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LA-UR--83-333

DE83 007546

TITLE: AN HEURISTIC INTRODUCTION TO GRAVITATIONAL WAVES

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SUBMITTED TO Proceedings of the Workshop on Science Underground
September 27-October 1, 1982

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AN HEURISTIC INTRODUCTION TO GRAVITATIONAL WAVES

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ABSTRACT

We describe in physical terms the phenomenon of gravitational waves. The philosophy of William Gilbert is used.¹

"Since in the discovery of secret things and in the investigation of hidden causes, stronger reasons are obtained from sure experiments and demonstrated arguments than from probable conjectures and the opinions of philosophical speculators of the common sort; therefore to the end that the noble substance of that great loadstone, our common mother (the earth), still quite unknown, and also the forces extraordinary and exalted of this globe may the better be understood ..."

INTRODUCTION

The gravitational interaction is the weakest of all known elementary physical interactions and on the size scale of particle physics it has been generally ignored. In the macroscopic world it is the force we deal with on a daily basis and it defines to a great extent our physiological and psychological conceptions of the world. On a cosmic scale it is the medium of the Universe itself. Gravity was the first physical interaction to be given an analytic treatment and Newton's synthesis of the world has remained valid over a very impressive range of phenomena. Einstein's general theory of relativity has extended this range to include the universe itself as a dynamic object and has brought forth a series of exotic objects, such as black holes, into our view of the world.

General relativity was constructed by Einstein through an enormous intellectual feat, based on carefully thought out comparisons of different areas of physics and by demanding a consistent and unified view. General relativity was produced theoretically. The experimental consequences of this theory of gravity only became known afterwards and then as a list of tests: the bending of light by the sun, the advance of the perihelion of the orbit of Mercury, and the gravitational redshift of light. The prediction of a dynamic universe was not accepted until Hubble's observations forced the idea.

The technological advances of experimental physics are affording us the possibilities of observing non-Newtonian gravitational fields. In addition to being interesting as tests of general relativity, detection of these fields will give rise to a new astronomy to be compared with the present electromagnetic and weak

interaction (e.g. neutrino) astronomies. Neutrino astronomy is exemplified by R. Davis's solar neutrino flux observations, discussed previously in these proceedings. The experimental searches for gravitational waves and inductive gravitational fields are described in W. Oelfke's, R. Spero's, and R. W. P. Drever's following articles. Experimental tests of terrestrial Newtonian gravity have been described by F. Stacey in the preceding session. The purpose of this present article is to provide a rough and somewhat heuristic theoretical background and introduction to gravitational radiation, its generation, and its detection based on Einstein's general theory of relativity. For details and further analysis the reader is invited to consult references 2, 3, or 4.

GENERAL RELATIVITY

General relativity theory describes the "gravitational field" as the "geometry" of a four dimensional mathematical manifold and the remaining interactions and sources are described by the fields (scalar, spinor, vector, tensor) that may be assigned to this manifold. The notion of a "geometry" is given an analytic realization in the form of the metric tensor field $g_{\mu\nu}(x)$ and its physical interpretation comes from the infinitesimal line element

$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu.$$

This line element determines either the temporal or spatial distance between two points with coordinates x^μ and $x^\mu + dx^\mu$, respectively separated by a timelike or spacelike interval. The metric field describes everything of physical interest. For example, test particle trajectories are given by the paths of extremal interval as determined by $g_{\mu\nu}(x)$. In flat spacetime, in what are usually referred to as Galilean coordinates, $ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$, $g_{\mu\nu}(x) = \eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$ and the extremal paths are "straight lines". This is Newton's law of inertia, for in a flat spacetime geometry we would say there is no gravitational field acting on the particle.

