Adaptive Contact Elements for Three-
Dimensional Explicit Transient Analysis*

by

R. F. Kulak

Reactor Analysis and Safety Division

Argonne National Laboratory

9700 South Cass Avenue

Argonne, Illinois

ABSTRACT

A finite element method was developed for treating the mechanics of contact between deformable bodies. The method uses a family of adaptive interface elements, which were based on the penalty method, to handle the changing contact configurations that can occur between discretized contacting bodies. The nodal connectivity of these interface elements was allowed to change during the computations in order to accommodate finite sliding. The infusion of these elements in the interface satisfies the stress equilibrium condition during contact. Explicit forms for the nodal internal forces are presented. The methodology has been coded and several sample problems are presented.

*This work was performed under the auspices of the U. S. Department of Energy, Office of Technology Support Program, under contract W-31-109-Eng-38.

MASTER

RESTRICTION OF THIS DOCUMENT IS OBSOLETE

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

I. INTRODUCTION

The need to develop efficient and accurate numerical methods for treating interface problems is evident by the number of computation mechanics papers [1-17] that have recently been devoted to problems involving contact, impact, and/or sliding. The majority of the above developments were restricted to infinitesimal sliding and two-dimensional geometries. In order to treat the interface problems that occur in reactor safety assessments, we cannot tolerate such restrictive assumptions on the formulation. A novel approach is developed here to handle the nonlinear kinematics that occurs at the interface of two contacting bodies of arbitrary geometry in three-dimensional space.

Several methods have been developed to treat the mechanics of contact/impact between two bodies. In varying degrees, these methods satisfy compatibility at the interface and conservation of momentum. The Lagrange Multiplier method (e.g., Hughes, et al. [1]) exactly enforces the compatibility condition, whereas conservation of momentum is accounted for with discrete impact and release conditions. Additional degrees of freedom are added to the problem, which increases the storage requirements. Nichols and Hirt [5] used a regularization procedure to smooth the discontinuous transition between pre-impact and post-impact velocities for rigid body impact with a fluid. Belytschko and Mullen [7] applied a regularization procedure to treat the impact of a deformable structure with an incompressible fluid. Regularization procedures conserve momentum and approximate the compatibility condition. The penalty method (e.g., Hallquist [4]) essentially inserts interface springs between the impacting surfaces. The penalty method conserves momentum exactly and does not require impact and release conditions. In contrast, the compatibility condition is only approximately satisfied.

In this paper, we present a finite element methodology for the analysis of interactions in which impact, contact, and/or finite sliding can occur. The approach uses a family of elements based upon the penalty method. It was determined that a family of three elements is needed in order to handle all the possible configurations that can occur between the discretized contacting bodies. The family consists of the following: a tetrahedral element, a triangular element and a line element. The inclusion of these elements between the common surfaces of the two bodies results in satisfying the force equilibrium condition during the interaction. The finite elements that are developed can be used (1) to simulate contact in which finite sliding and release can occur and (2) to model impact. In order to treat problems in which finite sliding occurs, a connectivity algorithm determines which nodes form each element. The hydrodynamic analyzer that is used for fluid domains was previously reported by Kulak [18,19], and the structural analyzers were reported by Belytschko and Marchertas [20] and Belytschko et al. [21]. These analyzers form the basis for our fluid-structure interaction finite element code called NEPTUNE [18,19]. The newly developed interface methodology has been embedded into the NEPTUNE code. The next section describes the interface mechanics and develops the general forms for the interface nodal forces. In the following section, the explicit forms are developed for a tetrahedral element, a triangular element, and a line element. Then the next section describes the adaptive connectivity scheme that is used to change the element connectivity when needed as the finite element mesh deforms. The next-to-last section outlines the explicit time-marching procedure that is used to integrate the semidiscretized equations of motion. The final section presents several illustrative examples. The works of Kennedy and Belytschko [12] and Marti [10] were seminal to our development.

II. INTERFACE MECHANICS

In this section, the interface conditions are first stated and then regularized by the penalty method. Next, the general form for the internal nodal force matrix of a contact element is obtained by invoking the principle of virtual power. Finally, the constitutive relation used with the contact elements is described.

Consider two bodies B^1 and B^2 (Fig. 1) with surfaces S^1 and S^2 , respectively, to have a common interface (i.e. a contact interface), S_c , given by $S_c = S^1 \cap S^2$. The size of S_c can vary during the interaction between the two bodies. The part of S^α that belonged to S_c was designated S_c^α , that is $S_c^\alpha = S_c \cap S^\alpha$. The superscript $\alpha = 1, 2$ refers to body B^α . Let $\underline{n}$ be the unit normal vector field along the contact interface, S_c , let $\underline{s}$ be the unit tangent vector field that lies in the direction of relative velocity (sliding) between the two surfaces, and let $\underline{m}$ be the unit vector orthogonal to both $\underline{n}$ and $\underline{s}$. Thus, $\underline{s}$ and $\underline{m}$ form a tangent plane to the contact surface. The normal vector field $\underline{n}^\alpha$ and the tangential vector field $\underline{s}^\alpha$ are defined on the contact surface S_c^α of each body as follows:

$$\underline{n} = \underline{n}^1 = -\underline{n}^2 \quad \text{and} \quad \underline{s} = \underline{s}^1 = -\underline{s}^2. \quad (1)$$

We define $\underline{\sigma}^\alpha$ to be the Cauchy traction vector acting on the contact surface S_c^α . The normal component of the traction is given by

$$\sigma_n^\alpha = \tilde{\sigma}^\alpha \cdot \tilde{n}^\alpha \quad (2)$$

$$\tilde{\sigma}_n^\alpha = \sigma_n^\alpha \tilde{n}^\alpha .$$

The tangential component of $\tilde{\sigma}^\alpha$ is determined from

$$\tilde{\sigma}_s^\alpha = \tilde{\sigma}^\alpha - \sigma_n^\alpha \tilde{n}^\alpha . \quad (3)$$

