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**A Basis for Calculations
in the Topological Expansion**

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by

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Abstract

Investigations aimed at putting the topological theory of particles on a more quantitative basis are described. First, the incorporation of spin into the topological structure is discussed and shown to successfully reproduce the observed lowest mass hadron spectrum. The absence of parity-doubled states represents a significant improvement over previous efforts in similar directions. This theory is applied to the lowest order calculation of elementary hadron coupling constant ratios. $SU(6)_W$ symmetry is maintained and extended via the notions of topological supersymmetry and universality. Finally, efforts to discover a perturbative basis for the topological expansion are described. This has led to the formulation of off-shell Feynman-like rules which provide a calculational scheme for the strong interaction components of the topological expansion once the zero-entropy connected parts are known. These rules are shown to imply a "topological asymptotic freedom". Even though the nonlinear zero-entropy problem cannot itself be treated perturbatively, plausible

general assumptions about zero-entropy amplitudes allow immediate qualitative inferences concerning physical hadrons. In particular, scenarios for mass splittings beyond the supersymmetric level are described.

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I. Introduction

During the early 1960's, the hopes of many physicists for a comprehensive theory of elementary particles resided in the bootstrap idea that S-matrix causality and unitarity could determine, through self-consistency, all observed properties of hadrons. Although this idea never lost its esthetic appeal, early investigators were thwarted by its seemingly impenetrable mathematical complexity. Coupled with the successes of constituent models, the S-matrix approach lost considerable favor by the end of the sixties.

In 1974 [1], a breakthrough in the bootstrap approach was achieved through the realization of the relevance to particle physics of combinatorial topology. The earliest version of a topological particle theory was a theory of mesons based on the embedding of Landau graphs (which provide a representation for the singularities of the S-matrix) in two-dimensional bounded, orientable surfaces. By considering the geometrical complexity of the embedding surface (genus and number of boundaries) every S-matrix connected part could be expanded as an infinite sum over contributions of increasing topological complexity,

$$M_{fi} = \sum_{\tau, \kappa} T_{M_{fi}}^{\kappa} \quad (1.1)$$

where τ is an index that specifies the topological complexity of the surface and κ specifies the association of the various particles

with different boundary components.

The special usefulness of this expansion arises from an analysis of discontinuity products. Replacing every connected component by its topological expansion, one obtains the equation

$$\text{disc } \sum_{\tau, \kappa} \tau_{fi}^{\kappa} = \sum_{\substack{\tau', \kappa' \\ \tau'', \kappa''}} \int d\mathbf{p} \tau'_{fi}{}^{\kappa'} \otimes \tau''_{fi}{}^{\kappa''} \quad (1.2)$$

The Landau connected sum on the right-hand side of this equation corresponds to a topological product of the two constituent surfaces τ', κ' and τ'', κ'' into an overall product surface of a particular complexity. Hence, the natural assumption that can be made is that this equation separates into an infinite number of independent equations, one for each topology.

$$\text{disc } \tau_{fi}^{\kappa} = \sum_{\substack{\tau', \kappa' \\ \tau'', \kappa''}} \int d\mathbf{p} \tau'_{fi}{}^{\kappa'} \otimes \tau''_{fi}{}^{\kappa''} \quad (1.3)$$

with $(\tau', \kappa') \otimes (\tau'', \kappa'') = (\tau, \kappa)$

Since Landau connected sums of surfaces have the property that the resultant genus can never be less than the sum of the genres of the constituent surfaces (a property referred to as entropy) the only components of the topological expansion that can be decomposed into a product of terms of the same complexity are those of minimal complexity. Thus, the tremendous simplification that the topological expansion allows is the isolation of the intrinsic nonlinearity at the most ordered, minimum complexity, "planar"

level.

To complete the logic of the topological expansion it is assumed that discontinuity formulas as shown in Eq. (1.3) can be combined with Cauchy-Riemann formulas to provide a means for calculating topological connected parts. This non-linear "bootstrap" system may be solved separately without invoking any higher order contributions. The calculation of higher complexity components is accomplished through linear equations from the lower entropy results.

This application of topology to S-matrix causality and unitarity resulted in an impressive understanding of mesons and meson interactions during the seventies [2]. This included all of the successes of the dual models; an explanation of the Okubo-Zweig-Iizuka (OZI) rule and a quantitative account of the degree to which the rule is broken; an understanding of the observed level of exchange degeneracy of leading Regge trajectories; approximate hadronic regularities such as the limitations of transverse momentum; the short range character of rapidity correlations.

However, this program, often referred to as "dual topological unitarization" (DTU), was deficient in several respects. First, the theory was confined only to the description of mesons. Also, although DTU was ostensibly a bootstrap theory, certain elements regarding internal quantum numbers and the particle spectrum were undetermined. Finally, there was no treatment of spin.

The incorporation of other hadrons into the theory beyond mesons has been accomplished by the elaboration of the DTU surface to a multisheeted structure. Although DTU prescribed a constituent

structure to mesons analogous to the quarks of more conventional theories, in order to incorporate particles with more than two constituents it was necessary to attach additional sheets to the embedding surface. In fact, such "feathered" surfaces then allowed the incorporation of particles interpretable as baryonium (2 quarks, 2 antiquarks) as well as baryons. Notably, although additional features of complexity are introduced in this scheme, it has been found that at the minimal complexity level the dynamical content is restricted to a single principal sheet and hence remains unaffected by the multisheeted structure.

The inability of DTU to determine the internal quantum numbers of hadrons through self-consistency has also been overcome via the introduction of a more elaborate topological formalism. The principal new feature of the topological bootstrap is an additional surface--a two-dimensional closed surface transverse to the DTU surface [3]. For each connected graph, the two surfaces are arranged in such a manner that their intersection consists of the boundary of the DTU surface. At the lowest "zero entropy" level of complexity the topological configuration consists of a plane (DTU surface) circumscribed by a sphere (quantum surface).

Complementary to the classical momentum-energy type of information carried by the DTU ("classical") surface, the second so-called "quantum" surface carries a triangulation whose orientations represent internal quantum numbers. The appeal of the quantum surface is that it has allowed a topological solution to the minimal complexity bootstrap equations.

The picture that has emerged from the topological scheme consists of a definite and restricted set of elementary particles. On the quantum surface, particles are represented by areas that are built from triangle constituents. It is natural to identify these triangles as quarks which carry three colors, eight flavors and half-integral spin. Topological quarks, however, do not carry momentum. Confinement emerges as a natural consequence of S-matrix self-consistency.

Investigations of the quantum surface have also led to the extension of the topological formalism to the electroweak interactions [4,5,6]. While orientable quantum surface areas are invoked for hadrons, a non-orientable structure is associated with the electroweak particles. Although leptons and photons are not generated by the zero-entropy bootstrap in the sense of hadrons, there is a common topological framework. It now seems likely that the unitarity of the theory leads to a Weinberg-Salam type of structure.

The incorporation of spin into the topological expansion has been accomplished by Stapp [7]. Using a two-component M function formalism Stapp has shown how spin can be introduced as indices on graphs analogous to Chan-Paton factors. In fact, the search for the topological representation of spin provides the springboard for all of our subsequent investigations aimed at establishing a calculational basis for the topological expansion.

The remainder of this paper is organized as follows: In Chapter II we give a more detailed description of elements of the

classical surface and, in particular, the notion of "thickened" Feynman graphs where each elementary hadron momentum line is accompanied by 2,3, or 4 quark lines. This shorthand exhibits "color" while concealing the topological meaning of quark isospin and generation. For strong interactions this deficiency is unimportant, as flavor may be represented as an index on each quark line.

In Chapter III we review the general principles involved in the description of spin. Following Stapp, the two-component M-function formalism is developed and applied to the topological expansion. We then translate this formalism into four component language.

Chapter IV consists of the application of these ideas to the calculation of coupling constant ratios. The idea of topological supersymmetry allows a comparison between meson and baryon coupling constants.

In Chapter V we formulate Feynman-graph rules for the strong interaction components of the topological expansion and see how a "topological asymptotic freedom" emerges. The considerations of this chapter are then applied to a discussion of mass calculations in Chapter VI and the current picture is summarized in Chapter VII.

II. The Classical Surface

A. Introduction

A precondition for the development of Feynman rules for topological amplitudes was the invention of a satisfactory graphical representation. The original DTU version of the theory turns out to encompass many of the desirable features. Acknowledging the more general usefulness of Feynman graphs as representations of the singularities of the S-matrix (a viewpoint emphasized by Landau), a consideration of these graphs provides the basis for the DTU approach to hadron physics. The topological connection is established by the embedding of the "Landau graphs" in two-dimensional, bounded, orientable surfaces.

At the zero-entropy level the classical surface consists of a disk in which a single-vertex Feynman graph is embedded (Fig. 1). Segments of the boundary are associated with elementary particles according to their attachment to external lines. The embedding process immediately establishes a significance to the order of the lines entering a vertex. For hadronic processes it is always possible to specify a global orientation for the classical surface. In constructing diagrams of topological amplitudes, the orientation is indicated by a circular arrow which may either be associated with the ordered vertex or with the surface boundary.

B. Representation of Internal Symmetries

The attachment of orienting arrows to the boundary of the classical surface suggests a constituent structure for the mesons . By redrawing the classical surface with a gap in the boundary at the endpoints of the Feynman arcs, a more familiar representation is obtained (Fig. 2). Boundary segments become clearly identified with Harari-Rosner (HR) arcs. Weissmann [8] was able to show that additively conserved internal quantum numbers may be consistently introduced into the zero-entropy topological S-matrix only as indices attached to HR lines. Hence, while boundary segments reflect singularity structure in continuous energy momentum variables (by conforming to Feynman graphs) the topological notion of orientation also provides the mechanism for the description of discrete "internal" variables.

A complete understanding of the origin of internal symmetries depends on quantum surface considerations. (Appendix A) The possibilities for orienting the triangular patches on the quantum surface specifies a fourfold generational structure. For the representation of charge, quantum considerations suggest the usefulness of the introduction of an additional arc on the classical surface [9]. Running parallel to HR arcs, such charge fibers may be oriented to either agree or disagree with the HR orientations, corresponding to either $q=1$ or $q=0$ charge options (Fig.3).

As is evident from Fig. 3, the picture that emerges from DTU is that of a constituent quark-antiquark flavor structure. In Appendix A it is shown how the above charge assignments are

integrated with the topological structure to produce the experimentally observed hadron electric charges. The essence of Stapp's invention is that spin may be treated in a parallel fashion to flavor. In this sense, a classical bootstrap leads to the quark structure of mesons.

C. Connected Sums and Entropy

One of the features most naturally incorporated into the topological approach is that of unitarity. Corresponding to the Landau connected sum that appears on the right-hand side of the discontinuity equation (Eq. 1.3) one can define a geometrical connected sum of the constituent surfaces. In combining two single-boundary single-vertex classical surfaces there is a joining of boundary segments belonging to intermediate elementary particles so as to create a unique surface with boundary segments corresponding only to the new overall external particles (Fig.4). The two ingredient global orientations are matched so that the two-vertex graph on the new surface has a coherent orientation and Feynman arcs, charge arcs and HR indices are connected in a consistent manner.

In forming connected sums of classical surfaces it is easy to obtain graphs that can no longer be embedded in a plane. As an example, Fig. 5 shows a connected sum of two single-vertex graphs whose product is a surface with the topology of a cylinder. Particles (A,B) and (C,D) belong to two distinct boundary components and clearly require a surface more complex than a plane

in order to embed the graph without the crossing of internal lines.

The association of an amplitude with a surface allows an unambiguous determination of its complexity. For the classical surfaces of DTU there are two relevant indices of topological complexity--the number of handles on the surface, and the boundary structure. As shown in Reference 2, the number of handles needed to embed a graph can be calculated immediately upon knowledge of the edge and vertex structure from the Edmond's rule. This allows a definition of the genus of the surface, g_1 , as twice the number of handles. Boundary information is conveyed by means of an index g_2 , defined by

$$g_2 = b + g_1 - 1 \quad (2.1)$$

where b is the number of separate boundary components. The lowest level of topological complexity is uniquely characterized by the conditions $g_1 = g_2 = 0$.

The most significant feature of the indices g_1 and g_2 is their non-decreasing character under Landau connected sums. In the formation of the product of any two surfaces, $\Sigma = \Sigma' \# \Sigma''$, the index g_1 satisfies a so-called "strong" entropy condition

$$g_1 \geq g_1' + g_1'' \quad (2.2)$$

while g_2 satisfies the "weak" entropy condition

$$g_2 \geq \max (g_2', g_2'') \quad (2.3)$$

It is as a consequence of these entropy rules that the possibility exists for identifying a minimal subset of zero-entropy topological connected parts to which the non-linear bootstrap conditions are confined. As mentioned in the introduction a fundamental assumption of the theory is that the zero-entropy problem may be solved independently without the calculation of any connected parts outside this subset. In fact, in practice, due to the intractability of the zero-entropy problem, most analyses have been devoted to the development of a scheme by which higher components of the topological expansion may be calculated through linear equations once the lower entropy components are known. As will be shown in the following chapters a plausible calculational scheme may be invented based only on very general assumptions regarding the zero-entropy content of the theory.

D. Contractions

One of the major complications in theories of strong interactions is the difficulty one encounters in formulating a satisfactory definition of an interaction vertex. Hadronic vertices are complex objects that exhibit duality and other composite properties. A graphical representation of duality was discovered independently by Harari and Rosner [10] in 1969 and corresponds to the well-known contractions illustrated in Fig. 6. DTU

incorporates the Harari-Rosner contractions and extends them to Feynman graphs. Two kinds of contractions are allowed. "Parallel" internal Feynman arcs may be contracted to a single arc (Fig. 7(a)) and any internal arc connecting two distinct vertices may be shrunk to a point (Fig. 7(b)). Although the form of a totally contracted Feynman graph may depend on the order in which the contractions are performed, it is a topologically unique operation. Contractions do not change the entropy indices and non-contracted and contracted graphs are topologically equivalent. Although a fully contracted Feynman graph will always have a single surviving vertex, certain internal arcs may remain as a reflection of non-zero entropy (Fig. 8).

