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UNIVERSAL SCALING FUNCTION FOR FEL GAIN*

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Abstract

We have developed an analytic calculation of FEL gain in the exponential regime taking into account the finite emittance, energy spread, focusing and betatron oscillation of the electron beam, and the diffraction and guiding of the radiation. The gain is expressed in terms of a universal scaling function with only three independent parameters. Excellent agreement is found with results of numerical simulation.

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We have recently developed an analytic calculation [1] of the FEL gain valid in the regime of exponential growth before saturation, based upon a dispersion relation [2,3] derived from the Vlasov-Maxwell equations. Our treatment includes all of the effects of energy spread, emittance, and focusing of the electron beam, and the diffraction and guiding of the radiation. The dispersion relation is solved using a variational approximation [4]. With a reasonable choice of frequency detuning yielding near maximum gain, we have shown that the e-folding length is a universal function of three dimensionless variables (emittance to wavelength ratio, scaled electron focusing strength, and scaled electron energy spread), and a dimensionless parameter measuring transverse current. The importance of dimensionless scaling variables was discussed in refs. [5] and [6]. Our scaling relations [1] are new results, and we have presented the first calculation of the universal scaling function determining FEL gain. Some graphs of the universal function are given in Figs. 1a,b,c. These graphs can be used for quick estimates of the gain for quite general parameters.

Until now, only large simulation codes [7,8,9] requiring long CPU running times on fast computers could include all the effects determining FEL gain. Our analytic approach has several advantages: rapid computation time, elucidation of the dependence of the gain on the large number of system parameters, and the ease of design optimization. We have checked that the variational approximation yields agreement to within 1% with exact results [10] which we have derived for a parallel electron beam with finite beam size and energy spread. In this note, we present a comparison of our analytic results with several cases with nonvanishing emittance run on the simulation codes FELEX and TDA. Agreement of the calculated gain length is found to within a few percent.

In our calculation, the electron beam's energy distribution $h(\gamma)$ is Gaussian, with average energy $\gamma_0 mc^2$, and rms spread $\gamma_0 \sigma$. The static wiggler magnetic field has period length λ_w , wavenumber $k_w = 2\pi/\lambda_w$. The resonant radiation frequency $\omega_r = k_r c$ of the FEL is determined by $k_r = 2\gamma_0^2 k_w / (1 + K^2)$, where $K = eB_{rms}/k_w mc$ and B_{rms} is the rms value of the wiggler magnetic field (mks units). We assume either a helical wiggler, or a flat wiggler with parabolic pole faces. After averaging over the fast wiggler oscillations, the transverse electron motion is described by harmonic betatron oscillations [11] in the transverse displacement $d^2\bar{R}/dz^2 = -k_\beta^2 \bar{R}$. In the absence of external focusing, the betatron wavenumber [11] $k_\beta = Kk_w/(\gamma\sqrt{2})$. Natural focusing does not cause longitudinal velocity modulation. For external focusing, we ignore the longitudinal velocity modulation.

Initially, we assume the electron beam has a uniform longitudinal density, and a uniform "waterbag" distribution inside a four-dimensional sphere in the four-dimensional transverse phase space $\bar{R} = (x, y)$, $\bar{R}' = d\bar{R}/dz = (x', y')$:

$$U(\bar{R}, \bar{R}') = \frac{n_0}{\pi k_\beta^2 R_o^2} \theta(k_\beta^2 R_o^2 - k_\beta^2 R^2 - R'^2), \quad (1)$$

where the step function $\theta(v) = 1$ for $v > 0$ and $\theta(v) = 0$ for $v < 0$. Integrating $U(\bar{R}, \bar{R}')$ over the angular deviation $\bar{R}'$, one obtains the parabolic transverse density profile:

$g(\bar{R}) = n_0(1 - R^2/R_o^2)$ for $R < R_o$, and $g(\bar{R}) = 0$ for $R > R_o$. The peak electron density is n_0 , and the electron beam current is $I_0 = en_0 c \pi R_o^2 / 2$. The rms transverse emittance ϵ of the matched electron beam is defined by

$$\epsilon = (\langle x^2 \rangle \langle x'^2 \rangle)^{1/2} = (\langle y^2 \rangle \langle y'^2 \rangle)^{1/2} = k_\beta R_o^2 / 6. \quad (2)$$

