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**A Parallel-Beam Correlation Technique for
Measuring Density Fluctuations in
Plasmas with Strong Magnetic Shear**

University of California

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A PARALLEL-BEAM CORRELATION TECHNIQUE FOR MEASURING DENSITY
FLUCTUATIONS IN PLASMAS WITH STRONG MAGNETIC SHEAR

by

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ABSTRACT

A laser diagnostic scheme is described which facilitates localization of density fluctuations along the line of sight. The method exploits both the generally observed anisotropy of density fluctuations in low-beta plasmas, as well as the twisting of the magnetic field which occurs across the minor diameter of reversed-field pinches, spheromaks, etc. Both interferometric and schlieren variations are discussed.

I. INTRODUCTION

Density fluctuations are frequently observed in laboratory and fusion plasmas. In a few cases, measurements have been sufficiently quantitative to test theories of cross-field particle and energy transport. Microwave scattering¹ on the FM-1 spherator indicated that anomalous diffusion in that device could be attributed to drift-wave turbulence. Similar measurements² on the PLT tokamak indicated the possibility of a turbulent electron cross-field heat flux much larger than neoclassical. Double Langmuir probe measurements on the Proto-Cleo torsatron³ have also shown that drift-wave turbulence may explain the experimental cross-field diffusion coefficient. In view of the importance of anomalous transport, measurements of plasma density fluctuations are useful programmatically as well as interesting physically.

Plasma density fluctuations may also be related to beneficial processes in certain confinement schemes. Laser forward scattering measurements^{4,5} on the Alcator tokamak have indicated the presence of a thin annulus of density turbulence in the vicinity of the limiter, which may be associated with that tokamak's ability to ingest gas from the plasma edge and thereby to achieve high-density operation. The reversed-field pinch (RFP) and spheromak may rely on turbulent processes⁶ in order to relax to a minimum energy state.⁷ Indeed,

the density fluctuation observed with double Langmuir probes on Zeta⁸ may be associated with this relaxation process in terms of the "tangled discharge" model.⁹

The turbulence which will be referred to in the remainder of this paper, like the examples mentioned above, is typified by cross-field scale lengths ($k_{\perp}^{-1}$) of several millimeters or longer. Furthermore, this paper will be concerned with measurements in high background densities, in the range $\bar{n}_e \approx 1-5 \times 10^{14} \text{ cm}^{-3}$, for which microwave scattering is problematic, so that shorter-wavelength probe beams are preferable. This combination of long plasma wavelengths ($2\pi k_{\perp}^{-1}$) and short radiation wavelengths gives Bragg scattering angles which are too small to allow good spatial resolution in present-day experiments.

The poor spatial resolution of small-angle scattering (including interferometry) is a serious problem during interpretation of data from inhomogeneous plasmas. Not only may such scattering measurements fail to locate the turbulent region of a plasma, but furthermore, a recorded spectrum (in frequency or wave vector) may actually be an artifact of summing contributions from more than one zone along the line of sight. For such reasons, techniques to improve the spatial resolution of near-forward scattering are needed.

A dramatic improvement in spatial resolution was obtained in the crossed-beam correlation experiments of C. M. Surko and R. E. Slusher.⁵ Their technique uses two CO₂ laser beams which cross each other at a finite angle; the intersection location may be scanned through the plasma volume. If the correlation length in the plane of the laser beams is sufficiently short, then the covariance between fluctuating signals belonging to the two beams is due to density disturbances localized near the intersection of the two beams. In this manner, if statistically reliable data are available, the fluctuation level may be measured as a function of minor radius using only restricted optical access along a minor diameter. In the experiments of Ref. 5, the fluctuating signals are either interferometer-like ("heterodyne") or schlieren-like ("homodyne"), that is, proportional either to $\int \tilde{n}_e d\ell$ or to components of $\int (\tilde{\nabla n}_e) \times d\vec{\ell}$, respectively.

