin{array}{l} \hat{\delta p}_\perp \\ \hat{\delta p}_\parallel \end{array} \right\} = 2^{7/2} i n_{hc} \int_{B_{\max}^{-1}}^{B^{-1}} d\alpha (1-\alpha B)^{1/2} \int_0^\infty dE \frac{E^{5/2} Q_h}{\omega(\omega - \bar{\omega}_{dh})} \bar{J} \left\{ \frac{\alpha B}{2(1-\alpha B)} \right\} \quad (8b)$$

and

$$\bar{J} = \left(-\frac{1}{2} \alpha B \right) (\hat{e}_\parallel \times \nabla) \cdot \nabla \delta \phi \quad (8c)$$

This completes the kinetic derivation of the trapped particle response.

Since we are here going to consider only moderately large mode numbers and since $\omega \sim \bar{\omega}_{dh} \gg \omega_{*i}$, we may use fluid equations to describe the core plasma. For the particle drift motions we have

$$\hat{u}_{ij} = \frac{c}{B} (\hat{e}_\parallel \times \nabla \delta \phi) - \frac{c}{B n_j} \frac{\partial}{\partial t} \nabla_\perp \delta \phi + \frac{c}{\alpha_j N_j B} (\hat{e}_\parallel \times \nabla \cdot \hat{\delta p}_j) \quad (9)$$

where we may neglect the polarization drift of electrons. Using Maxwell's equations for the parallel current we have

$$\frac{\partial j_\parallel}{\partial t} = \frac{c^2}{4\pi} \hat{e}_\parallel \cdot (\Delta \nabla_\perp \hat{\psi} - \nabla \Delta_\perp \hat{\psi}) \quad (10)$$

From the quasineutrality condition, $\nabla \cdot \vec{j} = 0$, we obtain an eigenvalue equation after inserting Eq. (9) for the perpendicular part and Eq. (10) for the parallel part. The driving pressure force is obtained from the divergence of the diamagnetic flux as

$$\vec{\nabla} \cdot \left[\frac{c}{q_j B} (\hat{e}_\parallel \times \vec{\nabla} \cdot \delta \vec{p}_j) \right] = 2 \frac{c}{q_j B_0} (\hat{e}_\parallel \times \vec{k}) \cdot (\vec{\nabla} \cdot \delta \vec{p}_j) \quad .$$

For the core plasma we may now use only the convective pressure perturbation

$$\delta p_c = -i \frac{c}{\omega B} (\hat{e}_\parallel \times \vec{\nabla} \delta \phi) \cdot \vec{\nabla} p_c \quad . \quad (11)$$

From the anisotropic part of the pressure tensor of hot particles, we obtain

$$\vec{\nabla} \cdot (N \vec{v}_\pi)_h = \frac{c}{qB} (\hat{e}_\parallel \times \vec{k}) \cdot \vec{\nabla} (\delta p_\parallel - \delta p_\perp)_h$$

leading to the total hot particle pressure contribution

$$\nabla(N \vec{v}_{*h}) + \nabla \cdot (N \vec{v}_\pi)_h = \frac{c}{qB} (\hat{e}_\parallel \times \vec{k}) \cdot \vec{\nabla} (\delta p_\perp + \delta p_\parallel)_h \quad .$$

To proceed further, we assume shifted circular magnetic surfaces and introduce the well-known ballooning-mode eikonal description for the perturbed quantities, i.e.,

$$\delta \phi = \phi(r, \chi, t) \exp\{i[nS(r, \chi, \xi) - \omega t]\} \quad (12)$$

where r is the small radius, χ is the poloidal angle, ξ is the toroidal angle, and $S = \xi - \int^\chi v(\chi', r) d\chi'$ where $v(\chi, r)$ is the local χ -dependent safety factor

related to the safety factor q through $q = (1/2\pi) \oint v(\chi, r) d\chi$. In this description we have to leading order in n^{-1} ,

$$\Delta \delta \phi = -n^2 \left[\left(\frac{\partial s}{\partial r} \right)^2 + \frac{v^2}{r^2} + \frac{1}{R^2} \right] \delta \phi \quad (13)$$

where R is the large radius. Assuming a harmonic variation of v^1 with χ we then have, using the model equilibrium given in Refs. 14-16, $(\hat{e}_\parallel \times \vec{k}) \cdot \vec{k} = (k_\chi/R)g(\chi)$ where $g(\chi) = \cos \chi + (s\chi - \Delta \sin \chi) \sin \chi$, $s = (r/q)(dq/dr)$, $\Delta = \beta' q^2 R \sim O(\epsilon \beta_p)$, and $\Delta \sin \chi$ is due to the χ dependence of v .