E. Baryons

As mentioned in the introduction, the extension of DTU to encompass baryonic structure has entailed a modification of the classical surface to a triple-sheeted structure. The threefold structure is a consequence of quantum surface consistency requirements and allows the existence of baryonium (composed of two quarks and two antiquarks) as an elementary hadron in addition to mesons and baryons.

The present picture has emerged from a series of investigations beginning with the search for a simple solution to the purely topological problem of extending the meson scheme to three-quark baryons [11], then incorporating ideas based in QCD [12], and finally from attempts to establish consistent analyticity

and entropy properties within the context of a topological expansion [13,3].

The classical surface for a baryon consists of three smooth surfaces (sheets) joined along a common segment characterized as a junction line (Fig. 9). Each sheet is orientable and carries a coherent orientation which, as in DTU, can be associated with Harari-Rosner arcs along the boundary. Additionally, the HR orientations of the three sheets imposes an orientation on the junction lines. Overall consistency of orientation requires that all three sheets meeting along a junction line must specify a common orientation for that junction line.

As in DTU, flavor indices are associated with HR lines while charge structure is conveyed by additional charge arcs. The boundary of the classical surface (the belt) is sectioned into pieces corresponding to the ingoing and outgoing elementary particles appearing in Fig. 10. The foregoing features constrain the zero-entropy belt graph to an extremely simple form—a single-strand "beaded necklace" , as illustrated in Fig. 11.

The symmetry of the three sheets of the classical surface is lost upon introduction of the Feynman graph. In order to smoothly join the Feynman arcs of baryons and baryonium to the mesonic graphs of classical DTU it is necessary to single out a "principal" sheet of the classical surface (Fig. 12). This is a general feature of hadron topology, valid to all orders in the topological expansion.

In fact, at higher orders in the topological expansion the

apparent baryonic asymmetry is diminished. The notion of a Landau connected sum is easily extended to feathered classical surfaces. As in classical DTU, corresponding elementary particle pieces of the belt are joined and a coherent HR orientation is ensured if the HR orientations are opposite for a pair of matched belt segments. Accompanying the connected sum of the surfaces is a connected sum of the junction lines and charge fibers, and flavor identity must be respected. Contraction rules again stipulate the removal of closed HR loops and, in addition, closed junction lines may now be treated similarly.

The additional option now available beyond DTU is the possibility of constructing plugs between different sheets of the classical surface. The distinction of the principal sheet via the presence of the Feynman graph allows a consistent labeling of all three sheets of the classical surface. Evidently, there are six possible permutations in the formation of connected sums between two baryon surfaces. Bridges between different sheets increase classical surface complexity, with odd-permutation switches increasing g_1 (the genus of the classical surface) by one unit and even permutations incrementing g_1 by two units.

The precise rules for the construction of baryon plugs have been given by Finkelstein [14]. Finkelstein stresses the usefulness of attaching the concept of color to the three sheets. Assigning the principal sheet color #1, the other two sheets may be assigned by clockwise convention to colors #2 and #3. Mesons

which are always on the principal sheet, carry only color #1, while baryonia carry only colors #2 and #3. A simple consequence of this construction is that at the lowest level of complexity baryonium does not couple to mesons. However, "color switches" such as those described above will allow these transitions at higher order topologies. Topological color itself is a physically inaccessible degree of freedom since the construction of physical amplitudes entails a sum over all possible color permutations in the topological expansion.

In order to more accurately distinguish color transitions (and for later chirality interpretations), Finkelstein [14] has proposed an additional embellishment for the classical surface. Although classical sheets colored #2 and #3 do not carry Feynman arcs, by adding to these sheets a new line parallel to the quark charge arcs a graphical representation for color is obtained. These so-called "momentum-copy" or "color" arcs are thus located similarly to a baryon or baryonium line on sheet #1.

All switch plugs are manifestly unique and junction lines remain uncrossed by either charge or momentum arcs. In the formation of connected sums, bridges between different sheets are now accompanied by a reconnection of corresponding momentum arcs. An immediate consequence of this construction is that duality transformations (contractions) cannot move Feynman vertices past switches.

F. Thickened Feynman Graphs

The precise statement of color integrity thus attaches color to Feynman graphs, not to sheets of the classical surface. As shown in Ref. (3), the full content of strong interaction topology--which involves a pair of intersecting surfaces--can be transferred to a thickened momentum graph with associated quark lines. This two-dimensional representation of multisheeted structure provides a natural generalization of the original Harari-Rosner scheme.

Flavor-carrying quark lines running parallel to momentum lines distinguish different types of elementary hadron. With the convention that graph vertices are always oriented clockwise, the three categories of elementary hadron are shown in Fig. 13. Topological color is now represented by the relative position of the quark lines with respect to the associated momentum arc. By convention, a quark line by itself on one side of the momentum line carries color #1, and is directed in agreement with surface orientation. Quarks that appear in "diquark" pairs are colored #2 and #3, #2 being adjacent to the momentum line. Colors #2 and #3 flow in opposition to surface orientation.

Thickened Feynman graphs provide a concise representation for color switching. Designating by p a permutation of quarks colored #2 and #3 (a "diquark twist") and by p_+ a clockwise cyclic permutation, the six different baryon plugs correspond to the elements of the permutation group:

$$I, p, p_+, p_+^2, pp_+, pp_+^2 \quad (2.4)$$

where $p^2 = p_+^3 = 1$ and $p_+p = p p_+^2$. The graphical representation shown in Fig. 14 emphasizes the fact that although quarks may change color, each topological color is separately conserved (i.e., there is always one quark in each position relative to the momentum arc). Baryonium color switching possibilities, corresponding to the permutations $I, p, \bar{p}, p\bar{p}$ ($\bar{p}$ means interchange of the two antiquarks), are illustrated in Fig. 15.

In the following chapters, extensive use will be made of thickened Feynman graphs. More than just a convenient notational device for topological color, this representation will turn out to provide the proper framework for the complete formulation of Feynman rules.

III. SPIN AND THE TOPOLOGICAL EXPANSION

A. M-functions

The association of spin indices with HR graphs within the context of dual models was proposed by Mandelstam in 1970 [15]. According to this scheme, a single two-valued spin index is attached to the end of each HR arc in the manner of Chan-Paton [16] factors for internal symmetries. However, the simultaneous requirements of pole factorization and parity invariance in these early attempts (see also Bardakci and Halpern [17]) proved to be irreconcilable. The troubles, at the meson level, included the appearance of ghost parity-doublet partners for the pseudoscalar and vector mesons as well as an overall doubling of the entire meson spectrum. In 1980, Stapp [7] reinterpreted the spin formalism within the context of the topological expansion. One of the foremost achievements of the topological approach has been the successful incorporation of spin into a theory with a quark substrate and the physical spectrum of particles.

The natural framework within which to treat spin in an approach based on analyticity and unitarity is in terms of the so-called invariant M-functions. In the presence of spin, the Landau rules for singularities continue to have their spinless form if understood as applying to the M-functions. The relationship between the standard S-matrix elements and the M-function amplitudes may be symbolically represented as

$$\langle p_{of} | S | p_{oi} \rangle \equiv \langle p_f | M | p_i \rangle = \langle p_{of} | L(p) M L(p) | p_{oi} \rangle \quad (3.1)$$

where the state vectors are standard rest-frame state vectors for particles i, f in the S-matrix amplitude; $\langle p_f |$ and $| p_i \rangle$ are standard general momentum state vectors; and $L(p)$ represents the Lorentz velocity transformation ("boost") that converts the rest frame to the frame in which the particle has momentum p .

A more explicit representation is obtained by using the standard Pauli spin matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (3.2)$$

$$\begin{aligned} \sigma_\mu &= (\sigma_0, \sigma_1, \sigma_2, \sigma_3) = (1, \vec{\sigma}) \\ \bar{\sigma}_\mu &= (\sigma_0, -\sigma_1, -\sigma_2, -\sigma_3) = (1, -\vec{\sigma}) \end{aligned} \quad (3.3)$$

Then the Lorentz transformation has the representation

$$\begin{aligned} L(p) &= [\vec{\sigma} \cdot \mathbf{p}/m]^{1/2} \equiv [\vec{\sigma} \cdot \mathbf{v}]^{1/2} = \exp [-\theta/2 (\vec{\sigma} \cdot \hat{\mathbf{n}})] \\ &= \cosh \theta/2 - \vec{\sigma} \cdot \hat{\mathbf{n}} \sinh \theta/2 \end{aligned} \quad (3.4)$$

where $E = m \cosh \theta$, $p = m \sinh \theta$.

By making use of the identities

$$\begin{aligned}
 (v \cdot \sigma)(v \cdot \bar{\sigma}) &= 1 \\
 \sqrt{v \cdot \sigma} \sqrt{v \cdot \bar{\sigma}} &= 1
 \end{aligned}
 \tag{3.5}$$

one finds the relation

$$M(p) = \sqrt{v_f \cdot \sigma} S(p) \sqrt{v_i \cdot \sigma} \quad . \tag{3.6}$$

By a redefinition of the standard state vectors of the S-matrix the kinematic singularities are separated from the spinorial dependence leaving as the principal object of study a manifestly Lorentz invariant function with simple behavior under crossing [18].

In general, M-functions may be separated into the product of a covariant spin function and an invariant scalar function of momenta. The special usefulness of M-functions for the topological expansion stems from the correspondence one can establish between the spin dependence of the quark lines in graphs and spinorial M-function indices.

B. Finite Dimensional Representations of the Lorentz Group

To see explicitly how M-functions can provide specific calculational rules for the evaluation of topological graphs it is necessary to first recall some properties of the finite-dimensional representations of the Lorentz group (as opposed to the infinite dimensional unitary representations appropriate to field theory). It is well known that the representations of the Lorentz

group corresponding to half-integral spin are double-valued. This peculiarity reflects the inequivalence between the representations induced by a fundamental spinor and its complex conjugate. Thus, to avoid ambiguity, it is necessary to label Lorentz transformations not by the elements of the Lorentz group but by the elements of its universal covering group $SL(2, C)$ - the group of complex, unimodular, 2×2 matrices. The six linearly independent generators of $SL(2, C)$ (corresponding to rotations and boosts) can be linearly combined in such a manner that the Lie algebra appears as the trivial direct sum of two $SU(2)$ algebras. This allows the convenient labeling of the irreducible finite dimensional representations of the Lorentz group by the pair of $SU(2)$ Casimir eigenvalues (s_1, s_2) . This representation has dimensionality $(2s_1+1)(2s_2+1)$. The basic "first-kind" $(2s_1+1)$ -dimensional representation is conventionally denoted by $D^{0, s}$, and is defined as follows [19]:

If

$$g = \exp(1/2 \vec{a} \cdot \vec{\sigma}) \exp(-1/2 i b \cdot \vec{\sigma}) \in SL(2, C),$$

where a and b are real, then

$$D^{0, s}(g) = \exp(\vec{a} \cdot \vec{J}) \exp(-i \vec{b} \cdot \vec{J}) \quad (3.7)$$

where the three matrices $\vec{J}$ are the usual $(2s+1)$ -dimensional angular momentum matrices defined according to the convention in which J_1 and J_3 are real and J_2 is purely imaginary. With this

definition, $D^{0,1/2}$ coincides with $SL(2,C)$.

Using Eq. (3.7) three other important representations of $SL(2,C)$ may be constructed.

$$D^{0,s}(g^{T-1}) \quad (3.8)$$

$$D^{0,s}(g^{T-1}) = D^{0,s}(g)^{T-1} D^{s,0}(g) \quad (3.9)$$

$$D^{0,s}(g^*) D^{s,0}(g^{T-1}) \quad (3.10)$$

As usual, the significance of these representations is most clearly revealed when one considers the transformations of state vectors. Any set of measurable quantities which transform under $SL(2,C)$ according to one of the four representations defined in Eqs. (3.7)-(3.10) is called a spinor. The four different kinds of spinors corresponding to these representations are conventionally distinguished by the writing of their indices. A spinor which transforms in accordance with Eq.(3.7) is written with a raised index,

$$\eta^\alpha \xrightarrow{g} \sum_\beta \eta^\beta D_{\beta\alpha}^{0,s}(g) \quad (3.11)$$

By designating the spinor that transforms according to Eq.(3.8) with a lowered index, the contraction of an upper and a lower index corresponds to a Lorentz invariant object.

$$\eta_\alpha \xrightarrow{g} \sum_\beta \eta_\beta D_{\beta\alpha}^{0,s}(g) \quad (3.12)$$

In fact, these two representations are equivalent, for if one defines the matrix C as follows

$$C = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = -i\sigma_2 \in SL(2, C) \quad (3.13)$$

then for any g in $SL(2, C)$, $C^{-1} g C = g^T{}^{-1}$.

The corresponding statement about spinors is that C (C^{-1}) acts as a metric operator, lowering (raising) indices in spinor space.

If Eq.(3.11) defines the transformation law for a particular ket vector, then the corresponding bra will transform according to $D^{0,s}(g^*)$ of Eq.(3.10). Spinors transforming according to this complex conjugate representation are identified by upper dotted indices:

$$\eta^{\dot{\alpha}} \rightarrow \sum_{\dot{\beta}} \eta^{\dot{\beta}} D_{\dot{\beta}\alpha}^{0,s}(g^*) \quad (3.14)$$

An equivalent representation can again be constructed by using the metric operator C , defining lower dotted spinors that transform according to Eq.(3.9).