Recently the crossed-beam method has been implemented at shorter wavelength ($\lambda = 3.39 \mu\text{m}$) on the (ceramic liner version of the) ZT-40 toroidal RFP. The results were confusing, in that the density fluctuations were

located (according to the crossed-beam measurements) at smaller minor radius than indicated by simultaneous interferometric measurements along several chords. Further measurements,¹⁰ involving parallel-beam correlations (utilizing the seven simultaneous interferometer chords) showed that, in fact, one important prerequisite for using the crossed-beam method, mentioned explicitly in Ref. 5, was not met in ZT-40. The turbulence was highly anisotropic, with cross-field correlation length $\lambda_{\perp} \lesssim 2.5$ cm and along-field correlation length $\lambda_{\parallel} \gtrsim 12$ cm. Although tokamak density fluctuations are also observed to be elongated along the magnetic field,⁴ this need not cause a problem; if the plane containing the two crossed chords is normal to the toroidal direction, then the only relevant correlation length is the short one, $\lambda_{\perp}$. In an RFP, however, the magnetic field is highly sheared, transforming from being purely toroidal on axis to purely poloidal at the reversal layer, which is near the edge. As a result, it is not possible to place the chords in a plane in which they are everywhere connected across the short scale ($\lambda_{\perp}$) of the turbulence. In the crossed-beam measurements on ZT-40, the longer turbulence scale ($\lambda_{\parallel} \gtrsim 12$ cm) connected the chords in the vicinity of the reversal surface, in a manner which was insensitive to whether the intersection of the two chords lay near that surface. This is an example of how the crossed-beam method is problematic in the presence of extreme shear of the magnetic field combined with extreme anisotropy in the density fluctuations.

That density fluctuations are anisotropic in toroidal discharges seems to be a general experimental fact. The Zeta double Langmuir probe measurements⁸ found $\lambda_{\perp} \approx 5$ cm and $\lambda_{\parallel} > 30$ cm. In the (ceramic liner version of the) ZT-40 RFP, the correlation lengths were measured by interferometric techniques¹⁰ to be $\lambda_{\perp} \lesssim 2.5$ cm and $\lambda_{\parallel} \gtrsim 12$ cm. In tokamaks, also, observed density fluctuations are approximately two-dimensional. In the Alcator CO₂ laser scattering measurements,⁴ typical wave numbers were $k_{\perp} \approx 14$ cm⁻¹ and $k_{\parallel} \lesssim 1$ cm⁻¹. The double Langmuir probe measurements³ on the Proto-Cleo torsatron indicated an anisotropy $k_{\perp}/k_{\parallel} \gtrsim 10$ at all frequencies observed, in the turbulent edge region.

The observations of anisotropy cited above are not at variance with theory. In the observations involving RFPs,^{8,10} the fluctuations appeared to be associated with plasma flows mainly parallel to the local magnetic field. Given the low beta of these plasmas ($\beta_{\text{local}} < 0.1$), the plasma can only weakly

perturb the field, and this is consistent with $k_{\perp} \gg k_{\parallel}$, or equivalently for a turbulent state, $\lambda_{\parallel} \gg \lambda_{\perp}$. A more general theoretical argument¹¹ regarding nonlinear fluid turbulence in the limit of low beta also indicates that fluctuations ought to be highly elongated parallel to the field. For drift-wave turbulence as well, there exist arguments¹² in favor of a similar anisotropy.

II. A PARALLEL-BEAM CORRELATION TECHNIQUE FOR PLASMAS WITH STRONG SHEAR

A. General

The problems (discussed above) in applying the crossed-beam correlation technique to the ZT-40 RFP suggest an alternative means of achieving spatial localization in minor radius using only limited optical access along a minor diameter. The alternative technique will use parallel (rather than crossed) beams and will make use of the fact that the magnetic field direction is sheared, so that anisotropic turbulence within a localized layer in minor radius is recognizable by its characteristic direction of elongation. The many such layers all contribute to each chord-integrated signal; comparing two such signals and selecting signal components of a given "polarization" will then provide spatial localization in minor radius. The selected polarization is determined by the vector direction of separation of the two beams.

It must be stressed that the parallel-beam idea owes very much to the crossed-beam method.⁵ In a practical sense, only during an application of the crossed-beam technique to an inappropriate plasma by this author did the parallel-beam method suggest itself. Furthermore, much of the formalism of Ref. 5 is directly useable in the parallel-beam technique, although the model calculations to follow will use a correlation function approach exclusively, rather than use the wave vector structure factor. This difference in formalism does not imply a difference in the physics but only an alternative notation (deriving from interferometry and turbulence instead of scattering and Bragg diffraction).

