Neutronic Analysis of LMFBRs During Severe Core Disruptive Accidents

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NEUTRONIC ANALYSIS OF LMFBRs DURING SEVERE CORE DISRUPTIVE ACCIDENTS*

E. T. Tomlinson

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TABLE OF CONTENTS

	Page
LIST OF FIGURES	v
LIST OF TABLES	vii
ABSTRACT	1
INTRODUCTION	1
CALCULATIONAL METHODS	2
SMALL HOMOGENEOUS CORE CONFIGURATIONS	3
Case 1 - The Reference Core	3
Case 2 - The Sodium Void Model	5
Case 3 - The Post-Clad Motion Pre-Fuel Motion Model	7
Case 4 - The Post-Fuel Motion Pre-Shutdown Model	8
Case 5 - The Post-Fuel Motion Model	8
LARGE HETEROGENEOUS CORE CONFIGURATIONS	8
Case 6 - Reference BOL Case	10
Case 7 - The Sodium Void Model	13
Case 8 - The Post-Clad Motion Pre-Fuel Motion Model	14
Case 9 - The Post-Fuel Motion Internal Blanket Void Model	14
Case 10 - Post-Internal Blanket Void Model	14
NUMERICAL RESULTS AND ANALYSIS	14
Calculated k-eff's	14
Homogeneous Core	14
Heterogeneous Core	20
Reactivity Changes	22
Homogeneous Core	22
Heterogeneous Core	27
SUMMARY AND CONCLUSIONS	30
ACKNOWLEDGMENTS	31
REFERENCES	33
APPENDIX	35
APPENDIX REFERENCES	43

LIST OF FIGURES

	Page
1. Material Map for Disrupted Core Case 1	3
2. Zone Map and Mesh Description for the Small Homogeneous Core Configurations	4
3. Material Map for Disrupted Core Case 2	5
4. Material Map for Disrupted Core Case 3	7
5. Material Map for Disrupted Core Case 4	9
6. Material Map for Disrupted Core Case 5	10
7. Material Map for Disrupted Core Case 6	11
8. Zone Map and Mesh Description for Heterogeneous Core Configurations	12
9. Material Map for Disrupted Core Case 7	13
10. Material Map for Disrupted Core Case 8	15
11. Material Map for Disrupted Core Case 9	16
12. Material Map for Disrupted Core Case 10	17
13. The Dependence of k_{eff} on the Choice of Diffusion Coefficients in the Voided Region of the Small Homogeneous Core	19
14. A Comparison of the Fission Densities Calculated at the Core Midplane of the Heterogeneous Core Based on Diffusion and Transport Calculations	21
A.1. The 350 MW(e) Calculational Model	35
A.2. Comparison of the Sodium Transport Cross Section Calculated Using XSDRNP, AXMIX, and MC ² -2	39

LIST OF TABLES

	Page
1. Energy Boundaries for the Seven-Group Homogeneous Core Library	2
2. Energy Boundaries for the Nine-Group Heterogeneous Core Library	2
3. Disrupted Configurations for the Small Core Study	6
4. Calculated k_{eff} for the Various Disrupted Core Configurations for the Small Core Models	18
5. Calculated k_{eff} for the Large Core Disrupted Geometries Using Various Calculational Approximations	22
6. Reference Case Transport Diffusion Comparison for the Inner Two Rows of Subassemblies	23
7. Calculated Reactivity Changes for the Various Disrupted Configurations for the Homogeneous Core	23
8. Reactivity Changes Calculated Using Transport Perturbation Theory	24
9. Reactivity Changes Calculated Using Diffusion Perturbation	25
10. Relative Error in Reactivity Contributions Based on Diffusion Perturbation Theory	25
11. Reactivity Changes Calculated Using Diffusion Theory Fluxes and Currents in a Transport Perturbation Calculation for the Homogeneous Core	26
12. Calculated Reactivity Change for the Large Heterogeneous Core Using k_{eff} Difference Techniques	27
13. Large Heterogeneous Core Voided ± 50.96 cm About the Core Midplane	27
14. Calculated Reactivity Change in the Large Heterogeneous Core Using Various Perturbation Techniques	29
15. Relative Error in Reactivity Contribution Based on Diffusion Perturbation Theory	29
A.1. Calculated Reactivity Changes for the Various Disrupted Configurations	36
A.2. Comparison of k_{eff} for Various Calculational Models and Cross-Section Sets for a 350 MW(e) LMFBR Core	36

NEUTRONIC ANALYSIS OF LMFBRs DURING SEVERE CORE DISRUPTIVE ACCIDENTS

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ABSTRACT

A number of numerical experiments were performed to assess the validity of diffusion theory and various perturbation methods for calculating the reactivity state of a severely disrupted liquid metal cooled fast breeder reactor (LMFBR). The disrupted configurations correspond, in general, to phases through which an LMFBR core could pass during a core disruptive accident (CDA). Two reactor models were chosen for this study, the two zone, homogeneous Clinch River Breeder Reactor and the Large Heterogeneous Reactor Design Study Core. The various phases were chosen to approximate the CDA results predicted by the safety analysis code SAS3D. The calculational methods investigated in this study include the eigenvalue difference technique based on both discrete ordinate transport theory and diffusion theory, first-order perturbation theory, exact perturbation theory, and a new hybrid perturbation theory. Selected cases were analyzed using Monte Carlo methods. It was found that in all cases, diffusion theory and perturbation theory yielded results for the change in reactivity that significantly disagreed with both the discrete ordinate and Monte Carlo results. These differences were, in most cases, in a non-conservative direction.

INTRODUCTION

The determination of the reactivity state of a liquid metal cooled fast breeder reactor (LMFBR) during a core disruptive accident (CDA) is of crucial importance in the estimation of the accident's energetics. Traditionally, material worths are calculated using two- or three-dimensional diffusion theory for use in point kinetics calculations in reactor safety analysis codes such as SAS3D.¹ The purpose of this study is to investigate the adequacy of two-dimensional diffusion theory and various perturbation techniques for the calculation of these worths in both a small 350 MW(e) homogeneous reactor design such as Clinch River Breeder Reactor,² and in a large 1200 MW(e) heterogeneous reactor design such as the Large Heterogeneous Reactor Design Study Core.³ The calculational methods investigated in this study include the eigenvalue difference technique based on both discrete ordinate transport and diffusion theory calculations, first-order perturbation theory, exact perturbation theory and a new hybrid perturbation technique.⁴ Selected cases were calculated using Monte Carlo methods to assess the validity of the discrete ordinate results.

CALCULATIONAL METHODS

The small homogeneous core calculations were performed using a seven energy group cross-section library, and the large heterogeneous core calculations were performed using a nine energy group cross-section library. The MINX SPHINX AMPX¹ code systems were used to generate these weighted broad group libraries. The energy boundaries for the seven-group library are given in Table 1 and the nine-group library in Table 2. A discussion of cross section validation is presented in the appendix.