Having now defined our geometry, we may evaluate the expression for $\bar{J}$. Recalling that $v_\parallel^2/v_\perp^2 \sim \epsilon < 1$ for trapped particles, we may take $\alpha_B = 1$ in $\bar{J}$. We then have

$$\bar{J} = \frac{ik_\chi}{2R} \frac{\chi}{g(\chi) \delta \phi} \quad (14)$$

The corresponding eigenmode equation is then given in the following form

$$\frac{d}{d\chi} r(\chi) \frac{d\phi}{d\chi} + \Omega^2 r(\chi) \phi + \Delta g(\chi) \phi + g(\chi) \widetilde{\Delta}_h \phi = 0 \quad ; \quad (15)$$

where, including an average well term (c.f. Ref. 16),

$$g(\chi) = \cos \chi + (s\chi - \Delta \sin \chi) \sin \chi - \epsilon \left(1 - \frac{1}{q^2} \right) \quad , \quad (16)$$

$$r(\chi) = 1 + s^2 \chi^2 \quad , \quad (17)$$

$$\widetilde{\Delta}_h \phi = 2^{7/2} \frac{\pi^2 q^2 m_h}{B^2} \int_{B_{\max}^{-1}}^{B^{-1}} \frac{B d\alpha}{(1-\alpha B)^{1/2}} \int_0^\infty dE \frac{E^{5/2} Q_h}{\omega - \bar{\omega}_{dh}} g(\chi) \phi \quad (18)$$

We note that Eq. (15) is a differential-integral eigenmode equation and, thus, it in general must be solved numerically. In some limiting cases, however, analytical dispersion relations may be derived and these will be discussed in the next section.

III. DERIVATION OF A DISPERSION RELATION

A. Weak shear limit

In order to be able to solve the eigenvalue equation Eq. (15), we shall now assume that $s < 1$ so that we may distinguish two separate spatial scales,¹⁴⁻¹⁶ χ and $z = s\chi$. We then write

$$\frac{d}{d\chi} = \frac{\partial}{\partial\chi} + s \frac{\partial}{\partial z}$$

and introduce the following subsidiary orderings

$$\Omega^2 \sim \Delta^4 \sim \Delta_h \sim |\varepsilon(1 - q^{-2})| \sim s^2 \quad . \quad (19)$$

We obtain to $O(1)$

$$\frac{\partial}{\partial\chi} (1 + z^2) \frac{\partial\phi_0}{\partial\chi} = 0 \quad ; \quad (20)$$

i.e., $\phi_0 = \phi_0(z)$ does not vary on the fast scale χ . To $O(\Delta)$, we have

$$(1 + z^2) \frac{\partial^2 \phi_1}{\partial\chi^2} + \Delta g_0 \phi_0 = 0$$

where g has been expanded in its appropriate orders and $g_0 = \cos \chi + z \sin \chi$.

Integrating twice, we have

$$\phi_1 = \frac{\Delta g_0}{1+z^2} \quad (21)$$

To $O(\Delta^2)$, we have

$$(1 + g^2) \frac{\partial^2 \phi_2}{\partial \chi^2} + \Delta g_0 \phi_1 + \Delta g_1 \phi_0 = 0$$

where $g_1 = -\Delta \sin^2 \chi$. Substituting g_1 , we find

$$\phi_2 = \frac{1}{4} \frac{\Delta^2 \tilde{g}_0^2}{(1+z^2)^2} \phi_0 \quad (22)$$

where $\tilde{g}_0^2$ is the second harmonic part of g_0^2 . To $O(\Delta^3)$, we obtain

$$(1 + z^2) \frac{\partial^2 \phi_3}{\partial \chi^2} + 2zs \frac{\partial \phi_1}{\partial \chi} + 2(1+z^2) \frac{\partial^2 \phi_1}{\partial z \partial \chi} + \Delta g_0 \phi_2 + \Delta g_1 \phi_1 = 0$$

