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QCD Working Group Summary

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ABSTRACT

The status of Monte Carlo and other perturbative QCD calculations for signals and backgrounds in hadronic collisions is reviewed, and possibilities for improvements are discussed.

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QCD Working Group Summary

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ABSTRACT

The status of Monte Carlo and other perturbative QCD calculations for signals and backgrounds in hadronic collisions is reviewed, and possibilities for improvements are discussed.

1. Introduction

The QCD Working Group concentrated primarily on signatures and backgrounds for large- Q^2 processes in hadronic collisions. The cross sections for such processes can be calculated in QCD perturbation theory. It is believed that these calculations are theoretically reliable, although in practice there are uncertainties on the order of factors of two because the running coupling constant $\alpha_s(Q^2)$ is not very small. The study of experimental detection, however, generally depends at least in part on the complete structure of the events, and this involves all Q^2 scales from that of the primary hard scattering down to that at which hadrons form. At present this complete structure can be investigated only by using Monte Carlo simulation based on a combination of QCD perturbation theory and models for soft QCD, so Monte Carlo programs were extensively discussed.

The Monte Carlo programs mainly used at this workshop, ISAJET¹ and PYTHIA,² have not significantly changed in their treatment of QCD since the 1986 Madison Workshop, so this is reviewed only briefly. Both programs have recently added a more correct treatment of Higgs production, which is discussed. While these programs generally describe existing data reasonably well, it is important to stress that they are not complete. One important limitation is that coherence effects known to exist in QCD perturbation theory are not properly implemented, so that the amount of soft QCD radiation, and hence the multiplicity, is overestimated. Some progress on understanding this problem has recently been made by Ellis, Marchesini, and Webber.³ The other important known limitation is that QCD multijet cross sections are reproduced only approximately. The approximation is probably adequate for estimating backgrounds, but it is certainly not adequate for testing QCD in a quantitative way.

QCD perturbation theory is straightforward if there is only one large scale in the problem, although calculations beyond the leading order are extremely difficult technically. The application of perturbation theory is more subtle if there is more than one large scale or if the scale is very small compared to the total energy. A good example of this is the "minijet" phenomenon: at Sp $\bar{p}$ S energies the cross sections for jets with p_T of a few GeV are several millibarns, and at SSC energies they become comparable to the total cross section. A closely related issue is the behavior of the structure functions as $x \rightarrow 0$, which is discussed separately by W.K. Tung.⁴ Certain measurements at the Tevatron Collider should help to understand these problems better.

2. Simulation of QCD

Since the 1986 Madison Workshop,⁵ there have been no fundamental changes in the treatment of QCD by ISAJET and PYTHIA, although there have been some changes in detail. Both programs perform basically the same functions. First, they generate a primary hard scattering using the appropriate QCD-improved parton model cross section. Next they add QCD radiation in the initial and final states with the leading-log QCD matrix elements and exact non-collinear kinematics. This radiation is stopped at a cutoff t_c and nonperturbative models are then used to create the observed hadrons from both the QCD partons and the spectator beam jets.

While the discussion here concentrates on ISAJET and PYTHIA, which are the programs that have been most used at this Workshop, one should not forget that other programs exist. In particular, Fox and Wolfram⁶ pioneered the use of QCD branching for final state jets, Odorico⁷ wrote the first usable program for initial state QCD branching, and Marchesini and Webber⁸ have developed a more correct treatment of coherent effects in QCD showers.

2.1 Cross sections

The first step in simulating an event is to generate the partons in the primary hard interaction according to the appropriate cross section. In QCD perturbation theory the hadronic cross section for a hard process $i + j \rightarrow k + l$ is given by

$$\frac{d\sigma}{d\hat{s}d\hat{y}} = \frac{1}{\hat{s}} \hat{\sigma}_{i+j \rightarrow k+l} f_i(x_1, Q^2) f_j(x_2, Q^2) \quad (2.1)$$

$$x_1 x_2 = \hat{s}/s, \quad x_1/x_2 = e^{2\hat{y}}$$

where $Q^2 \sim \hat{s}$ is the hard scale and $\hat{\sigma}$ is the elementary cross section, which is calculable as a series in $\alpha_s(Q^2)$. The first term in this series is simply given by the lowest order Feynman graphs treating the quarks and gluons as free particles. Individual higher order graphs produce $\log(Q^2/\mu^2)$ factors from the infrared and mass singularities of perturbation theory. If these were to survive in the final answer, then the expansion in $\alpha_s(Q^2)$ would be invalid even for $Q^2 \rightarrow \infty$. However, the infrared singularities cancel between real and virtual processes. The mass singularities arise from parton lines going on mass shell and so factorize and can be absorbed into universal parton distributions defined by their connection to deep inelastic scattering. It is primarily the need to carry out this cancellation which makes higher order QCD calculations difficult. The terms remaining after the factorization give higher order corrections. Since these are approximately proportional to the lowest order cross section, they are usually described as K factors.