In an RFP the magnetic field twists from purely toroidal on axis to purely poloidal near the edge. Experimental RFP magnetic field profiles can be reasonably approximated (for the purposes of this illustration) by the force-free Bessel function model, in which the toroidal field varies as $J_0(kr)$ and the poloidal field as $J_1(kr)$. Since experiments are usually performed

with only slight reversal and with the reversal layer near the wall ($r = r_{\text{wall}}$), it is reasonable to choose $k = 2.405/r_{\text{wall}}$, so that J_0 has its first node at the edge. The resultant field profiles are shown in Fig. 1, along with the angle between the local magnetic field vector and the toroidal direction. This angle is nearly a linear function of minor radius; the angle is odd with respect to the axis ($r = 0$) and thus extends to -90° on the other half of a minor diameter. This monotonic and nearly linear twisting of the magnetic field vector, through a range of approximately 180° during one transit of the minor diameter, makes possible the parallel-beam correlation method.

The model calculations below will consider two parallel probe beams, parallel to, and in the vicinity of, a vertical minor diameter through the plasma minor cross section. A rectangular coordinate system will be used, with the minor diameter along the z -axis, and with the separation between the beams being $S = [(\Delta x)^2 + (\Delta y)^2]^{1/2}$, with $S \ll 2b$ (the minor diameter) (see Fig. 2). The beams thus are oriented almost at right angles to the equilibrium flux surfaces, which may therefore be treated as parallel slabs, each at constant- z . Within a slab, all quantities are considered constant, including the mean-square fluctuation in the electron density, $\langle \tilde{n}^2(z) \rangle$. The diameter ($2a_0$) of either beam will be assumed small compared to the density fluctuations' transverse correlation length $\lambda_\perp$. The beam separations will be small compared to the parallel correlation length $\lambda_\parallel$. Finally, the Salpeter phase-screen approximation¹³ will be invoked: the typical turbulent element's Rayleigh length, $R_L \equiv 2\pi\lambda_\perp^2/\lambda$ (where λ is the wavelength of the probing light), is much larger than the minor diameter. This has the important consequence that diffraction of light by the turbules is unimportant. Furthermore, if the typical phase shift $\delta\phi$ (experienced by the light) due to a turbule is sufficiently small, that is, $\delta\phi \ll R_L/(2b)$, then refractive bending of the beams is unimportant.

The ordering of parameters may be summarized:

- (1) $S \ll \lambda_\parallel, 2b$
- (2) $\lambda_\perp \ll \lambda_\parallel, 2b$
- (3) $2a_0 \ll \lambda_\perp$
- (4) $2\pi\lambda_\perp^2/\lambda \gg 2b$
- (5) $\delta\phi \ll 2\pi\lambda_\perp^2/(2b\lambda),$

Fig. 1.

Toroidal (B_ϕ) and poloidal (B_θ) magnetic fields as a function of minor radius, in the Bessel function model with $B_\phi(r_{\text{wall}}) = 0$. Also shown is the local angle between the magnetic field vector and the toroidal direction.

Fig. 2.

Coordinate systems used in the model calculations. The probe beams (shown as darkened circles) are separated by a vector $x\Delta x + y\Delta y$ shown as a dashed line. The local magnetic field direction is $\hat{e}_\parallel$, rotated by an angle α (about the z-axis) from the x-axis. The width ($2a_0$) of either beam is small compared to the interbeam separation. The viewing direction (the z-axis) is normal to the plane of the diagram.

Measurements¹⁰ on (the ceramic liner version of) ZT-40 (for which $2b = 40$ cm) indicate $\lambda_\perp \lesssim 2.5$ cm and $\lambda_\parallel > 12$ cm. Thus for $\lambda = 10.6$ μm (CO_2 laser) and $2a_0 = 0.5$ cm, conditions (1)-(4) would be easily satisfied. Furthermore, condition (5) would only require $\delta\phi \ll 600$ radians and would thus be automatically satisfied as well.

In the sections to follow, model results will be obtained using a gaussian point-to-point correlation function for the density fluctuations. The measurements on both Zeta⁸ and Alcator⁵ are consistent with gaussian correlation functions, within experimental error. Measurements on (the ceramic liner version of) ZT-40¹⁰ indicate only slight departure from a gaussian, with a zone of shallow negative correlation at greater separation

than a correlation length. In the light of these prior experimental results, it appears that a gaussian correlation function would be a reasonable choice for model calculations whose primary purpose is to illustrate the properties of a diagnostic method.