Due to the freedom of choice of coordinates to label the points of the manifold, the unambiguous determination of equivalent gravitational fields is difficult. Metric fields with different functional forms may be transformable into one another by a coordinate transformation. The coordinates are only labels and in general carry no physical significance beyond being labels. The Galilean coordinates of flat spacetime possess additional geometric properties that makes inertial motion look simple in terms of them. This is reflected in the simple form the flat spacetime metric field takes when written in terms of Galilean coordinates (i.e. writing the spatial terms in spherical-polar coordinates). An invariant description of a geometry is given in terms of the curvature, a fourth rank tensor field derived from $g_{\mu\nu}(x)$ and its first and second derivatives. Geometrically this tensor measures

the "curvatures" of two-dimensional surface elements and its vanishing provides the necessary and sufficient conditions for the spacetime to be flat. Physically the curvature tensor is the manifestation of a "real" gravitational field. (It is the appropriate analog of the electromagnetic field). Consider two nearby observers following their metrically defined "straight" paths (i.e. two geodesics infinitesimally close). Physically they correspond to two freely falling observers. The curvature tensor describes how the distance they measure between them varies as they move through the spacetime. (In fact this equation of geodesic deviation takes on the form of Newton's second law with one observer measuring the local acceleration of the neighboring observer and putting this equal to a term involving the curvature tensor and the observers' four-velocities.) This distance is coordinate independent and provides the physical manifestation of the gravitational field through the curvature.

Einstein's field equations describe how the density of stress and energy in spacetime curves and bends the geometric properties of the spacetime. They are a set of second order nonlinear partial differential equations for the $g_{\mu\nu}(x)$ with source terms constructed from the stress-energy tensors of the "matter" fields. An interesting consequence of Einstein's equations is that they comprise an overdetermined set. They are consistent, however, because they satisfy a number of constraints. These constraints impose conditions upon the matter field sources that become the equations of motion or field equations for the sources themselves.

A general procedure for solving the field equations does not exist due to their enormous complexity. The exact solutions that exist are based on exploiting some symmetry to simplify the field equations. There are exact solutions known for the astrophysically important cases such as the Robertson-Walker metrics that describe open and closed homogeneous and isotropic cosmologies and the Kerr-Newman metrics that describe electrically-charged rotating-black-hole solutions. (The Schwarzschild black hole is the neutral, nonrotating case of the later.)

In spite of their complexity, it is possible to make predictions about the physical processes that go on. The approach is to use a perturbation analysis about one of the exact solutions. The dominant strong-field nonlinear features are then taken care of at lowest order and smaller deviations, that can be calculated, may be used to describe the variations. This way, for example, the dynamical evolution of a star undergoing gravitational collapse can be followed and estimates of the radiation it generates (electromagnetic, gravitational, neutrino) can be made.

GRAVITATIONAL WAVES

If the field equations are expanded about the flat spacetime solution a set of equations describing a spin-2 massless field that propagates as a wave traveling at the speed of light results. These waves are transverse and have two polarization states. They manifest themselves as shears in the geometry. Consider a set of

freely falling observers on which a gravitational wave is impinging. If one observer is selected as the reference, then, as the wave passes, the distance he measured to other observers will oscillate in time with the frequency of the wave, and the magnitude of the change in distance will be directly proportional to the original baseline distance, i.e., the wave's amplitude is described by a strain. If the observer measures the distance to a ring of surrounding observers, he will find that at any given instant two diametrically-opposite observers will be moving towards him and at a right angle to them two other similarly situated observers will be moving away from him. The motion of the ring of observers is that of a pulsating ellipse oscillating from a condition of maximum eccentricity, through a circle, to minimum lateral extent, and then reversing. The area of the ring will remain constant; only its eccentricity will change. The observers will find that their displacement is transverse to the direction of propagation of the wave. For any such wave the observers' motions can be described as a linear superposition of two elliptical figures, one rotated 45° relative to the other. These represent the two states of linear polarization of a gravitational wave. As with an electromagnetic wave states of circular polarizations can be defined by retarding one state of a linearly polarized wave a quarter wavelength and adding it to the other linearly polarized state. The result is an ellipse that appears to rotate either clockwise or counterclockwise.