The fact that there exists no similarity transformation of the form CDC^{-1} relating $D^{0,s}(g)$ to $D^{0,s}(g^*) \equiv D^{s,0}(g^{T-1})$ underlies the fundamental inequivalence of these two representations. In fact, the transformation so-defined is closely related to the parity operation. A parity operator has the property of converting

upper to lower and dotted to undotted indices [20]. Hence, the above inequivalence may be seen as the impossibility of constructing a two-dimensional representation for the parity operator. It is thus related to the difficulties mentioned earlier that were encountered in establishing spin formalisms for HR graphs. As will be shown below, this "doubling" has a natural correlation with topological structure which simultaneously resolves all the parity-associated difficulties.

C. Topological Structure and Stapp's Two-Component Formalism

The additional topological structure necessary to accommodate the spin and parity degrees of freedom is a patchwise orientation of the classical surface. For the orientable and globally oriented classical surfaces of strong interactions one can consistently make a comparison between the local and the global orientations. The labels ortho and para have been used to identify the situations in which the patch orientations agree or disagree, respectively, with the global (HR) orientation. Then, as shown in Fig. 16, each quark line is associated unambiguously with either an ortho or a para patch.

This degree of freedom is believed to be inaccessible for the strong interactions. At the zero entropy level one must include contributions in which each quark line is independently allowed both ortho and para possibilities. Stapp's [7] suggestion was to assign index types according to the ortho/para character of the quarks.

Incorporating general results from earlier S-matrix investigations [18], Stapp was able to construct a complete set of rules for integrating spin into the topological expansion. The elements of this formalism may be summarized as follows [7]:

1) The two-component M function formalism is adopted and each M-function is represented as the product of a scalar function $f(p_1, p_2, \dots, p_n)$ of the invariant products of momentum-energy vectors p_i , and a spinorial factor $M(p)$.

2) The spinorial factor $M(p)$ is most generally a Lorentz invariant function of four-vectors $a_1, \dots, a_{2n}$ of the following form:

$$\text{Tr } a_1 \cdot \sigma a_2 \cdot \bar{\sigma} a_3 \cdot \sigma \dots a_{2n} \cdot \bar{\sigma} \quad (3.15)$$

where the indices of σ_μ and $\bar{\sigma}_\mu$ are specified as

$$\sigma_\mu \rightarrow \sigma_{\mu\alpha\dot{\beta}} \quad \bar{\sigma}_\mu \rightarrow \bar{\sigma}_{\mu}^{\dot{\beta}\alpha} \quad (3.16)$$

3) By considering the no-scattering part of the S-matrix for a single spin 1/2 particle

$$S(p) = \sigma_0 (2\pi)^3 \delta^3(p_i + p_f) 2 \omega_i \quad (3.17)$$

and utilizing the relationship between the M-function and S-matrix embodied in Eq.(3.6), one finds that each quark line contributes a "propagator" factor to $M(p)$ of the form $\sigma \cdot v$. More specifically, taking into account the "parity-transform" relation between ortho

and para quarks, each ortho quark contributes a factor $u_a \cdot \sigma_{\alpha\dot{\beta}}$ while a para quark is associated with the factor $-u_b \cdot \sigma_{\alpha\dot{\beta}}$. The index α here is associated with the leading end of the directed quark line, $\dot{\beta}$ is the spinor index associated with the trailing end of the quark line, u_a is the mathematical covariant velocity p_a/m_a of the physical particle associated with the leading end of the quark line (and is thus equal to [the sign of p_0][the physical velocity v]) and u_b is the velocity p_b/m_b associated with the particle at the trailing end of the quark line.

4) Each hadron contributes a spin wave function that may be constructed as a direct product of the appropriate number of two-dimensional spinors. For mesons, these spin factors may be explicitly taken into account by writing for each particle vertex a factor

$$1/\sqrt{2} \, \delta \cdot s \quad (3.18)$$

where for a scalar particle, the spin four-vector s is just the mathematical velocity $s = u$, $s^2 = 1$, while for a vector meson s satisfies the relations $s^2 = -1$ and $s \cdot p = 0$.

5) An overall negative sign is included to take into account the odd number of permutations necessary to take the quark variables from their original no-scattering ordering to an order in which the pair of indices α_j and $\dot{\beta}_j$ corresponding to the physical particle j stand adjacent in the order $(\dot{\beta}_j, \alpha_j)$.

These rules, which apply to all graphs at the zero-entropy

level of the topological expansion are illustrated by the example of Fig. 17.

The above procedure incorporates a number of attractive features. By associating ortho and para quarks with dotted and undotted indices one is immediately led to an interpretation of the parity operation as classical patch orientation inversion. Individual zero-entropy amplitudes are not parity invariant. However, since all patch orientations occur symmetrically at zero-entropy, and since all higher order hadron terms are constructed from zero-entropy components, parity is ensured as a symmetry of the strong interactions.

The presence of quarks with both ortho and para character at zero-entropy implies the existence of four "parity partners" for each meson. This is exactly the situation of Mandelstam [15] and Bardakci and Halpern [17]. However, as will be discussed further below, by explicitly performing the summation over all zero-entropy contributions, only "ortho plus para" quarks with negative intrinsic parity will survive. The resulting spectrum of particles then precisely corresponds to the observed physical hadrons.

Stapp's construction implies that at zero-entropy spin factors can be completely factored out of the discontinuity equations associated with the scalar functions $f(p_i)$. By removing the spin complications from the non-linear bootstrap, the possibility arises for a topological supersymmetry [21]. As noticed by Chew [22], Stapp factorization is consistent with the assumption that an N-line zero-entropy connected part is associated with a momentum

function that is independent of external quark lines. A zero-entropy symmetry between states of spin 0, $1/2$, 1, $3/2$, and 2 is thereby implied. In the following chapter, where the spin formalism is applied to the calculation of hadronic coupling constant ratios, this phenomenon will be used to obtain a connection between meson-meson-meson and meson-baryon-baryon coupling constants. When further supplemented by an assumption about universality [22], a value may be obtained for the ratio of the strong-interaction coupling constants to the electromagnetic coupling constant as well.

D. Discontinuity Products and Chiral Complexity

The principal intention of Stapp's original investigations was to develop a formalism for spin that possessed the same simple product properties enjoyed by the Chan-Paton factors. The importance of this feature for the topological expansion arises in the analysis of the discontinuity equations. By requiring that spin and isospin factors of any zero-entropy amplitude reproduce themselves in the formation of discontinuity products, one ensures the existence of a solution to the spin and isospin part of the zero-entropy bootstrap conditions. As a byproduct of this parallel treatment of spin and isospin, $SU(6)_W$ symmetry of the hadronic coupling constants automatically emerges at the lowest topological level.

A further consequence of the above approach is a simplification of the rules for the treatment of the spin dependence of discontinuity products. A straightforward calculation of the M-function for

the product of amplitudes shown in Fig. 18 would consist of the calculation of the spin dependence for the individual factors followed by an explicit sum over intermediate spins. Hence, denoting by s_{ci} the four possible physical orthogonal spin vectors for the intermediate particle where $i = 0, 1, 2, 3$ and $s_{c0} = v_c$, the discontinuity of Fig. 18 is associated with the following function:

$$M = (1/\sqrt{2})^6 \sum_{i=0}^3 (\text{Tr } \sigma \cdot u_C \bar{\sigma} \cdot s_B \sigma \cdot u_A \bar{\sigma} \cdot s_A \sigma \cdot u_C \bar{\sigma} \cdot s_{Ci}) \\ \times (\text{Tr } \bar{\sigma} \cdot s_{Ci} \sigma \cdot u_D \bar{\sigma} \cdot s_D \sigma \cdot u_D \bar{\sigma} \cdot s_E \sigma \cdot u_E) \quad (3.19)$$

Stapp [7] has shown how the spin sum may be performed to yield

$$M = (1/\sqrt{2})^4 (\text{Tr } \sigma \cdot u_C \bar{\sigma} \cdot s_B \sigma \cdot u_A \bar{\sigma} \cdot s_A \sigma \cdot u_C \bar{\sigma} \cdot v_C \sigma \cdot u_D \bar{\sigma} \cdot s_D \sigma \cdot u_D \\ \times \bar{\sigma} \cdot s_E \sigma \cdot u_E \bar{\sigma} \cdot v_C) \quad , \quad (3.20)$$

corresponding to the insertion of spin sum factors $\bar{\sigma} \cdot v_C$ into a continuous calculation around the four-point function illustrated in Fig. 19.

The evaluation of such discontinuity products introduces two new elements into the formalism. First, as illustrated by Fig. 20, the possibility arises for the formation of closed internal loops. As in Feynman theory, the internal loop introduces a

negative trace into the amplitude. The formula associated with Fig. 20(b) is

$$[-(1/\sqrt{2})^2 \text{Tr } \bar{s}_A \sigma \cdot u_B \bar{s}_B \sigma \cdot u_D \bar{s}_D \sigma \cdot u_C \bar{s}_C \sigma \cdot u_A] \\ \times [-\text{Tr } \bar{s}_B \sigma \cdot u_C \bar{s}_C \sigma \cdot u_B] \quad (3.21)$$

In fact, in both Eq. (3.20) and Eq. (3.21), use of the product property embodied in Eq.(3.5) allows a simplification corresponding to the contraction rules discussed in Chapter II. One can thus write in place of Eq.(3.20) and Eq.(3.21),

$$M = (1/\sqrt{2})^4 \text{Tr } \bar{s}_B \sigma \cdot u_A \bar{s}_A \sigma \cdot u_D \bar{s}_D \sigma \cdot u_D \bar{s}_E \sigma \cdot u_E \quad (3.20')$$

$$M = (1/\sqrt{2})^2 \text{Tr } \bar{s}_A \sigma \cdot u_D \bar{s}_D \sigma \cdot u_A \times \text{Tr } I \quad (3.21')$$

where in Eq.(3.21') the inner loop has been completely contracted to the trace of the identity.

The possibility of the above contractions depends on a consistent matching in the formation of discontinuity products of ortho to ortho quarks and para to para quarks. Applying Stapp's rules to the graph illustrated in Fig.21 one obtains the spin dependence

$$M = -(1/\sqrt{2})^4 \text{Tr } \bar{s}_A \sigma \cdot u_C \bar{s}_C \sigma \cdot u_C \bar{s}_D \sigma \cdot u_D \bar{s}_E \sigma \cdot u_E \bar{s}_E \sigma \cdot u_C \bar{s}_C \sigma \cdot u_B \bar{s}_B \sigma \cdot u_A$$

or,

$$M = -(1/\sqrt{2})^4 \text{Tr } \delta \cdot s_A \sigma \cdot u_C \delta \cdot s_D \sigma \cdot u_D \delta \cdot s_E \sigma \cdot u_E \delta \cdot s_B \sigma \cdot u_A \quad (3.22)$$

In contrast to Eq.(3.20'), the spin dependence on the intermediate particle C may not be eliminated in this case. The contraction of Feynman graphs is only possible within individual patches. Thus, the formation of discontinuity products introduces a new element of spin complexity into the topological expansion. Such ortho-para transitions may be indicated as in Fig.21(b). In fact, the number of points where a transition arc touches a Feynman arc constitutes a new index of entropy, g_3 . Like the entropy index g_1 , this index satisfies the strong-entropy condition

$$g_3 \geq g_3' + g_3'' \quad (3.23)$$

The integration of this spin complexity into the general topological expansion framework will be one of the principle elements of Chapter V.

The correlation between quark indices and ortho-para character remains somewhat obscure in Stapp's approach which avoids references to states in favor of the underlying M-functions. It should be noted though that there do exist schemes in which the connections ortho $\rightarrow$ undotted and para $\rightarrow$ dotted are more explicit. However, since all of the calculations described in this paper were conducted within a more conventional four-component framework, it is to the construction of this formalism that we now turn.

E. Four-component Formalism

A fundamental accomplishment of the four-component formalism is the automatic incorporation of parity invariance. In order to translate Stapp's rules into a four-component language, it is necessary to unveil the separate parity non-invariant components. This suggests the usefulness of working in the Weyl representation, in which basis the chiral operators ($1 \pm \gamma_5$) are known to have a diagonal form. The Dirac matrices then take the form,

$$\begin{aligned}
 \gamma_\mu &= \begin{pmatrix} 0 & \sigma_\mu \\ \bar{\sigma}_\mu & 0 \end{pmatrix} \\
 \sigma_i &= \begin{pmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix} \quad (i = 1, 2, 3) \\
 \beta &= \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \\
 \gamma_5 &= \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix} \quad (3.24) \\
 1 + \gamma_5 &= \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} \quad 1 - \gamma_5 = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}
 \end{aligned}$$

and the free-particle spinors appear as

$$u(p,s) = 1/\sqrt{2} \begin{Bmatrix} \sqrt{\sigma \cdot p/m_0} \chi(s) \\ \sqrt{\bar{\sigma} \cdot p/m_0} \chi(s) \end{Bmatrix} = 1/\sqrt{2} \begin{Bmatrix} \sqrt{\sigma \cdot v} \chi(s) \\ \sqrt{\bar{\sigma} \cdot v} \chi(s) \end{Bmatrix} \quad (3.25)$$

$$v(p,s) = 1/\sqrt{2} \begin{Bmatrix} \sqrt{\sigma \cdot p/m_0} \chi(s) \\ -\sqrt{\bar{\sigma} \cdot p/m_0} \chi(s) \end{Bmatrix} = 1/\sqrt{2} \begin{Bmatrix} \sqrt{\sigma \cdot v} \chi(s) \\ -\sqrt{\bar{\sigma} \cdot v} \chi(s) \end{Bmatrix} \quad (3.26)$$

where χ is a normalized 2-component Pauli spin vector, $p_0 > 0$,
the usual normalizations

$$\begin{aligned} \bar{u}(p,s)u(p,s) &= 1 \\ \bar{v}(p,s)v(p,s) &= -1 \end{aligned} \quad (3.26)$$

are assumed to apply, and

$$\chi^c(s) = C \chi(s) = i \sigma_2 \chi(s) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \chi(s) \quad (3.27)$$

Stapp's rules allow one to construct the following S-matrices
for the scattering of the ortho and para type Dirac particles shown
in Fig. 22:

$$S_{fi} = \chi_a \sqrt{\bar{\sigma} \cdot p_a/m} \sigma \cdot p_a/m \sqrt{\bar{\sigma} \cdot p_b/m} \chi_b \quad \text{Ortho} \quad (3.28)$$

$$S_{fi} = \chi_a \sqrt{\bar{\sigma} \cdot p_a/m} \sigma \cdot p_b/m \sqrt{\bar{\sigma} \cdot p_b/m} \chi_b \quad \text{Para} \quad (3.29)$$

(all momenta are physical)

It is easy to verify that these same forms result from the following
constructions:

$$\bar{u}(p,s) (1 + \gamma_5) u(p,s) \quad (3.28')$$

$$\bar{u}(p,s) (1 - \gamma_5) u(p,s) \quad (3.29')$$

The natural assumption to make, then, is that the analogues of the two-component ortho and para propagator factors are the operators $1 + \gamma_5$ and $1 - \gamma_5$, respectively.