We consider the linear region before saturation, and write the electric field of the fundamental guided laser mode of frequency ω in the form:

$$E(\vec{R}, \omega) e^{-i\mu k_w z} e^{-i\omega(t-z/c)\hat{e}} + c.c., \quad (3)$$

where $\hat{e}$ is either the helical polarization vector $(\hat{x} + i\hat{y})/\sqrt{2}$, or the linear polarization vector $\hat{x}$. The function $E(\vec{R}, \omega)$ describes the transverse mode profile. The factor $\exp(-i\mu k_w z)$ describes the deviation from free space propagation and $2\pi \text{Im}\mu$ is the growth rate per wiggler period. The e-folding length L_G of the power of the amplified mode is given by

$$\frac{1}{2L_G} = k_w \text{Im}\mu \quad (4)$$

Assuming $\gamma_o \gg 1$, so space charge effects are negligible, Vlasov-Maxwell equations have been used to derive a dispersion relation [2] determining μ and $E(\vec{R})$. From this dispersion relation, we have shown [1] that the e-folding length L_G of the amplified power can be expressed in the scaled form:

$$\frac{1}{2k_w L_G} = \text{Im}\mu = DG(k_r, \epsilon, \frac{\sigma}{D}, \frac{k_p}{k_w D}, \frac{\omega - \omega_r}{\omega_r D}). \quad (5)$$

The scaling parameter D is a measure of the transverse electron current,

$$D = \left(\frac{4eZ_o}{\pi mc^2} \frac{K^2}{1+K^2} \frac{I_o}{\gamma_o} \right)^{1/2} [JJ], \quad (6)$$

where $Z_o = 377\Omega$ is the impedance of free space and $[JJ] = 1$ for a helical wiggler and for a flat wiggler,

$$[JJ] = J_0(K^2/2(1+K^2)) - J_1(K^2/2(1+K^2)).$$

Utilizing the variational approximation, we have determined [1] the universal gain function $G = \text{Im}\mu/D$. Because of the scaling law, the entire physical parameter space can be described by the dependence of G on four dimensionless scaled variables. In Figs. 1a,b,c we plot $G = \text{Im}\mu/D$ against $2k_r\epsilon = 4\pi\epsilon/\lambda_r$, for several values of the scaled focusing strength $k_\beta/(k_w D)$, corresponding to scaled energy spread (a) $\sigma/D = 0$, (b) $\sigma/D = 0.1$, (c) $\sigma/D = 0.2$. For each point $(2k_r\epsilon, k_\beta/(k_w D), \sigma/D)$, the scaled detuning is chosen to be

$$\frac{\omega - \omega_r}{\omega_r D} = -3\left(\frac{k_\beta}{k_w D}\right)k_r\epsilon, \quad (7)$$

which we have found to yield near maximum gain. This detuning corresponds to the reduction of the average longitudinal velocity due to the nonvanishing emittance and focusing by the wiggler. Figs. 1a,b,c cover most of the practical range of the FEL parameters, and they can be used to obtain quick estimates of the e-folding length.

To check the accuracy of our results, we have compared our calculation with several simulation codes [7,8,9]. In Table 1, we compare our calculation with FELEX, as calculated by J. Goldstein [12]. The parameters for case 1 are the following: $\gamma_0 = 362$, normalized rms emittance $\epsilon_n = 7.74$ mm-mrad, rms relative energy spread $\sigma = 6.6 \times 10^{-4}$, current $I = 300$ Amp, the peak on-axis magnetic field $B_w = 0.75$ Tesla, wiggler period $\lambda_w = 1.6$ cm. The natural betatron wavelength for these parameters is $\lambda_\beta = 10.3$ m, and the matched edge electron beam size is 0.459 mm. The radiation wavelength is $\lambda = 995.4$ Å. Our result for power gain length is $L_G = 108$ cm, as compared to 105 cm, given by J. Goldstein. The case 2 is different from case 1 in that the normalized emittance is doubled, and correspondingly the gain length is

increased to 271 cm, as compared to the simulation value 268 cm. The case 3 is obtained from case 1 by doubling the energy spread, and the gain length is 164 cm, as compared to the simulation value 166 cm. In cases 4 and 5, the betatron wavelength has been decreased by a factor of 2 and 3 respectively, relative to the case 1, and for both cases the gain lengths only change slightly. Table 1 shows our results agree with simulation to within a few percent.

In Fig. 2, we compare the power gain length for different detuning from case 1. The simulation used the code TDA as provided by J. Wurtele from MIT [9]. For small deviation from the minimum gain length, the agreement is within a few percent. For very large detuning from the minimum gain length, it becomes difficult to determine the gain length by simulation, because it is difficult to identify the domination of a single mode. The errors become larger. The figure also shows that Eq. (7) accurately gives the detuning for minimum gain length.

