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TITLE ANALYTICAL AND NUMERICAL SOLUTION OF ONE- AND TWO-DIMENSIONAL
STEADY HEAT TRANSFER IN A COLDPLATE

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ANALYTICAL AND NUMERICAL SOLUTION OF ONE- AND TWO-DIMENSIONAL STEADY HEAT TRANSFER IN A COLDPLATE

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Abstract

We develop analytical models for steady-state, one- and two-dimensional heat transfer in a single-material, flat-plate coldplate. Discrete heat sources are mounted on one side of the plate and heat transfer to a flowing fluid occurs on the other. The models are validated numerically using finite differences. We propose a simple procedure for estimating maximum coldplate temperature at the location of each heat source which includes thermal interaction among the sources. Results from one model are compared with data obtained for a composite coldplate operated in the laboratory. We demonstrate the utility of the models as diagnostic tools to be used for predicting the existence and extent of void volumes and delaminations in the composite material that can occur with coldplates of this type. Based on our findings, recommendations for effective coldplate design are given.

Nomenclature

a Coefficient for heat generation function, equation (8)
 d coldplate thickness
 D thickness for fluid-flow channel
 f heat generation function, equation (8)
 F Dimensionless temperature distribution in heated part of coldplate
 $\bar{h}$ average convective heat transfer coefficient
 H term defined in equation (15c)
 k thermal conductivity
 L_1 smallest length between two parallel boundaries for a single element
 L_2 largest length between two parallel boundaries for a single element
 $m_1 = L_1 \sqrt{\bar{h}/k_p d}$
 $m_2 = L_2 \sqrt{\bar{h}/k_p d}$
 $\bar{Nu}$ average Nusselt number, $\bar{h}D/k_f$
 Pr Prandtl number
 q heat flux, $Q/t_1 t_2$
 Q heat flow rate
 Re Reynolds number, $\bar{u}D/2$
 t_1 length of heat source measured in direction of L_1
 t_2 length of heat source measured in direction of L_2
 T temperature
 $\Delta T = T - T_{f1}$
 $\bar{u}$ average fluid flow speed
 x coordinate measured from center of heat source in direction of L_1
 y coordinate measured from center of heat source in direction of L_2
 u ratio of heat transfer coefficient of gap to $\bar{h}$
 β term defined in equation (15b)
 $\gamma_1 = t_1/L_1$
 $\gamma_2 = t_2/L_2$

$\eta = x/L_1$
 ν kinematic viscosity for fluid
 $\xi = y/L_2$
 cc heat capacity
 $c = T - T_{f1}/(q/\bar{h})$

Subscripts

e edge
 f fluid
 $f1$ fluid at inlet to coldplate
 l linear heat generation
 m maximum value
 p plate
 u uniform heat generation
 $1d$ from one-dimensional model
 $2d$ from two-dimensional model

Superscript

$*$ refers to non symmetrical coldplate element

1. Introduction

Thermal analysis of electronic components and their supporting hardware has developed considerably over the past decade. The driving force for this development is the increase in component heat-flux caused first, by reduced area for heat transfer from the component to the thermal sink and second, by increased component power dissipation. The development of new, more effective heat transfer processes has not kept pace, and heat transfer by conventional forced or natural convection continues to be the most commonly used ultimate means for cooling. Consequently, junction temperatures have increased significantly. Improved methods for accurately predicting temperature distributions are necessary to ensure component reliability, performance, and lifetime at these elevated temperatures, and for optimization of electronic systems design.

Several approaches have been used for predicting temperature distributions of electronic components and supporting hardware including analytical¹⁻³, numerical⁴⁻⁶, and semi-analytical⁷⁻⁹ methods. The analytical and semi-analytical approaches have the advantage of generally being less time consuming and simpler to implement than the numerical approach since fine discretization is not necessary and computer run times are relatively small. On the other hand, analytical and semi-analytical methods often suffer from lack of generality and sometimes accuracy if applied out of their range of applicability. If the problem being considered is naturally restrictive or if extreme accuracy is not required, as is the case for preliminary design calculations, the analytical approach offers many advantages over the others.

We develop analytical models for steady-state, one- and two-dimensional heat transfer in a single-material, flat-plate coldplate. Discrete heat sources are mounted on one side of the plate and heat transfer to a flowing fluid occurs on the other. The models are validated numerically using finite differences. We propose a simple procedure for estimating maximum coldplate temperature at the location of each heat source which includes thermal interaction among the sources. Results from one model are compared with limited data obtained for a composite coldplate operated in the laboratory. We demonstrate the utility of the models as diagnostic tools to be used for predicting the existence and extent of gas bubbles and de-laminations in the composite material that can occur with coldplates of this type.