When the two surfaces are not in contact, the normal tractions, $\tilde{\sigma}_n^\alpha$, at the interface are given by

$$\tilde{\sigma}_n^1 = \tilde{\sigma}_n^2 = 0 . \quad (4)$$

When contact occurs, then the normal tractions satisfy the relation

$$\tilde{\sigma}_n^1 = - \tilde{\sigma}_n^2 , \quad (5)$$

and the normal velocities, $\tilde{v}_n^\alpha$, are equal, that is

$$\tilde{v}_n^1 = \tilde{v}_n^2 . \quad (6)$$

The tangential components of the velocity vectors, $\tilde{v}_t^\alpha$, is determined from

$$\tilde{v}_t^\alpha = \tilde{v}^\alpha - v_n^\alpha \tilde{n}^\alpha . \quad (7)$$

Since adhesion is not permitted along the interface, only negative normal tractions, σ_n , are developed; thus,

$$\sigma_n < 0 \quad \text{on } S_c . \quad (8)$$

Also, the two bodies are not permitted to penetrate into each other; this defines the unilateral contact condition

$$g_n > 0 \quad \text{on } S_c , \quad (9)$$

where g_n is the gap normal to the interface. In the approach presented here, we have infused the contact interface, S_c , with elements capable of treating the mechanics of contact. We defined h to be the current height of the element and h_c to be the height of the element when contact occurs. Thus, the normal gap is computed from

$$g_n = h - h_c \quad \text{on } S_c . \quad (10)$$

For the case of impact between the two bodies, we must consider the following additional interface relations. Prior to impact the normal velocities of the points on the contact surfaces of the two bodies will not be equal. Immediately after impact, however, the normal velocities will be equal and are obtained as follows. By invoking the balance of linear momentum at the contact interface the jump conditions are given by

$$[\tilde{\sigma}^\alpha] = - \rho^\alpha \tilde{c}_n^\alpha [\tilde{v}^\alpha] , \quad \alpha = 1, 2 , \quad (11)$$

where ρ is the density, $\bar{c}_n$ is the appropriate wave speed, and $\underline{v}$ is the velocity of a point on the contact surface. The brackets, [], are used to indicate a jump across a singular surface; thus

$$[\underline{v}] = \underline{v}^+ - \underline{v}^- , \quad (12)$$

where the + and - superscripts indicate pre- and post-impact values, respectively. The normal tractions at each contact point of the two surfaces are obtained from Eq. (11) by taking the dot product with $\underline{n}_{\alpha}$ which yields

$$\sigma_n^{\alpha} = (-1)^{\alpha} \rho^{\alpha} \bar{c}_n^{\alpha} (\underline{v}_n^{\alpha} - \underline{v}^{\alpha} \cdot \underline{n}) . \quad (13)$$

Since the normal tractions at the contact points must be equal, the normal velocity is obtained from Eq. (13) and found to be

$$\underline{v}_n = \frac{\rho^{1-1} \bar{c}_n^{1-1} \underline{v}_n^{1-1} + \rho^{2-2} \bar{c}_n^{2-2} \underline{v}_n^{2-2}}{\rho^{1-1} \bar{c}_n^{1-1} + \rho^{2-2} \bar{c}_n^{2-2}} . \quad (14)$$

Since we consider the sliding only case, the tangential post-impact velocities at each point in S_c are obtained from

$$\underline{v}_t^+ = \underline{v}_t^- - \underline{v}_n \underline{n} . \quad (15)$$

Pires and Oden [11] pointed out that the addition of the penalty term $\sigma_n g_n$ to the total energy of the system is equivalent to replacing the contact conditions, Eqs. (8) and (9), by the relation

$$\sigma_n = -\frac{1}{\delta} g_n, \quad (16)$$

where δ is the penalty parameter. Thus, the unilateral contact condition, Eq. (9), is enforced as δ approaches zero. Note, the reciprocal of the penalty parameter acts like a modulus, E .

The normal traction is computed from an appropriate constitutive equation. To date, we have only used a constitutive equation that is a function of the current height of the element and a user specified contact height, h_c . Thus, if the height of the element is greater than the contact height, there is no contact between the two bodies, and the resistance traction is zero; otherwise, a normal traction is generated. For a linear resistance function we have

$$\dot{\sigma}_n = \begin{cases} E\dot{h} & \text{if } h < h_c \\ 0 & \text{if } h > h_c \end{cases}, \quad (17)$$

where E is a resistance modulus.

The updated stress is computed from

$$\sigma_n(t+\Delta t) = \sigma_n(t) + \dot{\sigma}_n \Delta t, \quad (18)$$

where t is the time, and Δt is the time step. We found that a rate-form of traction-gap relation produces less numerical noise.

Once contact occurs, frictional tractions, σ_s , will develop between the two surfaces in the direction of the relative velocity (i.e., sliding), v_s , which is given by

$$v_s = v_t^2 - v_t^1, \quad (19)$$

$$v_s = \dot{g}_s s,$$

where $\dot{g}_s$ is the magnitude of the sliding.

A comprehensive treatment of dynamic friction is presented by Oden [16]. A general approach for modelling tangential surface traction between two bodies would consider two modes: sticking and sliding. However, we feel that the sticking mode would not be important for our class of dynamic problems and, therefore, at this time we have elected to consider only the sliding mode. Our future work will include implementation of a more complete friction model. By using a Coulomb friction relationship, the tangential tractions take the following form

$$\sigma_s^\alpha = (-1)^\alpha \sigma_s s, \quad \alpha = 1, 2, \quad (20)$$

where

$$\sigma_s = \mu \sigma_n, \quad (21)$$

$$s = \frac{v_s}{||v_s||},$$

and μ is the coefficient of dynamic friction.