The forward and adjoint flux calculations were performed using the DOT IV² two-dimensional discrete ordinate transport code using both the diffusion and transport theory options. The recently developed weighted-difference formulation³ was used in all of the transport calculations to minimize the effect of model switching on the convergence of the problem. The Monte Carlo code KENO⁴ was used to perform a few confirmatory calculations. All perturbations were performed using a number of inhouse code modules.

Table 1. Energy Boundaries for the Seven-Group Homogeneous Core Library

Group	Upper Energy Boundary (eV)
1	1.7333×7
2	9.1973×6
3	5.4981×6
4	2.4660×6
5	4.9787×5
6	1.2341×3
7	4.1399×1

Table 2. Energy Boundaries for the Nine-Group Heterogeneous Core Library

Group	Upper Energy Boundary (eV)
1	1.7333×7
2	6.0653×6
3	1.1080×6
4	4.9787×5
5	4.0868×4
6	3.3546×3
7	2.6126×3
8	1.0110×2
9	1.0000×1

SMALL HOMOGENEOUS CORE CONFIGURATIONS

Five "snapshot" configurations were chosen for detailed neutronic analysis. Each configuration approximated, within the limits of R-Z geometry, a distinct phase in either the unprotected transient overpower (TOP) or transient undercooling (TUC) accident scenario. The configurations were defined as follows:

- Case 1 - normal beginning of life (BOL) 350 MW(e) core.
- Case 2 - initial sodium voiding phase of a TUC accident.
- Case 3 - post-clad motion pre-fuel motion phase of a TUC accident.
- Case 4 - post-fuel motion pre-shutdown phase of a TUC accident.
- Case 5 - post-fuel motion phase of a TOP accident.

Case 1 - The Reference Core

The reference BOL core is depicted in Fig. 1. The calculational model was extended through the shield in the radial direction (167.842 cm from the centerline) where a non-return boundary condition was imposed. The axial model extended from the non-return boundary at the lower edge

ORNL-DWG 77-8516

Fig. 1. Material Map for Disrupted Core Case 1.

of the lower attachment, upward through the upper fission gas plenum, (a distance of 367.436 cm) to another non-return boundary. The interior mesh spacing and zone map are shown in Fig. 2. A total of 3672 mesh points were used to describe this and all subsequent disrupted geometries for the homogeneous core.

An extra row of control rods was included in the model presented in Fig. 1. This was done in an attempt to model the partially inserted control rods on the corners of the hex that approximates the inner core. The safety rods on the flats of this hex were parked in the upper axial blanket. This is a standard modeling technique employed by Westinghouse (W-ARD) in modeling of the Clinch River Breeder Reactor.

Fig. 2. Zone Map and Mesh Description for the Small Homogeneous Core Configurations.

Case 2 - The Sodium Void Model

The intent of the sodium void configuration was to model the initial phases of sodium voiding. In this configuration, the sodium was voided from the high power-to-flow ratio channels, namely the centermost subassemblies and the first row of subassemblies in the outer core. The sodium voiding was assumed to occur in the active core region and to extend up through the upper axial blanket. The material changes and the zones involved in this perturbation are presented in Table 3 and illustrated in Fig. 3.

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Fig. 3. Material Map for Disrupted Core Case 2.

Table 3. Disrupted Configurations for the Small Core Study

Problem ID	Effect of Interest	Quantity	Zones Involved
Case 1*	Reference		
Case 2*	Sodium void	100% from	28,29,35,36,42,43,50,51 61,62 33,40,47,57,66
Case 3*	Sodium void	100% from	21,22,23,26,28,29,30,31,33, 35,36,37,38,40 42,63,64,65,67,50,51,53,54 57,61,62,63,64,66
	Move SS	100% from	28,29,30,35,36,37,42,43,44
		40% to	50,51,53
		60% to	21,22,23
	Move SS	100% from	33,40,47
		40% to	57
		60% to	26
Case 4*	Same as 3 and Void Fuel	100% from	42
		5% to	50
		80% to	35
		10% to	28
		5% to	21
	Void Fuel	100% from	47
		5% to	57
		80% to	40
		10% to	33
		5% to	26
Case 5*	Move Fuel	20% from	28,35,42
		20% to	61
		20% from	33,40,47
		20% to	66
Case 6†	Reference Large Core		
Case 7*	Sodium void	100% from	50,67,81,95 53,69,83,97 55,71,85,99 58,73,87,101
Case 8*	Move SS	100% from	67,69,71,81,83,85
		30% to	95,97,99
		70% to	50,53,55
Case 9*	Move fuel	5% from	67
		5% to	95
	Sodium void	100% from	66,80,68,82
Case 10†	Replace fuel		67
	Sodium void	100% from	66,80,68,82

*See Fig. 2.

†See Fig. 8.

Case 3 - The Post-Clad Motion/Pre-Fuel Motion Model

The third disrupted configuration is described in Table 3 and illustrated in Fig. 4. This configuration was representative of the core after a large fraction, 78%, of the inner core had voided in addition to the first row of subassemblies in the outer core. The steel clad in the first three rows of subassemblies in the inner core and the first row of subassemblies in the outer core was assumed to have melted throughout the entire active core region. This material was then postulated to move upward and downward into the upper and lower axial blankets. Approximately 40% of the clad was assumed to freeze uniformly in the lower 11.913 cm of the upper axial blanket and 60% in the upper 11.895 cm of the lower axial blanket.

ORNL-DWG 77-8517

Fig. 4. Material Map for Disrupted Core Case 3.

Case 4 – The Post-Fuel Motion Pre-Shutdown Model

The fourth disrupted configuration is described in detail in Table 3 and illustrated in Fig. 5. This configuration was representative of a core shortly after the initiation of fuel slumping in the center six subassemblies of the inner core. The upper 36.681 cm of the unclad fuel was assumed to redistribute itself uniformly into the lower portion of the subassembly and into the lower and upper axial blankets. The majority of the fuel was assumed to slump under the force of gravity redistributing itself as follows: 80% in the center 19.225 cm of the active fuel region, 10% in the lower 36.132 cm of the active fuel region and 5% in the upper 11.895 cm of the lower axial blanket. The remaining 5% was postulated to move upward and freeze on or near the steel plug in the lower 11.913 cm of the upper axial blanket.

Case 5 – The Post-Fuel Motion Model

The fifth and final disrupted configuration was representative of the post-TOP accident fuel relocation and is described in Table 3 and illustrated in Fig. 6. It was postulated that 20% of the fuel was expelled from the fuel pins in the center subassemblies of the inner core and the first row of subassemblies in the outer core. It was then swept up and out of the core into the upper 23.825 cm of the upper axial blanket, where it was assumed to freeze. The steel clad and sodium in the axial blanket were assumed to remain in place.

LARGE HETEROGENEOUS CORE CONFIGURATIONS

Again for the large heterogeneous core studies, five "snapshot" configurations were chosen for detailed analysis. These configurations approximated distinct phases of a TUC accident scenario. The configurations were defined as follows:

- Case 6 normal BOL 1200 MW(e) core.
- Case 7 initial sodium voiding of the driver subassemblies.
- Case 8 post-clad motion pre-fuel motion phase.
- Case 9 Case 8 plus sodium voiding in internal blankets and initial fuel motion in drivers.
- Case 10 Case 8 plus sodium voiding in internal blankets.

