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BENCHMARKING REPORT

for

WIGGLE: A One-Dimensional Transient Diffusion Theory Code

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SUMMARY

WIGGLE is a static/transient one-dimensional diffusion theory calculation written to estimate the axial power profile while safety rods are falling during a scram. The code is used in the LOCA Limits Analysis Package (LLAP)¹, a part of the SRS system for calculating thermal-hydraulic limits. Since WIGGLE was designed to be implemented through LLAP and not as a stand-alone code, it consists entirely of subroutines; the problem data must be passed to it from a driver routine.

This project concerned the verification of WIGGLE, which limited it to the determination that WIGGLE is correctly implementing the transient 1D diffusion equation. The approach was to compare the results of the code with three analytic solutions:

- (1) A static homogeneous calculation of the pre-accident power profile (without end-fittings);
- (2) A static heterogeneous calculation of the pre-accident power profile (includes end-fittings); and
- (3) A transient calculation designed to test the time-dependent calculational ability.

The results of all three calculations were essentially identical to the analytical solutions, thus giving us confidence that WIGGLE is correctly solving the one-dimensional time-dependent diffusion equation.

INTRODUCTION

WIGGLE² is a one-dimensional, two energy group diffusion theory calculation which is used to provide transient axial power profiles following a safety rod scram. The coding contains both a static and time-dependent solution to the two-group diffusion equation; the calculational flow consists of:

- A static "search" calculation in which fission correction factors (multiplicative factors which modify the number of neutrons emitted per fission for each spatial point) are found which cause the calculated fission power profile to duplicate an input axial power shape; and
- A time-dependent calculation in which user-supplied thermal absorption is added to the problem (from the top) according to the user-supplied time-dependent safety rod position following scram.

Details of the theory and implementation are given in Reference 2.

Since WIGGLE was designed to be implemented through LLAP and not as a stand-alone code, it consists entirely of subroutines; the problem data must be passed to it from a driver routine. The input data which must be passed to the code consist of:

- Neutron cross sections and height of the core material
- Neutron cross sections and height of the top and bottom endfittings
- Absorption cross sections for the safety rods
- Transient neutronic data

- Number of precursors
- Precursor decay constants and yield fractions
- Average neutron velocities by energy group;
- Initial normalized axial power profile in the core;
- Safety rod positions, decay power fractions, and deposited power fractions during the rod drop; and
- Computational parameters such as time step size, number of axial regions, and convergence criteria.

With this information, WIGGLE is not only able to calculate fission correction terms to allow the one dimensional diffusion theory static calculation to duplicate the initial axial power profile. In addition, using the same information, WIGGLE is able to calculate the time-dependent axial power profile during and after a safety rod scram.

The simple geometrical situation modeled by WIGGLE allowed us to benchmark the code against analytical solutions. Three test problems were calculated and compared to analytical solutions:

- (1) A test of the **static** calculation consisting of a comparison of the WIGGLE solution of a homogeneous reactor core (without endfittings) to the analytic two-group solution of the same problem.
- (2) A test of the **heterogeneous static** calculation consisting of a comparison of the WIGGLE solution of a reactor core with endfittings to the analytic one-group solution of the same problem.
- (3) A test of the **time-dependent** calculation consisting of a comparison of the WIGGLE solution of the response of the configuration in step 1 instantaneously faced with a fully inserted safety rod; the spatial power shape for this problem was sinusoidal, but the time behavior was compared to the analytical solution.

Each of these tests will now be described.

[Note: Since WIGGLE is used as part of an engineering package, the dimension of length used in this report is inches, except for cross sections for which the traditional cm⁻¹ is used.]

SPATIAL HOMOGENEOUS BENCHMARK TEST

For the first spatial benchmarking calculation, a static calculation was run for the simple geometric situation illustrated in Figure 1. The configuration consisted of a homogeneous core region 150 inches high; the cross sections shown in the figure are the Mark 22 cluster averaged cross sections at beginning of life. The initial fission power profile was set to be sinusoidal across the core, which is the expected power shape for a homogeneous fissionable slab. With this sinusoidal shape, it was expected that the "fission correction" found in the static calculation of WIGGLE would then be spatially constant, with a value of $1/k_{eff}$ (to maintain criticality).