Since we are going to obtain an averaged equation to $O(\Delta^4)$ we will only need the first harmonic of ϕ_3 . Since, furthermore, terms of the type $g_0(\partial \phi_1 / \partial \chi)$ will not contribute to $O(\Delta^4)$, the second term will not contribute; while from the third term only the part coming from $\partial g_0 / \partial z$ when Eq. (21) is used for ϕ_1 will contribute. Then using

$$\tilde{g}_0^2 = \frac{1}{2} \cos 2\chi (1 - z^2) + z \sin 2\chi$$

$$(g_0 \tilde{g}_0^2)_1 = \frac{1}{4} (1 + z^2) g_0$$

and

$$(g_0^3)_1 = \frac{3}{4} (1+z^2)g_0$$

where $()_1$ denotes first harmonic, we may write the relevant part of ϕ_3 as

$$\begin{aligned} \phi_3 = & \left[\frac{1}{16} \frac{\Delta^3}{(1+z^2)^2} g_0 - \frac{1}{2} \frac{\Delta^3}{(1+z^2)^2} g_0 + \frac{2s\Delta}{1+z^2} \cos \chi \right. \\ & \left. + \frac{1}{4} \frac{\Delta^3}{(1+z^2)^2} \cos \chi - z \sin \chi \right] \phi_0 \end{aligned}$$

Finally to $O(\Delta^4)$, we obtain the averaged equation

$$(1+z^2)\Omega^2\phi + s^2 \frac{d}{dz}(1+z^2)\frac{d\phi}{dz} + \Delta\langle g_0\phi_3 + g_1\phi_2 + g_3\phi_0 \rangle + \langle g_0\hat{\Delta}_h\phi_0 \rangle = 0,$$

where $g_3 = -\epsilon(1 - 1/q^2)$ and $\langle \dots \rangle \equiv \oint (d\chi/2\pi)(\dots)$. Since $\phi_0 = \phi_0(z)$, it can be taken out of the average. We may then evaluate the averages and obtain

$$(1+z^2)\left(\frac{\Omega}{s}\right)^2\phi_0 + \frac{d}{dz}(1+z^2)\frac{d\phi_0}{dz} + u_c\phi_0 + u_h\phi_0 = 0 \quad (24)$$

where

$$u_c = \frac{\Delta_c}{2s^2} \left[2 \frac{s\Delta_c - (1/4)\Delta_c^3}{1+z^2} + \frac{1}{16} \frac{\Delta_c^3}{1+z^2} + \frac{1}{4} \frac{\Delta_c^3}{1+z^2} \frac{1-z^2}{1+z^2} - \epsilon(1 - q^{-2}) \right],$$

and

$$u_h = 2^{7/2} \left(\frac{\pi q}{Bs} \right)^2 m_h \int_{1-r/R}^{1+r/R} d(\alpha B) \int_0^\infty dE E^{5/2} \frac{K_2^2}{K_b} \left(\frac{\omega}{\omega - \bar{\omega}_{dh}} \right) \left(\frac{3}{3E} + \frac{\hat{\omega}_{*h}}{\bar{\omega}_{dh}} \right) F_{oh}$$

with $K_b = \oint (d\chi/2\pi)(1-\alpha B)^{-1/2}$ and $K_2 = \oint (d\chi/2\pi) \cos \chi (1-\alpha B)^{-1/2}$. Note also that only the core plasma contributes to u_c .

Equation (24) can be transformed to the form

$$\frac{\partial^2 \psi}{\partial z^2} + w(z) \psi = 0 \quad (25a)$$

by introducing $\psi = \phi(1+z^2)^{1/2}$. We then have

$$w(z) = \frac{\Omega^2}{s^2} + \frac{u_c}{1+z^2} - \frac{1}{(1+z^2)^2} + \frac{u_h}{1+z^2} \quad (25b)$$

We now observe that $\psi \sim \exp[i(\Omega/s)|z|]$ for large $|z|$. We will now make the additional assumption that $\Omega \sim \bar{\omega}_{dh}/\omega_A < s$. This means that $(\Omega/s)^2$ is small and we may use a constant- ψ approximation in integrating $w(z)\psi$. Matching $d \ln \psi / dz$ of the inner and outer regions, we have

$$i \frac{\Omega}{s} = \int_0^\infty \left[\frac{1}{(1+z^2)^2} - \frac{u_c}{1+z^2} - \frac{u_h}{1+z^2} \right] dz \quad (26)$$