This proposal leads to an interesting interpretation for the Dirac spinors appearing in Eq. 3.25. In this representation,

$$(1 + \gamma_5) u(p,s) = 2/\sqrt{2} \begin{pmatrix} 0 \\ \sqrt{\vec{\sigma} \cdot \vec{v}} \chi(s) \end{pmatrix} \quad (3.30)$$

$$(1 - \gamma_5) u(p,s) = 2/\sqrt{2} \begin{pmatrix} \sqrt{\vec{\sigma} \cdot \vec{v}} \chi(s) \\ 0 \end{pmatrix} \quad (3.31)$$

so that it is justified to regard the four-component Dirac spinors as the superposition of separate ortho and para components. Recalling the usual function of $\beta = \gamma_0$ as the parity operator, one finds

$$\begin{aligned} P[(1 + \gamma_5)u(p,s)] &= \gamma_0(1 + \gamma_5)u(p,s) = 2/\sqrt{2} \begin{pmatrix} \sqrt{\vec{\sigma} \cdot \vec{v}} \chi(s) \\ 0 \end{pmatrix} \\ &= (1 - \gamma_5)u(p,s) \\ P[(1 - \gamma_5)u(p,s)] &= \gamma_0(1 - \gamma_5)u(p,s) = 2/\sqrt{2} \begin{pmatrix} 0 \\ \sqrt{\vec{\sigma} \cdot \vec{v}} \chi(s) \end{pmatrix} \\ &= (1 + \gamma_5)u(p,s) \end{aligned} \quad (3.32)$$

maintaining the parity inversion relationship between ortho and para

sectors. Consequently, the states of definite parity are $P = +1$ and $P = -1$. The Dirac formalism thus shows that physical states appear only in the combination $P = +1$, automatically performing the zero-entropy sum of the topological expansion. In fact, the combination $P = -1$ is precisely that which appears in Eq.(3.25) for the antiparticle spinor $v(p,s)$. The apparent parity doubling of the Lorentz group can thus be interpreted simply as the prediction of particle-antiparticle pairs of opposite behavior under parity.

As in the two-component formalism, it is always a correct procedure to explicitly introduce spinors at the ends of quark lines. Physical particles can be constructed by taking proper spin and isospin combinations. However, for the mesons, one can again derive a convenient summed representation for the spin wave function. Proceeding in the most direct manner, one can construct meson states of definite spin by taking appropriate combinations of quark and antiquark spinors. For example, for an outgoing spin 0 particle with the Condon-Shortley convention (with which Eq.(3.27) is consistent) one needs to consider the wave function

$$\frac{1}{\sqrt{2}}[v(mv,s+)\bar{u}(mv,s+) - v(mv,s+)\bar{u}(mv,s+)] \quad (3.33)$$

The physical velocity, v , appearing in the quark and antiquark spinors is that of the meson. Topological quarks do not individually carry momentum. Then, in the rest frame of the meson, with the relationship prescribed in Eq.(3.27) between the quark and antiquark spin, one finds the following rest frame operator:

$$\begin{aligned}
& (1/\sqrt{2})^3 \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix} (0 \ 1 \ 0 \ 1) \begin{pmatrix} 0 & I \\ I & I \end{pmatrix} - (1/\sqrt{2})^3 \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} (1 \ 0 \ 1 \ 0) \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \\
& = (1/\sqrt{2})^3 \begin{pmatrix} -I & -I \\ I & I \end{pmatrix} = (1/\sqrt{2})^3 \begin{pmatrix} I & -I \\ -I & I \end{pmatrix} \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix} \quad (3.34)
\end{aligned}$$

which may be cast in the form of the general covariant spin 0 projection operator,

$$[(1 - \gamma \cdot v)/2\sqrt{2}] \gamma_5 \quad (3.35)$$

Similarly, for the $s_z = 0$ component of the spin 1 projection operator, one obtains

$$\begin{aligned}
& 1/\sqrt{2} [v(mv, s^+) \bar{u}(mv, s^+) + v(mv, s^-) \bar{u}(mv, s^-)] = \\
& (1/\sqrt{2})^3 \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix} (0 \ 1 \ 0 \ 1) \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} + (1/\sqrt{2})^3 \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} (1 \ 0 \ 1 \ 0) \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \\
& = (1/\sqrt{2})^3 \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & -1 \\ -1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} = (1/\sqrt{2})^3 \begin{pmatrix} I & -I \\ -I & I \end{pmatrix} \begin{pmatrix} 0 & \sigma_3 \\ -\sigma_3 & 0 \end{pmatrix} \\
& = [(1 - \gamma \cdot v)/2\sqrt{2}] \gamma_3 \quad (3.36)
\end{aligned}$$

General Lorentz invariant forms may be obtained by applying appropriate rotations and velocity transformations. The results may be summarized as follows:

$$\begin{aligned}
\text{Spin 0} & \quad [i(1 - \gamma \cdot u)/2\sqrt{2}] \gamma_5 \\
\text{Spin 1} & \quad [(1 - \gamma \cdot u)/2\sqrt{2}] \gamma \cdot s \quad (3.37)
\end{aligned}$$

In these equations, u is the mathematical velocity, defined to be equal to the physical velocity v ($v_0 > 0$) for the case of an outgoing particle, but negative the physical velocity for an incoming particle. The factor i in the pseudoscalar operator is necessary for consistency with unitarity [7]. *

The four component formalism may thus be used to calculate spinorial factors $M(p)$ for zero-entropy topological graphs according to the following rules:

- 1) Attach a Dirac spinor $u, v, \bar{u}$, or $-\bar{v}$ to the end of each quark line according to the usual Feynman prescription. The negative sign for $\bar{v}$ arises from the particle conjugation properties of the spinors [23].
- 2) For each ortho quark line include a factor $1 + \gamma_5$ while for each para line a factor $1 - \gamma_5$ is to be incorporated.
- 3) Construct required spin and flavor states by taking appropriate combinations of quark spinors.
- 4) For mesons, the spin projection operators displayed in Eq.(3.37) may be inserted. Calculation of $M(p)$ is then converted to a trace with an overall minus sign as in the two-component formalism.

The construction of discontinuity products is particularly

*These structures differ from those presented by Stapp in Ref.7 by a factor $1/2\sqrt{2}$. The factor of $1/2$ is due to the omission of a normalizing factor in Stapp's definition of the spinors u and v . The factor $1/\sqrt{2}$ was inadvertently dropped in the derivation of the relationship between the two- and four-component formalisms.

revealing in the four-component representation. Consider the meson junction schematically shown in Fig.23. To effect the sum over intermediate spin states analogous to Eq.(3.20), it is most convenient to deal directly with the spinors, rather than the spin projection operators. The sum over intermediate spin 0 and spin 1 states is equivalent to the following sum over independent spin states for the constituent quark and antiquark:

$$\begin{aligned}
 \sum_{\text{spins}} M O M' &= M_{\beta\alpha} v_{\alpha}(+) \bar{u}_{\beta}(+) \otimes u_{\gamma}(+) [-\bar{v}_{\delta}(+)] M'_{\delta\gamma} \\
 &M_{\beta\alpha} v_{\alpha}(+) \bar{u}_{\beta}(+) \otimes u_{\gamma}(+) [-\bar{v}_{\delta}(+)] M'_{\delta\gamma} \\
 &M_{\beta\alpha} v_{\alpha}(+) \bar{u}_{\beta}(+) \otimes u_{\gamma}(+) [-\bar{v}_{\delta}(+)] M'_{\delta\gamma} \\
 &M_{\beta\alpha} v_{\alpha}(+) \bar{u}_{\beta}(+) \otimes u_{\gamma}(+) [-\bar{v}_{\delta}(+)] M'_{\delta\gamma} \quad (3.38)
 \end{aligned}$$

Rearranging the factors in each of the four terms, one obtains

$$\begin{aligned}
 \sum_{\text{spins}} M O M' &= -u_{\gamma}(+) \bar{u}_{\beta}(+) M_{\beta\alpha} v_{\alpha}(+) \bar{v}_{\delta}(+) M'_{\delta\gamma} - u_{\gamma}(+) \bar{u}_{\beta}(+) M_{\beta\alpha} v_{\alpha}(+) \bar{v}_{\delta}(+) M'_{\delta\gamma} \\
 &- u_{\gamma}(+) \bar{u}_{\beta}(+) M_{\beta\alpha} v_{\alpha}(+) \bar{v}_{\delta}(+) M'_{\delta\gamma} - u_{\gamma}(+) \bar{u}_{\beta}(+) M_{\beta\alpha} v_{\alpha}(+) \bar{v}_{\delta}(+) M'_{\delta\gamma} \quad (3.39)
 \end{aligned}$$

Making use of the identities

$$\begin{aligned}
 \sum_{\alpha=1}^2 u_{\alpha}(v, \lambda) \bar{u}_{\beta}(v, \lambda) &= [(1 + \gamma \cdot v)/2]_{\alpha\beta} \\
 \sum_{\alpha=1}^2 v_{\alpha}(v, \lambda) \bar{v}_{\beta}(v, \lambda) &= [(-1 + \gamma \cdot v)/2]_{\alpha\beta} \quad , \quad (3.40)
 \end{aligned}$$

Eq.(3.39) may finally be written as the single trace

$$\sum_{\text{spins}} M \odot M' = \text{Tr} \left(\frac{1 + \gamma \cdot v_T}{2} \right) M \left(\frac{1 - \gamma \cdot v_T}{2} \right) M'$$

This is more general than a zero-entropy identity. To specify a particular zero-entropy product, one needs to insert chiral factors indicating the ortho or para character of each quark line. Considering once again the product shown in Fig.18, one can write the sum over spin 0 and spin 1 intermediate states as (external particles are treated as spin 1)

$$\begin{aligned} & \{-\text{Tr}(1 + \gamma_5) \left[\frac{(1 + \gamma \cdot v_A)}{2\sqrt{2}} \gamma \cdot s_A \right] (1 + \gamma_5) \left[\frac{(1 + \gamma \cdot v_B)}{2\sqrt{2}} \gamma \cdot s_B \right] (1 - \gamma_5) \left[\frac{(1 - \gamma \cdot v_C)}{2\sqrt{2}} \gamma \cdot s_C \right] \} \\ & \times \{-\text{Tr} \left[\frac{(1 + \gamma \cdot v_C)}{2\sqrt{2}} \gamma \cdot s_C \right] (1 - \gamma_5) \left[\frac{(1 - \gamma \cdot v_E)}{2\sqrt{2}} \gamma \cdot s_E \right] (1 - \gamma_5) \left[\frac{(1 - \gamma \cdot v_D)}{2\sqrt{2}} \gamma \cdot s_D \right] (1 + \gamma_5) \} \\ & + \sum_C \{-\text{Tr}(1 + \gamma_5) \left[\frac{(1 + \gamma \cdot v_A)}{2\sqrt{2}} \gamma \cdot s_A \right] (1 + \gamma_5) \left[\frac{(1 + \gamma \cdot v_B)}{2\sqrt{2}} \gamma \cdot s_B \right] (1 - \gamma_5) \left[\frac{(1 - \gamma \cdot v_C)}{2\sqrt{2}} \gamma \cdot s_C \right] \} \\ & \times \{-\text{Tr} \left[\frac{(1 + \gamma \cdot v_C)}{2\sqrt{2}} \gamma \cdot s_C \right] (1 - \gamma_5) \left[\frac{(1 - \gamma \cdot v_E)}{2\sqrt{2}} \gamma \cdot s_E \right] (1 - \gamma_5) \left[\frac{(1 - \gamma \cdot v_D)}{2\sqrt{2}} \gamma \cdot s_D \right] (1 + \gamma_5) \} \end{aligned} \quad (3.42)$$

Using the identity derived in Eqs.(3.38)-(3.41), this becomes

$$\begin{aligned} & -\text{Tr} \left\{ (1 + \gamma_5) \frac{(1 + \gamma \cdot v_C)}{2} (1 + \gamma_5) \left[\frac{(1 + \gamma \cdot v_A)}{2\sqrt{2}} \gamma \cdot s_A \right] (1 + \gamma_5) \left[\frac{(1 + \gamma \cdot v_B)}{2\sqrt{2}} \gamma \cdot s_B \right] \right. \\ & \left. \times (1 - \gamma_5) \frac{(1 - \gamma \cdot v_C)}{2} (1 - \gamma_5) \left[\frac{(1 - \gamma \cdot v_E)}{2\sqrt{2}} \gamma \cdot s_E \right] (1 - \gamma_5) \left[\frac{(1 - \gamma \cdot v_D)}{2\sqrt{2}} \gamma \cdot s_D \right] \right\} \end{aligned} \quad (3.43)$$

which, as a result of the properties of the chiral operator γ_5 ,

$$\begin{aligned}
\gamma_5^2 &= 1 \\
(1 + \gamma_5)^2 &= 2(1 + \gamma_5) \\
(1 - \gamma_5)^2 &= 2(1 - \gamma_5) \\
\gamma_5 \gamma_\mu &= -\gamma_\mu \gamma_5
\end{aligned} \tag{3.44}$$

may be rewritten as

$$\begin{aligned}
& -\text{Tr}[(1+\gamma_5) \left\{ \frac{(1+\gamma \cdot \mathbf{v}_A)}{2\sqrt{2}} \gamma \cdot \mathbf{s}_A \right\} (1+\gamma_5) \left\{ \frac{(1+\gamma \cdot \mathbf{v}_B)}{2\sqrt{2}} \gamma \cdot \mathbf{s}_B \right\} \\
& \times (1-\gamma_5) \left\{ \frac{(1-\gamma \cdot \mathbf{v}_E)}{2\sqrt{2}} \gamma \cdot \mathbf{s}_E \right\} (1-\gamma_5) \left\{ \frac{(1-\gamma \cdot \mathbf{v}_D)}{2\sqrt{2}} \gamma \cdot \mathbf{s}_D \right\}]
\end{aligned} \tag{3.45}$$

Since every zero-entropy product entails only ortho to ortho or para to para connections, the intermediate spin sum factor for zero-entropy is effectively only 1/2, maintaining the Chan-Paton transitivity and embodying the properties of zero-entropy contractions.