The analytical solution for this situation is a straight-forward solution of the two-group static diffusion equation:

$$(Eqn. 1) \quad -D_1 \frac{d^2 \phi_1(x)}{dx^2} + \sigma_{r1} \phi_1(x) = \frac{\nu \sigma_{f1} \phi_1(x) + \nu \sigma_{f2} \phi_2(x)}{k_{eff}}$$

$$(Eqn. 2) \quad -D_2 \frac{d^2 \phi_2(x)}{dx^2} + \sigma_{r2} \phi_2(x) = \sigma_{s1 \rightarrow 2} \phi_1(x)$$

where

- D_g = Diffusion coefficient for group g
- ϕ_g = Flux for group g
- σ_{rg} = Removal cross section for group g
- $\nu \sigma_{fg}$ = nu * Fission cross section for group g
- $\sigma_{s1 \rightarrow 2}$ = Scattering cross section from group 1 to group 2
- k_{eff} = K-effective eigenvalue

There is an elegant way to solve this equation (which involves converting it to a fourth order ODE and wrestling it to the ground), but luckily we don't have to go through it. Based on the knowledge that eigenvalue bare homogeneous slab problems result in sinusoidal fluxes, we can "guess" that:

$$\phi_1(x) = \phi_1 \sin(Bx)$$

$$\phi_2(x) = \phi_2 \sin(Bx)$$

Applying the boundary condition that the flux must go to zero at the edges gives us (for slab thickness a):

$$\sin(Ba) = 0$$

$$Ba = \pi$$

$$B = \frac{\pi}{a} = \frac{\pi}{(150)(2.54)} = 0.00824565 \text{ cm}^{-1}$$

Substituting these values into Equations (1) and (2) and dividing by $\sin(Bx)$ gives us:

$$(Eqn. 3) \quad D_1 B^2 \phi_1 + \sigma_{r1} \phi_1 = \frac{\nu \sigma_{f1} \phi_1 + \nu \sigma_{f2} \phi_2}{k_{eff}}$$

$$(Eqn. 4) \quad D_2 B^2 \phi_2 + \sigma_{r2} \phi_2 = \sigma_s^{1 \rightarrow 2} \phi_1$$

Solving Equation (4) for ϕ_1 yields:

$$\begin{aligned} \phi_1 &= \left(\frac{D_2 B^2 + \sigma_{r2}}{\sigma_s^{1 \rightarrow 2}} \right) \phi_2 \\ &= 2.437067 \phi_2 \end{aligned}$$

Substituting this into Equation (3) and solving for k-effective yields:

$$k_{eff} = \frac{\nu \sigma_n (D_2 B^2 + \sigma_{r2}) + \nu \sigma_{f2} \sigma_s^{1 \rightarrow 2}}{(D_1 B^2 + \sigma_{r1}) (D_2 B^2 + \sigma_{r2})}$$

Putting in our known cross sections and B^2 yields an eigenvalue of:

$$\begin{aligned} k_{eff} &= \frac{[0.0023]([0.93179][0.0082456]^2 + [0.01975]) + [0.02028][0.00813]}{([1.4163][0.0082456]^2 + [0.01054])([0.93179][0.0082456]^2 + [0.01975])} \\ &= 0.9986070 \end{aligned}$$

The problem was calculated using WIGGLE. The resulting fission corrections predicted by WIGGLE averaged 1.00139987, corresponding to an eigenvalue of 0.9986020, which is within round-off error of the analytical result. The correction factors are shown in Figure 2. They were practically flat; the maximum deviation occurred at the endpoints with a correction of 1.00152218 (which corresponds to an eigenvalue error of 0.01%).