We then arrive at the following dispersion relation

$$-i\Omega + \hat{\delta W}_c + \hat{\delta W}_k = 0 \quad (27a)$$

where

$$\hat{\delta W}_c = -\frac{\pi}{4} \left[\Delta^2 \left(1 - \frac{7}{32} \frac{\Delta^2}{s} \right) - s - 2\varepsilon \frac{\Delta}{s} \left(1 - \frac{1}{2} \right) \right]_c \quad (27b)$$

and

$$\hat{\delta W}_k = \frac{-\pi s u_h}{2} \quad (27c)$$

We note that since the dispersion relation, Eq. (17a) has functional dependences similar to those of the internal kink mode,¹⁰ it, therefore, can be expected that the stability properties will also be similar to those derived for the internal kink mode.^{10,17} The major qualitative difference is that, while the internal kink mode involves global quantities with the $q = 1$ surface, the MHD ballooning interchange mode considered here involves only local surface quantities. To be more specific, we shall, as in Ref. 10, assume the following model distribution for the slowing-down beam ions

$$F_{oh} = c_o E^{-3/2} \delta(\alpha - \alpha_o) \quad \text{for } 0 \leq E \leq E_m, \quad (28)$$

where $c_o(r) = \langle p_{h,t} \rangle / (\pi K_{bo} m_h B^2)^{3/2} E_m$, $\langle p_{h,t} \rangle$ being the surface-averaged pressure of the beam ions, and $K_{bo} = K_D(\alpha = \alpha_o)$. $\hat{\omega}_k$ then becomes

$$\hat{\omega}_k = \frac{\pi q^2}{2s} \langle p_{h,t} \rangle \hat{I}_o \frac{\omega}{\omega_{dh}} \ln \left(1 - \frac{\bar{\omega}_{dm}}{\omega} \right), \quad (29)$$

where

$$\hat{I}_o = \left(\frac{1}{2K_{bo}} \right) \left[\frac{\alpha_o I'(\alpha_o) + I(\alpha_o) \hat{\omega}_{*h}}{\omega_{dh}} \right], \quad (30)$$

$$I(\alpha_o) = \left(\frac{2R}{r} \right)^{1/2} \frac{[2E(k_o^2) - K(k_o^2)]^2}{\pi K(k_o^2)}, \quad (31)$$

$$K_{bo} = \left(\frac{2R}{r} \right)^{1/2} \frac{K(k_o^2)}{\pi}, \quad (32)$$

$k_o^2 = (1 + r/R - \alpha_o B_o)R/2r$, $E(k_o^2)$ and $K(k_o^2)$ being the complete elliptic integrals, $\hat{\omega}_{*h} = (nq/r\Omega_c) d\Omega_n \langle p_{h,t} \rangle / dr$, and

$$\omega_{dh} = -nq \frac{[2E(k_o^2)/K(k_o^2)-1]}{rR\Omega_c} \quad (33)$$

Equation (29) into Eq. (27a) then leads to a threshold for instability

$$\langle \beta_{h,t} \rangle > \langle \beta_{h,t} \rangle_{crit} = \frac{\bar{\omega}_{dm}}{\omega_A} \frac{2s}{\pi^2 q^2 I_o} \quad (34a)$$

which is similar to the threshold for the internal kink instability.¹⁰ Note that within the present model for F_{oh} , there is no threshold in $\delta \hat{w}_c$. We also note that $\hat{I}_o \sim O(K_2/K_b \epsilon) \sim O(1/\epsilon)$ and, from Eq. (27b), the critical value of β for purely MHD ballooning instability is $\beta_{crit} \sim O(\epsilon s^{1/2}/q^2)$. We thus, have

$$\langle \beta_{h,t} \rangle_{crit} \sim O(s^{1/2} \bar{\omega}_{dm}/\omega_A) \beta_{crit}$$

i.e., the trapped-particle induced branch has a corresponding threshold in β lower than that of the MHD branch. For more general F_{oh} , however, there will be a threshold due to positive finite $\delta \hat{w}_c$ as a Nyquist analysis of Eq. (27a) would indicate. In particular, one finds that Eq. (34a) holds for $\delta \hat{w}_c < \bar{\omega}_{dm}/\omega_A$. Meanwhile, for $\delta \hat{w}_c > \bar{\omega}_{dm}/\omega_A$, we have approximately

$$\langle \beta_{h,t} \rangle_{crit} = \delta \hat{w}_c \frac{2s}{\pi^2 q^2 I_o} \quad (34b)$$