11. Analysis

One-Dimensional Conjugate Fin

The problem at hand is shown in Fig. 1. It consists of a flat-plate coldplate having a number

Figure 1. Coldplate geometry

of two-dimensional discrete heat sources on the top side and heat transfer to a flowing fluid on the other. We assume that the plate is homogeneous and isotropic, and that the edges of the plate, bottom of the flow channel, and the areas between the heat sources on the top of the plate are insulated. The coldplate is divided into a number of parts referred to as "elements", each containing a single heat source located at the element center. Presently, we assume that the element boundaries are insulated, but later we include a feature that accounts for thermal interaction among elements having sources with different heat fluxes. The boundary of a typical element is shown as the dotted line in Fig. 1. We describe a procedure for constructing the boundaries of each element in a later section.

Consider first the simple case of a coldplate element having a heat source as shown in Fig. 2. The plate thickness, d , is taken to be smaller than t_1 so that we may approximate the coldplate as a fin. L_1 is much larger than the heat-source length in the x direction (t_1) so heat conduction in the coldplate is purely one dimensional in this direction. The variable heat generation rate, Q , is exactly balanced by convective heat transfer from plate temperature T_p to fluid temperature T_f

Figure 2. Coldplate element having a long heat source.

through the constant heat transfer coefficient $\bar{h}$. A steady energy balance on the plate is written in dimensionless form as

$$\frac{1}{\pi_1} \frac{d^2 \xi}{dx^2} - (\xi_{pid} - \xi_f) + \Gamma^* f(\eta^*/\gamma_1) = 0, \quad (1)$$

$$\text{where } \Gamma^* = \begin{cases} 1, & \frac{1}{2}(1-\gamma_1) \leq \eta^* \leq \frac{1}{2}(1+\gamma_1) \\ 0, & 0 \leq \eta^* < \frac{1}{2}(1-\gamma_1) \text{ and } \frac{1}{2}(1+\gamma_1) < \eta^* \leq 1 \end{cases} \quad (1)$$

and for the fluid

$$\frac{d\xi_f}{d\eta} - St(\xi_{pid} - \xi_f) = 0 \quad (2)$$

The variable π_1 is the usual fin parameter that characterizes heat transfer in a fin.

$$\pi_1^2 = \frac{\bar{h} L_1^2}{k_p d} \quad (3)$$

π_1^2 is the ratio of the convective conductance between the plate and the fluid, to the conductive conductance in the plate. A large value of π_1^2 leads to large temperature gradients in the plate with a rapid approach to the fluid temperature at a short distance from the edge of the heat source. Conversely, a small value of π_1^2 produces plate temperatures that are nearly uniform.

The variable St is a modified Stanton number that characterizes the temperature distribution in the fluid.

$$St = \frac{\bar{h} L_1}{(\rho c)_f \bar{U} D} = \frac{\bar{Nu}}{Re Pr} \left(\frac{L_1}{D} \right) \quad (4)$$

It is the ratio of the convective conductance from the plate to the fluid, to the rate of convection in the direction of fluid flow. Alternatively, it may be interpreted as the ratio of the characteristic temperature difference in the fluid, to the characteristic temperature difference in the plate; both measured in the direction of fluid flow.

The boundary conditions for the fluid and plate are

$$c_f(r^*=0)=0, \quad \left. \frac{dc_f}{dr^*} \right|_{r^*=0} = 0 = \left. \frac{dc_p}{dr^*} \right|_{r^*=1} \quad (5)$$

Also, because this is a conjugate heat transfer problem, the temperatures and heat fluxes at the two interfaces between the heated and unheated parts of the plate are matched (i.e., at $\eta=(1-\gamma_1)/2$ and $\eta=(1+\gamma_1)/2$).

The fluid temperature from equation (2) is written in integral form and combined with equation (1) to give a single integro-differential equation for the plate temperature.

$$\frac{1}{\pi^2} \frac{d^2 c_p}{dr^{*2}} - c_p + \Gamma^* f(r^*/\gamma_1) = -St e^{-St\eta^*} \int_0^{\eta^*} St c_p(\zeta) d\zeta \quad (6)$$

where the values for Γ^* are the same as in equation (1).

Consider the range of values for St . Conservatively, for a water flow rate through the coldplate channel of 0.1 gallons per min. per foot of coldplate width, and h corresponding to fully developed flow in a channel, St varies between 0.001 and 0.1 for a wide range of L_1/D . The value of St becomes smaller with increased fluid flow rate, decreased fluid inlet temperature (through Pr), and decreased $\bar{h}$, all of which are possible to occur in practice. Similar results are obtained for large air flow rates. Small St indicates an insignificant fluid temperature increase relative to the temperature rise in the plate (in the limit of zero St , a fluid temperature gradient of zero is predicted from solution of equation (2)). Thus heat transfer from the plate to the fluid occurs as if the fluid is at constant temperature equal to the inlet temperature. From the standpoint of coldplate thermal performance, it is desirable to maintain small St since increasing fluid temperature in the direction of flow causes elevated component temperatures downstream from the coldplate inlet.