Normalizing the flux to deliver a unit average fission rate:

$$\begin{aligned} 1.000 &= (\sigma_{f1} \bar{\phi}_1 + \sigma_{f2} \bar{\phi}_2) 150 / k_{eff} \\ &= (\sigma_{f1} \phi_1 + \sigma_{f2} \phi_2) 150 / k_{eff} \times 2/\pi \end{aligned}$$

$$= (\sigma_{f1} 2.437067 \phi_2 + \sigma_{f2} \phi_2) \frac{300}{\pi k_{eff}}$$

$$= ([0.00094][2.437067] + [0.00834]) \frac{300}{[3.141596][0.9986070]} \phi_2$$

yields:

$$\phi_2 = 0.9836838$$

The resulting two group fluxes are:

$$\phi_1(x) = 2.3973031 \sin(0.00824565x)$$

$$\phi_2(x) = 0.9836838 \sin(0.00824565x)$$

(x in centimeters)

Figure 3 shows a comparison of the two-group fluxes for the analytical solution and the WIGGLE run. The results for the two cases are essentially identical (too close to show up in the figure); a fairly uniform error of 0.003% occurred due to roundoff.

The FORTRAN listing of the WIGGLE test problem driver is given on the LAB1 VAX under srluser3:[t4579.wiggle]bench1.for; the output file is stored in the same directory as bench1.out. The analytical input and output are located in the same directory as anal1.for and anal1.out. (These test input and output members have been submitted to SCMS for archival.)

SPATIAL HETEROGENEOUS BENCHMARK TEST

The second benchmark was another spatial problem, but limited to a single energy group and including endfittings on the top and bottom of the reactor. The problem configuration and data are shown in Figure 4. The cross sections were chosen to simulate a normal core (with k-infinity of 1.00) reflected by a D2O-like moderator. The solution to this test is found by dividing the core by a midplane and setting up the one-group diffusion equation for the core:

$$-D_c \frac{d^2 \phi_c(x)}{dx^2} + \sigma_{ac} \phi_c(x) = \frac{\nu \sigma_{fc}}{k_{eff}} \phi_c(x)$$

with a solution of:

$$\phi_c(x) = A_1 \cos(Bx) + A_2 \sin(Bx)$$

where:

$$B^2 = \frac{\frac{v\sigma_{fc}}{k_{eff}} - \sigma_{ac}}{D_c}$$

For the endfitting, the equation is:

$$-D_o \frac{d^2 \phi_o(x)}{dx^2} + \sigma_{ao} \phi_o(x) = 0$$

which has a solution of:

$$\phi_o(x) = C_1 e^{-\frac{x}{L}} + C_2 e^{\frac{x}{L}}$$

where:

$$L^2 = \frac{D_o}{\sigma_{ao}}$$

Four boundary conditions were used:

(B. C. #1) The flux solution in the core is symmetric about the midplane,

$$\left. \frac{d\phi_c}{dx} \right|_0 = 0$$

This eliminates the sinusoidal solution and we then have:

$$\phi_c(x) = A_1 \cos(Bx)$$

(B. C. #2) The second boundary condition is that the flux goes to zero at the outside edge of the endfitting, i. e.

$$\phi_o(a_o) = C_1 e^{-\frac{a_o}{L}} + C_2 e^{\frac{a_o}{L}} = 0$$

which results in:

$$C_2 = -C_1 e^{-\frac{2a_o}{L}}$$

with a resulting endfitting flux profile of:

$$\phi_o(x) = C_1 \left(e^{-\frac{x}{L}} - e^{-\frac{(x-2a_o)}{L}} \right)$$

(B. C. #3) The third boundary condition is that the flux is continuous across the interface, i. e.

$$\phi_c(a_c) = \phi_e(a_c)$$

$$(Eqn. 5) \quad A_1 \cos(Ba_c) = C_1 \left(e^{-\frac{a_c}{L}} - e^{\frac{(a_c-2a_e)}{L}} \right)$$

which results in:

$$C_1 = \frac{A_1 \cos(Ba_c)}{\left(e^{-\frac{a_c}{L}} - e^{\frac{(a_c-2a_e)}{L}} \right)}$$

(B. C. #4) The final boundary condition is that the current is continuous across the interface. Taking advantage of the fact that our test problem has the same diffusion coefficient in the two material regions, this gives us:

$$\left. \frac{d\phi_c}{dx} \right|_{a_c} = \left. \frac{d\phi_e}{dx} \right|_{a_c}$$

$$(Eqn. 6) \quad A_1 B \sin(Ba_c) = \frac{C_1}{L} \left(e^{-\frac{a_c}{L}} + e^{\frac{(a_c-2a_e)}{L}} \right)$$