By combining the equations for the real and imaginary parts of Eq. (27a), we can derive the relations

$$\frac{\omega_r}{\omega_{dm}} = - \frac{\delta \hat{w}_c + (\gamma^2/\omega_r \bar{\omega}_{dm}) (\pi q^2 \langle \beta_{h,t} \rangle \hat{I}_o/2s) \ln |1 - (\bar{\omega}_{dm}/\omega)|}{(\pi q^2 \langle \beta_{h,t} \rangle \hat{I}_o/2s) \ln |1 - (\bar{\omega}_{dm}/\omega)|} \quad (35)$$

and

$$\frac{\gamma}{\bar{\omega}_{dm}} = - \frac{\delta \hat{W}_c [(\bar{\omega}_{dm}/\omega_A) - (\pi q^2 < \beta_{h,t} > \hat{I}_0 / 2s) \phi]}{[(\bar{\omega}_{dm}/\omega_A) - (\pi q^2 < \beta_{h,t} > \hat{I}_0 / 2s) \phi]^2 + [\pi q^2 < \beta_{h,t} > \hat{I}_0 / 2s \ln |1 - (\bar{\omega}_{dm}/\omega)|]^2} \quad (36)$$

where the phase ϕ is defined by $\cos \phi = \text{Re}[1 - (\bar{\omega}_{dm}/\omega)] / |1 - \bar{\omega}_{dm}/\omega|$ and $\omega = \omega_r + i\gamma$. An important parameter here is

$$A = - \frac{\delta \hat{W}_c}{(\pi q^2 < \beta_{h,t} > \hat{I}_0 / 2s)} ,$$

which is negative in the MHD stable case. At marginal stability, we find from Eq. (35a)

$$\frac{\omega_r}{\bar{\omega}_{dm}} \ln |1 - \frac{\bar{\omega}_{dm}}{\omega_r}| = A .$$

With $\omega_r < \bar{\omega}_{dm}$ this leads to

$$\frac{\bar{\omega}_{dm}}{\omega_r} = 1 + \exp\left(\frac{\bar{\omega}_{dm}}{\omega_r} A\right) \quad (37)$$

Close to MHD stability we then have $|A| \ll 1$ and $\omega_r \approx (1/2)\bar{\omega}_{dm}$. In the opposite limit, i.e., $|A| \gg 1$ we have $\omega_r \approx \bar{\omega}_{dm}$ and in general ω_r will take values between $(1/2)\bar{\omega}_{dm}$ and $\bar{\omega}_{dm}$ at marginal stability. Relaxing now the condition of marginal stability but assuming that $|A| \gg 1$, we find that Eq. (37) is modified to

$$\frac{\bar{\omega}_{dm}}{\omega} = 1 + \exp\left(\frac{\bar{\omega}_{dm}}{\omega} A\right) ,$$

i.e., $\omega \approx \bar{\omega}_{dm}$ and $\gamma \ll \omega_r$ when $|A| \gg 1$ and then $\delta \hat{W}_c \approx \pi q^2 < \beta_{h,t} > \hat{I}_0 / 2s \ln |1 - \bar{\omega}_{dm}/\omega|$.

Also, in this case $|\hat{\omega}_c| \gg |(\bar{\omega}_{dm}/\omega_A) - \phi(\pi q^2 < \beta_{h,t} > \hat{I}_O/2s)|$ and we may write Eq. (36) as

$$\frac{\gamma}{\bar{\omega}_{dm}} \approx \frac{(\bar{\omega}_{dm}/\omega) - \{\pi q^2 < \beta_{h,t} > \hat{I}_O/2s\} \phi}{\delta \bar{\omega}_c} \quad \text{when } |A| \gg 1.$$

Thus, far from MHD instability the growth rate is small. In general, we find from Eq. (36)

$$\gamma \approx \left\{ \frac{\omega_r \bar{\omega}_{dm} [A - (\omega_r/\bar{\omega}_{dm}) \ln |1 - (\bar{\omega}_{dm}/\omega)|]}{\ln |1 - (\bar{\omega}_{dm}/\omega)|} \right\}^{1/2}, \quad (38)$$

which shows that typically $\gamma \sim \omega_r \sim \bar{\omega}_{dm}$ when $A \sim 1$.