Cases 9 and 10 were of particular interest since they represent two possible configurations that could occur if the timing of events were to be switched (i.e., fuel motion in the driver subassemblies were to occur simultaneously with sodium voiding or after sodium voiding in the internal blankets).

ORNL-DWG 77-8518

	INNER CORE		OUTER CORE
	AXIAL BLANKET		RADIAL BLANKET
	RADIAL SHIELD		FISSION GAS PLENUM
	LOWER ATTACHMENT		CONTROL ROD
	CONTROL CHANNEL		SODIUM VOIDED INNER CORE
	SODIUM AND STEEL VOIDED INNER CORE		SODIUM VOIDED AXIAL BLANKET
	40% CORE STEEL + SODIUM VOIDED AXIAL BLANKET		60% CORE STEEL + AXIAL BLANKET
	VOID		VERY HIGH DENSITY INNER CORE FUEL
	HIGH DENSITY INNER CORE FUEL		VERY HIGH DENSITY OUTER CORE FUEL
	HIGH DENSITY OUTER CORE FUEL		40% CORE STEEL + 5% INNER CORE FUEL + SODIUM VOIDED AXIAL BLANKET
	40% CORE STEEL + 5% OUTER CORE FUEL + SODIUM VOIDED AXIAL BLANKET		60% CORE STEEL + 5% INNER CORE FUEL + AXIAL BLANKET
	60% CORE STEEL + 5% OUTER CORE FUEL + AXIAL BLANKET		

Fig. 5. Material Map for Disrupted Core Case 4.

Fig. 6. Material Map for Disrupted Core Case 5.

Case 6 - Reference BOI. Case

The reference BOI core is depicted in Fig. 7. The calculational model employed for the large core study included many of the same features incorporated in the small core model. The interior mesh and zone map are illustrated in Fig. 8. A total of 8134 mesh points were used to describe this and all subsequent disrupted geometries for the heterogeneous core.

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Fig. 7 Material Map for Disrupted Core Case 6

ORNL-DWG 78-5425

Fig. 8. Zone Map and Mesh Description for Heterogeneous Core Configurations.

Case 7 - The Sodium Void Model

In this configuration, the sodium was voided from the four innermost driver regions from 50.96 cm below the core midplane, upward 72.81 cm above the midplane into the upper axial blanket. The material changes and the zones involved in this perturbation are presented in Table 3 and illustrated in Fig. 9.

Fig. 9. Material Map for Disrupted Core Case 7.

Case 8 - The Post-Clad Motion Pre-Fuel Motion Model

This configuration is described in Table 3 and illustrated in Fig. 10. In this configuration, no additional sodium voiding was assumed to occur. The steel clad and structure were removed from the inner three rows of driver subassemblies from core midplane upward to the lower edge of the upper axial blanket. Of this material, 30% was assumed to freeze uniformly in the lower 11.85 cm of the upper blanket, and 70% was assumed to freeze uniformly in the 50.96 cm below the core midplane.

Case 9 - The Post-Fuel Motion Internal Blanket Void Model

This configuration is described in Table 3 and illustrated in Fig. 11. In this configuration, 5% of the fuel in the upper 10.00 cm of the innermost row of driver subassemblies was assumed to move upward and freeze in the lower 11.35 cm of the upper axial blanket. The innermost two rows of internal blanket subassemblies were assumed to be void of sodium from the core midplane upward for 60.96 cm.

Case 10 - Post-Internal Blanket Void Model

Case 10 is identical to Case 9, except the fuel was assumed to remain in place. The configuration is described in Table 3 and illustrated in Fig. 12.

These ten particular disrupted configurations were chosen to be representative of the spectrum of material redistribution in severe core disruptive accidents. They by no means represent all the possible configurations, nor do they necessarily represent the most reactive configurations.

NUMERICAL RESULTS AND ANALYSIS

A number of discrete ordinate transport theory k_{∞} eigenvalue calculations were performed for each configuration using various angular quadratures and scattering cross section expansions in order to assess the effects of anisotropic scattering and streaming in internal voids. Four calculations were done for the small core cases (S₁P₁, S₁P₂, S₂P₁, S₂P₂), and three for the large core cases (S₁P₁, S₁P₂, S₂P₁). The S₂ cases were not performed for the large core cases since the computing costs became prohibitive. The P₂ cross-section sets were transport corrected as described in the appendix of this report.

Calculated k_{∞} 's

Homogeneous Core

There was no significant change in the calculated k_{∞} due to varying either the quadrature or the order of scattering over the aforementioned range, with the exception of the S₂ calculations which resulted in a 0.32% and a 0.45% underprediction of k_{∞} for the homogeneous and heterogeneous reference cores, respectively. The calculated eigenvalues for these cases are presented in Table 4.

An assumption of a transport cross section of 1.0×10^{-4} for the voided regions in Case 4 was made so that the initial diffusion theory calculations could proceed. This led to an inordinately large diffusion coefficient which grossly overpredicted the neutron leakage from the core region into the

ORNL-DWG 78-5428

DRIVER ZONE	INTERNAL BLANKET
AXIAL BLANKET	CONTROL ROD
CONTROL CHANNEL	LOWER ATTACHMENT
RADIAL BLANKET	RADIAL SHIELD
FISSION GAS PLENUM	SODIUM VOIDED DRIVER
SODIUM VOIDED AXIAL BLANKET	
HIGH DENSITY STEEL + SODIUM VOIDED DRIVER	
HIGH DENSITY STEEL + SODIUM VOIDED AXIAL BLANKET	
DRIVER FUEL ONLY	

Fig. 10. Material Map for Disrupted Core Case 8.

ORNL-DWG 78-5430

DRIVER ZONE	INTERNAL BLANKET
AXIAL BLANKET	CONTROL ROD
CONTROL CHANNEL	LOWER ATTACHMENT
RADIAL BLANKET	RADIAL SHIELD
FISSION GAS PLENUM	SODIUM VOIDED DRIVER
DRIVER FUEL ONLY	SODIUM VOIDED INTERNAL BLANKET
LOW-DENSITY FUEL	HIGH DENSITY STEEL + SODIUM VOIDED AXIAL BLANKET + FUEL
SODIUM VOIDED AXIAL BLANKET	
HIGH DENSITY STEEL + SODIUM VOIDED DRIVER	
HIGH DENSITY STEEL + SODIUM VOIDED AXIAL BLANKET	

Fig. 11. Material Map for Disrupted Core Case 9.