The resulting normalized flux shapes in the core and endfitting are:

$$\phi_c(x) = \cos(Bx)$$

$$\phi_e(x) = \frac{\cos(Ba_c)}{\left(e^{-\frac{a_c}{L}} - e^{\frac{(a_c-2a_e)}{L}} \right)} \left(e^{-\frac{x}{L}} - e^{\frac{(x-2a_e)}{L}} \right)$$

By dividing Eqn. 6 by Eqn. 5, we find that the buckling obeys the transcendental equation:

$$B \tan(Ba_c) = \frac{\left(e^{-\frac{a_c}{L}} + e^{\frac{(a_c-2a_e)}{L}} \right)}{L \left(e^{-\frac{a_c}{L}} - e^{\frac{(a_c-2a_e)}{L}} \right)}$$

Using the cross sections for this second benchmark problem (from Figure 4), we have:

$$a_c = 75 \text{ in} = 190.5 \text{ cm}$$

$$a_e = 95 \text{ in} = 2413 \text{ cm}$$

$$L = \sqrt{\frac{1}{0.0002}} = 70.7107 \text{ cm}$$

$$B \tan(190.5B) = 0.022960623 \text{ cm}^{-1}$$

which solves to get:

$$B = 0.006745642$$

$$k_{eff} = \frac{0.024}{(0.006745642)^2 + 0.024} = 0.9981076$$

$$C_1 = 5.469478647$$

and the resulting fluxes are:

$$\begin{aligned} \phi(x) &= \cos(0.006745642 x) , \quad 0 \leq x \leq 190.5 \\ &= 5.469478647 e^{-0.01414214x} - 0.005941395 e^{0.01414214x} \\ &\quad 190.5 \leq x \leq 241.3 \end{aligned}$$

In the WIGGLE calculation, we gave it an initial unnormalized fission power profile of:

$$P(x) = \cos(0.006745642x) , \quad 0 \leq x \leq 190.5$$

in anticipation that WIGGLE would deliver a constant fission correction profile of $1/k_{eff} = 1.0018960$. The actual fission correction profile is shown in Figure 5. It deviates from the predicted only slightly at the endpoints; the average correction factor is 1.0018963, which is effectively identical with the predicted value.

The analytical flux profile is compared with the WIGGLE-produced flux profile in Figure 6. The differences are almost too slight to show up on the graph.

The FORTRAN listing of the WIGGLE test problem driver is given on the LAB1 VAX under srluser3:[t4579.wiggle]bench2.for; the output file is stored in the same directory as bench2.out. The analytical input and output are located in the same directory as anal2.for and anal2.out. (These test input and output members have been submitted to SCMS for archival.)

TIME DEPENDENT BENCHMARK TEST

The third benchmark case was a pure time-dependent test consisting of the same configuration used in the Static Homogeneous Benchmark subjected to an instantaneous full insertion of the safety rods at the beginning of the transient. Use of an instantaneous insertion causes the spatial configuration to be homogeneous throughout the transient, which guarantees that the sinusoidal spatial shape is preserved while the reactor power decays.

The analytical solution to this problem is obtained from the transient two-group diffusion equation with six precursor groups:

$$(Eqn. 7a) \quad \frac{1}{v_1} \frac{\partial \phi(x, t)}{\partial t} = \frac{(1 - \beta)}{k_{eff}} (v \sigma_{r1} \phi_1(x, t) + v \sigma_{r2} \phi_2(x, t)) - \sigma_{r1} \phi_1(x, t) \\ + \sum_{i=1}^6 \lambda_i C_i(x, t) + D_1 \nabla^2 \phi_1(x, t)$$

$$(Eqn. 7b) \quad \frac{1}{v_2} \frac{\partial \phi_2(x, t)}{\partial t} = - \sigma_{r2} \phi_2(x, t) + D_2 \nabla^2 \phi_2(x, t) + \sigma_s^{1 \rightarrow 2} \phi_1(x, t)$$

$$(Eqn. 7c) \quad \frac{\partial C_i(x, t)}{\partial t} = \frac{\beta_i}{k_{eff}} (v \sigma_{r1} \phi_1(x, t) + v \sigma_{r2} \phi_2(x, t)) - \lambda_i C_i(x, t)$$

$$i = 1, 6$$

where:

- C_i = Density of precursor i
- λ_i = Decay constant of precursor i
- v_g = Velocity of neutrons in group g
- β_i = Precursor fraction for precursor i
- β = Sum of the β_i

The solution strategy consists of finding and using separable space and time solutions of Eqns. (7a-7c) in terms of flux and precursor concentration constants ϕ_g and C_i :

$$\phi_g(x, t) = \phi_g f(x) g(t)$$

$$C_i(x, t) = C_i f(x) g(t), \quad g=1,2; i=1,6$$

Substituting these into Eqns. (7a-7c) and dividing by $f(x)g(t)$ gives us:

$$\frac{\phi_1}{v_1} \frac{g'(t)}{g(t)} = \frac{(1 - \beta)}{k_{eff}} (v \sigma_{r1} \phi_1 + v \sigma_{r2} \phi_2) - \sigma_{r1} \phi_1$$

$$+ \sum_{i=1}^6 \lambda_i C_i + D_1 \phi_1 \frac{f''(x)}{f(x)}$$

$$\frac{\phi_2}{v_2} \frac{g'(t)}{g(t)} = -\sigma_{r2} \phi_2 + D_2 \phi_2 \frac{f''(x)}{f(x)} + \sigma_s^{1 \rightarrow 2} \phi_1$$

$$C_i \frac{g'(t)}{g(t)} = \frac{\beta_i}{k_{eff}} (v\sigma_{f1} \phi_1 + v\sigma_{f2} \phi_2) - \lambda_i C_i, i=1,6$$

Since a time dependent left-hand-side is equal to a space-dependent right-hand-side, both sides must be equal to a constant. Let us set:

$$\frac{g'(t)}{g(t)} = \alpha$$

which gives a normalized shape of:

$$g(t) = e^{\alpha t}$$

and

$$\frac{f''(x)}{f(x)} = -B^2$$

which (along with the boundary conditions) gives us a set of normalized solutions:

$$f(x) = \sin B_m x$$

$$B_m = \frac{\pi(m+1)}{150 \cdot 2.54}$$

(The cosine terms have disappeared because of zero flux boundary conditions.)

Recognizing that each spatial eigenvalue B_m will give rise to eight associated time eigenvalues α_{mn} , we have a set of full space/time eigenmode solutions to Eqns. (7a-7c), each of which will have an associated vector of coefficients:

$$\underline{\Phi}_{mn} = \begin{bmatrix} \phi_{mn1} \\ \phi_{mn2} \\ C_{mn1} \\ C_{mn2} \\ C_{mn3} \\ C_{mn4} \\ C_{mn5} \\ C_{mn6} \end{bmatrix}, \quad n = 1, 8 \quad m = 0, \infty$$

The full eigenmodes for each (B_m, α_{mn}) pair are:

$$\phi_{mng}(x, t) = \phi_{mng} \sin(B_m x) e^{\alpha_{mn} t}, \quad g = 1, 2$$

$$C_{mni}(x, t) = C_{mni} \sin(B_m x) e^{\alpha_{mn} t}, \quad i = 1, 6$$

$$m = 0, \infty; n = 1, 8$$

Substituting these into Eqns. 7 and dividing out the exponential and sinusoidal terms gives us:

$$\begin{aligned} (\text{Eqn. 8 a}) \quad \frac{\alpha_{mn} \phi_{mn1}}{v_1} &= \frac{(1 - \beta)}{k_{eff}} (v \sigma_{r1} \phi_{mn1} + v \sigma_{r2} \phi_{mn2}) - \sigma_{r1} \phi_{mn1} + \sum_{i=1}^6 \lambda_i C_{mni} \\ &\quad - D_1 B_m^2 \phi_{mn1} \end{aligned}$$

$$(\text{Eqn. 8 b}) \quad \frac{\alpha_{mn} \phi_{mn2}}{v_2} = -\sigma_{a2} \phi_{mn2} - D_2 B_m^2 \phi_{mn2} + \sigma_s^{1 \rightarrow 2} \phi_{mn1}$$