After invoking symmetry about the heat source center-line as a consequence of small St , equation (6) becomes

$$\frac{1}{\pi^2} \frac{d^2 c_p}{dr^{*2}} - c_p + \Gamma f(2r^*/\gamma_1) = 0, \quad (7)$$

$$\text{where } \Gamma = \begin{cases} 1 & , 0 \leq \eta < \frac{\gamma_1}{2} \\ 0 & , \frac{\gamma_1}{2} < \eta \leq \frac{1}{2} \end{cases}$$

where we note that the problem depends upon only two parameters, π , and the aspect ratio of the heat source γ_1 .

We consider two distributions for the heat generation rate. The first is uniform heat generation, referred to as UHG, and the second (LHG) is one that decreases linearly from a maximum at the center of the source to zero at its edge. Both distributions are shown in Fig. 3.

Figure 3. Heat generation distributions considered. UHG is uniform heat generation, LHG is linear heat generation.

The LHG case is included because it was suggested that this and the case for quadratic heat generation distribution are important. The quadratic case is more difficult to include in the analytical model than the LHG case. The LHG case considered here produces smaller maximum coldplate temperatures than the quadratic case, whereas the UHG case produces larger maximum coldplate temperatures than either one. Thus, the UHG and LHG cases bracket the quadratic case so that it is not necessary to consider a quadratic heat generation profile at this time. The heat generation function is

$$f(\eta/\gamma_1/2) = 1 + \frac{2a}{\gamma_1} \eta \quad (8)$$

$$f(\xi/\gamma_2/2) = 1 + \frac{2a}{\gamma_2} \xi$$

where $a=0$ for UHG, and $a=-1.0$ for LHG.

The solution of equation (7) obtained for the heated part of the plate is designated as F_1

$$F_1(r) = 1 + \frac{2a}{\gamma_1} - \frac{2a}{\gamma_1 m_1} \left(\sinh m_1 r - \frac{\cosh m_1 r}{\sinh \frac{m_1}{2}} \left[\cosh \frac{m_1}{2} - \cosh \frac{m_1}{2} (1 - \gamma_1) \right] \right) - \frac{(1+a) \sinh \frac{m_1}{2} (1 - \gamma_1) \cosh m_1 r}{\sinh \frac{m_1}{2}} \quad (9)$$

For the case of UHG, $a=0$ and equation (9) becomes

$$F_U(r) = 1 - \frac{\sinh \frac{m_1}{2} (1 - \gamma_1) \cosh m_1 r}{\sinh \frac{m_1}{2}} \quad (10)$$

The maximum temperature occurs at $r=0$ directly beneath the heat source and is written as

$$\phi_{mid} = 1 + \frac{2a}{\gamma_1 m_1} \frac{\cosh \frac{m_1}{2} - \cosh \frac{m_1}{2} (1 - \gamma_1)}{\sinh \frac{m_1}{2}} - (1 + a) \frac{\sinh \frac{m_1}{2} (1 - \gamma_1)}{\sinh \frac{m_1}{2}} \quad (11)$$

for LHG. For the UHG case, equation (11) applies with $a = 0$.

Dimensionless maximum temperatures from equation (11) are shown in Figs. 4 (for the UHG case), and 5 (for LHG) for ranges of m_1 and γ_1 values. Also

Figure 4. Dimensionless maximum coldplate temperature as a function of γ_1 ($=t_1/L_1$) with m_1 as a parameter for UHG. One-dimensional heat conduction.

Figure 5. Dimensionless maximum coldplate temperature as a function of γ_1 ($=t_1/L_1$) with m_1 as a parameter for LHG. One-dimensional heat conduction.

shown in the figures are numerical solutions obtained to verify the analytical solution. Details of the numerical method are given in a later section. In all cases, agreement between the analytical and numerical solutions is excellent. For the UHG case, we note from Fig. 4 that the maximum plate temperature ranges between 1 and γ_1 corresponding to the limiting cases of infinite m_1 and zero m_1 , respectively. Infinite m_1 occurs when the ratio of plate conductance to fluid conductance approaches zero, whereas for zero m_1 the ratio is large. From the definition of ϕ_{mid} we get

$$\frac{Q}{L_2} = \frac{1}{\phi_{mid}} \bar{h} t_1 [T_m - T_{f1}] \quad (12a)$$

Thus, ϕ_{mid} is a "plate-effectiveness factor", where $\gamma_1 \leq \phi_{mid} \leq 1$. For $\phi_{mid} = 1$, equation (12a) becomes

$$\frac{Q}{2L_2} = \frac{\bar{h} t_1}{2} [T_m - T_{f1}] \quad (12b)$$

which shows that heat transfer occurs through only the area of the heat source ($L_2 t_1/2$) and not from the unheated part of the plate. The lack of sensitivity of maximum temperature to γ_1 for large values of m_1 is consistent with this conclusion. For $\phi_{mid} = \gamma_1$, equation (12a) is

$$\frac{Q}{2L_2} = \frac{\bar{h} L_1}{2} [T_m - T_{f1}] \quad (12c)$$

indicating that all of the plate area ($L_2 L_1/2$) contributes to the heat transfer. This result is reflected in the fact that plate temperatures are lower for this limit than for the other. For the LHG case shown in Fig. 5, we see evidence of trends similar to those for UHG. However, maximum plate temperatures are smaller for LHG than for UHG because of smaller rates of total heat generation.