Understanding the role of mass singularities is important not only for calculating K factors but also for using Monte Carlo programs. The factorization of mass singularities defines the way in which different processes can be combined without double counting. Consider for example the production of a heavy quark Q . The lowest order processes are

$$g + g \rightarrow Q + \bar{Q} \quad (2.2a)$$

$$q + \bar{q} \rightarrow Q + \bar{Q} \quad (2.2b)$$

But there are additional processes — also included in the programs — that produce heavy quarks, e.g.

$$g + g \rightarrow g + g \rightarrow Q + \bar{Q} + g \quad (2.3a)$$

$$g + Q \rightarrow g + Q \quad (2.3b)$$

The precise meanings of these additional processes are determined by the way in which they are generated. Process (2.3a) is produced by the leading log evolution of the jet, which contains the correct $g \rightarrow Q\bar{Q}$ matrix element in the collinear limit, so it correctly describes a high p_T $Q\bar{Q}$ pair balanced by a gluon jet. Process (2.3b) uses the heavy quark content in the leading-log QCD parton distributions, so it correctly describes a single high- p_T Q balanced by a gluon, with the $\bar{Q}$ at low p_T . In the appropriate regions these processes should simply be added to (2.2a) and (2.2b), and they are in fact essential for describing the data, e.g. the ratio of single muons to dimuons from b production.⁹ But for $p_T \rightarrow 0$ the processes are no longer distinct. The cross sections for (2.3a) and (2.3b) are divergent because of the mass singularities, so naively adding them gives absurd results for the total cross section. The same is true for other processes. Unfortunately, while the correct prescription for adding processes is understood in principle, it is so complicated in practice that it is not implemented in either ISAJET or PYTHIA. Hence the user must exercise care.

2.2 Higher order QCD

The next step in simulating an event is to add QCD radiation from both initial and final partons. Such radiation plays an essential role in the structure of the events. Without it, for example, jets would have an angular width scaling like $1/p_T$ rather than $1/\log p_T$. QCD radiation is most important when the partons are almost collinear, that is, near a mass singularity. In this region the quantum mechanical QCD perturbation theory is equivalent to a classical branching process, with the probability of one additional branching $p_0 \rightarrow p_1 + p_2$ being given by a simple factor:

$$\sigma = \sigma_0 \times \frac{\alpha_s(Q^2)}{2\pi p_0^2} P_{0 \rightarrow 1+2}(z) \quad (2.4)$$

where z is the momentum fraction and $P(z)$ is an Altarelli-Parisi function which reflects the structure of the elementary coupling:

$$P_{q \rightarrow qg}(z) = \frac{4}{3} \left(\frac{1+z^2}{1-z} \right) \quad (2.5a)$$

$$P_{g \rightarrow gg}(z) = 6 \left(\frac{z}{1-z} + \frac{1-z}{z} + z(1-z) \right) \quad (2.5b)$$

$$P_{g \rightarrow q\bar{q}}(z) = \frac{1}{2} \left(z^2 + (1-z)^2 \right) \quad (2.5c)$$

The branching approximation consists of using Eq. (2.4) for the cross section but keeping exact, noncollinear kinematics. Since this approximation involves only probabilities, it can be naturally implemented in a Monte Carlo program, and it is the basis of both ISAJET and PYTHIA. By construction it reproduces the leading mass singularities and hence the leading order QCD scaling violations both for parton distributions and for jet fragmentation. Since a jet has $M^2 \sim \alpha_s p_T^2 \ll p_T^2$, it also reproduces the correct shape for jets. Finally, it turns out to give the correct QCD three-jet cross sections within about a factor of two over all of phase space.