In order to obtain an equivalent set of nodal forces from the distributions of σ_n and σ_s over the contact surface, we employ the principal of virtual power; thus,

$$\int_{S_c} \sigma_n \dot{g}_n dS_c = f_{iI}^n v_{iI}, \quad \int_{S_c} \sigma_s \dot{g}_s dS_c = f_{iI}^s v_{iI}, \quad (22)$$

where f_{iI}^n and f_{iI}^s are the components of the nodal force due to the normal and tangential tractions, respectively, at node I in the i th global coordinate direction, v_{iI} are the velocity components, and $\dot{g}_n$ and $\dot{g}_s$ are the rate-of-change in gap and the rate of sliding, respectively. We have assumed that $\dot{g}_n$ and $\dot{g}_s$ can be expressed in linear forms given by

$$\dot{g}_n = B_{iI}^n v_{iI}, \quad \dot{g}_s = B_{iI}^s v_{iI}. \quad (23)$$

Using Eqs. (1) and (4) in Eq. (6) and noting the arbitrariness of the nodal velocities, the internal forces are found to be

$$f_{iI}^n = \int_{S_c} \sigma_n B_{iI}^n dS_c, \quad f_{iI}^s = \int_{S_c} \sigma_s B_{iI}^s dS_c. \quad (24)$$

It should be noted that because we used the penalty method to solve contact problems we do not have to explicitly solve Eq. (13) for the contact stress or Eq. (14) for the normal velocities. This task can be quite difficult when treating three-dimensional geometries in which general contact can

occur. If both bodies were discretized such that node-to-node contact between the two bodies was assured, then Eqs. (13) and (14) could be easily used. Also, in order to properly use Eqs. (13) and (14) it would be necessary to determine the exact time of contact, τ , within a computational step, Δt , where $0 < \tau < \Delta t$, and to adjust all kinematic quantities to this time level. Since the penalty method regularizes the impact, it was not necessary to alter the basic time-stepping algorithm of the computer program. With our approach, the only change to an existing finite element program, which use explicit temporal integration, was the addition of the internal force subroutine for the interface element.

III. CONTACT/IMPACT ELEMENTS

It was determined that a family of three elements are needed in order to handle all the possible topological configurations that can occur between the discretized contacting bodies.

Prior to presenting the derivation of the specific forms for the internal forces for each of the contact elements, we will, for the sake of clarity, describe some of their common attributes. It will be helpful to define an interface vector triad ($\underline{n}$, $\underline{s}$, $\underline{m}$) for each contact element such that $\underline{n}$ is normal to the base, $\underline{s}$ is tangent to the base and in the direction of relative surface sliding, and $\underline{m}$ is also tangent to the base and orthogonal to $\underline{n}$ and $\underline{s}$. The rate-of-change in gap, $\dot{g}_n$, is measured along $\underline{n}$ and the relative sliding velocity, $\dot{g}_s$, between the base and apex is measured along $\underline{s}$. Each element is considered to consist of a base, which may be defined by one or more nodes, and an apex node (Fig. 2). The base is always affixed to one body and the apex node belongs to the other. The height of the element, h , is the distance from the base to the apex node measured along a normal to the base. The rate-of-change in element height, $\dot{h}$, is given by the following linear form

$$\dot{h} = B_{iI} v_{iI} \quad , \quad (25)$$

where v_{iI} is the component of velocity in the i th global coordinate direction at node I , and B_{iI} is a matrix which must be determined for each element. This form for the rate-of-change of element height was used by Kennedy and Belytschko [12] for two-dimensional geometries.

A. Tetrahedral Element

Here we develop the specific form of B_{iI} for a tetrahedron. The tetrahedron is a simplex element in three-dimensional space. The local non-dimensional coordinates, say (ξ, η, ζ) , which range from zero to one, are shown in Fig. 3a. The element interpolation functions, $H_I(\xi, \eta, \zeta)$, are independent of the geometry of the element in the global system (Fig. 3b). For the tetrahedron the interpolation functions are

$$H_1 = 1 - \xi - \eta - \zeta$$

$$H_2 = \xi$$

(26)

$$H_3 = \eta$$

$$H_4 = \zeta$$

The global coordinates (x, y, z) can be expressed in terms of the local coordinates and nodal coordinates by

$$x = \sum_{I=1}^4 H_I(\xi, \eta, \zeta) x_I$$

$$y = \sum_{I=1}^4 H_I(\xi, \eta, \zeta) y_I$$

(27)

$$z = \sum_{I=1}^4 H_I(\xi, \eta, \zeta) z_I$$

The volume of the element is given by

$$V = \frac{1}{6} |J| , \quad (28)$$

where $|J|$ is the determinant of the Jacobian matrix of the coordinate transformation, Eq. (27). The area of the base is given by

$$A = \frac{1}{2} \left| \vec{L}_{21} \times \vec{L}_{31} \right| , \quad (29)$$

where $\vec{L}_{21}$ and $\vec{L}_{31}$ are vectors that lie along the side of the base from nodes 1 to 2 and from nodes 1 to 3, respectively.

Since we always position the tetrahedron so that nodes 1, 2, and 3 (Fig. 4) lie on one impacting surface and node 4 lies on the other, we see that the height is measured from the base formed by nodes 1-2-3 to the apex at node 4. The height is given by

$$h = 3V/A , \quad (30)$$

where V is the volume of the tetrahedron and A is the area of the base. We obtain the form for the matrix B_{iI} defined in Eq. (25) by substituting Eqs. (28) and (29) into Eq. (30) and then differentiating. Using Eq. (24), we find that the internal force expression for the tetrahedron is given by the simple form

$$f_{iI} = f B_{iI} , \quad (31)$$

where

$$B_{iI} = \frac{0.5}{A_{123}} (x_{jI} x_{k4K} + x_{jK} x_{kI4} + x_{j4} x_{kKJ}) - \frac{1.5}{A_{123}^2} (A_j x_{kKJ} + A_k x_{jJK}) \quad (I = 1, 3), \quad (32a)$$

and for $I = 4$ (i.e. the apex node)

$$B_{i4} = \frac{0.5}{A_{123}} (x_{j1} x_{k23} + x_{j2} x_{k31} + x_{j3} x_{k12}), \quad (32b)$$

where, for example, x_{kKJ} is defined to be the difference between the k -th coordinate of nodes K and J . The appropriate node numbers are given in the permutation table (Table 1). The subscripts (i,j,k) take cyclical permutations of (x,y,z) . A_{123} is the area of face 1-2-3, and A_j is the projected area in the j -th direction. The coefficient f is computed from

$$f = \sigma_n A_{123}, \quad (33)$$

in which the normal resistance traction, σ_n , is computed from Eq. (18).