B. Variational analysis

In arriving at the dispersion relation Eq. (27a), we have used a systematic perturbation expansion for small s , Δ , and Δ_h . As indicated by analyses on the usual MHD ballooning modes, this expansion scheme is accurate only for $s \lesssim 0.2$. It is, therefore, desirable to relax the shear value to $s \lesssim 1$. At such moderate shear values, the two-scale expansion breaks down for $\chi \sim 0(1)$ and effects due to overlapping between the scales, χ and $s\chi$, become important.¹⁹ However, if we formally treat $\Delta \sim s \sim 1$ and $\Delta_h \sim \Omega \sim \epsilon < 1$, then for $\chi \sim 0(1)$, the eigenmode equation, Eq. (15), is variational in the lowest order. It, thus, suggests that we can follow Ref. 19 and employ a trial function approach to provide better estimates of $\delta \omega_{MHD}$ and $\delta \omega_K$.

To proceed, we let $\psi = (1 + s^2 \chi^2)^{1/2} \phi$ and rewrite Eq. (15) as

$$\frac{d^2 \psi}{d\chi^2} + \left[\Omega^2 - \frac{s^2}{(1 + s^2 \chi^2)^2} + \frac{\Delta g(\chi)}{1 + s^2 \chi^2} \right] \psi + \frac{g(\chi)}{(1 + s^2 \chi^2)^{1/2}} \frac{\widetilde{\Delta_h \psi}}{(1 + s^2 \chi^2)^{1/2}} = 0, \quad (39)$$

where $g(\chi) = \cos \chi + s\chi \sin \chi - \Delta/2$ and the average well is ignored here since $\Delta \sim s \sim 1$. Performing $(1/2) \int_{-\infty}^{\infty} d\chi \psi^*$ on Eq. (39), we obtain the dispersion functional $D[\psi]$ as

$$D[\psi] = \delta W_{\text{MHD}} + \delta I + \delta W_K,$$

where

$$\delta W_{\text{MHD}} = + \int d\chi \left[\left| \frac{d\psi}{d\chi} \right|^2 + \left[\frac{s^2}{(1+s^2\chi^2)} - \frac{\Delta g(\chi)}{1+s^2\chi^2} \right] |\psi|^2 \right],$$

$$\delta I = -(\Omega^2/2) \int d\chi |\psi|^2,$$

and

$$\delta W_K = 2^{7/2} \frac{\pi^3 q^2 m_h}{B^2} \int \frac{1+r/R}{1-r/R} d(\alpha B) \int_0^\infty dE E^{5/2} K_D \frac{Q_h}{\bar{\omega}_{dh} - \omega} \left| g(\chi) \psi (1+s^2\chi^2)^{-1/2} \right|^2. \quad (40)$$

Let $D = D_I + D_S$, where D_I and D_S correspond to contributions from the ideal [$|\chi| \sim O(1)$] and inertial [$|\chi| \sim O(1/|\Omega|)$] regions, respectively. For $|\chi| \sim O(1)$, δI is ignorable and, since $\delta W_K \sim O(\epsilon)$, D_I is thus variational to $O(\epsilon)$. Formally treating $\epsilon^2 < 1$ in Eq. (39), the lowest-order trial function is then given by

$$\psi_{t,I} = 1 + \tilde{B}(\chi)(\cos \chi + s\chi \sin \chi), \quad (41a)$$

where

$$\tilde{B}(\chi) = \frac{\Delta}{1+s^2\chi^2} \left[1 - \frac{s^2}{(1+s^2\chi^2)^2} - \frac{\Delta/2}{1+s^2\chi^2} \right]. \quad (41b)$$

We then have

$$D_I = D_I[\psi_{t,I}] = \delta W_{MHD}[\psi_{t,I}] + \delta W_K[\psi_{t,I}] \equiv \hat{\delta W}_{MHD} + \hat{\delta W}_K \quad (42)$$

Meanwhile, for $|\chi| \sim 0(1/\Omega)$, only the inertial and field-line bending terms are important, i.e.,

$$D_S = \int_{\chi_0}^{\infty} d\chi \left(\left| \frac{d\psi_S}{d\chi} \right|^2 - \Omega^2 |\psi_S|^2 \right) \quad (43)$$

where $i \ll \chi_0 \ll 1/|\Omega|$ and ψ_S satisfies

$$\frac{d^2 \psi_S}{d\chi^2} + \Omega^2 \psi_S = 0 \quad (44)$$

Employing the causality condition and matching to $\psi_{t,I}$, we have $\psi_S = \exp(i\Omega|\chi|)$. Thus

$$D_S[\psi_S] = \psi_S \frac{d\psi_S}{d\chi} \Big|_{\chi_0}^{\infty} = -i\Omega \quad (45)$$