The branching approximation reduces QCD to a classical evolution process. It is still necessary to deal with infrared and mass singularities. The mass singularities are handled by introducing a cutoff and assuming that all radiation softer than the cutoff is included in the nonperturbative hadronization model. The infrared singularities are

handled by treating the Attarelli-Parisi functions as distributions with $\delta(1-z)$ terms such that

$$\int dz [P(z) + c\delta(1-z)] = 0. \quad (2.6)$$

This is equivalent to the usual Bloch-Nordsieck cancellation and is also what is required for the energy-momentum sum rules to be satisfied. It is then possible to sum up the soft radiation and to calculate the probability that in either the initial or the final state an evolution occurs from an initial mass t to a final mass t' with only soft radiation. This same probability also determines the distribution for the emission of the first observable radiation. Additional radiation is generated by iterating the whole procedure for each of the new partons.

While the branching approximation reproduces the main features of QCD, it has important defects. It treats the radiation from partons incoherently. This is correct for hard gluons but not for soft ones, which see only the net color charge of the entire system, so it overestimates the soft radiation and hence of the multiplicity. PYTHIA by default includes coherence for the final state radiation. Also, the branching approximation gives only approximately the correct multijet cross sections.

2.3 Hadronization

Below some scale t_c perturbation theory ceases to be valid, and it is necessary to use a model for the nonperturbative production of hadrons. For high- p_T partons the factorization theorem implies that the hadronization is the same as in e^+e^- annihilation at the same scale. ISAJET uses the simplest possible model for this hadronization, essentially the original independent fragmentation model of Field and Feynman.¹⁰ In this model, a quark-antiquark pair is generated, the antiquark is combined with the original quark to form a meson, and the process is iterated. PYTHIA uses the Lund string model,¹¹ which is more sophisticated and provides a better fit to the PETRA and PEP data. In the string model the confining QCD field is treated as a classical relativistic string, and the breaking of this string results in particle production. A gluon is described as a kink in the string, so that the model behaves smoothly if a parton splits into two collinear ones.

For the beam jets there is no such factorization theorem, and indeed the beam jets appear to be different in hard interactions than in minimum bias ones. The striking features of the data are approximate scaling in rapidity and large, KNO-like, multiplicity fluctuations. These features are most naturally explained by the existence of multiple chains, each having independent particle emission as in either the independent fragmentation model or the Lund string model. ISAJET uses such a picture based on the work of Gribov¹² and collaborators, who call the chain a "cut Pomeron." In PYTHIA the chains are taken to be Lund strings. At low energy, say $\sqrt{s} \sim 20\text{GeV}$, a single string agrees reasonably well with the data, but at higher energy both the average multiplicity and the fluctuations from a single string are much too small. Reasonable agreement¹³ can be obtained, however, by including multiple chains generated by additional parton scatterings at low p_T .

Unitarity probably plays an important role in low- p_T physics; since the proton is close to being black, absorption is important. While unitarity can be implemented by using an eikonal formalism, absorption involves negative probabilities, which are not easily implemented in a Monte Carlo program. Although the ISAJET model for beam jets is based on a model which uses unitarity, only positive contributions to cross sections

are included; there is no absorption. Likewise, PYTHIA includes a sum of cross sections for 1, 2, 3, ..., scatterings with no absorption. Fortunately, the properties of beam jets seem to vary only logarithmically, so a smooth extrapolation from existing data may not be too unreliable even if important elements of the physics are not properly included.

3. Higgs Production

The major change in both ISAJET and PYTHIA over the last year has been the incorporation of a rather complete treatment of standard Higgs production. In the standard model there is a single $SU(2)$ doublet of Higgs fields, leaving after symmetry breaking one physical scalar particle H . In hadronic collisions the dominant mechanisms for producing the H are^{14,15}

$$g + g \rightarrow H \text{ with a } t \text{ loop} \quad (3.1a)$$

$$t + \bar{t} \rightarrow H \quad (3.1b)$$

$$W^+ + W^- \rightarrow H, \quad Z^0 + Z^0 \rightarrow H \quad (3.1c)$$

Since the longitudinal $W^\pm$ and Z^0 are essentially Higgs bosons and have couplings proportional to mass, the Higgs decays predominantly into W^+W^- and Z^0Z^0 if it can with a width proportional to $G_F m_H^3$:

$$\Gamma_{W^+W^-} = \frac{G_F m_H^3}{8\pi\sqrt{2}} (1 - x_W + .75x_W^2) \quad (3.2a)$$

$$\Gamma_{Z^0Z^0} = \frac{G_F m_H^3}{16\pi\sqrt{2}} (1 - x_Z + .75x_Z^2) \quad (3.2b)$$

$$x_V = \frac{4m_V^2}{m_H^2}$$

Otherwise, it decays into fermion pairs with branching ratios proportional to mass.