ORNL-DWG 78-5429

DRIVER ZONE	INTERNAL BLANKET
AXIAL BLANKET	CONTROL ROD
CONTROL CHANNEL	LOWER ATTACHMENT
RADIAL BLANKET	RADIAL SHIELD
FISSION GAS PLENUM	SODIUM VOIDED DRIVER
SODIUM VOIDED AXIAL BLANKET	SODIUM VOIDED INTERNAL BLANKET
HIGH DENSITY STEEL + SODIUM VOIDED DRIVER	
HIGH DENSITY STEEL + SODIUM VOIDED AXIAL BLANKET	
DRIVER FUEL ONLY	

Fig. 12. Material Map for Disrupted Core Case 10.

Table 4. Calculated k_{eff} 's for the Various Disrupted Core Configurations for the Small Core Models

	S_4P_1	S_4P_2	S_4P_3	S_2P_1	Coarse Mesh ² Diffusion	Fine Mesh ² Diffusion	KENO P_1
Case 1	0.985797	0.985950	0.987033	0.985091	0.970650	0.967050	0.98190 ± 0.00255
Case 2	0.985891	0.986057	0.987215	0.985091	0.969481		
Case 3	1.00017	1.00051	1.00155	0.994259	0.979777		
Case 4	1.00381	1.00915	1.01636	1.00707	0.993561 0.968576	0.989703 0.963693	1.00529 ± 0.00293
Case 5	0.979032	0.979203	0.980214	0.977960	0.963520		

¹54 x 68 mesh.

²108 x 136 mesh.

³ $D = R/3$ cm

⁴ $D = 333.33$ cm in void

axial blanket regions. In the blanket, neutrons were less important, and consequently, an underprediction of k_{∞} resulted. Further diffusion calculations were performed to assess the effects of the assumed value of the diffusion coefficient on k_{∞} .

The dependence of k_{∞} on the value chosen for the diffusion coefficient in the internally voided regions is shown in Fig. 13. The calculated multiplication factor is extremely sensitive to the choice of diffusion coefficients, even though the voided region represented only 0.74% of the active core region. An investigation of the shape of the curve in Fig. 13 revealed that for small values of the diffusion coefficient, D , the curve approximates the shape corresponding to Eq. (1)

$$k_{eff} \propto \frac{k^{\infty}}{1 + B^2 D / \Sigma_a} \quad (1)$$

where the terms have their standard definitions. As the value of the diffusion coefficient was increased, the diffusion length, $(D \Sigma_a)^{1/2}$, also increased. Consequently, the spatial variation of the flux through the void in the axial direction became flat while the radial component remained virtually unchanged. This implied that the axial current, which was proportional to $\nabla \Phi$, approached zero and, therefore, the axial DB^2 also approached zero. As a result, Eq. (1) became a constant, which explains the flattening of the curve for large diffusion coefficients.

A more rational approach for changing a diffusion coefficient was based on the assumption that the transport mean free path can be approximated by the mean chord length of the voided region,

$$\bar{R} = \frac{4V}{S} \quad (2)$$

Fig. 13. The Dependence of k_{eff} on the Choice of Diffusion Coefficients in the Voided Region of the Small Homogeneous Core.

where

V is the volume of the void, and

S is the surface area of the void.

The multiplication factors calculated using this approach are shown in Fig. 13 and in Table 4. This approximation yielded good results compared to the expected value based on transport calculations (within 0.10% of the k_{eff} value needed to predict the same reactivity change as predicted by transport theory). Although the use of the mean chord length in the void resulted in a substantial improvement in the eigenvalue calculated using diffusion theory, significant errors in the pointwise fluxes persisted.

The k_{eff} eigenvalues for the small core, calculated using discrete ordinate transport theory, diffusion theory, and Monte Carlo are presented in Table 4. A comparison of the k_{eff} 's calculated using transport theory indicates that the effect of increasing either the number of angles in the quadrature or the order of scattering has little effect ($< 0.15\%$) on the results.

The adequacy of the mesh was examined by comparing the k_{eff} 's calculated with the Monte Carlo code KENO with their corresponding DOT IV values. This was done only for the most severely disrupted (most heterogeneous) configuration since this was where inadequacies in the mesh would most likely have occurred. The DOT IV results were in excellent agreement with the Monte Carlo results (within 2σ) thus providing a degree of confidence in the mesh used in the transport calculations.

Further examination of Table 4 indicated that diffusion theory consistently underpredicted k_{eff} from 1.62% for Case 5, to 2.06% for Case 3. Compared to transport theory, there was a 1.5% underprediction of k_{eff} for the reference Case 1. The results for Case 4 were purposely deleted from this comparison because of the arbitrary choice of diffusion coefficients in the voided regions. The percent underprediction in k_{eff} could have been much larger if these results were included.

A "fine mesh" diffusion calculation was performed on both the Case 1 and Case 4 disrupted configurations. The fine mesh was defined by uniformly doubling the original mesh. Diffusion coefficients of 3333.33 and 8.3 were assumed for the in-core voids. Based on these cases, there did not appear to be any significant advantage in using the fine mesh for the diffusion calculations. Therefore, the remainder of the configurations were calculated using "coarse mesh" diffusion theory.

Diffusion theory's continuous underprediction of k_{eff} was caused by an overprediction of the neutron leakage from the active core zone. The neutron leakage out of the core at the upper core axial blanket interface near the core center was determined to be approximately 15% higher in the diffusion calculation than in an equivalent transport calculation for the reference core. The magnitude of this discrepancy increased as the density of the core zone decreased (i.e., the diffusion coefficient increased). Consequently, no single bias factor could be determined *a priori* which, when applied to the diffusion theory results, would yield equivalent transport results.

The relatively large discrepancies between the diffusion and transport theory k_{eff} 's for the reference case were later investigated by Adams¹¹ at Argonne National Laboratory (ANL). It was found that better agreement could be achieved if a better approximation was made for the sodium transport cross section used in the control rod channels. A brief discussion of this work is described in the appendix. Follow up studies conducted by the author indicated that indeed the sodium transport cross section in the seven-group library was deficient. However, the sodium transport cross section in the nine-group library was of sufficient quality to be used in the heterogeneous core studies. A comparison of the various cross sections is presented in the appendix.

Heterogeneous Core

The results obtained from the large heterogeneous core calculations indicate the same trends in the value of k_{eff} as was seen for the small homogeneous configurations. Here again, the resultant k_{eff} 's had small sensitivities to the varying of the scattering order, see Table 5. The S_2 and diffusion results again underpredicted the system multiplication factor compared to the transport results.

The underprediction was less for the heterogeneous core than for the homogeneous core. The reason for this phenomenon is that, unlike in the homogeneous design where the major contributor to the underprediction of k_{eff} is the overprediction of the axial leakage, there are a number of competing contributions in the heterogeneous calculations that affect the final result. In all the fertile (blanket) regions, diffusion theory consistently gave a harder spectrum than transport theory. In all the fissile (driver) regions, diffusion theory consistently gave a softer spectrum than transport theory. The total flux and fission rate were depressed more in the fertile regions for the transport calculations than for the diffusion calculations. This effect can be seen in Fig. 14. It has been found by other investigators,^{12,13} that the discrepancy between the transport and diffusion theory calculated

TRANSPORT VS. DIFFUSION FISSION PROFILES

Fig. 14. A Comparison of the Fission Densities Calculated at the Core Midplane of the Heterogeneous Core Based on Diffusion and Transport Calculations.