B. Interferometric Correlations

In this configuration each probe beam is the scene path of a separate interferometer, and the correlation will be measured between the two fluctuating interferometric phases. (This variant of the parallel-beam technique is most similar to the heterodyne crossed-beam correlation scheme.⁵⁾

The covariance between the interferometric phase fluctuations on two parallel chords is proportional to the ensemble-averaged product of the two chordal integrals of fluctuating electron density.

$$\begin{aligned}
 \langle \tilde{\phi}(x,y) \tilde{\phi}(x+\Delta x, y+\Delta y) \rangle &\sim \langle \int \tilde{n}(x,y,z) dz \int \tilde{n}(x+\Delta x, y+\Delta y, z') dz' \rangle \\
 &= \iint \langle \tilde{n}(x,y,z) \tilde{n}(x+\Delta x, y+\Delta y, z') \rangle dz dz' \\
 &\approx \int \langle \tilde{n}^2(z) \rangle \left[\int C_n(S_\perp) dz' \right] dz
 \end{aligned} \tag{1}$$

where the point-to-point density correlation function is

$$C_n(S_\perp) = \frac{\langle \tilde{n}(x,y,z) \tilde{n}(x+\Delta x, y+\Delta y, z') \rangle}{\langle \tilde{n}^2(x,y,z) \rangle}, \tag{2}$$

with the cross-field separation being

$$S_\perp = ((z-z')^2 + (\Delta y \cos \alpha - \Delta x \sin \alpha)^2)^{1/2}.$$

The relationship between the point-to-point correlation C_n and the measured correlation (between the phase fluctuations on nearby chords) has been extensively discussed by Chandrasekhar¹⁴ in the context of atmospheric

fluctuations and by Salpeter¹³ in the context of interplanetary plasma fluctuations.

Here α , the angle between the local magnetic field vector $\vec{B}$ and the x-axis, is a function of z and is assumed approximately constant over a range Δz equal to the correlation length $\lambda_{\perp}$. This is ensured by the inequality $\lambda_{\perp} \ll 2b$, since the shear length (in which α changes by $\pi/2$) is the minor radius b , in an RFP. It is also assumed that the point-to-point correlation function $C_n(S_{\perp})$ is independent of position, although the mean-square density fluctuation $\tilde{n}^2(z)$ is a function of z .

The significance of Eq. (1) is that the phase covariance $\langle \tilde{\phi}(x,y) \tilde{\phi}(x+\Delta x, y+\Delta y) \rangle$ consists of a path integral of the local mean-square density $\langle \tilde{n}^2(z) \rangle$ with weighting factor given by

$$\begin{aligned} W_{\phi}(z) &= \int C_n(S_{\perp}) dz' \\ &= \int C_n \{ [(z-z')^2 + (\Delta y \cos \alpha - \Delta x \sin \alpha)^2]^{1/2} \} dz' \quad . \end{aligned} \quad (3)$$

The weighting factor selects a portion of the path. For example, for a gaussian point-to-point correlation function, the weighting factor is:

$$\begin{aligned} W_{\phi}(z) &\approx e^{-\frac{(\Delta y \cos \alpha - \Delta x \sin \alpha)^2}{\lambda_{\perp}^2}} \int_{-\infty}^{\infty} e^{-\left(\frac{z-z'}{\lambda_{\perp}}\right)^2} dz' \\ &= \pi^{1/2} \lambda_{\perp} e^{-\frac{(\Delta y \cos \alpha - \Delta x \sin \alpha)^2}{\lambda_{\perp}^2}} \quad . \end{aligned} \quad (4)$$

The selectivity due to the weighting factor $W_{\phi}(z)$ is in angle α . However, for realistic profiles of the magnetic fields (see, for example, Fig. 1) this transforms approximately linearly into position z . Various curves of $W_{\phi}(z(\alpha))$ vs α are shown in Fig. 3, for the case of $\Delta x = 0$. Each curve corresponds to a different separation between the two beams, the separation being normalized to the cross-field correlation length. The resolution in α becomes tighter as the separation Δy increases. With

$\Delta y = 3 \lambda_{\perp}$ the selectivity function e-folds in 18° , corresponding to roughly 20% of the minor radius (see Fig. 1). For ZT-40, taking $\lambda_{\perp} = 2$ cm, such an aperture (of 6 cm diameter) would be feasible. If the beams were separated in the major-radial direction, then the selectivity function would peak at the flux surface where B_{ϕ} reverses. If, on the other hand, the beams were separated in the toroidal direction, then the selectivity function would peak on axis. In general, any given beam separation axis would select for fluctuations which are located at that region where the local magnetic field is aligned with the separation axis.