The sliding velocity [Eq. (19)] between the base and apex of the element is taken to be the difference between the tangential velocity of the apex minus the average tangential velocity of the base; thus

$$\hat{v}_{\sim s} = \hat{v}_{\sim t4} - \frac{1}{3} \sum_{I=1}^3 \hat{v}_{\sim tI}, \quad (34)$$

$$\hat{v}_{\sim s} = \dot{g}_s s_{\sim},$$

where the symbol " $\wedge$ " indicates that the velocity is referred to the interface coordinate system whose vector basis is $\underline{n}$, $\underline{s}$, and $\underline{m}$. The relative sliding, $\dot{\underline{g}}_s$, is given by

$$\dot{\underline{g}}_s = \hat{B}_{iI}^s \hat{v}_{iI} \quad , \quad (35)$$

where

$$\{\hat{B}^s\}^T = [0 \ -\frac{1}{3} \ 0 \ 0 \ -\frac{1}{3} \ 0 \ 0 \ -\frac{1}{3} \ 0 \ 0 \ 1 \ 0] \quad , \quad (36)$$

and

$$\{\hat{v}\}^T = [\hat{v}_{n1} \ \hat{v}_{s1} \ \hat{v}_{m1} \ \hat{v}_{n2} \ \hat{v}_{s2} \ \hat{v}_{m2} \ \hat{v}_{n3} \ \hat{v}_{s3} \ \hat{v}_{m3} \ \hat{v}_{n4} \ \hat{v}_{s4} \ \hat{v}_{m4}] \quad . \quad (37)$$

The nodal forces due to sliding are expressed in the interface coordinate system by

$$\hat{f}_{iI}^s = A_{123}^s \sigma^s \hat{B}_{iI}^s \quad . \quad (38)$$

Global components for the nodal forces are obtained through standard coordinate transformation procedures.

B. Triangular Element

Now we will apply the foregoing procedure to a triangular element in three-dimensional space. We will position the triangle so that nodes 1 and 2 (Fig. 5) define the line base of the triangle and node 3 defines the apex.

The height is measured from the base formed by nodes 1 and 2 to the apex at node 3; thus the height is given by

$$h = \frac{A}{L_{21}} , \quad (39)$$

where A is the area of the triangle and L_{21} is the length of the base. As before, the rate-of-change of the height is related to nodal velocities through matrix B_{iI} , which is defined in Eq. (25). Now we will derive the explicit form for that matrix. The area of the triangle is given by

$$A = \frac{1}{2} | \underline{\tilde{L}}_{21} \times \underline{\tilde{L}}_{31} | , \quad (40)$$

where $\underline{\tilde{L}}_{21}$ and $\underline{\tilde{L}}_{31}$ are vectors that lie along the sides defined by nodes 2 and 1 and by nodes 3 and 1, respectively. Thus, the edge vector $\underline{\tilde{L}}_{21}$ is defined by

$$\underline{\tilde{L}}_{21} = X_{21} \underline{\tilde{i}} + Y_{21} \underline{\tilde{j}} + Z_{21} \underline{\tilde{k}} , \quad (41)$$

where, for example, X_{IJ} is defined to be the difference between the X -coordinates of nodes I and J (i.e. $X_{IJ} = X_I - X_J$) and $\underline{\tilde{i}}, \underline{\tilde{j}}, \underline{\tilde{k}}$ are the base vectors in the global coordinate system. Using Eqs. (40), (41), and (25), we obtain the following explicit form for the B_{iI} matrix for our triangular contact element

$$b_{iI} = \frac{0.5}{A L_{21}} (A_j X_{kKJ} + A_k X_{jJK}) - \frac{A}{L_{21}^3} X_{iML} \quad (I = 1, 2) , \quad (42a)$$

and

$$b_{i3} = \frac{0.5}{A L_{21}} (A_j x_{k21} + A_k x_{j12}) \quad (I = 3) , \quad (42b)$$

where A_i is a projected area. The resistance force, f , is computed from Eq. (33). The appropriate node numbers are given in the permutation table (Table 2) for the triangular element.

C. Line Element

The final element that we need to complete our family of contact elements is a two-noded line element (Fig. 6), which is the simplest of the three. One node is defined to be the base node, which is usually positioned on the structure, and the other is defined to be the apex node, which is usually positioned on the fluid. The height of this element is equal to the distance between the two nodes and is given by

$$h = [x_{21}^2 + y_{21}^2 + z_{21}^2]^{1/2} . \quad (43)$$

The rate-of-change of the height thus becomes

$$\dot{h} = \frac{1}{h} \begin{Bmatrix} x_{21} \\ y_{21} \\ z_{21} \end{Bmatrix}^T \begin{Bmatrix} v_{x21} \\ v_{y21} \\ v_{z21} \end{Bmatrix} , \quad (44)$$

where, for example, v_{x21} is the difference in velocities in the X-coordinate direction between nodes 2 and 1. The nodal internal forces for this line element are given by

$$f_{11} = f_{B_{11}} = \frac{f}{h} \begin{Bmatrix} x_{21} \\ y_{21} \\ z_{21} \end{Bmatrix} , \quad (45)$$

where, as before, the resistance force is computed from Eq. (32).

IV. ADAPTIVE CONNECTIVITY

In this section, we describe the algorithm that is employed to adapt the connectivity of the contact elements so that we can model problems with finite sliding in which element boundaries are crossed. It should be emphasized that there is not a unique method for handling this aspect of the problem. The algorithm we propose is simple, efficient, and capable of treating the contact problems that we had to deal with to date. Other more complicated and computationally expensive algorithms could be developed, if needed. During the discretization of a problem, potential contact elements are defined. Each of these elements consists of a base and an apex node. The specific node(s) that form the base of the element are defined by the user and remain fixed throughout the numerical simulation. In contrast, the apex node of the element can change during the simulation. In fact, it is possible that during some point in the simulation an element will not have an apex node and would be considered a null element, that is an element which does not participate in the calculations. Thus, we see that a specific apex node can be associated with different contact elements at different times during the simulation.

In order to reduce the computational time required when a search is performed to identify the element that an apex node is associated with, we utilized a contact element adjacency table. For each tetrahedral contact element, the table contains the element numbers of all the contiguous tetrahedral contact elements. Thus, when a specific apex node fails to project itself onto the base that it was previously identified with, the adjacency table contains the information needed to reconnect the apex node to another base.

