PEP Note 263
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Estimate of Spin Polarization for PEP
Using Generalized Transformation Matrices

The spin polarization for PEP has been estimated before by using simplified models.^{1,2} The main difficulty in the previous estimates is that the strength of depolarization effects caused by various electromagnetic field errors could not be specified accurately. To overcome this difficulty, a matrix formalism for depolarization calculation was developed recently.³ One basic ingredient of this theory is to represent an electron by an 8-dimensional state vector, $X = (x, x', y, y', z, \delta, \alpha, \beta)$ where the first six coordinates are the usual transverse and longitudinal canonical coordinates, while α and β are the two components of the electron's spin vector perpendicular to the equilibrium direction of polarization $\hat{n}$. The degree of depolarization is specified by $\frac{1}{2}(\alpha^2 + \beta^2)$. The state vector X will be transformed by an 8x8 matrix as the electron passes through a beam-line element such as a bending magnet or an rf cavity. From any position s , one multiplies successively the 8x8 matrices around one revolution of the storage ring to obtain the total transformation $T(s)$. Any impulse perturbation ΔX to the electron's state vector occurring at s will be transformed repeatedly by $T(s)$ as the electron circulates around the storage ring. Another basic ingredient of Ref. 3 is to decompose ΔX into 8 eigenstate components with eigenvectors determined from $T(s)$. Six of these eigenstate components corresponding to the space states will be damped out by the usual radiation damping.⁴ The projections of ΔX onto the remaining two spin eigenstates are directly related to the loss of polarization due to the impulse perturbation ΔX . Depolarization effects can thus be calculated directly once all perturbations are specified.

The matrix method developed in Ref.3 is applied to PEP in this note. Two types of depolarization mechanisms are considered:

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- (i) In the presence of a static EM field error in the storage ring, the equilibrium direction of spin polarization $\hat{n}$ does not coincide with the direction of bending magnetic field $\hat{y}$. Since the spin polarization built up during synchrotron radiation is necessarily along $\hat{y}$, and only a projection along $\hat{n}$ will be kept in the storage ring, the net polarization must be reduced by a factor $\hat{n} \cdot \hat{y}$. This depolarization mechanism is particularly important if the spin tune $\nu = \gamma \left(\frac{g-2}{2} \right)$ is close to an integer. That is, it drives the integer depolarization resonances.
- (ii) When the electron emits a photon during synchrotron radiation, the state vector is perturbed. These perturbations cause diffusion of spin direction away from the equilibrium direction $\hat{n}$ and drives side-band depolarization resonances at $\nu \pm \nu_{x,y,s} = \text{integer}$, where ν_x , ν_y and ν_s are the horizontal betatron tune, the vertical betatron tune and the synchrotron tune, respectively.⁵

We have not included nonlinear resonances here since they cannot be treated by using the matrix formalism. An analysis of the important nonlinear depolarization mechanism of the beam-beam collision below the stochastic limit can be found in Ref.6. The model calculation of depolarization at or beyond the beam-beam stochastic limit discussed in Ref.2 can also be treated by the matrix method. Although the present approach looks different from that of Ref.2, it gives identical results. We have also ignored effects due to tune spreads of the beam. All particles are assumed to have the same tunes: ν_x , ν_y and ν_s .

In the present numerical calculation, the beam-line elements for the ideal PEP lattice include horizontal bending magnets, quadrupole magnets, sextupole magnets, rf cavities and drift spaces. The 24 strong quadrupoles in the interaction regions and all bending magnets are split in halves to improve accuracy of calculation. Without field errors, the ideal lattice produces an equilibrium polarization of 92.4%. To simulate field errors, we introduce a distribution of vertical orbit kickers. The resulting vertical closed orbit makes sextupoles behave like skew quadrupoles and quadrupoles behave like additional vertical kickers. In

the presence of these field errors, both types of the above-mentioned depolarization mechanisms are driven.

Figs. 1 and 2 show the expected equilibrium polarization P_0 as a function of the beam energy E_0 for two different distributions of vertical kickers. The lattice used is that of the PEP "normal" configuration: $\nu_x = 21.15$, $\nu_y = 18.75$, $\nu_s = .049$, $\beta_x^* = 2.8\text{m}$, $\beta_y^* = .11\text{m}$ and $\eta_x^* = -491\text{m}$. The strengths of the vertical kickers are normalized such that the rms closed orbit is $\Delta y_{\text{rms}} = 1.2\text{mm}$ in both cases. For a different strength of the same kicker distribution, the depolarization strength scales roughly quadratically with Δy_{rms} .

It should be mentioned that the integer resonances driven by the depolarization mechanism of type (i) are rather weak. The widths of these resonances in spin tune are typically less than 10^{-3} . Sideband resonances driven by mechanism (ii), on the other hand, are stronger. In particular, resonances with $\nu \pm \nu_y$ close to multiples of the machine periodicity (6 for PEP) are especially harmful for polarization. It is, therefore, desirable that the PEP lattice be designed to be flexible enough to avoid these resonances by, e.g., being able to shift ν_y by an integer.

In Figs. 1 and 2, resonance locations are indicated by arrows at the bottom. The beam is completely depolarized at those locations. Each integer resonance is surrounded by six sidebands. In most cases, the two synchrotron sidebands and the integer resonance overlap to show a single dip. Furthermore, by a closer inspection of the fine structure of these figures, it is possible to observe interference effects around a resonance dip. When this happens, polarization may become substantially stronger than the background value. One such effect can be seen on Fig. 2 at $E \approx 13.77\text{ GeV}$.

In Fig. 3, we present a similar calculation for a PEP high energy configuration with a reduced luminosity and $\nu_x = 27.79$, $\nu_y = 18.69$, $\nu_s = 0.04129$, $\beta_x^* = 3.6\text{m}$, $\beta_y^* = 0.2\text{m}$, $\eta_x^* = -.285\text{m}$. The field errors are again introduced through a vertical closed orbit with $\Delta y_{\text{rms}} = 1.2\text{mm}$.

For high energy physics experiments, it is sometimes desirable to have the spin polarization along the longitudinal direction at the interaction point. One possible way to achieve this is to insert vertical bending

magnets of proper strengths on both sides of the interaction point. Unfortunately, such arrangement also introduces perturbations to the equilibrium polarization. To study this effect, we have turned on a set of vertical bending magnets for longitudinal polarization around one of the six interaction points. As a consequence, both depolarization mechanisms (i) and (ii) are driven. Results of this calculation without any field errors other than those of vertical bending magnets, for the PEP "normal" configuration, are plotted in Fig.4 on a semilogarithmic scale. The dotted curve represents the polarization reached assuming only mechanism (i) and, therefore, shows only integer depolarization resonances. To include the effect of field errors, Fig.4 should be combined with Fig. 1 and 2.

References

1. A.W. Chao and R.F. Schwitters, PEP-217/SPEAR-194 (1976), and all references quoted or not quoted herein.
2. A.W. Chao and R.F. Schwitters, PEP-237 (1977).
3. A.W. Chao, PEP-257/SPEAR-208 (1977).
4. M. Sands, "The Physics of Electron Storage Rings", SLAC-121 (1970).
5. Notice that the energy diffusion mechanism does not drive integer depolarization resonances.
6. A.M. Kondratenko, Sov. Phys., JETP, Vol.39, No.4, 592 (1974).
7. R.F. Schwitters and B. Richter, SPEAR-175 (1974).

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Fig. 1

Fig. 2

Fig. 3

Fig. 4