Two-Dimensional Conjugate Fin

Consider the element shown in Fig. 1. Because it is small, thermal symmetry exists about the center of the heat source and we need only consider one quadrant of the element (Fig. 6). The steady-state energy balance for this geometry is

$$\frac{1}{2} \left\{ \frac{\partial^2 \phi_{2d}}{\partial \eta^2} + \left(\frac{L_1}{L_2} \right)^2 \frac{\partial^2 \phi_{2d}}{\partial \xi^2} \right\} - \phi_{2d} + \Gamma(2\eta/\gamma_1) f(2\xi/\gamma_2) = 0 \quad (13)$$

where, as for the one-dimensional fin, Γ is 1 for any point within the heated part of the plate and zero elsewhere. The heat generation function is taken to be a product of $f(2\eta/\gamma_1)$ and $f(2\xi/\gamma_2)$ where f is defined in equation (8).

Figure 6. One quadrant of coldplate element bounded by dotted lines in Fig. 1.

For two-dimensional heat conduction, we see in equation (13) that the element aspect ratio, L_1/L_2 , enters into the problem as a coefficient of the conduction term in the ξ direction. In addition, the aspect ratio of the heat source, γ_2 , becomes a boundary variable in the ξ direction. Equation (13) can be solved exactly by classical methods by dividing the geometry shown in Fig. 6 into four sub-regions, solving for the temperature distribution in each sub-region in terms of a Fourier series, and evaluating the coefficients of each series by matching temperatures and heat fluxes across the boundaries of each sub-region. Rather than following this cumbersome approach, we attempt an approximate solution. We assume that the element aspect ratio (L_1/L_2) is less than 1 so that, to first order, the area for heat conduction from sub-region II to IV through the plate across line a-b may be neglected in favor of conduction from sub-region III to IV across line a-c. Under this assumption, the temperature distribution in sub-region II becomes quasi one dimensional, i.e., one-dimensional in the η direction with its

dependence on ξ imposed by the temperature distribution in sub-region I.

Except for L_1/L_2 very close to one, this approximation is expected to be quite accurate and the results obtained from the model will show this to be true when compared with detailed numerical solutions. For L_1/L_2 greater than 1, the coordinates of the element are simply interchanged to produce a ratio of less than 1. Thus, L_1 is always taken to be the smaller length of the element.

Temperature distributions in sub-regions I, III, and IV are then obtained by solving equation (13) for each and by matching temperatures and heat fluxes across the boundaries of two connected sub-regions. Each temperature distribution can be written as a separable function of η and ξ because of the separable heat generation function, and because the outside boundary conditions of the element are not functions of η or ξ .

The maximum dimensionless temperature occurs at η and ξ equal to zero and is written

$$\phi_{m2d} = F_U(0) [1 - H] \quad (14a)$$

where

$$G(\eta) \equiv 1 + F_U(\eta) \frac{\tanh \frac{m_2}{2} (1 - \gamma_2)}{\tanh \frac{m_2 \gamma_2}{2}} \quad (15a)$$

$$\beta \equiv 1 + \frac{\tanh \frac{m_1 \gamma_1}{2}}{\tanh \frac{m_1}{2} (1 - \gamma_1)} \quad (15b)$$

$$H \equiv \frac{\tanh \frac{m_2}{2} (1 - \gamma_2) \left[F_U(0) - \frac{F_U(\gamma_1/2)}{[1 + \beta G(\gamma_1/2)] \cosh \frac{m_1 \gamma_1}{2}} \right]}{G(0) \sinh \frac{m_2 \gamma_2}{2}} \quad (15c)$$

and $F_U(0)$ is from equation (10). We note that equation (14a) is just the product of the solution for one-dimensional heat conduction and a term that accounts for conduction in the ξ direction, $(1 - H)$, which is one or less. Both $F_U(0)$ and H are based on uniform heat generation from the source. Rather than developing a different expression for the LMG case, we attempt to apply equation (14a) to this case by replacing $F_U(0)$ with $F_1(0)$

$$\phi_{m2d} = F_1(0) [1 - H] \quad (14b)$$

where $F_1(0)$ is from equation (9). Thus, equation (14a) applies to uniform heat generation, and equation (14b) to linear heat generation.