Returning to Fig.23 one can now uncover the elements of the zero-entropy spectrum. The eight possible zero-entropy products appear as

$$\begin{aligned}
& \text{Tr} M(1+\gamma_5) \frac{(1-\gamma \cdot \mathbf{v}_I)}{2\sqrt{2}} \left\{ \begin{smallmatrix} 1\gamma_5 \\ \gamma \cdot \mathbf{s} \end{smallmatrix} \right\} (1+\gamma_5) \text{Tr}(1+\gamma_5) \frac{(1+\gamma \cdot \mathbf{v}_I)}{2\sqrt{2}} \left\{ \begin{smallmatrix} 1\gamma_5 \\ \gamma \cdot \mathbf{s} \end{smallmatrix} \right\} (1+\gamma_5) M' \\
& \text{Tr} M(1+\gamma_5) \frac{(1-\gamma \cdot \mathbf{v}_I)}{2\sqrt{2}} \left\{ \begin{smallmatrix} 1\gamma_5 \\ \gamma \cdot \mathbf{s} \end{smallmatrix} \right\} (1-\gamma_5) \text{Tr}(1-\gamma_5) \frac{(1+\gamma \cdot \mathbf{v}_I)}{2\sqrt{2}} \left\{ \begin{smallmatrix} 1\gamma_5 \\ \gamma \cdot \mathbf{s} \end{smallmatrix} \right\} (1+\gamma_5) M' \\
& \text{Tr} M(1-\gamma_5) \frac{(1-\gamma \cdot \mathbf{v}_I)}{2\sqrt{2}} \left\{ \begin{smallmatrix} 1\gamma_5 \\ \gamma \cdot \mathbf{s} \end{smallmatrix} \right\} (1+\gamma_5) \text{Tr}(1+\gamma_5) \frac{(1+\gamma \cdot \mathbf{v}_I)}{2\sqrt{2}} \left\{ \begin{smallmatrix} 1\gamma_5 \\ \gamma \cdot \mathbf{s} \end{smallmatrix} \right\} (1-\gamma_5) M' \\
& \text{Tr} M(1-\gamma_5) \frac{(1-\gamma \cdot \mathbf{v}_I)}{2\sqrt{2}} \left\{ \begin{smallmatrix} 1\gamma_5 \\ \gamma \cdot \mathbf{s} \end{smallmatrix} \right\} (1-\gamma_5) \text{Tr}(1-\gamma_5) \frac{(1+\gamma \cdot \mathbf{v}_I)}{2\sqrt{2}} \left\{ \begin{smallmatrix} 1\gamma_5 \\ \gamma \cdot \mathbf{s} \end{smallmatrix} \right\} (1-\gamma_5) M' .
\end{aligned} \tag{3.46}$$

By simplifying and recombining terms one finds the following effective spin structures:

$$\begin{aligned}
 & \text{Tr}(MI)\text{Tr}(IM') \quad \text{Tr}(MY_5)\text{Tr}(\gamma_5 M') \quad \text{Tr}(MY \cdot v_I)\text{Tr}(\gamma \cdot v_I M') \\
 & \text{Tr}(MY_5 \gamma \cdot v_I)\text{Tr}(\gamma_5 \gamma \cdot v_I M') \quad \text{Tr}(MY \cdot s_I)\text{Tr}(\gamma \cdot s_I M') \\
 & \text{Tr}(MY_5 \gamma \cdot s_I)\text{Tr}(\gamma_5 \gamma \cdot s_I M') \quad \text{Tr}(MY \cdot v_I \gamma \cdot s_I)\text{Tr}(\gamma \cdot v_I \gamma \cdot s_I M') \\
 & \text{Tr}(MY_5 \gamma \cdot v_I \gamma \cdot s_I)\text{Tr}(\gamma_5 \gamma \cdot v_I \gamma \cdot s_I M') \quad (3.47)
 \end{aligned}$$

This is precisely the Bardakci-Halpern [17] multiplicity. While individual zero-entropy amplitudes are factorizable, they are not invariant under the parity operation. The complete zero-entropy spectrum includes eight elementary mesons for each isospin structure.

By performing the sum over the ortho and para possibilities for each quark line individually, the spin structures that appear for intermediate particles are reduced to only $\frac{(1-\gamma \cdot v)}{2\sqrt{2}}\gamma$'s and $\frac{(1-\gamma \cdot v)}{2\sqrt{2}}\gamma_5$. This is the result derived explicitly in Eq.(3.37) where the chiral sum is performed automatically by the use of the Dirac equation solutions. Although previous investigators used four-component spinors, they did not impose the restrictions on the form of these spinors mandated by the Dirac equation. Hence, the reduction to only a single spin 0 particle and a single spin 1 particle was not observed.

The independent summation over ortho and para quarks and antiquarks on both sides of a discontinuity product introduces non-zero entropy elements of the topology. Ortho-para transitions receive

contributions only from the $\gamma \cdot v_I$ term of the intermediate spin sum factors displayed in Eq.(3.41). As before, these factors then disallow contractions and provide a convenient signature for the higher complexity nature of the interaction.

The spin scheme developed in this chapter successfully represents a theory based on the topological expansion. After summing over the lowest levels of complexity there results a factorizable, parity invariant theory with only a physical particle spectrum. The application of this formalism to the calculation of physical effects will be the objective of the following chapters.

IV. HADRON COUPLING CONSTANTS

A. Introduction

The spectrum of hadrons at the zero-entropy level is determined from the zero-entropy non-linear bootstrap conditions. The resulting degenerate collection of elementary particles shares a single mass, m_0 , and stands in one-to-one correspondence with certain physical mesons (spin 0 and spin 1), baryons (spin 1/2 and spin 3/2) and baryonium (spin 0, spin 1, spin 2). A zeroth approximation to the physical three-hadron coupling constants may be obtained through the application of the formalism developed in the previous chapter [24].

A key consideration in this venture is the notion of topological supersymmetry [22]. Stapp's separation of the spin and momentum dependence of the M-functions together with the product properties of the spin factors (like those for flavor and color) reduces the form of the discontinuity equations to those of a spinless, flavorless, colorless, planar theory. Consequently, the same dynamical amplitude is to be associated with a given zero-entropy Feynman graph irrespective of the number and character of the attached quark lines. For example, one expects the same momentum function $M_3(p_1, p_2, p_3)$ to be associated with all three of the amplitudes represented in Fig.24.

At zero-entropy, then, elementary mesons, baryons, and baryoniums share the single mass m_0 and couple to each other through the single dimensionless coupling constant which may be defined by

$$g_0 = M_3/m_0 \quad . \quad (4.1)$$

S-matrix unitarity for the physical amplitudes specifies that each zero-entropy component contributes with an equal weight of unity. Thus, physical hadronic coupling constants may be approximated by summing over the relevant zero-entropy topologies, incorporating Clebsch-Gordan coefficients appropriate to the physical particles involved. This prescription turns out to encompass all the predictions of Mandelstam's model [15]--which was already an extension of $SU(6)_W$ symmetry--and adds a supersymmetry prediction for the ratios of baryon, meson, and baryonium couplings.

B. Meson-Meson-Meson Coupling Constants

We begin by computing an elementary three meson coupling constant as shown in Fig.25. The first task is to enumerate the compatible zero-entropy topologies, associating the M-function value $g_0 m_0$ to any single topology. According to the rules formulated in Chapter III, the spin dependence of the graph represented in Fig.25 may be written as

$$g_0 m_0 \bar{u}_\alpha(p_B)(1+\gamma_5)u_\alpha(p_A)\bar{v}_\gamma(p_A)(1+\gamma_5)v_\gamma(p_C)\bar{u}_\beta(p_C)(1-\gamma_5)v_\beta(p_B) \quad . \quad (4.2)$$

To describe physical mesons one must sum over all possible patchwise orientations. This effectively associates a factor $(1+\gamma_5) + (1-\gamma_5) = 2$ with each quark line, allowing one to write for Eq.(4.2),

$$2^3 g_o m_o \bar{u}_\alpha(p_B) u_\alpha(p_A) \bar{v}_\gamma(p_A) v_\gamma(p_C) \bar{u}_\beta(p_C) v_\beta(p_B) \quad (4.2')$$

For mesons, the spin state projection operators derived in Eq. (3.37) may be utilized although care may be necessary to include multiplicities arising from required flavor superpositions.

Consider then the specific example of the $\rho^+ \pi^- \pi^0$ coupling. The $u\bar{u}$ and $d\bar{d}$ composition of the π^0 introduces two equal contributions (see Fig. 26) with an isospin factor of $1/\sqrt{2}$. Effecting the ortho-para summation one can write the amplitude

$$\begin{aligned} (F^0)_{\rho^+ \pi^- \pi^0} &= 2(2)^3 m_o g_o \text{Tr} \left(\frac{1+\gamma \cdot v_\pi}{2} \frac{1+\gamma \cdot v_\rho}{2} i\gamma_5 \frac{(1-\gamma \cdot v_\pi)}{2} i\gamma_5 \frac{(1-\gamma \cdot v_\rho)}{2} \right) \gamma \cdot s_\rho \\ &= -3g_o s_\rho \cdot (p_{\pi^-} + p_{\pi^0})^* \end{aligned} \quad (4.3)$$

Where we've used the relations

$$\begin{aligned} v_{\pi^0} &= v_{\rho^+} + v_{\pi^-} \\ v_{\rho^+} \cdot v_{\pi^-} &= -1/2 \\ v_{\rho^+} \cdot v_{\pi^0} &= 1/2 \end{aligned} \quad (4.4)$$

The corresponding resonance width is

* This expression differs from that recorded in Ref.24 by a factor $(1/2) \cdot (1/\sqrt{2})^3$. The $(1/\sqrt{2})^3$ arises from the spin projection operators and was inadvertently omitted from the earlier work. The other factor of 2 difference is due to a misunderstanding of the significance of the patchwise orientation for the ortho-para quark assignments.

$$\Gamma_{\rho \rightarrow 2\pi} = 4/6(3)^2 (g_\rho^2 / 4\pi) (k_{cm}^3 / m_\rho^2). \quad (4.5)$$

from which one may identify the effective coupling constant

$$\frac{g_{\rho \pi \pi}^2}{4\pi} = (3)^2 \left(\frac{g_\rho^2}{4\pi} \right) \quad (4.6)$$

By performing a similar calculation for the $\omega\rho\pi$ amplitude, the $SU(6)_W$ ratio

$$\frac{g_{\omega\rho\pi}^2}{g_{\rho\pi\pi}^2} = \frac{4}{m_\rho^2} \quad (4.7)$$

is confirmed [25]. All three-meson coupling ratios are in agreement with $SU(6)_W$ symmetry.

