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Heavy Quarks

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Decay Constants of Pseudoscalar Mesons Containing
Heavy Quarks

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Abstract

The QCD sum-rules of Shifman et al. for n^{th} order moments are applied to the determination of the decay constants of pseudoscalar mesons containing a heavy quark (c or b). We consider the general case when Q^2 , the squared momentum transfer, is non-zero. We discuss the stability of the sum-rules against variations in both Q^2 and n .

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The decay constants of heavy mesons have played an important role in several recent calculations such as the non-spectator contribution to charmed meson decays¹, the estimation of the branching ratios² of decay modes like $F \rightarrow \tau \bar{\nu}_\tau$, $B \rightarrow \tau \bar{\nu}_\tau$, etc. Accordingly, they have been the subject of much investigation recently and many different techniques have been used in their estimation³⁻⁵. Unfortunately, these methods have led to a wide spectrum of values.

Recently, a promising approach has been developed by Shifman et al.⁶ who have found a way to apply Quantum Chromodynamics (QCD) to resonance physics. Here, starting with the polarization tensor of suitable currents, one writes, on the one hand, a dispersion representation for it, and on the other, the operator product expansion (OPE). Usually OPE is valid for large momentum transfers, but Shifman et al. have argued that for heavy quarks the short distance expansion can be valid even at zero momentum transfers. For the current J_μ , defining

$$\begin{aligned} \pi_{\mu\nu}(q) &= i \int d^4x e^{iq \cdot x} \langle 0 | T^*(J_\mu(x) J_\nu(0)) | 0 \rangle \\ &= -g_{\mu\nu} \pi_1(Q^2) + q_\mu q_\nu \pi_2(Q^2) \quad , \quad Q^2 = -q^2 \end{aligned} \quad (1)$$

one can then write down expansions for the polarization functions $\pi_{1,2}$ utilizing OPE. For currents involving heavy quarks, sum-rules for moments of the polarization functions

$$\frac{1}{n!} \left(-\frac{\partial}{\partial Q^2} \right)^n \pi(Q^2) \Big|_{Q^2=0} = \frac{1}{\pi} \int \frac{\text{Im } \pi(s)}{s^{n+1}} ds \quad (2)$$

can then be derived at $Q^2=0$. Using perturbative QCD in the calculation of the coefficient functions in OPE, and saturating the dispersion integral by low lying states, Shifman et al.^{4,6,7} have obtained very interesting results with appropriate choices of n .

Recently Reinders et al.⁸ have extended this technique by considering $Q^2 \neq 0$, and have successfully calculated the charmonium spectrum. In this paper, we calculate the pseudoscalar decay constants of D, F and the B-mesons using a similar approach. We also discuss the choice of n and Q^2 from considerations of the stability of the sum-rules.

For axial vector currents involving a heavy quark, utilizing OPE, we obtain from Eq. (1)

$$\begin{aligned} \pi_{\mu\nu}(q) = & C_{\mu\nu}^I(q) + C_{\mu\nu}^G(q) \langle 0 | G_{\alpha\beta}^a G_{\alpha\beta}^a | 0 \rangle \\ & + C_{\mu\nu}^M(q) \langle 0 | \bar{\psi} M \psi | 0 \rangle + \dots \end{aligned} \quad (3)$$

where the notation is the same as in Ref. (6). In perturbative QCD only the $C_{\mu\nu}^I(q)$ term, arising from the unit operator in the expansion, survives. Although Shifman et al. have advocated that the vacuum expectation values of higher dimensional operators need not vanish as a consequence of the non-perturbative structure of the vacuum, we will in this paper drop all terms on RHS of Eq. (3) except the first, because the successive terms in the series are suppressed by higher and higher powers of the heavy quark mass. Furthermore, since asymptotic freedom is realized with heavy quarks even for low values of the momentum transfer, we calculate $C_{\mu\nu}^I(q)$ using QCD only in the lowest order. Writing $\pi_{\mu\nu}$ in terms of its longitudinal and transverse components

$$\pi_{\mu\nu}(q) = \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu} \right) \pi_t(Q^2) + \frac{q_\mu q_\nu}{q^2} \pi_\ell(Q^2) \quad , \quad (4)$$

standard lowest order QCD calculations then determine π_t and π_ℓ . In our work here, we are interested in π_ℓ which gets contributions only from pseudoscalar states. If M and m are the masses of the heavy and light quarks, we obtain

(4)

$$\pi_\ell(Q^2) = -\frac{3}{4\pi^2} \int_0^1 dx [Mm + m^2x + M^2(1-x)] \ln \left[1 + \frac{Q^2x(1-x)}{m^2x + M^2(1-x)} \right] \quad (5)$$

We now consider the n^{th} moment of $\pi_\ell(Q^2)$ for arbitrary Q^2 , using a dispersion representation for π_ℓ with $\leq n$ subtractions.