III. Numerical Solution

The geometry shown in Fig. 6 was divided into 50 nodes in both the x and y directions. The nodes in any direction were equally spaced, but the node spacing in the x and y directions differed depending on the element aspect ratio. Equation (13) was solved for each internal node by successive over-relaxation where the relaxation factor was varied between 1.25 and 1.9 interactively to accelerate convergence. Convergence was checked at the origin and at nodes surrounding the interface between the heated and unheated portions of the plate. Convergence occurred when temperatures at the same node agreed to within 0.1 percent for two successive iterations. About 150 iterations were required for convergence for cases having small m_1 values (5 and smaller) whereas for large m_1 values (60 and greater), only about 30 iterations were necessary. The reason for this is that the entire plate is involved in the heat transfer for small m_1 whereas only the area directly beneath the heat source participates for large m_1 as discussed above.

IV. Results and Discussion

Uniform Heat Generation (UHG)

A comparison among results from the analytical (equation (14)) and numerical solutions for two-dimensional heat conduction in a coldplate are shown in Figs. 7-9. The figures show the ratio of

Figure 7. Dimensionless maximum coldplate temperature for UHG. Two-dimensional heat conduction.

maximum dimensionless plate temperature, Φ_{m2d} to γ_1 as a function of γ_1 for m_1 values of 5, 20 and 60, and for fixed values of L_1/L_2 and γ_2 . The ratio Φ_{m2d} to γ_1 does not contain γ_1 explicitly. Also shown in the figures is a limiting-case solution for large m_1 wherein all heat transfer occurs between the plate area directly beneath the heat source and the fluid. For this case $Q = h t_1 t_2 (T_m - T_{f1})$ so that $\Phi_{m2d}/\gamma_1 = 1/\gamma_1$. For comparison, we also show the solution from the one-dimensional conjugate fin analysis (equation (11) with $a=0$) under the label "Simple Analytical Sol'n."

Figure 8. Dimensionless maximum coldplate temperature for UHG. Two-dimensional heat conduction. Simple analytic and complete analytic solutions coincide.

Figure 9. Dimensionless maximum coldplate temperature for UHG. Two-dimensional heat conduction.

First, consider the results for L_1/L_2 of 0.9 and γ_2 of 0.5 shown in Fig. 7. For all values of m_1 , Φ_{m2d} decreases with increasing γ_1 , an effect that is more pronounced at large m_1 . This reflects the near-unity value of Φ_{m2d} as seen in Fig. 4 for large m_1 and a wide range of γ_1 . For $\gamma_1 > 0.6$, maximum temperatures are only weakly sensitive to m_1 and are nearly independent of γ_1 for all m_1 . The same trend is expected for $\gamma_2 > 0.6$ because of geometric similarity. This result is explained by recognizing that as the product $\gamma_1 \gamma_2$ increases, the heat flux from the source decreases accordingly and the maximum dimensional temperature for the element is reduced. This produces smaller temperatures at the boundary separating the heated and unheated parts of the plate so that the unheated parts of the plate become less important in transferring heat to the fluid. The overall sensitivity of the problem to dimensions beyond the heated part of the plate is thus reduced. We note that this behavior (of a conjugate fin having a constant heat flow input

over a fraction of its length) differs significantly from that for the classical fin treated in most heat transfer textbooks. This is one which is driven by a constant temperature input at one end.

In all cases, agreement between the analytical solution from the complete model and the numerical solution is excellent. This is true even in light of the fact that the element aspect ratio is close to 1; a condition for which the model is supposed to be invalid. This restriction appears to be less stringent than originally thought. Consistent with the observations made in the above paragraph, there is not much conduction into sub-region IV from either sub-regions II or III. Rather, most of the heat transfer from the source to the fluid occurs from sub-region I directly with just small contributions from sub-regions II and III.

Agreement is also good between the results for m_1 of 60 and the limiting-case solution for large m_1 . Except for very small γ_1 this case behaves as if m_1 was infinite. The same is also true for m_1 of 20 for γ_1 of 0.25 and greater.

We also see that the results from the simple analytical model agree with those from the complete model to within a line width for the two larger m_1 values, and with only minor differences for m_1 of 5. Maximum plate temperatures for $L_1/L_2 = 0.1$ and $\gamma_2 = 0.9$ are shown in Fig. 8. Agreement between the analytical model and numerical results are very good for this case with differences of 10 percent or less between the two.

The effect of small γ_2 is illustrated in Fig. 9 for L_1/L_2 of 0.9 and γ_2 of 0.1. Here, agreement between the analytical and numerical results is good except for small m_1 and small γ_1 . The striking difference between this case and the previous two is the inaccuracy of the simple analytical solution. This solution does not account for heat transfer in the ξ direction in the unheated part of the element. For the case shown in Fig. 9, there is a significant heat transfer occurring from sub-region III to the fluid since γ_2 is small (i.e., dimensional maximum plate temperatures are large as discussed above). The complete analytical model includes this effect and produces accurate results, whereas the simple analytical model does not.