Combining Eqs. (42) and (45), we then obtain, to $O(\epsilon^2)$, the following dispersion relation,

$$-i\Omega + \hat{\delta W}_{MHD} + \hat{\delta W}_K = 0 \quad (46)$$

Here, for model slowing-down beam ions, $\hat{\delta W}_K$ is of the same form as Eq. (29), but with $\hat{I}_0$ modified due to the inclusion of the oscillating part of ψ in the bounce average. This will give a modification of order 1. The MHD part may be integrated to give

$$\hat{\omega}_{\text{MHD}} = \frac{\pi}{2s} \left[\frac{1}{2} s^2 - \frac{1}{2} \Delta^2 \left[s - \frac{3}{4} s^2 - \frac{5}{4} s^3 - \frac{1}{2} \Delta^2 \left(1 + \frac{3}{2} s \right) \right] \right. \\ \left. - e^{-1/s} \Delta \left(\frac{3}{2} - s \right) \right] , \quad (47)$$

where the coefficient of $\exp(-1/s)$ is only accurate near the first stability region. Equation (47) shows that MHD stability imposes the scaling $s \sim \Delta^2$ for small s and Δ^2 and that this is replaced by $s^2 \sim 3\Delta \exp(-1/s)$ for moderate $s \leq 1$. The transition is found to occur for $s \approx 0.2$. The lowest-order coefficients in Eq. (47) are very close to those obtained in Ref. 19. The main difference is that Ref. 19 has the coefficient $3/4$ in the driving $\Delta^2 s$ term instead of $1/2$ obtained here. The result obtained here is, however, supported by more recent calculations.²⁰ The slight difference in the coefficient for Δ^4 (i.e., the second stability term) as compared to our previous perturbation analysis is due to the neglect of the second harmonic part of ψ in the variational analysis. The coefficient obtained in the perturbation calculation is very close to that obtained in Ref. 20. Note that Eq. (46) is formally the same as Eq. (27a) derived in the $s < 1$ limit. It, therefore, can be expected that, except for the appropriate replacements of $\hat{\omega}_c$ and $\langle \hat{\delta}_{h,t} \hat{i}_0 \rangle$, the stability properties would remain unchanged.

IV. CONCLUSIONS AND DISCUSSIONS

We have shown in this work that trapped energetic particles introduce a new unstable branch to the MHD ballooning modes. This trapped particle-induced ballooning instability is excited via resonances between the mode and trapped particle precessional drifts. The instability mechanism is, thus, similar to that of the "fishbone" instability.¹⁰ An analytical expression for

the dispersion relation is derived and a threshold condition in terms of $\langle \beta_{h,t} \rangle$ is obtained. Typically, both the real frequency and the growth rate of this instability are comparable to the trapped particle precession frequency.

We now consider the implications of the present theory to the high-frequency (50-150 kHz) MHD-type oscillations observed in PDX during high-power perpendicular neutral injection.¹¹ Summarizing briefly, these high-frequency oscillations have the following general features:

$$(i) f_{h.f.}/f_{f.b.} = n_{h.f.}$$

where $n_{h.f.}$ ($\approx 2-6$) is the toroidal mode number of the high-frequency oscillations, $f_{h.f.}$ and $f_{f.b.}$ are, respectively, the frequencies of the high-frequency and the $n = 1$ "fishbone" oscillations.

(ii) The oscillations tended to occur only at higher values of beam power and $\varepsilon \beta_p$, while the "fishbone" events could occur at lower such values.

(iii) The oscillations were predominantly located in the inner half core of the tokamak; approximately around $q \approx 1$.

(iv) For events where both types of oscillations were present, the high-frequency oscillations generally occurred at the beginning stage of the "fishbone."

(v) In cases where only the high-frequency oscillations were observed, sawteething generally was either absent or had small amplitude.