$$M_n \equiv \frac{1}{n!} \left[-\frac{\partial}{\partial Q^2} \right]^n \pi_\ell(Q^2) = \frac{1}{\pi} \int \frac{\text{Im}\pi_\ell(s)}{(s+Q^2)^{n+1}} ds \quad (6)$$

Separating out the lowest lying pseudoscalar meson contribution, we obtain

$$M_n = \frac{f_p^2 M_p^2}{(M_p^2 + Q^2)^{n+1}} + \text{continuum} \quad (7)$$

where M_p is the mass of the heavy pseudoscalar meson and f_p its decay constant. Dropping the continuum contribution and calculating the n^{th} moment of π_ℓ from Eq. (5), we get the desired relation

$$\frac{f_p^2 M_p^2}{(M_p^2 + Q^2)^{n+1}} = \frac{3}{4\pi^2} \frac{1}{n} \int_0^1 dx \frac{x^n (1-x)^n [Mm + m^2x + M^2(1-x)]}{[m^2x + M^2(1-x) + Q^2x(1-x)]^n} \quad (8)$$

Eq. (8) can determine the decay constant f_p if we know the quark masses and M_p . However, an important question is what do we use for n and Q^2 ?

To investigate this question, let us consider the restrictions on n and Q^2 imposed by the requirement that the terms neglected in obtaining the sum-rule (8) are small compared with the terms we have retained. Regarding the continuum corrections, simple models like replacing the continuum by effective poles or approximating the continuum by parton model, show that such corrections are small compared with the contribution of the lowest lying pseudoscalar meson for large n and small Q^2 . On the other hand, the neglect of higher order QCD corrections and the non-perturbative terms

in the RHS of Eq. (8), is more justified for small⁶ n and large Q^2 . Clearly, the validity of Eq. (8) rests on the possibility of finding a compromise choice for n and Q^2 . In practice whether such a choice exists and what it is, can be studied by first calculating f_p from Eq. (8) for a wide set of values of n and Q^2 . Now, if there exists a domain for n and Q^2 where f_p is sensibly a constant, so that the sum-rule is reasonably stable, we have found the compromise choice.

In Fig. 1, we have plotted f_D for several values of n and Q^2 . In computing f_D we have used the masses $M_D = 1.86$ GeV, $m_u \approx m_d \approx 0.01$ GeV and $M_c = 1.27$ GeV.⁹ To illustrate our arguments described above, consider Fig. 1(a). Clearly for $n = 3$ or 4 , f_p is reasonably flat with respect to variations in $\sqrt{Q^2}$ over the range shown. Similarly Fig. 1(b) shows that for $\sqrt{Q^2} \sim 0.8$ GeV, f_p is sensibly constant for a range of values of n . We thus find that the sum-rule (8) works best around $n = 3-4$, $\sqrt{Q^2} \sim 0.8$ GeV. To pinpoint the 'best' values of n and $\sqrt{Q^2}$, all one has to do is to look at the value of $\sqrt{Q^2}$ in Fig. 1(a) where the dispersion in f_p for different n values is the least, and similarly to look at the value of n in Fig. 1(b) where the f_D -curves for different $\sqrt{Q^2}$ values converge the most. This fixes the 'best' values at $n = 3$, $\sqrt{Q^2} = 0.9$ GeV for which we get $f_D \approx 144$ MeV.

In Figs. 2-5 we do the same analysis for the other f_p 's where $P = F, B_u, B_s$ and B_c , respectively. The masses of some of these mesons are not known, but for our purposes here we choose them to be $M_F = 2.03$ GeV,¹⁰ $M_{B_u} \approx 5.2$ GeV,¹¹ $M_{B_s} \approx 5.5$ GeV and $M_{B_c} \approx 6.6$ GeV. The mass of s and b quarks are taken as 0.15 GeV and 4.24 GeV⁹ respectively. We display the results for the 'best' values of n and $\sqrt{Q^2}$ in each case with the corresponding values of f_p in the table. Our results for f_D and f_F are not very different from those obtained from the $Q^2=0$ sum-rules,⁴ but the

stability considerations discussed here for arbitrary n and Q^2 provide extra reliability.