C. Schlieren Correlations

Improved spatial resolution may be obtained if the chordal measurements are schlieren rather than interferometric, that is; if each fluctuating signal is proportional to a component of $\int \tilde{n}_e \times d\vec{l}$ rather than to $\int \tilde{n}_e d\vec{l}$. This is particularly important when access is limited (see Fig. 3). This variant of the parallel-beam technique is most similar to the homodyne crossed-beam correlation scheme.⁵

The schlieren covariance will be calculated with the gradient direction being the x-axis. The schlieren covariance is the ensemble-average product of the two beams' fluctuating phase gradients.

$$\begin{aligned}
 & \left\langle \frac{\partial \tilde{\phi}(x, y)}{\partial x} \frac{\partial \tilde{\phi}(x+\Delta x, y+\Delta y)}{\partial (x+\Delta x)} \right\rangle = \\
 & \lim_{\substack{\delta_x \rightarrow 0 \\ \delta'_x \rightarrow 0}} \left\langle \left[\frac{\tilde{\phi}(x+\delta_x, y) - \tilde{\phi}(x-\delta_x, y)}{2\delta_x} \right] \cdot \left[\frac{\tilde{\phi}(x+\Delta x+\delta'_x, y+\Delta y) - \tilde{\phi}(x+\Delta x-\delta'_x, y+\Delta y)}{2\delta'_x} \right] \right\rangle \\
 & = \lim_{\substack{\delta_x \rightarrow 0 \\ \delta'_x \rightarrow 0}} \frac{1}{4\delta_x \delta'_x} \{ \tilde{\phi}(x+\delta_x, y) \tilde{\phi}(x+\Delta x+\delta'_x, y+\Delta y) \\
 & \quad + \tilde{\phi}(x-\delta_x, y) \tilde{\phi}(x+\Delta x-\delta'_x, y+\Delta y) \\
 & \quad - \tilde{\phi}(x+\delta_x, y) \tilde{\phi}(x+\Delta x-\delta'_x, y+\Delta y) \\
 & \quad - \tilde{\phi}(x-\delta_x, y) \tilde{\phi}(x+\Delta x+\delta'_x, y+\Delta y) \} .
 \end{aligned}$$

Defining the phase correlation

$$C_{\phi}(\Delta x, \Delta y) \equiv \frac{\langle \tilde{\phi}(x, y) \tilde{\phi}(x+\Delta x, y+\Delta y) \rangle}{\langle \tilde{\phi}^2(x, y) \rangle} = \frac{\langle \tilde{\phi}(x, y) \tilde{\phi}(x+\Delta x, y+\Delta y) \rangle}{\langle \tilde{\phi}^2 \rangle}, \quad (5)$$

the schlieren covariance may be re-expressed as

$$\begin{aligned} & \frac{\langle \frac{\partial \tilde{\phi}(x, y)}{\partial x} \frac{\partial \tilde{\phi}(x+\Delta x, y+\Delta y)}{\partial (x+\Delta x)} \rangle}{\langle \tilde{\phi}^2 \rangle} \\ &= \lim_{\substack{\delta_x \rightarrow 0 \\ \delta'_x \rightarrow 0}} \frac{\langle \tilde{\phi}^2 \rangle}{4 \delta_x \delta'_x} \{ C_{\phi}(\Delta x + \delta'_x - \delta_x, \Delta y) \\ & \quad + C_{\phi}(\Delta x - \delta'_x + \delta_x, \Delta y) \\ & \quad - C_{\phi}(\Delta x - \delta_x - \delta'_x, \Delta y) \\ & \quad - C_{\phi}(\Delta x + \delta'_x + \delta_x, \Delta y) \} \\ &= \langle \tilde{\phi}^2 \rangle \lim_{\substack{\delta_x \rightarrow 0 \\ \delta'_x \rightarrow 0}} \left\{ \frac{C_{\phi}(\Delta x + \delta'_x - \delta_x, \Delta y) - C_{\phi}(\Delta x - \delta'_x - \delta_x, \Delta y)}{4 \delta'_x \delta_x} \right. \\ & \quad \left. + \frac{C_{\phi}(\Delta x - \delta'_x + \delta_x, \Delta y) - C_{\phi}(\Delta x + \delta'_x + \delta_x, \Delta y)}{4 \delta'_x \delta_x} \right\} \\ &= \lim_{\delta_x \rightarrow 0} \frac{\langle \tilde{\phi}^2 \rangle}{2 \delta_x} \left\{ \frac{\partial C_{\phi}(\Delta x - \delta_x, \Delta y)}{\partial \Delta x} - \frac{\partial C_{\phi}(\Delta x + \delta_x, \Delta y)}{\partial \Delta x} \right\} \\ &= -\langle \tilde{\phi}^2 \rangle \frac{\partial^2 C_{\phi}(\Delta x, \Delta y)}{\partial (\Delta x)^2} \\ &= -\frac{\partial^2}{\partial (\Delta x)^2} \langle \tilde{\phi}(x, y) \tilde{\phi}(x+\Delta x, y+\Delta y) \rangle. \quad (6) \end{aligned}$$