Linear Heat Generation (LHG)

Results from the complete analytical (equation (14)), simple analytical (equation (11), with $a = -1.0$), and numerical solutions are shown in Fig. 10. In Fig. 10, the ratio of the dimensionless maximum temperature distribution to γ_1 is plotted as function of γ_1 for m_1 of 5 and 20 and, for $L_1/L_2 = 0.9$ and $\gamma_2 = 0.5$. The agreement between the complete analytical and numerical solutions is not as good for LHG as it was for UNG especially at $\gamma_1 < 0.2$. At $\gamma_1 = 0.2$, the complete analytical solution overpredicts maximum temperatures by about 20 percent and 30 percent for m_1 of 20 and 5, respectively. Agreement improves significantly as γ_1 increases. The reason for the large discrepancies is that the two-dimensional analytical model was developed assuming uniform heat generation in the source. After development, it was applied to the LHG case as discussed above.

Figure 10. Dimensionless maximum coldplate temperature for LHG. Two-dimensional heat conduction.

Thus, even though the term $F(0)$ in equation (14b) accounts for linear heat generation, the term H does not. Attempts at replacing F with F_1 in the expression for H did not improve the results.

The model for linear heat generation is not entirely unsatisfactory however. Even though the relative agreement between the predictions from the complete analytical model and the numerical model for small γ_1 is poor, the dimensional temperatures for the LHG case at small γ_1 are very close to the fluid temperature and thus extreme accuracy is not so critical. This is because: 1) the total heat generation is small due to the linear decrease in local heat generation to zero at the edge of the source; and 2) γ_1 multiplies the ordinate value from Fig. 10 to arrive at the dimensional temperature. For example, for $\gamma_1 = 0.2$ and $m_1 = 20$, $(T_m - T_{fi}) = 0.45 \text{ q/h}$, where q/h ranges to a maximum of about 6 °R for water. $T_m - T_{fi}$ is indeed small.

V. Experiment

Description

We are attempting to develop a flat-plate coldplate for the U. S. Navy that can accommodate many configurations of electronic components. One preliminary design that was considered and tested in the early stages of the program consisted of two ALFA-LAVAL⁸ corrugated heat-exchanger plates casted in thermally-conductive epoxy and carbon fabric/epoxy layers. The design is shown in Fig. 11. The corrugated flow passage provided very good convective heat transfer and the composite layers ensured a uniformly smooth surface for mounting the heat sources.

Several thermal performance tests were conducted on the coldplate in the low Stanton number regime using water as the heat-transfer fluid. The tests were configured as shown in Fig. 12. Temperatures were measured by calibrated copper-constantan thermocouples at 19 locations on the coldplate and at the water inlet and outlet, along with water flow rate, and pressure drop. Water flow rate was fixed at 0.4 gal/min. Wire-wound precision resistors were used to simulate heat loads from

Figure 11. Plan view and cross-sectional view of composite coldplate tested. Dimensions are in inches.

electronic components. The resistors and thermocouples of interest are located on the coldplate as shown in Fig. 13. Each of the six heat sources was composed of four resistors positioned side-by-side, mounted on an aluminum plate, and bonded to the coldplate with thermally-conductive epoxy. During the tests, each source dissipated 25 W as measured by a precision volt-ohm meter, and each produced a heat flux of 0.86 W/cm^2 (2731 Btu/h/ft^2).

Figure 12. Test configuration for composite coldplate.

Results and Comparison with Predictions

Temperature data from eight thermocouples surrounding heat sources S-3 and S-4 shown in Fig. 13 are compared with predictions from the analytical model in Fig. 14. Values for the pertinent variables used in the model are $m_1=16.5$, $m_2=47.0$, $\gamma_1=0.43$, $\gamma_2=0.30$, and $h=541 \text{ Btu/h/ft}^2/^\circ\text{R}$ and are obtained from thermal conductivity data for the composite materials, coldplate dimensions, and heat transfer data from ALFA-LAVAL. Because of the expected uniformity of heat flux over the heat source areas, the analytical results are from equation (14a) with $a=0$.

Figure 13. Locations of heat sources and thermocouples for composite coldplate. Dimensions are in inches.

Figure 14. Temperature data from composite-coldplate test and predictions from analytical model.

We note that the agreement between data and predictions is relatively good, especially in the y direction where temperatures from thermocouples T-14, T-17, and T-18 are almost perfectly matched. For the x direction, predictions from the model correspond well for temperatures at T-8 and T-19 and underpredict those at T-20 and T-21.

In order to obtain the analytical results, the model (which was developed assuming a single-material coldplate) was modified to account for the composite material. We assumed that the coldplate was composed of two parallel conducting plates, the upper one consisting of thermally conducting epoxy (see Fig. 11), and the lower one of carbon/epoxy and the steel ALFA-LAVAL corrugated plate. In between the two plates there is a gap having a uniform heat transfer coefficient equal to αh . Because of the large values for m_1 and m_2 for this design, a simple one-dimensional heat transfer analysis shows that the temperature

distribution predicted from equation (9) for a single-material coldplate can be used for a two-material composite coldplate if it is multiplied by $(1+a)/a$. The predictions shown in Fig. 14 include this term.