Now we address the task of dynamically adapting the connectivity of the contact elements. At this point, it is worth emphasizing the fact that an

element's connectivity only changes during a small percentage of the computational steps. At the end of each computational cycle, the connectivity of each tetrahedral contact element is checked to see if its apex node continued to project onto its current base. The node projection is performed in the following manner. As previously defined, $\underline{n}$, is the unit normal vector to the base of the contact element. Let point P (see Fig. 7) be the projection of the apex node onto the plane that contains the base of the element. It is obtained from

$$\underline{R}_{p1} = \underline{R}_{41} - h\underline{n} \quad (46)$$

where $\underline{R}_{IJ}$ is the vector from point "J" to point "I". We use the following approach to determine if point P lies within the triangular base. The area vectors, $\underline{A}_I$, of the three sub-triangles (Fig. 8) are computed from

$$\underline{A}_I = \frac{1}{2} \underline{R}_{KJ} \times \underline{R}_{PJ} = A_I \underline{n} \quad (47)$$

where A_I is the magnitude of the area of sub-triangle I and the subscripts of the $\underline{R}$ vectors are obtained from the permutations given in Table 2. Next, the area coordinates, L_I , are computed from

$$L_I = \frac{A_I}{A_{123}} \quad (48)$$

When all three of the coordinates lie within the base, that is $L_I \in [0,1]$, then the apex node projects onto the triangular base, and the element connectivity remains unchanged.

However, if an apex node is in the vicinity of an edge or vertex of the tetrahedron's triangular base, it receives special treatment in order to provide a smoother transition from its current tetrahedral base to its next tetrahedral base. This special treatment is not needed when the bases are attached to flat surfaces. However, when the mesh configuration is such that the bases lie on a convex surface, then it is possible for the apex node to lie in regions that do not belong to any element projections. For example, there is a "trough" shaped region, as shown in Fig. 9, that lies along the common line connecting the triangular bases of adjacent tetrahedral contact elements. In addition, a "pyramid" shaped region (Fig. 10) exists at the vertices of adjacent elements. When the bases lie on a concave surface, the opposite problem occurs; the apex node may project onto more than one base.

To resolve this issue, we use the following procedure. When the following conditions hold,

$$L_I < \delta_s, \quad L_J > 1 - \delta_n, \quad \text{and} \quad L_K < 1 - \delta_n, \quad (49)$$

where δ_s and δ_n are small numbers, we generate a so-called phantom triangular contact element along side "I" of the triangular base of the tetrahedral element. The line base of the triangular element is defined by nodes "J" and "K" and the apex node for the triangular element is the apex node of the tetrahedral element. The interaction between the two bodies is now computed from Eq. 41 instead of Eq. 31. For this situation, we can think of the tetrahedral contact element as being replaced by a temporary triangular element, and in this localized region of S_c the contact between the two bodies is a node-to-line contact. Since this element does not exist in the problem's connectivity array, it is thought of as a phantom element. It became visible only when the

above logic path was transversed. Similarly, when the following condition is true,

$$L_I > 1 - \delta_n \quad (50)$$

we activate a phantom line contact element with node "I" as the base node. For this condition the localized contact between the two bodies is a node-to-node contact, and Eq. 45 is used in place of Eq. 31.

For the condition in which the area coordinates are outside the interval, that is $L_I \notin [0,1]$, a search is made using the information from the adjacency table to find the new base upon which the apex node projected. When the apex node is found to project onto the base of another tetrahedron, the connectivity of the elements associated with the new and old bases are changed. The apex node is removed from the connectivity of the old element and added to the connectivity of the new element. If no other apex node projected onto the old base, then that element becomes a null element in the sense that it did not contribute to the internal force calculations.

If the apex node did not project onto the triangular base of an adjacent tetrahedron, then the apex node must lie in either the "trough" or "pyramid" regions previously described. Conditions are first checked to see if it projected onto the line base of the phantom element associated with the current tetrahedron. If this check failed, a projection check is made with the line bases of the phantom elements of the adjacent tetrahedrons. The following conditions are necessary and sufficient for an apex node to project onto a line base connecting generic nodes 1 and 2:

$$\tilde{R}_{13} \cdot \tilde{R}_{12} > 0 \quad ,$$

(51)

$$\tilde{R}_{23} \cdot \tilde{R}_{21} > 0 \quad .$$

If a line base cannot be found, then the apex node projects directly onto a node base.

V. TIME INTEGRATION

The semi-discretized equations of motion for the hydrodynamic, continuum, and structural domains can be expressed by

$$m_{iI} \partial_t (v_{iI}) + f_{iI}^{int} = f_{iI}^{ext} , \quad (52)$$

where m_{iI} is the nodal mass of node I associated with the i th direction, and f_{iI}^{int} , f_{iI}^{ext} are the internal and external nodal forces, respectively, at node I in the i th direction.

The contact elements contribute, as do the hydrodynamic, continuum, and structural elements, to the internal forces, f_{iI}^{int} . Equation (52) is explicitly integrated in time. We used the central difference integrator to time-advance the velocities and displacements according to

$$\begin{aligned} v_{iI}(t + 0.5\Delta t) &= v(t - 0.5\Delta t) + \Delta t(\partial_t v_{iI}(t)) \\ u_{iI}(t + \Delta t) &= u_{iI}(t) + \Delta t v_{iI}(t + 0.5\Delta t) , \end{aligned} \quad (53)$$

and the acceleration $\partial_t v_{iI}(t)$ is obtained from Eq. (52). After the new configuration is computed, the connectivity of all the contact elements is checked and adjusted, if required, according to the scheme previously outlined. Then the code returns to the beginning of the computational cycle for the next step. Thus, it is seen that our treatment of contact/impact does not disturb the basic architecture of an explicit solution algorithm.

VI. EXAMPLES

The following examples are presented to illustrate the use of our newly developed numerical formulation for treating the interface mechanics between two bodies in which impact, contact, and/or sliding take place.

1. Two Impacting Bars

Our first example is taken from Hughes et al. [1], and it is the longitudinal impact of two identical elastic bars. Although this problem is relatively simple, it proved to be very useful in checking out the interface element. The analytical solution which is available is used to benchmark our numerical results. We assumed that bar 1 (Fig. 11.a) had an initial velocity and that bar 2 was initially at rest. The data for this problem are given in Table 3; note that we use dimensionless quantities. Our finite element mesh is shown in Fig. 11.b, and it consists of 20 axial elements per bar with a tetrahedral gap-contact element between them.