An alternate form of the scaling relation is:

$$\frac{1}{2k_w L_G} = \text{Im} \mu = \rho F(\tilde{a}, \frac{\sigma}{\rho}, \frac{k_p}{k_w \rho}, \frac{\omega - \omega_r}{\omega_r \rho}), \quad (8)$$

which follows from Eq. (6) upon noting that $D = 2\rho\tilde{a}$, where ρ is the Pierce parameter [13,1] corresponding to the peak density n_0 , and the scaled electron beam radius $\tilde{a}$ is defined by [14],

$$\tilde{a}^2 = 2\rho(2k_w k_r R_0^2) = 24(k_r \epsilon)(\rho k_w / k_p). \quad (9)$$

Up to a multiplicative constant $\tilde{a}^2 \propto k_r R_0^2 / L_G^{1D}$, the ratio of the Rayleigh range to the one-dimensional gain length $L_G^{1D} \propto (2\rho k_w)^{-1}$.

For a parallel electron beam with parabolic transverse density profile:

$g_o(R) = n_o(1-R^2/R_o^2)$ for $R < R_o$ and $g_o(R) = 0$ for $R > R_o$, we have obtained [10] exact results as a function of electron beam size and energy spread (Gaussian distribution). We have compared the variational approximation for the universal scaling function of Eq. (12) with the exact results. In Fig. 3, we plot the scaled growth rate $\text{Im}\mu/2\rho$ versus the scaled energy spread $\sigma/2\rho$, for the scaled electron beam radius $\tilde{a} = 1$ and zero detuning $\omega = \omega_r$. The dashed line is the exact solution derived in ref. [10] using confluent hypergeometric functions. The solid line is calculated from the variational approximation. The agreement is good to about 1%. The same good agreement holds for the full range from large to small beam size ($\tilde{a} \gg 1$ to $\tilde{a} \ll 1$).

The limit of a parallel beam corresponds to $k_\rho \rightarrow 0$, $\epsilon \rightarrow 0$ with $\epsilon/k_\rho = R_o^2/6$ finite. The gain length goes over to the one-dimensional value for $\tilde{a}$ large.

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DISCLAIMER

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References

1. L.H. Yu, S. Krinsky and R.L. Gluckstern, Phys. Rev. Lett. 64, 3011 (1990).
2. L.H. Yu and S. Krinsky, Phys. Lett. A129, 463 (1988).
3. A dispersion relation of this type was considered by K.J. Kim, Phys. Rev. Lett. 57, 1871 (1986).
4. M. Xie and D.A.G. Deacon, Nucl. Instrum. Methods A250, 426 (1986).
5. C. Pellegrini, Nucl. Instrum. Methods A272, 364 (1988).
6. W.A. Barletta and A.M. Sessler, in Proc. INFN International School on Electromagnetic Radiation and Particle Beam Acceleration, Varenna, Italy (1988).
7. J.C. Goldstein, T.F. Wang, B.E. Newnam and B.D. McVey, Proc. 1987 Particle Accelerator Conference, Washington, D.C., p. 202.
8. E.T. Scharlemann and W.M. Fawley, Proc. SPIE 642, 2 (1986).
9. T.M. Tran and J.S. Wurtele, Computer Phys. Comm. 54, 263 (1989).
10. S. Krinsky, L.H. Yu and R.L. Gluckstern, unpublished.
11. E.T. Scharlemann, J. Appl. Phys. 58, 2154 (1985).
12. J.C. Goldstein, private communication.
13. R. Bonifacio, C. Pellegrini and L.M. Narducci, Opt. Commun. 50, 373 (1984).
14. S. Krinsky and L.H. Yu, Phys. Rev. A35, 3406 (1987).

Table 1. Gain Lengths Calculated by Simulation and Solving Dispersion Relation

	<u>FELEX (J. Goldstein)</u>	<u>Dispersion Relation</u>
Case 1	105 cm	108 cm
Case 2 ($\epsilon_n \times 2$)	268 cm	271 cm
Case 3 ($\sigma \times 2$)	166 cm	164 cm
Case 4 ($\lambda_\beta/2$)	107 cm	104 cm
Case 5 ($\lambda_\beta/3$)	115 cm	108 cm

Figure Captions

- Fig. 1. Scaling function vs. scaled emittance for several values of $k_\beta/(k_w D)$, corresponding to scaled energy spreads (a) $\sigma/D = 0$, (b) $\sigma/D = 0.1$, (c) $\sigma/D = 0.2$, with the detuning as given in Eq. (7).
- Fig. 2. Comparision of power gain lengths calculated from dispersion relation and from the TDA simulation code for the case 1 of Table 1, as a function of detuning.
- Fig. 3. Comparison of scaled growth rate $\text{Im}\mu/2\rho$ calculated from the variational approximation (solid curve) and from the exact solution (dashed curve).

Fig. 1c

Fig. 2

Fig. 3

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