$$(\text{Eqn. 8 c}) \quad \alpha_{mn} C_{mni} = \frac{\beta_i}{k_{eff}} (v \sigma_{r1} \phi_{mn1} + v \sigma_{r2} \phi_{mn2}) - \lambda_i C_{mni},$$

$$B_m = \frac{\pi(m+1)}{150 \cdot 2.54}$$

The transient solution of a problem in which the material composition does not change in time (which is the case for Benchmarks 2 and 3) can be found by expanding the initial shapes into the spatial eigenmode set:

$$\Phi(x, 0) = \sum_m \phi_m \sin(B_m x)$$

where:

$$\Phi(x, t) = \begin{bmatrix} \phi_1(x, t) \\ \phi_2(x, t) \\ C_1(x, t) \\ C_2(x, t) \\ C_3(x, t) \\ C_4(x, t) \\ C_5(x, t) \\ C_6(x, t) \end{bmatrix}$$

and then expanding the spatial coefficient vectors into the time eigenmode coefficient vectors. This corresponds to finding the coefficients a_{mn} for the initial (t=0) conditions such that:

$$\phi_m = \sum_n a_{mn} \Phi_{mn} e^{\alpha_{mn} 0} = \sum_n a_{mn} \Phi_{mn}$$

The transient solution is then:

$$(Eqn. 9) \quad \Phi(x, t) = \sum_m \sum_n a_{mn} \Phi_{mn} e^{\alpha_{mn} t} \sin(B_m x)$$

For the third benchmark problem, we only are concerned with the fundamental sinusoidal shape (m=0), which doesn't change during the transient. The initial values of the flux are therefore the same as the Static Benchmark results and the precursor concentrations can be found by solving Eqn. 8c (reduced to the fundamental mode coefficients) with $\alpha_{0n}=0$:

$$0 = \beta_i (\nu \sigma_n \phi_{01} + \nu \sigma_{i2} \phi_{02}) - \lambda_i C_{0i}$$

$$C_{0i} = \frac{\beta_i}{\lambda_i} (\nu \sigma_n \phi_{01} + \nu \sigma_{i2} \phi_{02}), \quad i = 1, 6$$

[Note that the "two subscript" coefficients correspond to initial (t=0) conditions, whereas the "three subscript" versions of the same coefficients have expanded them into time eigenmode components.]

The resulting initial conditions are:

$$\underline{\Phi}(x, 0) = \begin{bmatrix} \phi_{01} \\ \phi_{02} \\ C_{01} \\ C_{02} \\ C_{03} \\ C_{04} \\ C_{05} \\ C_{06} \end{bmatrix} \sin(B_0 x) = \begin{bmatrix} 2.39730310 \\ 0.983683787 \\ 2.35660645 \times 10^{-2} \\ 1.75595382 \times 10^{-3} \\ 2.63165905 \times 10^{-4} \\ 2.64291995 \times 10^{-4} \\ 1.67785092 \times 10^{-5} \\ 1.33494460 \times 10^{-6} \end{bmatrix} \sin(B_c y)$$

$$= \underline{\phi}_0 \sin(B_0 x)$$

Using the full safety rod insertion absorptions, we can find the time eigenvalues and modes by solving the homogeneous set Eqn. 8 for $m=0$. The 8 eigenvectors and eigenmodes are given by:

$$\underline{A}_0 = [\underline{\Phi}_{01} \underline{\Phi}_{02} \underline{\Phi}_{03} \underline{\Phi}_{04} \underline{\Phi}_{05} \underline{\Phi}_{06} \underline{\Phi}_{07} \underline{\Phi}_{08}]$$

with a corresponding eigenvalue set:

$$\underline{\alpha}_0 = \begin{bmatrix} \alpha_{01} \\ \alpha_{02} \\ \alpha_{03} \\ \alpha_{04} \\ \alpha_{05} \\ \alpha_{06} \\ \alpha_{07} \\ \alpha_{08} \end{bmatrix}$$

The actual eigenvalues and associated eigenmodes are:

For $\alpha_{01} = -1.05113658 \times 10^{-4}$

$$\underline{\Phi}_{01} = \begin{bmatrix} 1.00 \\ 3.83067470 \times 10^{-1} \\ 5.26616428 \times 10^0 \\ 6.97293356 \times 10^{-4} \\ 1.04156954 \times 10^{-4} \\ 1.04542944 \times 10^{-4} \\ 6.63510019 \times 10^{-6} \\ 5.27881628 \times 10^{-7} \end{bmatrix}$$