Table 5. Calculated k_{eff} 's for the Large Core Disrupted Geometries Using Various Computational Approximations

	S_4P_3	S_4P_0	S_2P_0	DIF
Case 6	0.999176	0.999682	0.995135	0.993433
Case 7	1.00240	1.00285	0.998095	0.995664
Case 8	1.00498	1.00550	1.00060	0.997372
Case 9	1.00554	1.00596	1.00106	0.997738
Case 10	1.00594	1.00638	1.00144	0.998120

power shapes increases when gamma transport and gamma heating effects are considered in the calculation. The power shape in the heterogeneous core was also noted to change more than in the homogeneous core calculations as the relative degree of disruption increased. This tends to somewhat invalidate the assumption that point kinetics are sufficiently accurate for transient calculations.

The axial leakage at the core blanket interface was overpredicted by factors of 2 to 6 by diffusion theory. The radial leakage between adjacent rows of subassemblies was slightly underpredicted (approximately 3%) in the diffusion calculations. The magnitude of these effects for the two innermost rows of subassemblies is presented in Table 6. The magnitude of this discrepancy tended to increase with increasing diffusion coefficients and increased core heterogeneity (i.e., more disrupted geometries). As a result, no single bias factor could *a priori* be determined and applied to the diffusion results to yield equivalent transport results.

Reactivity Changes

Homogeneous Core

Behavior similar to that observed for the multiplication factors was observed for the calculated reactivity changes between configurations for the homogeneous core cases, shown in Table 7. These reactivity changes were calculated using the direct k difference techniques.

Diffusion theory consistently predicted the system to be less reactive than transport theory. Unfortunately, this was always in the nonconservative direction. As was the case in the k_{eff} calculations, the majority of the error in the calculated reactivity change was in the calculation of the leakage component.

The reactivity changes calculated using various transport approximations were all in excellent agreement with each other (within a few cents). This indicates that neutron streaming and anisotropic scattering could be accounted for in these models by using low order transport methods

Table 6. Reference Case Transport Diffusion Comparison for the Inner Two Rows of Subassemblies

	Internal Blanket		Transport Diffusion
	Diffusion	Transport	
fission	1.48991-3	1.38055-3	0.93
absorption	6.85520-3	6.69809-3	0.98
top leakage	9.02739-3	1.4337-3	0.16
bottom leakage	9.08536-3	1.5014-3	0.17
right leakage	3.78086-2	3.89087-2	1.03
left leakage	0.0	0.0	1.0

	Driver		Transport Diffusion
	Diffusion	Transport	
fission	2.98271-2	2.99272-2	1.00
absorption	1.89638-2	1.9023-2	1.00
top leakage	1.83027-2	3.03136-3	0.17
bottom leakage	1.84411-2	3.21034-3	0.17
right leakage	7.50613-2	7.72056-2	1.03
left leakage	4.37263-2	4.47992-2	1.02

Table 7. Calculated Reactivity Changes for the Various Disrupted Configurations for the Homogeneous Core

					δρ[%]		Transport Diffusion
	S ₄ P ₁	S ₄ P ₂	S ₄ P ₃	S ₅ P ₂	Coarse Mesh Diffusion	Fine Mesh Diffusion	
Case 2	0.031	0.034	0.058	0.000	-0.256		-0.12
Case 3	4.556	4.619	4.620	4.180	3.064		1.49
Case 4	7.231	7.237	7.310	6.923	7.393 -0.771	7.210 -1.131	6.758 ± 1.224 0.98
Case 5	-2.190	-2.184	-2.201	-2.320	-2.416		0.91

$$\beta_{eff} = 0.0032$$

¹₅₄ x 68 mesh

²₁₀₈ x 136 mesh

³_D = $\bar{R}/3$ cm

⁴_D = 3333.33 cm] in void

such as S₂P. The S₂P. results underpredict the reactivity change by as much as 0.44\$. The discrete ordinate results lie within 1σ of the Monte Carlo calculated reactivity change, which gave an air of confidence to the results.

A closer comparison of the diffusion and transport results revealed that the underprediction of $\delta\rho$ ranged from 0.22\$ for Case 2 to 1.55\$ for Case 3. Again, Case 4 was not considered because of the uncertainties in the calculated reactivity changes for the arbitrary choice of diffusion coefficients.

Since perturbation theory is a standard procedure for determining material worths for safety calculations, the validity and accuracy of a number of perturbation techniques were investigated. Both transport and diffusion theory perturbation methods were used to calculate the reactivity changes between cases.

The transport perturbation calculations were performed using the S₂P forward and adjoint fluxes because S₂ and S₄ results were of comparable accuracy, and S₂ calculations were less costly to compute. First-order and exact perturbation calculations were performed for each of the four disrupted configurations. The results of these calculations are presented in Table 8.

First-order transport perturbation continuously overpredicted the magnitude of the reactivity change, for both positive and negative changes. This overprediction was conservative for positive reactivity changes, but not for negative reactivity changes (i.e., the system was calculated to be further subcritical than it actually was). The use of exact perturbation theory yielded reactivity changes that were within a few percent of the direct k_{eff} difference calculations. The difficulty with the use of exact perturbation theory was that the perturbed adjoint must be calculated for each disrupted configuration (*a priori* knowledge of the final configurations).

Table 8. Reactivity Changes Calculated Using
Transport Perturbation Theory

	$\delta\rho[\$]$		
	Direct	First-Order Perturbation	Exact Perturbation
Case 2	0.034	0.193	-0.044
Case 3	4.619	5.512	4.304
Case 4	7.287	9.150	6.970
Case 5	-2.184	-2.316	-2.321

First-order diffusion perturbation theory does not fair as well as first-order transport perturbation theory as can be seen in Table 9. The discrepancy for some cases was as large as a factor of 5 for the magnitude of the change, and in Case 2 even the sign of the reactivity change was incorrect. Again, exact diffusion perturbation theory could be used to calculate reactivity changes that were in good agreement with the direct diffusion k_{∞} difference results, but still not in agreement with direct transport results.

Table 9. Reactivity Changes Calculated Using
Diffusion Perturbation

	$\delta\rho[\$]$		
	Direct	First-Order Perturbation	Exact Perturbation
Case 2	-0.256	-0.743	-0.231
Case 3	3.064	-1.587	3.189
Case 4	7.393	1.458	7.495
Case 5	-2.416	-2.439	-2.420

A closer examination of the first-order versus exact perturbation theory is presented in Table 10. The largest discrepancy was in the calculation worth of the diffusion coefficient which had a maximum error of 43%. The maximum error occurred where a large amount of core material had been moved and the shape of the adjoint had drastically changed.