GRAVITATIONAL WAVE DETECTORS

In practice the strain induced in the spacetime geometry by a wave is measured by the displacement it induces in either an elastic bar or the change in optical path length of an interferometer.⁵ In either case the length to be measured is infinitesimal. For the expected gravity wave, strains of 10^{-17} from astrophysical sources the displacement of a one meter long aluminum bar is one hundredth the diameter of an atomic nucleus! This measurement has meaning only in the statistical sense of making position measurements averaged over the $\sim 10^{23}$ atoms on the face of the bar. Gravitational waves of kilohertz frequencies with strains on the order of 10^{-21} are still highly classical fields, however; the antenna systems being built to detect them are highly quantum mechanical systems. In an aluminum bar at a few millidegrees above absolute zero the first level of oscillation of the lowest normal mode of the bar has an energy of the same size as that associated with the displacement induced by a gravitational wave of strain amplitude 10^{-21} . This is referred to as the (apparent) "quantum limit of detectability". It can be circumvented by a more careful choice of measurement strategies than a simple position measurement.

The weakness of the interaction of a gravitational wave with an antenna and the subsequent attempts to make measurements on the antenna force us into treating the antenna as a quantum mechanical object. To an excellent approximation the antenna can be regarded as a simple harmonic oscillator that is driven by a classical force (the gravity wave) and by Nyquist forces (i.e. thermal noise). The

Nyquist forces may be quantified by the "Q" of the particular normal mode we are observing. This measures the coupling of this particular mode to all the other modes in terms of how quickly this mode loses energy. By using bars of crystalline material (e.g. sapphire) of good quality and by operating at cryogenic temperatures the Nyquist forces can (at least in principle) be made negligible. (This is a major technological feat and should not be underestimated!) The idealized model to be examined then consists of a quantum mechanical harmonic oscillator interacting with a classical force and a measuring apparatus. The interaction of the oscillator with the measuring apparatus will necessarily disturb the oscillator (for example a position measurement will impart an uncertainty in the oscillators' momentum) and depending upon the disturbance generally contaminate the result of any subsequent measurement. This appears as an irreducible "noise" produced by the quantum nature of the oscillator-measurement apparatus interaction. The key to its elimination is in the measurement interaction. Instead of measuring the position which disturbs the momentum and the subsequent evolution of the oscillator's motion, we enlarge the class of possible measurements by allowing time-dependent coupling of the measurement apparatus to the oscillator. By using a position transducer and a velocity transducer and modulating their couplings to the oscillator, we can arrange to couple to a dynamical variable of the oscillator that is not disturbed by the back reaction of the measurement interaction. It is in principle possible to measure in detail the excitation of the oscillator by the gravitational wave and the quantum mechanical nature of the oscillator is not a limiting principle. The study of these ideas has collected the name "quantum nondemolition measurements" and is elaborated in detail in reference 6.

FINAL COMMENTS

In addition to radiative solutions, the flat-spacetime background expansion also has solutions that correspond to magnetic like gravitational effects and weak corrections to Newton's equation for the gravitational field. These "non-Newtonian" fields provide additional interesting tests of general relativity. Experiments to detect gravitational waves and to measure departures from Newtonian gravity require a reasonable amount of isolation as would be offered by an underground laboratory. The specific type of disturbance and amount of isolation required are very experiment dependent. These are discussed in detail in the following papers.

REFERENCES

1. William Gilbert, On the Loadstone and Magnetic Bodies, translation into English contained in The Great Books (Univ. of Chicago Press, Chicago, 1952).
2. L. D. Landau and E. M. Lifshitz, The Classical Theory of Fields, 4th Ed. (Pergamon Press, N.Y. 1975).
3. C. W. Misner, K. S. Thorne, and J. A. Wheeler, Gravitation, (W. H. Freeman and Co, San Francisco, 1973).
4. S. Weinberg, Gravitation and Cosmology Principles and Applications of the General Theory of Relativity (John Wiley and Sons, N.Y., 1972).
5. L. Smarr, Editor, Sources of Gravitational Radiation, (Cambridge University Press, Cambridge, 1979).
6. C. M. Caves et al., "On the Measurement of a Weak Classical Force Coupled to a Quantum - Mechanical Oscillator I," Rev. Mod. Phys. 52, 341 (1980).