For $\alpha_{02} = -2.40921107 \times 10^{-2}$

$$\underline{\Phi}_{02} = \begin{bmatrix} 1.00 \\ 3.83069865 \times 10^{-1} \\ -4.09078333 \times 10^{-5} \\ 2.27666378 \times 10^{-2} \\ 1.31640494 \times 10^{-4} \\ 1.13438849 \times 10^{-4} \\ 6.75080631 \times 10^{-6} \\ 5.31176598 \times 10^{-7} \end{bmatrix}$$

For $\alpha_{03} = -1.12581686 \times 10^{-1}$

$$\underline{\Phi_{03}} = \begin{bmatrix} 1.00 \\ 3.83074470 \times 10^{-1} \\ -8.72412056 \times 10^{-6} \\ -1.96675640 \times 10^{-4} \\ 4.94862804 \times 10^{-3} \\ 1.65339075 \times 10^{-4} \\ 7.21488408 \times 10^{-6} \\ 5.43691188 \times 10^{-7} \end{bmatrix}$$

For $\alpha_{04} = -2.90275412 \times 10^{-1}$

$$\underline{\Phi_{04}} = \begin{bmatrix} 1.00 \\ 3.83083907 \times 10^{-1} \\ -3.38172736 \times 10^{-6} \\ -6.50088995 \times 10^{-5} \\ -6.82782323 \times 10^{-5} \\ 2.03377529 \times 10^{-3} \\ 8.37032064 \times 10^{-6} \\ 5.70690291 \times 10^{-7} \end{bmatrix}$$

For $\alpha_{05} = -1.37944932 \times 10^0$

$$\underline{\Phi_{05}} = \begin{bmatrix} 1.00 \\ 3.83141673 \times 10^{-1} \\ -7.11490650 \times 10^{-7} \\ -1.27395765 \times 10^{-5} \\ -9.46569249 \times 10^{-6} \\ -2.97954697 \times 10^{-5} \\ 4.52048481 \times 10^{-4} \\ 8.20361489 \times 10^{-7} \end{bmatrix}$$

For $\alpha_{06} = -3.85746816 \times 10^0$

$$\underline{\Phi_{06}} = \begin{bmatrix} 1.00 \\ 3.83273507 \times 10^{-1} \\ -2.54487578 \times 10^{-7} \\ -4.50386724 \times 10^{-6} \\ -3.19897611 \times 10^{-6} \\ -9.00822356 \times 10^{-6} \\ -3.78124770 \times 10^{-6} \\ 1.63085913 \times 10^{-4} \end{bmatrix}$$

For $\alpha_{07} = -5.01048712 \times 10^2$

$$\underline{\Phi}_{07} = \begin{bmatrix} 1.00 \\ 4.11673564 \times 10^{-1} \\ -2.07121797 \times 10^{-9} \\ -3.64226301 \times 10^{-8} \\ -2.52660548 \times 10^{-8} \\ -6.75431061 \times 10^{-8} \\ -1.96610571 \times 10^{-8} \\ -4.34562168 \times 10^{-9} \end{bmatrix}$$

For $\alpha_{08} = -8.95246950 \times 10^4$

$$\underline{\Phi}_{08} = \begin{bmatrix} 1.00 \\ -3.35579674 \times 10^{-2} \\ -1.76291204 \times 10^{-12} \\ -3.09995089 \times 10^{-11} \\ -2.15002362 \times 10^{-11} \\ -5.74542459 \times 10^{-11} \\ -1.66879660 \times 10^{-11} \\ -3.67035603 \times 10^{-12} \end{bmatrix}$$

The eigenmode coefficients are found by solving:

$$\underline{A}_0 \underline{a}_0 = \underline{\varphi}_0$$

to get:

$$\underline{a_0} = \begin{bmatrix} a_{01} \\ a_{02} \\ a_{03} \\ a_{04} \\ a_{05} \\ a_{06} \\ a_{07} \\ a_{08} \end{bmatrix} = \begin{bmatrix} 4.47577176 \times 10^{-3} \\ 7.78178611 \times 10^{-2} \\ 5.27725649 \times 10^{-2} \\ 1.21675372 \times 10^{-1} \\ 3.29456846 \times 10^{-2} \\ 7.20643184 \times 10^{-3} \\ 2.11223238 \times 10^0 \\ -1.18229690 \times 10^{-2} \end{bmatrix}$$