With the calculated values of f_p , we can estimate the τ -leptonic decay rates of the mesons F , B_u and B_c . For $P \rightarrow \tau \bar{\nu}_\tau$, the decay rate is given by

$$\Gamma(P \rightarrow \tau \bar{\nu}_\tau) = \frac{1}{8\pi} |U_{Qq}|^2 G_F^2 f_p^2 M_P m_\tau^2 (1 - m_\tau^2/M_P^2) \quad (9)$$

where U_{Qq} is an element of the Kobayashi-Maskawa mixing matrix.¹² Using the M_P 's quoted above and f_p from the table, we get

$$\begin{aligned} \Gamma(F \rightarrow \tau \bar{\nu}_\tau) &= 1.25 \times 10^{11} |U_{cs}|^2 \text{ s}^{-1} \\ \Gamma(B_u \rightarrow \tau \bar{\nu}_\tau) &= 4.88 \times 10^{12} |U_{bu}|^2 \text{ s}^{-1} \\ \Gamma(B_c \rightarrow \tau \bar{\nu}_\tau) &= 5.16 \times 10^{13} |U_{bc}|^2 \text{ s}^{-1} \end{aligned} \quad (10)$$

We may now use the favored values² of U_{Qq} : $|U_{cs}| = 0.97$, $|U_{bu}| = 0.07 - 0.12$, and $|U_{bc}| = 0.13 - 0.16$. The lifetime of F -meson¹⁰ is known with large uncertainties $\tau(F) = 2.2_{-1.0}^{+2.8} \times 10^{-13}$ s; for B -mesons, we may use the theoretical estimates^{2,13}: $\tau(B_u) \sim (5 - 10) \times 10^{-14}$ s and $\tau(B_c) \sim (1 - 3) \times 10^{-14}$ s.

Using these values in Eq. (10), we predict the branching ratios

$$\begin{aligned} \text{BR}(F \rightarrow \tau \bar{\nu}_\tau) &= 2.6_{-1.2}^{+3.3} \% \\ \text{BR}(B_u \rightarrow \tau \bar{\nu}_\tau) &= (0.12 - 0.7) \% \\ \text{BR}(B_c \rightarrow \tau \bar{\nu}_\tau) &= (0.9 - 4) \% \end{aligned} \quad (11)$$

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TABLE

P	'best' n	'best' $\sqrt{Q^2}$ (Gev)	f_p (Mev)
D	3	0.9	144
F	2	0.8	214
B _u	7	2.0	216
B _s	5	2.0	319
B _c	4	1.6	593

Table Caption

Calculated results for f_p for the 'best' values of n and $\sqrt{Q^2}$ obtained from the Figs. 1-5.

Figure Captions

- (a) f_p vs $\sqrt{Q^2}$ for a few selected values of n .
- (b) f_p vs n for different $\sqrt{Q^2}$.

References

1. M. Bander, D. Silverman, and A. Soni, Phys. Rev. Lett. 44, 7 (1980).
V. Barger, J. P. Leveille, and P. M. Stevenson, Phys. Rev. D22, 693 (1980).
2. V. Barger, J. P. Leveille, P. M. Stevenson, and R. J. N. Phillips, Phys. Rev. Lett. 45, 83 (1980).
3. N. G. Deshpande and D. Iskander, Phys. Rev. D19, 3457 (1979) and references therein.
S. S. Gershtein and N. Yu. Khlopov Pis'ma Zh. Eksp. Teor. Fiz 23, 374 (1976) [JETP Lett. 23, 338 (1976)]
4. V. A. Novikov, L. B. Okun, M. A. Shifman, A. I. Vainshtein, M. B. Voloshin, and V. I. Zakharov, Phys. Rev. Lett. 38, 626 (1977).
5. M. Suzuki, Nucl. Phys. B 177, 413 (1980).
6. M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. B 147, 385, 448 (1979).
V. A. Novikov, L. B. Okun, M. A. Shifman, A. I. Vainshtein, M. B. Voloshin, and V. I. Zakharov, Phys. Report 41, No. 1 (1978).
7. M. A. Shifman, A. I. Vainshtein, M. B. Voloshin and V. I. Zakharov, Phys. Lett. 77B, 80 (1978).
8. L. J. Reinders, H. R. Rubinstein and S. Yazaki, Phys. Lett. 95B, 103 (1980).
9. We have calculated the masses of the c and b quarks by using the lowest order sum-rules in Ref. 6 for $Q^2 \neq 0$, and the masses and e^-e^+ decay widths of the lowest four resonances of the ψ and the γ families, respectively. Strictly speaking, the quark mass depends on Q^2 , and is usually defined at some suitable Euclidean point. However, the difference between various renormalization points becomes

apparent only if higher order QCD terms are included.

10. K. Niu, Proceedings of the XX conference in High Energy Physics, American Institute of Physics, New York 1981.
N. Ushida et al., Phys. Rev. Lett. 45, 1053 (1980).
11. D. Andrews et al., Phys. Rev. Lett. 45, 219 (1980).
G. Finocchiaro et al., Phys. Rev. Lett. 45, 222 (1980).
12. M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).
13. K. Jagannathan, Ph.D. thesis, University of Rochester, 1980.

Fig 1-(a)

Fig 1-(b)

Fig 2-(a)

Fig 2-(b)

Fig 3 - (a)

Fig 3 - (b)

Fig 4 - (a)

Fig 4 - (b)

Fig 5 - (a)

Fig 5 - (b)