Thus the schlieren covariance is proportional to the appropriate second derivative of the phase covariance.

A model example of schlieren correlations will consider the gradient and separation axes being orthogonal to each other; that is, $\Delta x = 0$. The covariance in this case is (substituting Eq. (1) into Eq. (6))

$$\left\langle \frac{\partial \tilde{\phi}(x,y)}{\partial x} \frac{\partial \tilde{\phi}(x,y+\Delta y)}{\partial x} \right\rangle \sim - \left\{ \frac{\partial^2}{\partial (\Delta x)^2} \int \tilde{n}^2(z) W_\phi(z) dz \right\} \Big|_{\Delta x = 0},$$

where the weighting factor is

$$W_\phi(z) = \int C_n \{ ((z-z')^2 + (\Delta y \cos \alpha - \Delta x \sin \alpha)^2)^{1/2} \} dz'.$$

Using a gaussian point-to-point density correlation, the schlieren covariance becomes [using Eq. (4)]

$$\begin{aligned} \left\langle \frac{\partial \tilde{\phi}(x,y)}{\partial x} \frac{\partial \tilde{\phi}(x,y+\Delta y)}{\partial x} \right\rangle &= - \left\{ \frac{\partial^2}{\partial (\Delta x)^2} \int \langle \tilde{n}^2(z) \rangle \pi^{1/2} \lambda_\perp e^{-\left(\frac{\Delta y \cos \alpha - \Delta x \sin \alpha}{\lambda_\perp} \right)^2} dz \right\} \Big|_{\Delta x = 0} \\ &= -\pi^{1/2} \lambda_\perp \int \langle \tilde{n}^2(z) \rangle \left\{ \frac{\partial^2}{\partial (\Delta x)^2} e^{-\left[\frac{(\Delta y)^2 \cos^2 \alpha - 2\Delta y \Delta x \cos \alpha \sin \alpha + (\Delta x)^2 \sin^2 \alpha}{\lambda_\perp^2} \right]} \right\} dz \Big|_{\Delta x = 0} \\ &= \frac{2\pi^{1/2}}{\lambda_\perp} \int \langle \tilde{n}^2(z) \rangle \sin^2 \alpha \left\{ 1 - 2 \left(\frac{\Delta y}{\lambda_\perp} \right)^2 \cos^2 \alpha \right\} e^{-\left[\left(\frac{\Delta y}{\lambda_\perp} \right)^2 \cos^2 \alpha \right]} dz \\ &= \frac{2\pi^{1/2}}{\lambda_\perp} \int \langle \tilde{n}^2(z) \rangle W_s(z) dz, \end{aligned} \quad (7)$$

where the schlieren selectivity function is

$$W_S(z) = \sin^2 \alpha(z) \left[1 - 2 \left(\frac{\Delta y}{\lambda_{\perp}} \right)^2 \cos^2 \alpha(z) \right] e^{-\left(\frac{\Delta y}{\lambda_{\perp}} \right)^2 \cos^2 \alpha(z)} .$$

The schlieren selectivity function is shown in Fig. 4 for various beam separations. Comparison of Figs. 3 and 4 confirms that the parallel beam correlation technique using schlieren signals offers better spatial resolution, for a given beam separation. This advantage would be significant where diagnostic access is limited.

Fig. 3.

The interferometric selectivity function $W_{\phi}(\alpha)$ versus the angle α between the interbeam axis and the local magnetic field direction. Results for various interbeam separations (normalized to the cross-field correlation length) are shown. Larger interbeam separations yield improved spatial resolution.

Fig. 4.

Similar to Fig. 3, but for the schlieren selectivity function with $\Delta x = 0$ and with the gradient direction along x .

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