Table 10. Relative Error in Reactivity Contributions
Based on Diffusion Perturbation Theory

	$[\% \delta\rho(\$)]^a$			
	Case 2	Case 3	Case 4	Case 5
Diffusion	23.48	29.48	43.67	3.99
Removal	0.58	-0.52	2.12	0.97
Scatter	1.49	1.70	4.68	1.09
Fission	0.00	0.00	-0.51	0.72

$$^a \% \delta\rho(\$) = (\text{FOP-EXACT})/\text{EXACT} \times 100$$

An examination of the diffusion theory first-order perturbation results revealed that the net contribution of the removal and scattering terms was near zero, leaving a difference between the diffusion and fission terms for determination of the final reactivity change. Therefore, the large errors in the leakage worths were the dominant contributors to discrepancies in the total calculated reactivity change. It should be noted here that the leakage (diffusion) is implicitly calculated in the transport perturbation and, thus, does not contribute explicitly to errors in the calculated reactivity change. It is for this reason that first-order transport perturbation tends to yield reactivity changes that are in better agreement with the direct k_{∞} difference and exact perturbation results.

In view of the implicit nature of the leakage worth calculation in transport calculations, a hybrid diffusion transport perturbation technique was investigated. This method involved the calculation of the partial currents from the diffusion theory scalar fluxes for each interval. These partial currents were then treated as the components of the first flux moment and folded with the P_1 scattering cross section. The remainder of the calculation proceeded in the normal transport perturbation mode, thus accounting for the leakage implicitly.

This hybrid method was applied to each case using both the first-order and exact perturbation approximations. The results of these calculations are presented in Table II. The first-order hybrid perturbation results were significantly improved over those obtained using first-order diffusion theory perturbation. In fact, they were in excellent agreement with the first-order transport perturbation results. This indicated that the hybrid diffusion transport technique may be used to yield transport perturbation results with diffusion theory computing expense. The hybrid method by no means calculates the correct (direct k_{∞} difference) reactivity changes, but it does provide a substantial degree of improvement over standard first-order diffusion theory perturbation. No substantial improvement was noted for exact perturbation theory.

Table II. Reactivity Changes Calculated Using Diffusion Theory Fluxes and Currents in a Transport Perturbation Calculation for the Homogeneous Core

	Direct Transport	First-Order			Exact		
		Diffusion	Transport	Hybrid	Diffusion	Transport	Hybrid
Case 2	0.034	-0.743	0.193	0.151	-0.231	-0.044	-0.216
Case 3	4.619	-1.587	5.512	5.549	3.189	4.304	3.261
Case 4	7.287	1.458	9.150	9.139	7.495	6.970	6.539
Case 5	-2.184	-2.439	-2.316	-2.412	-2.620	-2.321	-2.420

Heterogeneous Core

The reactivity changes between the various disrupted configurations and the reference configuration for the large heterogeneous core are presented in Table 12. Here again, diffusion theory consistently underpredicts the reactivity changes between the various configurations. A cursory examination of Table 12 would indicate that a range of bias factors (T/D) beginning at 1.43 for the least disrupted case, Case 7, and increasing to 1.46 for the most disrupted case, Case 9, could be calculated for these configurations.

Table 12. Calculated Reactivity Change for the Large Heterogeneous Core Using k_{eff} Difference Techniques

	S_1P_1	S_1P_2	S_1P_3	Diffusion	CEC
Case 7	0.967	0.929	0.977	0.663	
Case 8	1.700	1.702	1.614	1.179	
Case 9	1.863	1.836	1.749	1.277	2.310 ± 0.302
Case 10	1.979	1.957	1.861	1.390	

$$\beta_{eff} = 0.0015$$

In order to investigate the phenomenon, two additional configurations were calculated. In the first, sodium was voided symmetrically about the core midplane (± 50.96 cm) in all the driver zones. The voiding did not extend into the axial blankets. This was done to minimize the axial leakage component of the sodium voiding reactivity. In the second configuration, all the internal blanket zones were voided of sodium to the same extent as were the driver zones in the first configuration. The results of these calculations are presented in Table 13.

Table 13. Large Heterogeneous Core Voided ± 50.96 cm About the Core Midplane

	k_{eff}			k_p		
	Diffusion	Transport	T/D	Diffusion	Transport	T/D
Reference	0.993433	0.999176	1.0057	--	--	--
Voided Driver	0.997307	1.004690	1.0074	1.150	1.616	1.41
Voided Blanket	1.00375	1.00950	1.0057	3.043	3.010	0.989

$$\beta_{eff} = 0.0015$$

The reactivity change calculated for the first configuration, the sodium voided driver zones, was 1.1505 from the diffusion solution versus 1.6165 for the transport solutions. This gave a bias factor of 1.41 which was similar to the previously calculated values. In the second case, the diffusion solution gave a reactivity change of 3.0435 which was slightly higher than the transport value of 3.0195. This results in a bias factor of 0.939. The overprediction of the reactivity in the voided blanket regions and the underprediction in the voided driver region was due totally to the difference in the neutron energy spectrum calculated by transport and diffusion theory. (The voiding pattern was chosen to minimize the leakage effects.) Diffusion theory predicted a harder spectrum in the internal blankets than transport theory; therefore, there were more fissions in ^{235}U . In the driver zone, where the spectrum predicted by diffusion theory was softer than transport theory, the spectral effects were less.

The ratio of the transport perturbation reactivity changes to the diffusion perturbation reactivity changes were calculated. This ratio was greater than unity and generally increased in magnitude as the reactivity change increased. On the other hand, perturbations in the internal blanket zones gave ratios that were less than unity. Perturbations that involved material motion in both the driver zones and in the internal blanket zones gave ratios that were not as large as would have been expected. This was based on cases where perturbations occurred solely in the driver zones.

Considering the vast number of possible combinations of perturbations involving the driver and the blanket zones and their compensating errors, there does not appear to be a method to *a priori* calculate a single bias factor which will yield transport results from diffusion calculations.

As in the homogeneous core study, the applicability of various perturbation techniques was assessed. First-order perturbation, exact perturbation and the previously discussed hybrid perturbation theories were employed. The results of the calculations are presented in Table 14. First-order diffusion perturbation theory underpredicts by as much as factors of three to five the reactivity changes calculated using transport and diffusion theory k_{eff} difference techniques (or exact perturbation theory). A breakdown of the differences in the various components (i.e., fission, removal, scatter, diffusion) that make up the reactivity change is presented in Table 15.

The diffusion term represented the largest discrepancy between the two perturbation methods, but was small in absolute magnitude compared to the removal and scatter terms. The same was true for the small homogeneous core studies. The differences in the removal, scatter, and fission terms were somewhat larger for the heterogeneous core case than those seen in the homogeneous core cases. These ranged from 2 to 10% in the heterogeneous case and 1 to 4% in the homogeneous cases. This was due to the more pronounced spectral hardening of the adjoint in the heterogeneous core than in the homogeneous core.