Our results are shown in Figs. 12-14. The displacement signature of the contact interface between the two bars (Fig. 12) shows that (1) prior to impact, bar 1 is in motion and bar 2 is at rest, (2) during contact, both bars are in motion, and (3) after impact, bar 1 is at rest and bar 2 is in motion. The velocity signature, which is shown in Fig. 13, indicates that the velocity of the interface during the impact is 0.05, which is in exact agreement with the analytical results. The initial contact and release however, are not as crisp as the theoretical solution. The interface force, Fig. 14, is also in good agreement with the analytical solution.

B. JSME Shipping Cask Benchmark Calculations

Recently, the Subcommittee on Structural Analysis of Nuclear Shipping Casks of the Japan Society of Mechanical Engineers (JSME) conducted a benchmark study to identify methods that are suitable for modeling the impact response of spent nuclear fuel shipping casks. Some of these results are reported by Yagawa et al. [22], and we decided to compare them with NEPTUNE calculations to help verify the interface element.

The second problem in the JSME study is the impact response of an assembly of a thin-wall stainless steel cylindrical shell that contained a lead cylinder. The assembly is dropped from a height of 9.0 m onto a rigid floor; the velocity at impact is 13.33 m/s. Note this problem can be treated as an axisymmetric problem because of its geometry and loading. However, since we are verifying a full three-dimensional finite element program, we developed a model for a 15-degree sector of the problem. It should be noted that large relative motions (sliding) occurred between the steel shell and the lead cylinder. Therefore, this problem provides a check on the ability of our newly developed interface element to model sliding between two bodies. The material properties for both the steel and lead are listed in Table 4.

Figure 15 is a perspective view of our model. The portion of the model that discretized the lead cylinder consists of fifty hexahedral continuum elements: five elements in the radial direction and ten elements in the vertical direction. The ten elements that lie along the vertical axis of the cylinder are degenerated into wedges by coalescing two pairs of nodes. The cylindrical shell is modeled with NEPTUNE's three-node triangular plate element. Interface elements are infused between the lead cylinder and steel shell in order to allow sliding and possible separation at the cylinder-shell interface. The interface is assumed to be frictionless. All the nodes along

the vertical axis are constrained to move along that axis, and all the nodes along the impacting surface are restricted from vertical motion in order to enforce the "no rebound" assumption. The remainder of the nodes are confined to motion within their sector planes. An initial downward vertical velocity of 13.33 m/s is applied to all the nodes.

The downward motion of the top of the lead cylinder is called head settlement, and it is used as a measure of the impact response. Figure 16 shows the history of the head settlement, which is the displacement of the top of the lead cylinder. The cask impacts against the floor at 0.0 ms. The maximum calculated value for head settlement was 6.5 cm and occurred at 5.5 ms. Yagawa et al. reported on the results from several organizations which used different computer programs, and Key [23] reported on his simulations of this problem. Table 5 shows the results for maximum head settlement and the time at which it occurred for numerical calculations performed by five different computer programs, including NEPTUNE. The range for maximum head settlement and the time at which it occurred is 5.8 to 7.25 cm and 4.3 to 6.25 m, respectively. Our results compare favorably with the above ranges. Figure 17 shows the deformed configuration of the cask at 5 ms; the large sliding between the thin shell and lead is clearly visible.

D. Oblique Impact of a Spherical-Bottomed Cylindrical Tank

The following problem is designed to check the performance of the element in a fairly complex impact situation. Since we constructed this problem, we have no other solution at this time with which to compare our results. Figure 18 is a schematic view of the problem. A 0.25 in. thick cylindrical tank with a spherical bottom is impacted against a rigid floor at an angle $\alpha = 15^\circ$. For simplicity, gravitational effects are neglected. The cylinder is 12.4 in. long with a diameter of 20 inches. The material properties are given in Table 6 below. The tank is assumed to have a velocity of 600 in/sec along the 15 deg. line. The finite element model (Fig. 19) consists of 350 quadrilateral plate elements and 91 tetrahedral impact elements.

The simulation is started with the closest part of the tank bottom being 0.10 in. above the floor. Figure 20 shows the deformed configuration at 2 ms into the simulation. Note the off-center indentation of the tank bottom. The history of the nodal displacement for node 12, as identified in Fig. 19, is plotted in Fig. 21. Figure 22 presents the evolution of the interaction between the impacting tank and the rigid floor. The total force is defined to be the sum of all the individual normal contact forces. The total force signature shows that the tank initially contacts the floor at 0.20 ms and rebounds off the floor at 3.80 ms. The force signature for contact element number 362, which is identified on the plot as CE362, shows that it makes contact at 0.82 ms and rebounds at 1.42 ms. The evolution of the total energy, kinetic energy, and strain energy is presented in Fig. 23. As with the previous problem we did not have other results to compare with, but the degree to which the total energy is conserved during the computations was a good indicator that the results are correct.

E. Impact Between Two Circular Plates

As with the previous problem, this problem is designed to assist us in checking the behavior of the interface element. This example is for two identical simply supported circular plates, positioned one over the other, initially separated by a uniform gap (Fig. 24). A step pressure is applied to the top surface of the upper plate causing it to deform downward and eventually impact against parts of the lower plate. The uniqueness of this problem is the fact that both the base and apex of the interface elements are attached to moving structures.

Because of geometric and loading symmetries, we only had to model one-quarter of each plate, as shown by the perspective view of the model in Fig. 25. A top view of the model (Fig. 26) shows the discretization which is used. Seventy-five quadrilateral Mindlin plate elements are used for each plate (150 elements total). Forty-seven interface elements are positioned in the central region between the two plates. Simply supported boundary conditions are prescribed along the outer diameter of both plates. The nodes along the symmetry planes are confined to motion in those planes. The properties of the plate material are listed in Table 5.