C. Baryon Couplings

The first stage in computing a meson-baryon-antibaryon coupling constant is similar to the foregoing. Considering the specific example of Fig. 27, after summing over the chirality for each of the four quark lines one obtains

$$2^4 m_0 g_0 [\bar{u}^u(p) u^u(n)] [\bar{u}^d(p) u^d(n)] [\bar{u}^u(p) v^u(\pi^-)] [\bar{u}^d(\pi^-) u^d(n)] \quad (4.8)$$

For baryons there does not exist the simple spin projection

representations that was possible for the mesons. Consequently, particular physical states must be especially constructed from the spins. Physical baryons, however, are not distinguished by the order of their three quark lines--the parameter previously characterized as "topological color". For each physical baryon, care must be taken to include the six possible distinct quark permutations with equal weight. A convenient procedure is to attach to each baryon location an ordinary three-quark spin-flavor SU(6) wave function, each quark carrying a (1,2,3) color label, but to assign the wave function a norm equal to six. This takes account of the fact that the usual SU(6) state wave functions include a sum over the six quark permutations with a corresponding factor of $1/\sqrt{6}$, which must be omitted if each permutation is to be given a weight +1. The properly normalized neutron and proton states are then as follows [26]:

$$\begin{aligned}
 |n, s_z = 1/2\rangle &= (\sqrt{6})(1/\sqrt{18}) [-2d+d+u+ -2d+u+d+ -2u+d+d+ +u+d+d+ \\
 &\quad +u+d+d+ +d+u+d+ +d+u+d+ +d+d+u+ +d+d+u+] \\
 |p, s_z = 1/2\rangle &= (\sqrt{6})(1/\sqrt{18}) [2u+u+d+ +2u+d+u+ +2d+u+u+ -u+d+u+ \\
 &\quad -u+u+d+ -u+d+u+ -d+u+u+ -d+u+u+ -u+u+d+] \quad (4.9)
 \end{aligned}$$

The evaluation of Eq.(4.8) is most easily accomplished in the Breit frame [27] which for equal mass particles is defined with the parameters

$$p_\pi = (0,0,0,q) \quad q = i m_0$$

$$\begin{aligned}
p_n &= (E, 0, 0, q/2) \\
p_p &= (E, 0, 0, -q/2) \\
E &= (m_0^2 - m_\pi^2/4)^{1/2} = \sqrt{3/4} m_0 \\
\bar{u}(p)u(n) &= E/m_0 = \sqrt{3/4}
\end{aligned} \tag{4.10}$$

This allows one to rewrite the neutron decay formula as

$$\begin{aligned}
F_{np\pi^-}^0 &= 2 m_0 g_0 \sqrt{3/4} \sqrt{3/4} \bar{u}(p) \frac{(1-\gamma_5 \gamma_\pi)}{2\sqrt{2}} \gamma_5 u^d(n) \\
&= 2 \frac{(2m_0 g_0)}{\sqrt{2}} (3/4) (3/4) \bar{u}(p) \gamma_5 u^d(n)
\end{aligned} \tag{4.8'}$$

which may be easily evaluated for the specific states displayed in Eq.(4.9) by virtue of the fact that

$$\frac{\bar{u}(p) \gamma_5 u^d(n)}{2m} \xrightarrow{\langle \chi_u | \vec{\sigma} \cdot \vec{q} | \chi_d \rangle} \frac{1 \langle \chi_u | \sigma_3 | \chi_d \rangle}{2}$$

The final amplitude is most conveniently expressed in the form

$$F_{np\pi^-}^0 = 30/(\sqrt{2}) g_0 (\bar{u}_p \gamma_5 \gamma_\pi u_n) \tag{4.11}$$

The relationship between g_0 and the well-known coupling constant

$g_{\pi^0 NN}^2 / 4\pi \approx (15)$ is

$$(30/\sqrt{2})^2 \frac{g_0^2}{4\pi} = \frac{g_{np\pi^-}^2}{4\pi} = 2 \frac{g_{\pi^0 NN}^2}{4\pi} \tag{4.12}$$

D. Supersymmetry Relations

All baryon-antibaryon-meson couplings agree with the $SU(6)_W$ ratios. Additionally, there is the potential now, through topological supersymmetry, for predicting the ratios between the three-meson and the baryon-antibaryon-meson coupling constants. In particular, the measured $g_{\pi^0 NN}^2/4 = 14.8$ predicts, through Eqs.(4.12) and (4.5), a ρ width of

$$\Gamma_{\rho \rightarrow 2\pi} = (2/75) (g_{\pi^0 NN}^2/4) (k_{cm}^3/m_\rho^2) = 31 \text{ MeV.}$$

This may be compared to the Breit-Wigner width listed in the Particle Data Group Tables of 158 MeV. The factor of five discrepancy with the zero-entropy calculation must be assumed to reflect the importance of higher order contributions in the topological expansion. The observed mass-breaking of supersymmetry tells us, of course, that such components are important. Apparently, the correction to zero-entropy that gives the large ratio between proton and pion masses also substantially affects the coupling constants.

A measure of the supersymmetry breaking is the disparity that exists in the effective zero-entropy coupling constant $g_0^2/4\pi$ resulting from mesonic or baryonic considerations. If one inserts the Breit-Wigner width into Eq. (4.5), one obtains the relation $g_0^2/4\pi \approx 1/3$. However, the pion-nucleon coupling constant of Eq. (4.12) suggests that $g_0^2/4\pi \approx 1/15$. An interesting proposal by Chew [22] is that a universality condition actually relates strong, weak, and electromagnetic interactions. Chew originally conjectured that the value

of g_0 determined from the zero-entropy bootstrap should be precisely e. Later considerations showed such a simple relation to be unlikely but maintained the order of magnitude. The involvement of electroweak interactions introduces an uncertain element since the techniques for handling the zero-mass internal lines of electroweak graphs are not fully developed. However, it is not unreasonable to anticipate additional supersymmetry breaking from infrared electroweak boson contributions.

Topological parameters systematically make $B\bar{B}M$ coupling constants larger than MMM coupling constants by a factor of 4 (16 in the square of the coupling constants). One factor of 2 arises from the chirality of the extra quark line, and one factor of 2 from the always-allowed permutation of the two quark lines that do not touch the meson. This systematic factor of four is obscured by the attachment of baryon wave functions but it is not removed.

The coupling of baryonium to $B\bar{B}$ introduces one further factor of four due to the one additional quark line and its additional permutation possibilities. The experimental failure to find narrow baryonium thereby becomes understandable. Even if a baryonium lies below the $B\bar{B}$ threshold so that decay to mesons must proceed via an Okubo-Zweig-Iizuka rule-violating higher component of the topological expansion, the OZI suppression mechanism has to overcome a factor of sixteen in rate as well as the large phase space available to the final mesons. Although the OZI mechanism for baryonium decay has not yet been understood in detail, the large baryonium coupling

constants would make it difficult for baryonium to have widths sufficiently narrow as to be detectable by straightforward experiments.

Fundamental to the calculations described in this chapter is an assumption regarding the relevance of zero-entropy solutions for the physical world. The justification is, to a certain extent, the accuracy of the results. Unsurprisingly, the above calculations indicate greater corrections to those predictions based on meson-baryon symmetry than for those reflecting the $SU(6)_W$ content of the theory. A more systematic consideration of the hierarchical order of effects will be developed in Chapter V. There a scheme will be displayed in which zero-entropy vertices maintain significance throughout higher order perturbative effects.

V. FEYNMAN RULES AND TOPOLOGICAL ASYMPTOTIC FREEDOM

A. Hadronic Vertices

The development of specific calculational rules for the topological expansion depends on an understanding of the interrelationships between the various components described in the previous chapters. The different elements of complexity that need to be woven together include color switching, ortho-para transitions, genus and boundary structure and electroweak contributions. This chapter will describe a proposal for a hadronic calculational scheme based on the construction of Feynman-like rules that automatically perform a sum over certain lowest levels of complexity [23]. The rules turn out to incorporate a qualitative feature that may be identified as "topological asymptotic freedom" (TAF) by virtue of its similarity to the asymptotic freedom of QCD. Therein lies the basis for a perturbative approach to hadron dynamics with a zeroth order that exhibits topological supersymmetry.

The essential ingredients in the development of Feynman-graph rules are the concepts of a vertex and a propagator. Rules for associating amplitudes with electroweak-boson-lepton topologies [4,5,6] may be similar to those of a Lagrangian field theory inasmuch as the elementary vertex functions are without any structure--these topologies do not admit duality transformations (contractions). In contrast, the duality features of strong-interactions require unorthodox Feynman rules with structured vertices. A central feature

of the theory described in this chapter is the construction of scattering functions that associate vertices not with the point couplings of local field theory but with fully contracted zero-entropy amplitudes.

Attempts to more precisely characterize the amplitudes of the topological expansion have led Chew [29] to distinguish between three general types of Feynman-graph vertices. The identity of a vertex is specified by the lines to which it is attached. Any vertex touched by a line corresponding to a weak elementary particle is defined to be explicitly weak. For strong interactions it is useful to distinguish between hadron lines which exhibit switching complexity along the accompanying quark lines ("clothed") and those lines from which such complexity is absent ("naked"). Strong vertices are identified by their exclusive attachment to hadronic lines. However, naked internal hadron lines beginning and ending at the same vertex ("tadpoles") in non-adjacent positions define a distinct non-contractible class of vertices suggestively labeled "implicitly weak".(Fig.28)

As emphasized by Chew [29], amplitudes corresponding to implicitly weak vertices are naturally suppressed relative to their strong interaction counterparts. Additionally, neither vertices touched by a weak line nor by naked noncontractible strong tadpoles can be evaluated as zero-entropy amplitudes--i.e., through elementary hadrons. The former vertex has no singularities while the latter has singularities in addition to the elementary hadrons. In fact, for the particular naked cylinder shown in Fig. 28, Chew has argued [30] that the vertex contains among its poles a Higgs scalar, forcing

introduction into the theory of the Weinberg-Salam electroweak boson family.

The Feynman rules developed in this chapter apply solely to strong vertices. For purely strong amplitudes, all naked internal lines may be contracted out, and this is assumed to be effected in what follows. One can then adopt a Feynman rule that assigns to each vertex function the value of a zero-entropy (strong) amplitude. All of the singularities of such vertices correspond to elementary hadrons. The use of soft vertices in the evaluation of Feynman graphs provides a potential convergence mechanism for the individual higher order components of the topological expansion and, perhaps, for the overall expansion itself.

There is one additional special class of vertices that play an important role in the topological expansion. By considering the connected sum illustrated in Fig. 29 one sees the necessity for incorporating certain two-particle vertices into the theory. The elimination of such "trivial" vertices would sacrifice the accurate representation of the discontinuity structure that is present. Although such graphs cannot be directly built by connected sum from zero-entropy components, they must evidently be included at levels in which either chiral or color complexity is introduced.

Trivial vertices may be associated with the real analytic function $M_2^0(p^2)$ with branch points at $p^2 = (2m_0)^2, (3m_0)^2, \dots$. Comparison of the discontinuity formula satisfied by $M_2^0(p^2)$ with that satisfied by the familiar "mass operator" $\Sigma_0(p^2)$ yields the relation

$$M_2^0(p^2) = -\Sigma_0(p^2) \quad * \quad (5.1)$$

The analogy with the mass shift operator allows one to represent the trivial vertex through the dispersion integral

$$M_2^0(p^2) = \frac{(p^2 - m_0^2)^2}{\pi} \int_{4m_0^2}^{\infty} dx \frac{\text{Im } M_2^0(x)}{(x-p^2)(x-m_0^2)^2} \quad (5.2)$$

where $\text{Im } M_2^0(p^2) = -\text{Im } \Sigma_0(p^2) > 0$ and $\text{Im } M_2^0(p^2)$ may be represented schematically as in Fig. 30 as the sum of absolute-value squares of zero-entropy vertex functions integrated over the mass shell.

As in Feynman theory, it is assumed that zero-entropy three-point functions do not include the elementary particle poles at $p^2 = m_0^2$ (other $L=0$ poles arising, for instance, as the bound states of Bethe-Salpeter types of equations do appear in the vertices). In the following, this condition will be integrated with general renormalization principles to yield consistency relations that have been incorporated into Eq. (5.2),

$$M_2^0(m_0^2) = 0$$

$$\left(\frac{d}{dp^2} M_2^0(p^2) \right) \Big|_{p^2 = m_0^2} = 0 \quad (5.3)$$

* Recall that positive scattering amplitudes correspond to negative mass shifts.

B. Intermediate State Plug Operators

Since zero-entropy amplitudes are all contractible to a single vertex, it is clear that the development of the notion of a propagator must involve non-zero elements of complexity. Graphically, an internal line with O-P transitions and/or color switches may be expected to be associated with the propagator. A source of puzzlement to those conversant with Lagrangian field theory may be the fact that each elementary hadron has only one momentum line but more than one spin 1/2 "quark" line. Nonetheless, because quarks do not individually carry momentum it is possible to devise a consistent representation for a hadron propagator with the same general structure as for the familiar case of a lepton.

Guidance in the necessary construction may be obtained from the results derived in Chapter III for connected sum operators. In the analysis of the general connected sum shown in Fig.23 it was shown that a sum over intermediate meson states may be replaced by effective plug operators on individual quark lines. Quarks yielded the factor $1 + \gamma \cdot p / m_0$ while antiquarks were associated with the factor $1 - \gamma \cdot p / m_0$. In these operators the unit elements correspond to zero-entropy attachments while the $\gamma \cdot p / m_0$ terms represent a chiral transition. An effective meson plug operator may thus be cast in the form

$$(1 + \gamma)_{\text{meson}} = (1 + \gamma_q \cdot p / m_0)(1 - \gamma_{\bar{q}} \cdot p / m_0) \quad . \quad (5.4)$$

The space in which this operator acts is the Dirac quark-antiquark

direct product space of $4^2 = 16$ dimensions.

Formula (5.4) corresponds to the four transition possibilities illustrated in Fig. 31. A cross on a quark line associates with a matrix $\gamma \cdot p/m_0$. By writing Eq. (5.4) in the form $(1 + \chi)$, one isolates the zero-entropy element from the three non-contractible intermediate states. These consist of: (1) a chiral transition on the quark line alone; (2) a chiral transition on the antiquark line alone; (3) chiral transitions on quark and antiquark lines simultaneously.