The hybrid diffusion-transport perturbation method performed better than classic diffusion theory perturbation. The results obtained using this method all fell between the diffusion and transport results. The inclusion of the first scattering moment accounted for approximately 75% of the reactivity difference between the diffusion and transport solutions. As before, this method represents an improvement over classic diffusion perturbation but does not supply a complete remedy.

Table 14. Calculated Reactivity Change in the
Large Heterogeneous Core Using Various
Perturbation Techniques

	Diffusion		Hybrid		Transport	
	FOP	EXACT	FOP	EXACT	FOP	EXACT
Case 7	0.626	0.672	0.731	0.678	0.928	0.896
Case 8	0.341	1.186	1.046	1.192	1.255	1.663
Case 9	0.399	1.290	1.115	1.303	1.329	1.791
Case 10	0.494	1.407	1.120	1.414	1.417	1.914

$$\beta_{\text{eff}} = 0.0034$$

Table 15. Relative Error in Reactivity Contribution
Based on Diffusion Perturbation Theory

	[% $\delta\rho$ \$] ^a			
	Case 7	Case 8	Case 9	Case 10
Diffusion ^b	1.42	21.27	20.94	19.32
Removal	-1.76	-8.71	-9.95	-10.67
Scatter	-1.71	-7.52	-8.75	-9.49
Fission ^b	0.00	0.00	-16.97	0.00

$$^a \% \delta\rho(\$) = \frac{\text{FOP} - \text{EXACT}}{\text{EXACT}} \times 100$$

^b The absolute value of this effect, in \$'s, is 1.5 to 2.0 orders of magnitude less than the removal and scattering terms.

First-order transport perturbation theory underpredicts the reactivity change compared to k_{eff} difference methods by approximately 25%, except the sodium void case, Case 9, where the agreement was much better, within 2.1%. Similar results were seen for the homogeneous core cases.

SUMMARY AND CONCLUSIONS

In all cases studied, diffusion theory underpredicted the effective system multiplication factor compared to an equivalent discrete ordinates transport calculation. The underprediction for the homogeneous cases was a direct result of the overprediction of the axial neutron leakage from the active core zone. This was an effect that increased as the degree of core disruption increased. On the other hand, for the large heterogeneous core, the underprediction was fostered more by the harder spectrum calculated in the internal blankets and a softer spectrum in the driver zones than by the axial leakage. This is not to say that the axial leakage was not overpredicted.

As a result of the studies done at ANL concerning the sodium transport cross section, two interesting effects were uncovered (see appendix). There was a high sensitivity of the transport-diffusion k_{eff} difference to the sodium transport cross section in the central channels of the homogeneous core. This was of particular interest since the cross sectional area (leakage surface) represented by these channels was a small fraction of the total surface of the core.

The second point was that the difference between the diffusion and transport k_{eff} 's can greatly be reduced by modifying the geometric model used in the calculation. As expected, the more homogeneous a reactor can be made to look (i.e., large zones of a single material), the closer the diffusion theory solution agrees with the transport theory calculation. This approach can be used for the analysis of homogeneous cores or clean critical assemblies, but its application to heterogeneous cores may be difficult.

In the large heterogeneous core, diffusion theory has difficulty predicting the neutron energy spectrum. Diffusion theory predicts a harder spectrum in the internal blankets than transport theory. In the drivers, diffusion theory predicts a softer spectrum than transport theory. The total flux is depressed more in the fertile regions in the transport calculation than in the diffusion calculation.

Diffusion theory consistently underpredicted the magnitude of the reactivity change due to material motion in the homogeneous core. The same was true for the heterogeneous core in cases where material motion occurred only in the driver zones. When material motion was assumed to occur in the blanket zone, diffusion theory overpredicted the reactivity change between configurations. This was a direct consequence of the harder spectrum in the blankets as predicted by diffusion theory. When the material motion was assumed to occur in both zones simultaneously, the two effects came into competition making an *a priori* prediction of a suitable bias factor impossible.

First-order perturbation theory consistently underpredicts the reactivity change between configurations for both the homogeneous and the heterogeneous cores. First order diffusion perturbation results were in error to the point of being almost useless. Transport perturbation results, although underpredicting the reactivity change, were in better agreement with the k_{eff} difference results. The hybrid diffusion-transport perturbation method predicted reactivity changes between the diffusion and transport results (usually closer to the first-order transport perturbation results).

The results of this study seem to indicate that indeed some attempt should be made to account for neutron transport effects in disrupted core physics calculations. It was also found that the difference between transport and diffusion theory results was highly sensitive to the transport cross sections used in the diffusion calculations and to the calculational model used for each configuration. Some effort should be made to investigate the latter in more detail.

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APPENDIX

Recently a study was undertaken to evaluate the validity of diffusion theory for the calculation of the reactivity state of various disrupted LMFBR core configuration, i.e., various phases that a reactor core could pass through during a hypothetical core disruptive accident. A model of a 350 MW(e) core, shown in Fig. A.1, was used for this analysis. A number of disrupted geometries were then defined based on this reference configuration. The only configuration of interest for this letter was the first which was a sodium void model - the sodium was removed from the inner core region between the central and first row of control rods and from the inner-outer core interface to the first row of control rods in the outer core.

ORNL-DWG 77-8516

Fig. A.1. The 350 MW(e) Calculational Model.

The k_{∞} 's and resultant reactivity changes, based on various calculational approximations, are shown in Table A.1. As can be seen from these results, there was a significant underprediction of both k_{∞} and $\delta\rho$ when the diffusion approximation was employed. It should be kept in mind that two-dimensional diffusion calculations are the standard method used in fast reactor safety codes for the calculation of material worths. The rather large discrepancy between the transport and diffusion results prompted investigators at ANL (C. H. Adams and D. R. Ferguson) to re-evaluate the results obtained at ORNL.

Table A1. Calculated Reactivity Changes for the Various Disrupted Configurations
 $\Delta\rho[\$]$

	S_6P_1	S_6P_2	$\frac{\text{Linner/Step}}{S_6P_3}$	S_6P_6	Coarse Mesh ¹ Diffusion	Fine Mesh ² Diffusion	KENO P_0
Case 2	0.031	0.034	0.318	0.058	- 0.256		
Case 3	4.556	4.619		4.620	3.064		
Case 4	7.231	7.287	7.382	7.310	7.393 ³ - 0.723 ⁴	7.210 ³ - 1.131 ⁴	6.758 $\pm$ 1.224
Case 5	- 2.190	-2.184		- 2.201	- 2.416		

$$\beta_{eff} = .0032$$

¹54 = 68 mesh

²108 = 136 mesh

³D = 8/3 cm

⁴D = 3333.33 cm

in void

The seven broad group cross-section library and the calculational model used in the DOT IV² calculations were transmitted to ANL where they proceeded to perform some confirmatory calculations using DIFF3D¹ and TWOTRAN⁴ for the diffusion and transport calculations, respectively. The results of these calculations using the ORNL cross sections and model are given in Table A.2. The ANL results are in excellent agreement with the ORNL results.