A uniform pressure of 600 psi is applied to the upper surface of the top plate at the beginning of our simulation ($t = 0.0$ sec) and maintained for the duration. The displacement history of the center of the upper and lower plates are shown in Fig. 27. It is seen that impact occurs at 0.34 ms after the application of the step pressure. Figure 28 shows the deformed configurations of both plates at 1.0 ms; note the large deformations that occurred in both plates. The excellent agreement between the external work performed by the pressure load and the total energy deposited into the two plates (see Fig. 29) is a good indicator that the impact simulation is reasonable. The

kinetic energy signature suggests that most of the external work is transformed into plastic strain energy.

ACKNOWLEDGMENTS

The author wishes to thank Drs. S. H. Fistedis and J. M. Kennedy for their support and Mr. C. Fiala for his computer related assistance. This work is part of the Engineering Mechanics Program of the Reactor Analysis and Safety Division of Argonne National Laboratory and was supported by the U. S. Department of Energy.

REFERENCES

- [1] T. J. R. Hughes, R. L. Taylor, J. L. Sackman, A. Curnier, and W. Kanoknukulchai, A finite element method for a class of contact-impact problems, *Comput. Meths. Appl. Mech. and Engrg.* 8, (1976) 249-276.
- [2] G. R. Johnson, Three-dimensional analysis of sliding surfaces during high velocity impact, *J. of Appl. Mech.* 44(4) (1977) 771-773.
- [3] S. W. Key, Z. E. Beisinger, and R. D. Krieg, HONDO-II - A finite element computer program for the large deformation dynamic response of axisymmetric solids, SAND 78-0422, Sandia National Laboratories, Albuquerque, New Mexico (1978).
- [4] J. O. Hallquist, A numerical treatment of sliding interfaces and impacts, in *Computational Techniques for Interface Problems*, American Society of Mechanical Engineers, AMD-30 (1978), 117-133.
- [5] B. D. Nichols, and C. W. Hirt, Numerical simulations of hydrodynamic impact loads on cylinders, EPRI NP-824 Interim Rep., Electric Power Research Institute, Palo Alto, California (1978).
- [6] H. D. Hibbitt, and B. I. Karlsson, Analysis of pipe whip, A.S.M.E. Paper No. 79-PVP-122, United Engineering Center, New York, (1979).
- [7] T. B. Belytschko, and R. Mullen, Two-dimensional fluid-structure impact computations with regularization, *Comput. Meths. Appl. Mech. and Engrg.*, 27 (1981), 139-154.
- [8] R. A. Brockman, Numerical solution of large deformation problems involving surface contact and impact, in *Penalty-Finite Element Methods in Mechanics*, American Society of Mechanical Engineers, AMD-51 (1982), 123-136.

- [9] G. L. Goudreau, and J. O. Hallquist, Recent developments in large-scale finite element lagrangian hydrocode technology, *Comput. Meths. Appl. Mech. and Engrg.*, 33 (1982), 725-757.
- [10] J. Marti, PR3D documentation manual, Tech. Rpt. No. PR-TN-4.32, (1983), Principia Mechanical Ltd., London, UK.
- [11] E. B. Pires, and J. T. Oden, Analysis of contact problems with friction under oscillating loads, *Comput. Meths. in Appl. Mech. and Engrg.*, 39 (1983), 337-362.
- [12] J. M. Kennedy, and T. B. Belytschko, Buckling and post-buckling behavior of the ACS support columns, *Nucl. Engng. Design*, 75(3) (1983), 323-342.
- [13] J. T. Oden, and E. B. Pires, Nonlocal and nonlinear friction laws and variational principles for contact problems in elasticity, *Journal of Applied Mechanics*, 50 (1983), 67-76.
- [14] J. Padovan, and R. Moscarello, Pantographing self adaptive gap elements, *Computers and Structures*, 20(4) (1985), 745-758.
- [15] J. C. Simo, P. Wriggers, and R. L. Taylor, A perturbed lagrangian formulation for the finite element solution of contact problems, *Comput. Meths. Appl. Mech. Engrg.*, 50 (1985), 163-180.
- [16] J. T. Oden, and J. A. C. Martins, Models and computational methods for dynamic friction phenomena, *Comput. Meths. Appl. Mech. Engrg.*, 52 (1985), 527-634.
- [17] R. F. Kulak, Adaptive contact elements for three-dimensional fluid-structure interfaces, *Fluid-Structure Dynamics*, American Society of Mechanical Engineers, PVP - 98-7 (1985), 159-166.
- [18] R. F. Kulak, A finite element formulation for fluid-structure interaction in three-dimensional space, *J. of Pressure Vessel Tech.*, 103(2) (1981), 183-190.

- [19] R. F. Kulak, A finite element quasi-eulerian method for three-dimensional fluid-structure interactions, *Computers and Structures*, 18 (2) (1984), 319-332.
- [20] T. Belytschko, and A. H. Marchertas, Nonlinear finite element method for plates and its application to the dynamic response of reactor fuel subassemblies, *J. of Pressure Vessel Tech.*, 96(4) (1974), 251-257.
- [21] T. Belytschko, J. I. Lin, and C. S. Tsay, Explicit algorithms for nonlinear dynamics of shells, *Comput. Meths. Appl. Mech. Engrg.*, 42, (1984), 225-251.
- [22] G. Yagawa, H. Ohtsubo, H. Takeda, Y. Toi, T. Aizawa, and T. Ikushima, A round robin on finite element calculation for impact problems, *Nuclear Engineering and Design*, 78 (1984), 377-388.
- [23] S. W. Key, A comparison of recent results from HONDO III with the JSME nuclear shipping cask benchmark calculations, *Nuclear Engineering and Design*, 85 (1985), 15-23.

LIST OF TABLES

<u>No.</u>	<u>Title</u>
1.	Permutation Table for Tetrahedral Contact Element
2.	Permutation Table for Triangular Contact Element
3.	Data for the Impact of Two Identical Elastic Bars
4.	JSME Study Material Properties
5.	Comparison of Numerical Results from Several Codes for Impact Response of a Cladded Cylinder
6.	Material Properties Used in Tank-Floor Impact Problems and in Circular Plates Impact Problem

Table 1.
Permutation Table for Tetrahedral Contact Element

I	J	K
1	2	3
2	3	1
3	1	2

Table 2.
Permutation Table for Triangular Contact Element

I	J	K	L	M
1	2	3	1	2
2	3	1	2	1

Table 3.