The value for a that produces the best match between data and predictions is 0.037. This indicates that the combined conductive resistance of the composite and thermal resistance of the gap is 27 times larger than the convective resistance at the plate-fluid interface. The thermal resistance posed by heat conduction through the plate thickness accounts for only about 55 percent of this. We conclude that the remainder of the resistance is caused by de-laminations and the presence of gas bubbles in the composite. Heat transfer occurs by conduction through the gas and by radiation at the gap boundaries; the combination of which is not as effective as conduction through the solid composite alone.

In this way, we see that the proposed analytical model may be used in conjunction with test data as a diagnostic tool to determine thermal penalties caused by the presence of voids and de-laminations in composite coldplates. Should the thermal penalty be much larger than that attributed to conduction through the plate, the coldplate design or fabrication procedure can be changed accordingly and the plate re-tested. We note that the same information could be determined without the model, but many more temperature measurements at both sides of the coldplate would be needed.

VI. A Method for Predicting Maximum Coldplate Temperatures

The Method

Referring to Fig. 15, begin the procedure by constructing boundaries for the elements. Divide the coldplate up so that element boundaries are equally spaced between neighboring sources. If

$$\frac{\phi_{e1d}}{\phi_{m1d}} = \frac{\frac{2a}{m_1 \gamma_1} [1 - \cosh \frac{m_1 \gamma_1}{2}] + (1+a) \sinh \frac{m_1 \gamma_1}{2}}{\sinh \frac{m_1}{2} + \frac{2a}{\gamma_1 m_1} [\cosh \frac{m_1}{2} - \cosh \frac{m_1}{2} (1 - \gamma_1)] - (1+a) \sinh \frac{m_1}{2} (1 - \gamma_1)} \quad (17)$$

there are two neighbors in the same direction (one on one side of the source under consideration and one on the opposite side such as source B with A and C in Fig. 15), the smallest distance between the neighbors is used to locate the boundary on one side of the source and the opposite side. The same applies to a source having a coldplate boundary as the closest neighbor (see the y-direction boundary for source A or B in Fig. 15.) Identify L_1 as the smallest distance between two parallel boundaries for the same element. L_2 is the distance between the remaining two. Likewise, t_1 is the dimension of the heat source in the direction of L_1 , and t_2 is the remaining heat-source dimension.

The remaining steps are:

2. Identify $\bar{h}$, k_p , d , and Q_m . For uniform heat

generation in a source, $Q_m = Q$. Calculate $m_1^2 = \bar{h} L_1^2 / k_p d$, $m_2^2 = \bar{h} L_2^2 / k_p d$, $\gamma_1 = t_1 / L_1$, $\gamma_2 = t_2 / L_2$, and $q/\bar{h} = Q_m / t_1 t_2 \bar{h}$.

3. Using equation (14a) for uniform heat generation, or equation (14b) for linear heat generation, calculate ϕ_{m2d} and $\Delta T_m = T_m - T_{f1} = \phi_{m2d} q/\bar{h}$. T_m is the maximum coldplate temperature for the source being considered.

4. If the value for $\bar{h}$ is large, values for ΔT_m from step 3. will be small enough so that thermal interaction among the heat sources need not be considered. In this case, the temperature calculations are completed. For cases where $\bar{h}$ is small (say 100 Btu/h/ft²/°R or smaller), maximum temperatures will be larger and thermal interaction may become important. In general, the interaction takes place at a common boundary for two elements. The temperature at this boundary, or "edge", determined from the temperature distribution for one element may differ from that calculated for the other. We assume that the correct edge temperature is the average of the two, and use this improved temperature to re-estimate ΔT_m for each element. The corrected temperature difference ΔT_m for the j th heat source when interacting with the i th heat source is obtained from

$$\Delta T_{mj(i)} = \frac{\Delta T_m}{2} \left\{ 1 + \frac{\left(\frac{\phi_{e1d}}{\phi_{m1d}} \right)_i \Delta T_{mi}}{\left(\frac{\phi_{e1d}}{\phi_{m1d}} \right)_j \Delta T_{mj}} \right\} \quad (16)$$

where the ratio ϕ_{e1d}/ϕ_{m1d} is from

Figure 15. Example of idealized coldplate with heat sources to illustrate method for predicting maximum coldplate temperatures.