Table A2. Comparison of k_{eff} for Various Calculational Models
 and Cross-Section Sets for a 350 MW(;) LMFBR Core

(Model) (# of Groups) (Control Channel)	Diffusion	Transport	Diffusion Transport
CRBR	.9699(DIFF3D)	.9884(TWOTRAN)	.0185
7 GP	.9706(DOT)	.9870(DOT)	.0164
Na			
CRBR	.9593*	.9693*	.0100
9 GP			
Na			
CRBR	.9611*	.9682*	.0071
9 GP			
Na + SS			
Homogenized Core	1.0269	1.0301	.0032

In an effort to resolve the discrepancy, a new nine-group cross-section set (from MC²-2)¹ was substituted for a seven-group library, and the calculations were redone. The results of these calculations are also presented in Table A.2. These results indicate an improvement in the agreement between the diffusion and transport results. Closer investigation revealed that the major difference between the ORNL seven-group library and the ANL nine-group library was in the transport cross section used for the sodium in the control rod channels. These channels were assumed to be pure sodium in all calculations. The seven-group library underestimated the sodium transport cross section by as much as a factor of three near the sodium resonance at 2.85 KeV. The cross sections for the other isotopes were within a few percent of the MC²-2 values. A discussion of this discrepancy will follow later in this appendix.

A third set of calculations was performed in which stainless steel was mixed with the sodium in the control rod channels. This more closely represents the control rod followers and the real situation. As can be seen from the results presented in Table A.2, the k_{∞} calculated using diffusion theory increased while the k_{∞} calculated using transport theory slightly decreased, as would be expected due to the increase in neutron absorption in the control rod channels. The diffusion results indicate that the increase in absorptions had less effect on k_{∞} than the decrease in leakage through the control channels in the bottom of the core, (i.e., the stainless steel in the sodium acted more like a reflector than an absorber).

In the fourth set of calculations, the control rods and control channels were smeared into the adjacent fuel regions and the nine-group cross sections were used. This is representative of the methods used by ANL-IDAHO for analysis of critical experiments. As expected for large homogeneous region problems, diffusion theory was in good agreement with the transport results.

The conclusions from this study indicate that, first, diffusion theory can be made to give equivalent transport results and, second, that the difference between transport and diffusion results is extremely sensitive to the transport cross sections (i.e., the diffusion coefficient) in the sodium channels. Even though the channels represent only a small fraction of the lower surface of the core, the leakage out of these channels is extremely important.

The explanation for the high sensitivity of the calculated results to the transport cross section for diffusion theory and not for transport theory can best be seen by examining the two equations as solved in DOT IV.

$$\Omega \cdot \nabla \phi + \Sigma_{TR} \phi = \int_{\Omega'} \Sigma_s^{g \rightarrow g'} \phi(\Omega') d\Omega' + F + S + \text{INSCATTER} \quad (\text{A.1})$$

$$-\left(\frac{1}{3\Sigma_{TR}}\right) \nabla^2 \phi + \Sigma_{TR} - \Sigma_s^{g \rightarrow g} \phi = F + S + \text{INSCATTER} \quad (\text{A.2})$$

In the first equation, the P₁ transport equation, the transport cross section is only used to calculate an effective group removal cross section, and the leakage is calculated implicitly through the $\Omega \cdot \nabla \phi$ term. In the diffusion equation, Eq. (A.2), the transport cross section is used not only to calculate an

effective group removal cross section, but also a diffusion coefficient. Here the leakage is calculated explicitly (D7²6); therefore, a change in the transport cross section directly affects the calculated leakage.

Based on the results of this brief study, an effort was made at ORNL to understand the difference between the MC²-2 nine-group transport cross sections for sodium and the ORNL seven-group cross sections. Also of major concern was the validity of the new nine-group library which was used in the large heterogeneous core studies reported in this document.

Five different "transport" cross sections were calculated using MC²-2, AXMIX,⁶ and XSDRNPM⁷ in three different energy group structures. The various cross sections are presented in Fig. A.2. The MC²-2 cross sections were obtained using the consistent P₁ or "inscatter" approximation that is presently available in that code. The ORNL cross sections that are labeled "AXMIX" were obtained using the transport correction option in the AXMIX code. This is defined by the following equation for each broad group.

$$\sigma_{TR_{i,g}} = \sigma_{T_{i,g}} - \bar{\mu} \sigma_{S_{i,g}}$$

where

(A.3)

$$\bar{\mu} \sigma_{S_{i,g}} = \frac{1}{3} \frac{\sigma_{S_g^1}}{\sigma_{S_g^0}} .$$

The cross sections labeled "XSDRNPM" were calculated in XSDRNPM using 126 energy-group cross sections weighted with the absolute value of the currents. The zone weighting option was used in XSDRNPM with the "inscatter" approximation defined as follows:

$$\sigma_{TR,G} = \sigma_{T,G} - \frac{1}{3J_G} \sum_{g \in G} \sum_{g'} J_{g'} \sigma^1(g' \rightarrow g) . \quad (A.4)$$

This approximation is equivalent to the one used in MC²-2, the difference being that MC²-2 uses a B₀ calculation to obtain the currents to weight the P₁ scattering cross section, and XSDRNPM uses an S₀ calculation to obtain the weighting function.

Fig. A.2. Comparison of the Sodium Transport Cross Section

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Calculated Using XSDRNPM, AXMIX, and MC²-2.

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The transport cross sections calculated in NSDRNPM are in better agreement with those calculated using MC^2 than are the AXMIN transport cross sections. The reason the AXMIN transport correction is in considerable disagreement is that the neutron currents needed to correctly weight the P_1 scattering cross sections are not available in AXMIN. The AXMIN values are equivalent to a flux weighted transport value. This deficiency is particularly noticeable when a poor group structure is chosen, as in the seven-group library; see Fig. A.2. However, when more care is taken to account for the sodium scattering resonance, the discrepancy between the AXMIN and NSDRNPM approximation decreases with the AXMIN values still too approximate for accurate use.

A second noticeable feature of the NSDRNPM cross sections is the large value (compared to the MC^2 value) for the transport cross section in the lowest energy group, $E < 0.1$ eV. The difference is due to different schemes for treating inelastic scattering. The anisotropy of inelastic scattering is treated in AMPX by including scattering matrices of arbitrary order. NSDRNPM uses the P_1 inelastic cross section as it would any other P_1 cross section; see Eq. (A.4). Approximately 70% of the calculated transport cross section in this group comes from inelastic scattering (all of the P_1 coefficients are negative; thus, they tend to increase the calculated transport cross section). On the other hand, MC^2 accounts for anisotropic inelastic scattering by using a P_1 "transport approximation" equivalent to an outscatter approximation. This causes the transport cross section disagreement, since the "outscatter" approximation is seen in source groups, while the "inscatter" will be seen in sink groups.

In conclusion, it is recommended that the use of AXMIN to produce transport cross sections be made only as a last resort and then only when the group structure is fine enough that the approximation is reasonable. The "inscatter" approximation in NSDRNPM should be used whenever possible, since it has a better theoretical basis than the alternate choice.

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