The analytical time-dependent flux solution is found by putting these a_{0n} into Eqn.9:

$$\Phi(x, t) = \sum_n a_{0n} \Phi_{0n} e^{a_{0n} t} \sin(B_0 x)$$

The total power can then be found by combining group fluxes multiplied by their σ_{fg} values:

$$P(x, t) = \sum_g \phi_g(x, t) * \sigma_{fg}(x)$$

[Note: The negative value for the coefficient of mode 8, which is the fastest decaying element, might suggest that the initial response of the system would be positive (i.e. as a dominant negative mode decays). A closer analysis shows, however, that mode 8 is dominated by mode 7 and the total power has a negative derivative throughout the transient.]

The corresponding WIGGLE problem involved the same static calculation as in Benchmark #1, with the transient calculation including an instantaneous full safety rod drop at $t=0$ sec. Because of the extremely fast decay times of the last two eigenmodes, the time range of the solution was set at 0 to 0.01 seconds. The spatial shape of the flux and power remained sinusoidal during this interval. The fast nature of the transient required that we run a series of WIGGLE calculations at decreasing time steps and extrapolate to an asymptotic value; this asymptotic total power versus time is compared with the analytical solution in Figure 7. The agreement between the two is extremely good.

The FORTRAN listing of the WIGGLE test problem driver is given on the LAB1 VAX under srluser3:[t4579.wiggle]bench3.for; the output file is stored in the same directory as bench3.out. The analytical solution and resulting output are located in the same directory as anal3.for and anal3.out. (These test input and output members have been submitted to SCMS for archival.)

CONCLUSIONS

WIGGLE has been compared to analytical diffusion theory solutions for three problems which test the space- and time-dependent calculational abilities of the code. In all three cases, the WIGGLE solutions were essentially identical to the analytical solutions.

REFERENCES

1. M. V. Gregory, Users Guide to LLAP (LOCA Limits Analysis Package), WSRC-RP-89-998, October 1989.
2. R. E. Pevey, "Axial Power Profiles in SRP Reactors During LOCA," DPST-87-511, June 26, 1987.

Figure 1

Geometry and Physics Data for Benchmarks #1 and #2

Group Constants

	<u>Group 1</u>	<u>Group 2</u>	
Diffusion	1.4163	0.93179	cm
Removal	0.01054	0.01975	cm ⁻¹
Capture	0.00147	0.01141	cm ⁻¹
Fission	0.00094	0.00834	cm ⁻¹
nu*Fission	0.0023	0.02028	cm ⁻¹
Scat to grp 1	0	0	cm ⁻¹
Scat to grp 2	0.00813	0	cm ⁻¹
Velocities	9918079.	339764.	cm/sec
SR absorption	0	0.00141	cm ⁻¹

Precursor data

<u>No.</u>	<u>β</u>	<u>λ (1/sec)</u>
1	9.732E-5	1.053E-4
2	1.7113E-3	2.485E-2
3	1.1869E-3	1.150E-1
4	3.1717E-3	3.060E-1
5	9.2123E-4	1.400E+0
6	2.0261E-4	3.870E+0

Figure 2
Correction Factors for Benchmark #1

Figure 3
Axial Flux Profiles in Benchmark #1

Figure 4

Geometry and Physics Data for Benchmark Problem #2

Group Constants

	<u>Core</u>	<u>Endfitting</u>	
Diffusion	1.00	1.00	cm
Absorption	0.024	0.0002	cm ⁻¹
Capture	0.014	0.0002	cm ⁻¹
Fission	0.01	0.	cm ⁻¹
nu*Fission	0.024	0.	cm ⁻¹

Figure 5
Fission Correction Factors for
Benchmark #2

Figure 6
Comparison of Flux Profiles
for Benchmark #2

Figure 7
Comparison of Power Decay
in Benchmark #3

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