Data for the Impact of Two Identical Elastic Bars

	Bar 1	Bar 2
Length, L	10.0	10.0
Young's Modulus, E	100.0	100.0
Density, ρ	0.01	0.01
Area, A	1.0	1.0
Initial velocity, v_0	0.1	0.0

Table 4.
JSME Study Material Properties

	Lead	Steel
Density, $\text{kgf s}^2/\text{mm}^4$	1.13×10^{-9}	8.0×10^{-10}
Young's Modulus, kgf/mm^2	19.5	1.96×10^4
Poisson's Ratio	0.42	0.33
Yield Stress, kgf/mm^2	3.02	31.6
Tangent Modulus, kgf/mm^2	1.85	195.0

Table 5.

Comparison of Numerical Results from Several Codes
for Impact Response of a Cladding Cylinder

Code	Maximum Head Settlement (Cm)	Time (msec)
ABAQUS	7.25	6.25
ANSYS	5.80	4.30
HONDO III	6.27	5.30
MARC	6.35	5.70
NEPTUNE	6.50	5.50

Table 6.
Material Properties Used in Tank-Floor Impact Problems
and in Circular Plates Impact Problem

Density, lb-sec ² /in ⁴	2.45×10^{-4}
Young's Modulus, lb/in ²	10.5×10^6
Poisson's Ratio	0.3
Yield Stress, lb/in ²	24,000
Tangent Modulus, lb/in ²	0.21×10^6

LIST OF FIGURES

<u>No.</u>	<u>Title</u>
1.	Contacting Bodies
2.	Generic Interface Element with a Base and Apex
3.	Coordinate System for a Tetrahedral Interface Element
4.	Typical Interface with a Tetrahedral Interface Element
5.	Triangular Interface Element
6.	Line Interface Element
7.	Definition of Quantities Used in Adaptive Connectivity Scheme for a Tetrahedral Contact Element
8.	Area Coordinates for a Triangular Base
9.	Trough Region that Lies Along the Common Side of Adjacent Triangular Bases which were Embedded on a Convex Surface
10.	Pyramid Region that was Located at the Vertices of Contiguous Triangular Bases which were Embedded on a Convex Surface
11.	(a) Two Impacting Bars (b) Finite Element Model of Impacting Bars
12.	Displacement Signature of Contact Interface
13.	Velocity Signature of Contact Interface
14.	History of Interface Force
15.	Perspective View of a Finite Element Model for the Impact of a Cladded Cylinder
16.	Temporal History of Head Settlement of the Cladded Cylinder
17.	Deformed Configuration of the Cladded Cylinder at 5 ms after Impact
18.	Schematic View of Oblique Impact of a Cylindrical Tank with a Spherical Bottom onto a Rigid Floor
19.	Finite Element Model for Oblique Impact of a Cylindrical Tank onto a Rigid Floor

LIST OF FIGURES
(Contd.)

<u>No.</u>	<u>Title</u>
20.	Deformed Configuration of the Tank after Rebounding from the Floor
21.	The Temporal History of the Displacement of Node 12 (see Fig. 19) as Predicted by NEPTUNE
22.	Temporal Evolution of Impact Forces
23.	Temporal Signature of Total Energy, Kinetic Energy, and Strain Energy as Predicted by NEPTUNE
24.	Schematic View of Two Identical Simply Supported Circular Plates Loaded by a Step Pressure Applied to the Topside of the Upper Plate
25.	Perspective View of a Finite Element Model for One-Quarter of Two Simply Supported Circular Plates
26.	Top View of a Finite Element Model for One-Quarter of Two Simply Supported Circular Plates
27.	Displacement History of the Center of the Upper and Lower Plates
28.	Deformed Configuration of the Plates at 1.0 ms after Application of the Step Pressure
29.	Temporal Signature of Total Energy, Kinetic Energy, and External Work

Figure 1. Contacting Bodies.

Figure 2. Generic Interface Element with a Base and Apex.

Figure 3. Coordinate System for a Tetrahedral Interface Element.

Figure 4. Typical Interface with a Tetrahedral Interface Element.

Figure 5. Triangular Interface Element.

Figure 6. Line Interface Element.

Figure 7. Definition of Quantities Used in Adaptive Connectivity Scheme for a Tetrahedral Contact Element.

Figure 8. Area Coordinates for a Triangular Base.

Figure 9. Trough Region that Lies Along the Common Side of Adjacent Triangular Bases which were Embedded on a Convex Surface.

Figure 10. Pyramid Region that was Located at the Vertices of Contiguous Triangular Bases which were Embedded on a Convex Surface.

(a)

(b)

Figure 11. (a) Two Impacting Bars.

(b) Finite Element Model of Impacting Bars.

Figure 12. Displacement Signature of Contact Interface.

Figure 13. Velocity Signature of Contact Interface.

Figure 14. History of Interface Force.

Figure 15. Perspective View of a Finite Element Model for the Impact of a Clad Cylinder.

Figure 16. Temporal History of Head Settlement of the Clad Cylinder.

Figure 17. Deformed Configuration of the Cladded Cylinder at 5 ms After Impact.

Figure 18. Schematic View of Oblique Impact of a Cylindrical Tank with a Spherical Bottom onto a Rigid Floor.

Figure 19. Finite Element Model for Oblique Impact of a Cylindrical Tank onto a Rigid Floor.

Figure 20. Deformed Configuration of the Tank After Rebounding from the Floor.

Figure 21. The Temporal History of the Displacement of Node 12 (see Fig. 19) as Predicted by NEPTUNE.

Figure 22. Temporal Evolution of Impact Forces.

Figure 23. Temporal Signature of Total Energy, Kinetic Energy, and Strain Energy as Predicted by NEPTUNE.

Figure 24. Schematic View of Two Identical Simply Supported Circular Plates Loaded by a Step Pressure Applied to the Topside of the Upper Plate.

Figure 25. Perspective View of a Finite Element Model for One-Quarter of Two Simply Supported Circular Plates.

Figure 26. Top View of a Finite Element Model for One-Quarter of Two Simply Supported Circular Plates.

Figure 27. Displacement History of the Center of the Upper and Lower Plates.

Figure 28. Deformed Configuration of the Plates at 1.0 ms After Application of the Step Pressure.