Since both the present theory and that of the "fishbone" instability¹⁰ would predict $\omega \sim \bar{\omega}_{dm} \approx n \bar{v}_{d\xi}$, the theoretical results are consistent with (i). Now as to (ii), it may be accounted for by noting that both the critical $\varepsilon \beta_p$ value for marginal $\hat{\delta} \hat{w}_c$ and $\langle \beta_{h,t} \rangle \approx \bar{\omega}_{dm}/\omega_A \approx n$ are higher for the large- n ballooning modes than the $n = 1$ "fishbones." Noting that the q -profile inside

the inner half core is relatively flat, (iii) is, hence, consistent with the theoretical predictions that both $(\epsilon\beta_p)_{\text{crit}}$ and $\langle\beta_{h,t}\rangle_{\text{crit}}$ take on smaller values as the shear, s , is reduced. Finally, since $q(0)$ drops during the heating, (iv) may be explained by noting that the ballooning modes could occur for $q \geq 1$, and, yet, the "fishbones" being an internal kink mode require $q(0) < 1$. That ballooning modes occur for $q \geq 1$ is also consistent with (v). All together, we feel that the present theory is, at least, qualitatively consistent with the PDX observations.

We may also anticipate that similar instabilities may be driven by alpha particles in a burning fusion plasma. The saturation of the "fishbone" instability was shown to occur through ejection of the hot trapped particles resonant with the mode,¹⁶ i.e., those responsible for exciting the mode. This ejection leads to a decrease of the hot particle beta to below the threshold. A similar mechanism, but more of a quasilinear diffusion type, will be present for the ballooning mode. Since it is only the trapped particles that are important for these kinds of instabilities, a more efficient way to reach high beta will be by injecting particles at a small angle to the magnetic field.

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APPENDIX

We will here derive an expression for the parallel electric field. As it turns out, this must be done by using kinetic equations in the region $\omega \ll k_{\parallel} v_{Te}$. Moreover, since the trapped particles do not contribute to $E_{\parallel}$ (no parallel current), we will here consider only the core Maxwellian plasma. In this case the operator $\partial/\partial E$ is replaced by $-m/T$ in Eq. (1). Then neglecting $\delta A_{\parallel}$ and with $k_{\perp} v_{\perp}/\Omega_c \ll 1$ and $k_{\parallel} v_{\parallel} \gg \omega$, we obtain

$$\delta F = -\frac{\delta \phi}{T} F_0 + \delta H \quad (A1)$$

$$\delta H = -\frac{q}{T} \frac{\omega - \omega_A}{k_{\parallel} v_{\parallel} - \omega + \omega_d} \left(\delta \phi - \frac{v_{\parallel}}{c} \delta A_{\parallel} \right) F_0 \approx -\frac{q}{T} \frac{\omega - \omega_A}{k_{\parallel} v_{\parallel}} \left(1 + \frac{\omega - \omega_d}{k_{\parallel} v_{\parallel}} \right) \left(\delta \phi - \frac{v_{\parallel}}{c} \delta A_{\parallel} \right) F_0.$$

Then taking the first moment in the parallel direction we obtain

$$j_{\parallel} = -\frac{q^2 m n}{k_{\parallel} T} (\omega - \omega_A) \left[\delta \phi - \left(1 - \frac{\omega_d}{\omega} \right) \hat{\psi} \right] \quad (A3)$$

where $\hat{\psi} = \omega/k_{\parallel} c A_{\parallel}$. Now using Ampere's law, we obtain

$$j_{\parallel} = en k_{\perp}^2 s^2 \frac{k_{\parallel} v_A^2}{\omega} \frac{e \hat{\psi}}{T}. \quad (A4)$$

Then adding up Eq. (A3) for ions and electrons and making the sum equal to Eq. (A4), we obtain

$$\delta \phi - \hat{\psi} = k^2 s^2 \frac{D - k_{\parallel}^2 v_{\parallel}^2}{\omega^2 (1 + T_e/T_i)} \hat{\psi} \quad (A5)$$

where D is the driving pressure force, i.e., $D = (\omega_A \omega_{pe}/k^2 s^2)$. In a proper geometry D contains also the geometry factor $g(x)$ and $k_{\parallel}^2 \rightarrow k_c^2 \times (\partial/\partial x)r(x)(\partial/\partial x)$. We thus see that $\delta E_{\parallel} = ik_{\parallel}(\delta \phi - \hat{\psi})$ is an FLR effect that does not enter in our present calculations.

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