The operator χ then corresponds to the non-zero entropy elements of an intermediate state and one arrives at the following form for a meson propagator:

$$\frac{1}{p^2 - m_0^2} \chi \quad (5.5)$$

It should be noted that in writing Eq. (5.4) the factors of $1/2$ that appear in Eq. (3.41) have been omitted. Since we are considering the sum of all possible chiral transitions, these factors are effectively cancelled by the addition of ortho and para elements for each quark line, $(1 + \gamma_5) + (1 - \gamma_5) = 2$. An alternative, but equivalent [13], prescription is used in Ref. 23. In that paper, the Dirac spinors in the Weyl basis are taken to be

$$u'(p, s) = \begin{pmatrix} \sqrt{\sigma \cdot p/m_0} \eta(s) \\ \sqrt{\bar{\sigma} \cdot p/m_0} \eta(s) \end{pmatrix}$$

$$v'(p,s) = \begin{pmatrix} \sqrt{\sigma \cdot p/m_0} \, \eta(s) \\ -\sqrt{\sigma \cdot p/m_0} \, \eta(s) \end{pmatrix} \quad (5.6)$$

where η is the 2-component Pauli spin vector. One then has the normalizations

$$\begin{aligned} \bar{u}'(p,s)u'(p,s) &= 2 \\ \bar{v}'(p,s)v'(p,s) &= -2 \end{aligned} \quad (5.7)$$

reflecting the fact that ortho and para zero-entropy quark lines are always added together in building the S-matrix. Associating the spinors $u', \bar{u}', v', -\bar{v}'$ with the two intermediate-state quark plugs shown in Fig. 32 (according to the prescription of Chapter III) one has, in case (a), a factor $u'_\alpha(p,s)\bar{u}'_\beta(p,s)$ and in case (b) the factor $-v'_\alpha(p,s)\bar{v}'_\beta(p,s)$ (α and β label the four components of the Dirac space). Summing over spins and taking account of the normalization in Eq. (5.7), one arrives at the two terms displayed in Eq. (5.4),

$$\begin{aligned} \sum_{\pm s} u'_\alpha(p,s)\bar{u}'_\beta(p,s) &= (1 + \gamma_q \cdot p/m_0)_{\alpha\beta} \\ \sum_{\pm s} v'_\alpha(p,s)\bar{v}'_\beta(p,s) &= (1 - \gamma_q \cdot p/m_0)_{\alpha\beta} . \end{aligned} \quad (5.8)$$

In the above equations p is the physical energy momentum. Hence, as with the usual Feynman rules, intermediate quark lines may be uniformly associated with the factor $(1 + \gamma \cdot k/m_0)$ where k is the

momentum in agreement with the quark-line direction.

For the cases of baryons and baryoniums, intermediate state matrices involve products of three or four factors like those in Eq. (5.4). However, non-contractible intermediate lines may now also arise through the possibility of color transitions. It is thus convenient to add a color plug-matrix and to define for baryon and baryonium plugs,

$$\begin{aligned}
 (1+x)_{\text{baryon}} &\equiv (1+\gamma_1 \cdot p/m_0)(1+\gamma_j \cdot p/m_0)(1+\gamma_k \cdot p/m_0)(1+\pi_1 \dots \pi_5) \\
 (1+x)_{\text{baryonium}} &\equiv (1-\gamma_1 \cdot p/m_0)(1-\gamma_j \cdot p/m_0)(1+\gamma_k \cdot p/m_0) \times \\
 &\quad (1+\gamma_1 \cdot p/m_0)(1+\pi_6+\pi_7+\pi_8)
 \end{aligned} \tag{5.9}$$

where the quark labels correspond to the diagrams illustrated in Fig. 33. In these formulas the symbols $\pi_1 \dots \pi_5$ designate the five possible color switches for the baryon's three quarks (3 odd permutations and 2 even). The elementary-baryon plug matrix thus acts in a space of $4^3 \times 6$ dimensions. Similarly, π_6 , π_7 and π_8 represent the three possible quark color switches within baryonium and the elementary-baryonium plug matrix acts in a space of $4^4 \times 4$ dimensions. Stapp [7] has shown from unitarity arguments that the zero-entropy part of any elementary-hadron plug matrix is the unit matrix, so normalizing factors are not to be added to the forms of Eqs. (5.4), (5.9) and (5.10). The propagators corresponding to the operators displayed in Eqs. (5.9) and (5.10) are defined as in Eq. (5.5).

From the results of Chapter III one sees that, as in Feynman's

original rules, a closed quark loop leads to a factor of (-1) . However, the definition of a closed loop may now be obscured by the distinction between momentum lines and quark lines, and the possibilities for quark transitions. Stapp [13] has found that the correct counting of closed-loop (-1) factors is achieved by eliminating all color switches from the embellished graph and then counting quark loops. Thus, for example, each of the 3 graphs in Fig. 34 has two closed-loop factors of (-1) even though their spin-flavor-chirality multiplicity is different.

The foregoing development of propagator matrices is justified by Stapp's S-matrix analysis for the case when $p^2 = m_0^2$ -- "on shell". It is a fundamental assumption for us to assert that these matrices may be used off-shell in Feynman formulas as well, with $m_0 \equiv \sqrt{p^2}$. It is similarly essential that one be able to continue zero-entropy functions M_N^0 off mass shell.

C. The Propagator D_x

Although the trivial vertices, zero-entropy connected parts and quark plug matrices provide the basis for all hadronic calculations in the topological expansion, there remains the matter of integrating these elements into an overall consistent picture. Note that the form for our "clothed" propagator (Eq. 5.5) corresponds to a momentum line segment possessing a single instance of topological complexity. However, between any two zero-entropy connected parts the topological expansion always includes the infinite series of alternating switches and trivial vertices represented in Fig. 35. It is an appealing

notion to perform the sum indicated by this series before introducing any subsequent complications, thereby removing any explicit graphical dependence on trivial vertices and quark transitions.

Because all strong-interaction components of the topological expansion are built by connected sum of zero-entropy single-vertex components, it follows that exactly one switch intervenes between any pair of adjacent strong-interaction vertices. With each switch providing a factor

$$\frac{1}{p^2 - m_0^2} \chi \quad (5.11)$$

and each trivial vertex corresponding to $-i \Sigma_0(p^2)$, the sum indicated in Fig. 35 can in closed form be effected. Denoting the sum of the series by $iD_x(p)$ one has

$$\begin{aligned} D_x(p) &= \frac{\chi}{p^2 - m_0^2} \cdot \frac{1}{1 - \frac{\chi \Sigma_0(p^2)}{p^2 - m_0^2}} \\ &= \frac{1}{\frac{p^2 - m_0^2}{\chi} - \Sigma_0(p^2)} \quad (5.12) \end{aligned}$$

All subsequent reference to an elementary-hadron propagator will be understood to refer to the operator $D_x(p)$ defined by this equation.

It is essential that the unit part of the plug matrix has been omitted; each plug must contribute complexity to the topology. The

operator χ has been constructed as the sum over all plugs where some mismatch--either in color or in chirality or in both--has occurred.

One can now see the origin of the consistency conditions imposed on the trivial vertex $\Sigma_0(p^2)$ in Eq. (5.3). The zero-entropy theory contains the elementary particle pole at $p^2 = m_0^2$. However, the series depicted in Fig. 35 corresponds to a pole that is in general located at a different position than $p^2 = m_0^2$. The condition on $\Sigma_0(p^2)$ was chosen to eliminate this unphysical mass shift. A related remark is that the external lines of our Feynman graphs are always naked and vertices receiving external lines are always nontrivial. These considerations lead to the derivative condition stated in Eq. (5.3).

The required Feynman rules have now been given. Strong interaction components of the topological expansion are to be represented in terms of the zero-entropy connected parts M_N^0 and the elementary hadron propagator $D_x(p)$. Thus assuming the zero-entropy problem has been solved to provide the vertices, a complete prescription for calculating any purely hadronic interaction will have been furnished.

D. Topological Asymptotic Freedom

The Feynman rules described herein attribute all breaking of supersymmetry to the switching matrix χ and thus to the propagator D_x ; strong vertices are supersymmetric. We now argue that even $D_x(p)$ is asymptotically supersymmetric--in the limit $p \rightarrow \infty$ [23].

The creation of $0 + P$ color singlet states for hadrons is precisely the accomplishment of our operator $D_x(p)$. In particular, the color permutation operators that appear in the baryon and baryonium

plug matrices explicitly project out the symmetric states in six- and four-dimensional quark permutation spaces respectively. An alternative representation in terms of the "color-singlet" projection operator P ($P^2=P$) turns out to be convenient:

$$\begin{aligned}
 (1+\pi_1+\dots+\pi_5) &= 6P = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \\
 (1+\pi_6+\pi_7+\pi_8) &= 4P = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}
 \end{aligned} \tag{5.13}$$

The formulas (5.4), (5.9), and (5.10) for χ may then be written in a shorthand form as

$$\chi = \Gamma N P - 1 \tag{5.14}$$

where Γ represents the Dirac spin-chirality factors and N corresponds to the dimension of the quark permutation space. The projection operator P has one eigenvalue 1 and $N - 1$ eigenvalues 0. It is then possible to rewrite Formula (5.12) as

$$D_x = \frac{P}{\frac{p^2 - m_0^2}{\chi} - \Sigma_0} + \frac{1 - P}{\frac{p^2 - m_0^2}{\chi} - \Sigma_0}$$

or,

$$D_x = \frac{P}{\frac{p^2 - m_o^2}{\chi^p} - \Sigma_o} + \frac{1 - P}{-(p^2 - m_o^2) - \Sigma_o} \quad (5.15)$$

where

$$\chi^p = N \Gamma - 1 \quad (5.16)$$

and the projection operators have been replaced by their eigenvalues for the states P and $1 - P$. Equation (5.15) may be regrouped to obtain

$$D_x = D_x^0 + D_x^1 \quad (5.17)$$

where

$$D_x^0 \equiv \frac{1}{-(p^2 - m_o^2) - \Sigma_o} \quad (5.18)$$

$$D_x^1 \equiv P \left\{ \frac{1}{\frac{p^2 - m_o^2}{\chi^p} - \Sigma_o} - \frac{1}{-(p^2 - m_o^2) - \Sigma_o} \right\} \quad (5.19)$$

The absence of χ^p from D_x^0 makes it apparent that D_x^0 is supersymmetric while D_x^1 is not. However, let us examine D_x^1 as $p \rightarrow \infty$.

From Formula (5.2) and the condition that $\text{Im } \Sigma_o(x)$ is negative definite one sees that $\Sigma_o(p^2)$ behaves roughly linearly as $p^2 \rightarrow \infty$. Because $\Gamma_{\text{meson}} \sim p^2$, $\Gamma_{\text{baryon}} \sim p^3$, $\Gamma_{\text{baryonium}} \sim p^4$, the first term within the bracket of (5.19) asymptotically approaches $[-\Sigma_o(p^2)]^{-1}$. What

about the second term?

It is reasonable from experience both with dispersion relations and with Feynman integrals to expect *

$$\Sigma_0(p^2) \sim p^2 \ln p^2 \quad \text{as } p^2 \rightarrow \infty. \quad (5.20)$$

With this assumption, the second term in the bracket of Eq. (5.19) also approaches $[-\Sigma_0(p^2)]^{-1}$ and the two terms asymptotically cancel, yielding

$$D_x(p^2) \sim D_x^0(p^2) \quad \text{as } p^2 \rightarrow \infty. \quad (5.21)$$

The behavior here is in striking similarity to the QCD feature of large- p^2 gluon decoupling--the feature that has been called "asymptotic freedom". The parallel becomes even more compelling if one recalls that the complete topology (not just the thickened Feynman graph) associates color and chirality switching with lines on the classical surface that have been called "topological gluons" [14]--for reasons recognized prior to the development of the Feynman rules described here.

Even if p^2 is not extremely large the term D_x^1 appears to be

* Nontrivial zero-entropy vertex functions are expected to decrease strongly as $p^2 \rightarrow \infty$.

sufficiently small so as to be treated as a perturbation. Notice that the projection operator P brings a factor $1/N$ which depresses D_x^1 for baryons and baryoniums. While there is no such factor for mesons, internal closed loops of maximum weight do not contain mesons. One therefore expects supersymmetrical features to survive in the properties of physical hadrons. Detailed study is necessary to identify those physical quantities with the best chance for exhibiting approximate supersymmetry. The results of Chapter IV encourage attention to dimensionless coupling constants.

It should be noted here that the smallness of D_x^1 does not necessarily mean that zero-entropy is a good approximation. In particular, a lowest order calculation of mass shifts using a zeroth order approximation for D_x^1 yields a supersymmetric (ground state) hadron mass m'_0 which is not expected to be close to m_0 . Although an understanding of the mass-symmetry breaking mechanism in the topological expansion remains incomplete, the Feynman rules developed here provide a useful framework for analysis. In the final chapter we discuss the general features of the picture that is emerging from these investigations.

VI. CALCULATIONAL APPROXIMATIONS

A. Mass Renormalization

One of the most obvious applications for a system of Feynman like rules is the calculation of the mass renormalization that generates the physical spectrum. Although the rules that have been presented here provide a specific formalism that may be applied to the evaluation of any given graph, the question of which graphs are most important for mass shifts is a nontrivial consideration for the topological expansion.

An understanding of the origin of the mass shift requires a careful analysis of the analyticity structure of the theory. Elementary particle poles at $p^2 = m_0^2$ appear both in the propagator and in the zero-entropy connected part, i.e. in the vertex functions. Representations of these two contributions are shown in Fig. 36. Strictly speaking the cross on the line associated with the propagator D_x is redundant since our convention is to always consider an uncontracted internal strong interaction line to correspond to such a summation. We will retain the crossed notation in the following to emphasize the non-zero-entropy nature of the lines.

The branch point structure is associated in the first instance with the two-particle discontinuity depicted in Fig. 37(a). This may be incorporated into the entire planar non-trivial discontinuity which we represent by the crossed bubble σ_x in Fig. 37(b).

Our claim is that mass renormalization may be studied as the

iteration of these planar discontinuities with the two pole terms $D_x + (1/p_o^2 - m_o^2)$. Formally summing this series yields the renormalization given by Eq. (6.1).

$$\frac{1 + (p_o^2 - m_o^2) D_x}{p_o^2 - m_o^2 - \sigma_x [1 + (p_o^2 - m_o^2) D_x]} \quad (6.1)$$

The primary importance of the planar terms in the discontinuity σ_x is suggested by an unproven but as yet uncontradicted conjecture by Finkelstein [32]. Finkelstein noticed that when one attempts to add an additional line to a planar graph in a fashion that preserves the planarity, one has the potential of adding two free quark loops with their corresponding spin and flavor multiplicity. However, the addition of a particle line in a manner that is contrary to the planarity necessarily decreases the number of free closed quark loops by at least one. This domination of planar graphs could provide an important simplification to the topological scheme.