and is plotted in Fig. 16. If maximum coldplate temperatures for two non-neighboring heat sources differ significantly, this procedure may be

extended to include interaction between widely-spaced sources. Estimate new values of γ_1 and m_1 for each source using the distance between the center of the two sources as length L_1 . If the new values for m_1 are larger than m_{crit} determined from Fig. 17, thermal interaction can be neglected. Figure 17 is a plot of m_{crit} as a function of the newly estimated γ_1 that produces a critical dimensionless edge temperature of 10 percent. m_{crit} may also be determined by solving

$$\phi_{eid} = \frac{2a}{m_1 \gamma_1} \left[\frac{1 - \cosh \frac{m_1 \gamma_1}{2}}{\sinh \frac{m_1}{2}} \right] + (1+a) \frac{\sinh \frac{m_1 \gamma_1}{2}}{\sinh \frac{m_1}{2}}, \quad (18)$$

Figure 16. Ratio of coldplate dimensionless temperature at an element boundary, to maximum temperature for that element.

Figure 17. Critical value for m . For m values larger than m_{crit} , no thermal interaction occurs between the source under consideration and any other source.

For $m_1 = m_{crit}$, where ϕ_{eid} is set equal to 0.10.

For simultaneous thermal interaction among many sources, the above formulas can be programmed and, assuming thermal interaction, the corrected maximum temperatures determined by an iterative procedure.

The method described here is currently being tested for accuracy and range of applicability. A numerical example is given to illustrate its use.

A Numerical Example

Consider a water-cooled coldplate shown in Fig. 15. The dimensions and power levels given are slightly exaggerated to simplify the example and to maintain representative values for heat fluxes. $\bar{h}$ is assumed to be 300 Btu/h/ft²/°R corresponding to turbulent flow in the coldplate channel and $\sqrt{(\bar{h}/k_d)}$ is 1.9/inch. We assume that heat generation in each source is uniform. Following the steps described in the above method, we produce the results given in Table I. The maximum coldplate temperature differences range from only 5.8 °R to 3.8 °R as a consequence of the large value for $\bar{h}$. Thermal interaction is not necessary to consider.

VII. Conclusions and Recommendations

We have developed and validated analytical models for steady-state, one- and two-dimensional heat transfer in a single-material flat-plate coldplate having discrete heat sources. We compared results from one of the models to data for a water-cooled coldplate operated in the laboratory. The utility of the models as diagnostic tools for quantifying thermal resistance in a composite coldplate caused by voids and de-laminations is demonstrated. We proposed a simple procedure for estimating maximum coldplate temperature at the location of each heat source which includes the effect of thermal interaction among sources having different heat fluxes.

All of the models may be applied to either liquid or gas cooling at the coldplate-fluid interface. For gas-cooling, $\bar{h}$ will be somewhat smaller than for liquid cooling, and thus, m and $q/\bar{h}$ values will be affected. The latter influence will result in larger maximum coldplate temperatures and the former will necessitate thermal interaction calculations for nearly all cases. In addition, the models presented may be applied to coldplates and PCBs that are cooled on both sides by letting $\bar{h}$ be the sum of that for both sides of the plate. Of course, the restriction of small St (say, less than 0.5) still applies to these uses.

Based on our findings, significant increases in fluid temperature in the direction of fluid flow occur for St of 1 and greater, whereas for St less than 0.1, the fluid-temperature rise is imperceptible. Maintaining St at 0.5 or less will minimize the effect of increasing maximum coldplate temperatures because of this effect. This recommendation is valid for both liquid and gas cooling. As always, pressure-drop considerations will need to be considered in designing the optimum value for St.

It is desirable to maintain as small a value for m as possible to minimize maximum coldplate temperatures. m should be reduced by increasing

plate conductance and not by reducing $\bar{h}$. A reduced value for $\bar{h}$ decreases the dimensionless plate temperature but increases dimensional temperature which is undesirable.

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TABLE I.

Source	$q \left(\frac{\text{Btu}}{\text{h ft}^2} \right)$	$q/\bar{h} \text{ (}^\circ\text{R)}$	L_1	L_2	t_1	t_2	γ_1	γ_2	m_1	m_2	ϕ_{m2d}	$\Delta T_m \text{ (}^\circ\text{R)}$
A	2340	7.8	2.7	5.7	1.5	2.8	0.56	0.49	5.1	10.8	0.74	5.8
B	1554	5.2	3.0	8.9	1.5	5.9	0.50	0.66	5.7	16.9	0.77	4.0
C	2908	9.7	3.9	7.0	1.3	1.3	0.33	0.18	7.4	13.3	0.54	5.3
D	1938	6.5	3.1	4.4	1.9	3.2	0.61	0.73	5.9	8.4	0.82	5.3
E	1788	6.0	3.9	7.6	3.0	2.2	0.77	0.29	7.4	14.4	0.84	5.0
F	1512	5.0	3.3	8.0	1.5	5.2	0.45	0.65	6.3	15.2	0.76	3.8

Table I. Results from numerical example for cold plate given in Fig. 15. Dimensions for L_1 , L_2 , t_1 , and t_2 are in inches