It is expected that a useful approximation for the discontinuity that appears in Eq. (6.1) may be obtained by considering first only the contributions due to the two particle intermediate states (Fig. 37(a)). For an intermediate meson line, approximations to the propagator D_x^M may be based on the following rearrangement:

$$D_x^M = \frac{1}{\frac{p_o^2 - m_o^2}{x_M} - \Sigma_o} \approx \frac{1}{-\Sigma_o} \left(1 + \frac{p_o^2 - m_o^2}{\Sigma_o x_M} + \dots \right)$$

Inverting the chiral operator,

$$D_x^M \cong \frac{-1}{\Sigma_0} \left(1 + \frac{p^2 - m_0^2}{\Sigma_0} \frac{(1 - \gamma_1 \cdot p/m_0)(1 + \gamma_2 \cdot p/m_0)}{(1 - p^2/m_0^2)(1 - p^2/m_0^2)} + \dots \right) \quad (6.2)$$

A lowest order calculation involving baryon lines would simply use the supersymmetric operator D_x^0 identified in Eq. (5.18). In analogy to Eq. (6.2), successive higher order approximations may be based on the following expansion for D_x^1 :

$$\begin{aligned} D_x^1 &= P \left[\frac{1}{\frac{p^2 - m_0^2}{\chi^p} - \Sigma_0} - \frac{1}{-(p^2 - m_0^2) - \Sigma_0} \right] \\ &\approx \frac{-P(p^2 - m_0^2)}{(\Sigma_0)^2} \left[1 + \frac{(1 - \gamma_1 \cdot p/m_0)(1 - \gamma_2 \cdot p/m_0)(1 - \gamma_3 \cdot p/m_0)}{6(1 - p^2/m_0^2)^3} \right] \end{aligned} \quad (6.3)$$

Investigations have so far concentrated on the application of these ideas to the meson mass shifts. In particular, it was originally expected that the two-particle loop would provide the basis for the splitting that is observed between the ρ -mesons and pions. The realization that this is not, in fact, the case, has depended on the recognition of two important features of our Feynman rules.

B. The Off-Mass-Shell extension

As mentioned several times previously Stapp's original M-function

analysis was conducted entirely within an S-matrix , on-mass-shell context. Strictly speaking, the associated two-component formalism was only designed to apply to situations consistent with these assumptions. Implicit in our four-component formulation of Feynman rules is a presumed behavior under off-shell conditions as well. A careful evaluation of the simple graph represented in Fig. 38 reveals distinct differences between the predictions of the two- and four-component formalisms when the analysis is extended away from the pole.

A calculation of the amplitude corresponding to Fig. 38 for the case in which all the external particles have spin 0 is expected to reproduce the $\cos \theta$ dependence (p-wave) of the one contributing spin 1 intermediate state. The calculation may be performed using either the two- or four-component spin projection operators. In either case, at the pole, precisely the expected spin dependence emerges. Variation away from the pole, however, results in a differentiation between the two approaches. An asymptotic behavior proportional to s^4 is predicted according to the two-component formalism but only an s^3 dependence results when the four-component operators are used. The softer behavior of the four-component propagator confirms its desirability for computations involving mass loop integrations.

In fact, a totally consistent off-shell prescription must include a modification of the spin operators derived in Chapter III. Clearly, the masses that appeared due to a derivation that used the free particle spinors should be replaced by $\sqrt{p^2}$. The more general Dirac structure consequently takes the form

$$1 + \gamma \cdot p / \sqrt{p^2} \quad . \quad (6.4)$$

This modification eliminates any differences between the mass renormalizations of the spin 1 and spin 0 particles due to meson intermediate states. The omission of such off-mass-shell considerations has led previous investigators to identify these contributions as the source of ρ - π symmetry breaking [31].

C. Baryon Loops

The contributions of baryons to meson mass renormalization is similarly ineffective in lifting the ρ - π mass degeneracy. Although one might expect that the more complicated Dirac structure arising from baryon intermediate states would provide sufficient complexity, a special operator identity allows a reduction of the spin structure to the simple mesonic form. The non-trivial Dirac structure that arises in connection with the meson loop of Fig. 39(a) is simply (all possible chiral transitions included)

$$[\gamma_1 \cdot (p/2 + q) + m][\gamma_2 \cdot (p/2 - q) - m] \quad (6.5)$$

The color switching in Fig. 39(b) seems to correspond to a more elaborate spin structure:

$$[\gamma_1 \cdot (p/2 + q) + m][\gamma_1 \cdot (p/2 - q) - m][\gamma_1 \cdot (p/2 + q) + m][\gamma_2 \cdot (p/2 - q) - m] \quad (6.6)$$

However, the following identity is observed to hold:

$$[\gamma_1 \cdot (\frac{p}{2} + q) + m][\gamma_1 \cdot (\frac{p}{2} - q) - m][\gamma_1 \cdot (\frac{p}{2} + q) + m] = -4q^2 [\gamma_1 \cdot (\frac{p}{2} + q) + m] , \quad (6.7)$$

reducing Eq. (6.7) to the same form as Eq. (6.5). Evidently, the ρ - π mass splitting requires consideration of contributions of more complexity than just the two-component intermediate state. It is only the supersymmetry degeneracy between baryons and mesons that may be expected to be broken at this level of approximation.

D. Zero-entropy Vertices

A complete solution to any dynamical problem clearly requires an understanding of the detailed properties of the zero-entropy vertex. While this still awaits a solution to the nonlinear zero-entropy bootstrap, a number of features have already been discerned. Chief among these is the damping phenomenon hoped to aid the convergence of various calculations and perhaps the convergence of the topological expansion itself. Bethe-Salpeter-type analyses of series such as that indicated in Fig. 40 do exhibit the desired damping and encourage further approximation in that direction.

A useful form for the vertex may be obtained by writing a dispersion relation for the continuation off-shell in one leg.

$$\Gamma_0(p^2, m_0^2, m_0^2) = 1 + \frac{p^2 - m_0^2}{\pi} \int \frac{ds' \operatorname{Im} \Gamma_0(s')}{(s' - s)(s' - m_0^2)} \quad (6.8)$$

Assuming this may be written in the form

$$\Gamma_0 = 1 + a \frac{p^2 - m_0^2}{\bar{s} - x} \quad (6.9)$$

and imposing the condition that this vanish in the limit $p^2 \rightarrow \infty$ yields the approximate solution

$$\Gamma_0(p^2, m_0^2, m_0^2) = \frac{\bar{s} - m_0^2}{\bar{s} - p^2} \quad (6.10)$$

The Bethe-Salpeter analogy indicates the necessity of including excited state poles in Γ_0 , and allows one to rewrite Eq.(6.10) in the general form,

$$\Gamma_0(p^2, p'^2, p''^2) \sim \frac{(m_0^2 - m_1^2)^3}{(p^2 - m_1^2)(p'^2 - m_1^2)(p''^2 - m_1^2)} \quad (6.11)$$

The possibility that Bethe-Salpeter-type poles may also exist in the discontinuity function σ_x allows a hope of eventually making contact with bound-state models of hadrons. If the final renormalized poles are in fact controlled by σ_x then there exists the potential for a fully Reggeized Topological Theory of Particles.

VII. SUMMARY AND OUTLOOK

The formalism that has been developed here provides a viable basis for performing calculations in the topological approach to elementary particles. These rules begin the task of associating specific computational procedures with the elaborate structure that has emerged primarily from self-consistency considerations.

Especially attractive accomplishments of the framework that has been discussed include the covariant incorporation of spin into a theory with quark-like substructure; a sustenance of the physically observed elementary particle spectrum with no ghosts or parity doubling; maintenance of $SU(6)_W$ symmetry with supersymmetric and universality extensions; possible understanding of the origin of Regge recurrences.

In addition, the propagator D_x developed in Chapter V, when combined with the zero-entropy vertices and appropriate off-mass-shell extensions establishes the basis for a perturbative approach to hadron dynamics. It appears that, for certain physical questions, color and chirality complexity may be treated perturbatively through a power series in D_x^{-1} as defined by Formula (5.19). The zeroth-order terms of this expansion exhibit full topological supersymmetry even though they do not correspond to zero-entropy.

The suppression of color and chirality transitions in the large p^2 regime closely parallels the well-known asymptotic freedom behavior in QCD. Effectively, a very large number of switches restores the supersymmetry characteristic of zero-entropy (where there are no

switches). It should be noted that $\chi_{\text{meson}}(p) \rightarrow 0$ as $p \rightarrow 0$, suggesting that for some questions involving low-momentum mesons it may be profitable to treat switching as "weak" rather than "strong".

The comparison with the QCD notion of asymptotic freedom is made more compelling by Finkelstein's planarity conjecture. Chew [33] has noticed that a related fact to Finkelstein's conjecture is the tendency of external quark lines to remain exterior to complicated planar loops (Fig.41). Such behavior exhibits a strong resemblance to the free quark picture that is often invoked for parton model calculations in QCD.

A crucial problem that remains to be addressed is the development of reliable methods for solving the nonlinear zero-entropy conditions. While certain questions may be addressed by using the approximations discussed in Chapter VI, a determination of the overall strength of the hadronic and electroweak interactions awaits the complete bootstrap solution.

Finally, the role of the electroweak interactions needs to be assessed. Understanding of the manner in which the electroweak components are to be integrated into topological particle theory is rapidly progressing. One anticipates large supersymmetry breaking from electroweak boson contributions in the infrared region--topological theory's analogue of the Higgs mechanism. However, the techniques for handling zero-mass scalar and vector internal lines in Feynman graphs still remain incomplete, and it is not certain which physical questions may be approached without having first solved

the infrared problem. Nevertheless, we suppose that the application of common sense will allow certain interesting issues to be dealt with through the machinery described here. Surely the old idea -- that for certain purposes electroweak interactions are negligible-- cannot be completely wrong.

APPENDIX A

The quantum surface is a necessary element for the imposition of self-consistency requirements on internal quantum numbers [3].

Transverse to the classical surface, the intersection of the two surfaces constitutes the components of the belt. At zero-entropy the quantum surface is a sphere and completely determines the topology of the classical surface. (Higher orders of the topological expansion require an independent classical surface)

Self-consistent bootstrap requirements and compatibility with the zero-entropy classical surface has suggested a decomposition of the quantum surface into oriented triangular patches. The intersection between the classical surface and the quantum surface allows two kinds of basic triangles. Triangles intersected by a single sheet of the classical surface are referred to as I-triangles; Y-triangles, or core triangles, intersect three different sheets (see Fig. 42).

Contraction conditions restrict elementary hadrons to only the three triangular configurations shown in Fig. 43. It is natural to associate the I-triangles with quarks and to identify the triangular orientations with the particle and antiparticle distinction. Then so far as quark content is concerned the elementary hadrons correspond to conventional mesons, baryons, and baryoniums.

Flavor variability resides in orientations of the perimeters of particle disks. In terms of basic quantum triangles, this amounts to attaching directions to the two edges of the I-triangles that are

not intersected by the belt. The theory thus stipulates a fourfold flavor structure corresponding to the distinct combinations shown in Fig. 44.

As discussed in Chapter II, electric charge is assigned based on a comparison between charge fiber and HR orientations. Relative agreement or disagreement corresponds to $q=1$ or $q=0$ charge associations. Notice that the inclusion of Y-triangles in baryons with a fixed charge of -1 allows a reproduction of the observed hadron charges even though the quarks are integrally charged [4,5,6]. The charged and neutral possibilities for the four generations displayed in Fig. 44 predict, overall, an eightfold quark multiplicity.

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Figure Captions

1. Classical surface for zero-entropy terms in the topological expansion.
2. Classical surface of Fig.1 with quark structure explicit.
3. Inclusion of charge fibers.
4. The connected sum of two single-vertex single-boundary classical surfaces.
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11. Zero-entropy belt graph.
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15. Baryonium color switches.
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19. Graphical representation of spin sum shown in Fig.18.

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22. (a) Ortho quark.
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38. Elementary pole.

- 39. Contributions to meson mass renormalization.
- 40. Bethe-Salpeter type equation for the vertex.
- 41. Free quark behavior in planar graphs.
- 42. Quantum surface triangles and their classical surface intersections.
- 43. Elementary hadron areas on the quantum surface.
- 44. The four quark flavor generations.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9

Fig. 10

==== Junction line

Fig. 11

———— Belt, HR arcs

==== Junction line

———— Feynman line

~~~~~ Charge fiber

Fig. 12

Meson

Baryon

Baryonium

Fig. 13

P

P_+ P_+^2

I

PP_+ PP_+^2

Fig. 14

Fig. 15

Fig. 16

Fig. 17

$$M = -\left(\frac{i}{\sqrt{2}}\right)^3 \left(\frac{-i}{\sqrt{2}}\right) T_r (\sigma \cdot U_B \tilde{\sigma} \cdot S_B \sigma \cdot U_C \tilde{\sigma} \cdot S_C \sigma \cdot U_D \tilde{\sigma} \cdot S_D \sigma \cdot U_D \tilde{\sigma} \cdot S_A)$$

$$\times f(p_A p_B p_C p_D)$$

Fig. 18

Fig. 19

Fig. 20

Fig. 21

Fig. 22

Fig. 23

Fig. 24

Fig. 25

Fig. 26

Fig. 27

Fig. 28

Fig. 29

$$\text{Im } M_2^0(p^2) = \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} + \dots$$

Fig. 30

Fig. 31

Fig. 32

(a)

(b)

Fig. 33

Weight = 1

Weight = 32

Weight = $(32)^2$

Fig. 34

Fig. 35

(a)

(b)

Fig. 36

(a)

(b)

Fig. 37

Fig. 38

(a)

(b)

Fig. 39

Fig. 40

Fig. 41

I - triangle

Y - triangle

--- Belt

Fig. 42

Fig. 43

Fig. 44