The gg and $t\bar{t}$ fusion processes, (3.1a) and (3.1b) have been included in PYTHIA for some time and are now also included in ISAJET. The only subtlety is that the t quark is treated as a parton in the distribution functions and is used to generate the primary hard scattering; this corresponds to a summation of terms of the form $\alpha_s \log(m_H^2/m_t^2)$. The initial state QCD evolution converts the primary process $t + \bar{t} \rightarrow H$ into the actual physical process $g + g \rightarrow t + \bar{t} + H$. This treatment is fine if $m_t \ll m_H$, but it overestimates the cross section if $m_t \sim m_H$ because it does not take account of the extra energy needed to produce the t and the $\bar{t}$.

The WW fusion process, (3.1c), dominates for large m_H , and it requires more care. Because a heavy Higgs has a large width,

$$\Gamma_H \approx .5\text{TeV} \left(\frac{m_H}{1\text{TeV}} \right)^3 \quad (3.3)$$

the narrow resonance approximation is not valid. Above the resonance the parton cross section $\hat{\sigma}$ grows with $\hat{s}$ unless a complete gauge invariant set of graphs is included.¹⁶ Thus process (3.1c) must be combined with WW scattering to avoid unphysical results.

Fortunately it is not necessary to provide a complete treatment of WW production from all possible sources. Away from the Higgs resonance, the dominant source of

W^+W^- and Z^0Z^0 pairs is the lowest order quark-antiquark annihilation graphs, which have already been included in both ISAJET and PYTHIA. At the resonance for a light Higgs gg and $t\bar{t}$ production dominates. For a heavy Higgs only longitudinal WW fusion is important, and the effective W approximation can be used. Thus it is sufficient to use the effective W approximation¹⁵ and to include a complete gauge-invariant set of $WW \rightarrow WW$ amplitudes, including $W^\pm$, Z^0 , and γ exchange, H exchange, and the 4- W contact graph, with the initial W longitudinally polarized. For ISAJET these cross sections were computed for arbitrary final polarization; PYTHIA uses the published cross sections with only longitudinal final polarization.

The actual simulation of a Higgs event from WW fusion is quite similar in ISAJET and PYTHIA. It begins with the generation of a $WW \rightarrow WW$ scattering using the cross section just described. The parton distribution for a longitudinal W in a quark is

$$f_{W/q}(x) = \frac{c_V^2 + c_A^2}{4\pi^2} \left(\frac{1-x}{x} \right) \int dk^2 \frac{m_W^2}{(m_W^2 - k^2)^2} \quad (3.4)$$

Note that because the W is longitudinally polarized the k^2 integral is convergent and does not need to be cut off at the Higgs mass. Hence when the distribution of a W in a proton is calculated from

$$f_{W/p}(x) = \int \frac{dx'}{x'} f_{W/q}\left(\frac{x}{x'}\right) f_q(x', Q^2) \quad (3.5)$$

the appropriate Q^2 scale is m_W^2 and not m_H^2 , as it would be for a QCD process. It is amusing to observe that the EHLQ parameterization of the quark structure functions at a fixed Q^2 is a polynomial in x and $\log x$, so that the integrals in Eq. (3.5) can be done analytically. After the WW scattering is generated the final W 's are decayed with the appropriate density matrix, e.g. with a $\sin^2 \theta^*$ distribution for a pure longitudinally polarized W .

The next step in the simulation is to produce the initial and final quarks from the two branchings $q \rightarrow Wq'$. This is essentially identical to one step on each side of the initial state QCD radiation process. For each side values of k^2 and $z = x/x'$ are generated according to Eq. (3.4)-(3.5) and an azimuthal angle ϕ is chosen uniformly. Then the kinematics can be solved, identifying z with the ratio $\hat{s}/\hat{s}'$ of the energies. This results in a transverse momentum for the Higgs of order m_W and typical values for z of order 0.3. While this treatment is only approximate, it does include all the physics contained in the effective W approximation. ISAJET and PYTHIA agree well with each other and with more exact calculations.

Finally, the standard QCD evolution is carried out for the initial and final quarks generated from the $q \rightarrow Wq'$ branchings, using $Q^2 = m_W^2$ as the scale, and the resulting partons and beam jets are hadronized as usual.

4. Possible Monte Carlo Improvements

The most important defects of the existing programs are the inadequate treatment of coherence in QCD radiation and the lack of a way to generate multijet cross sections exactly.

4.1 Coherence

The leading-log branching approximation described so far provides a very natural basis for Monte Carlo simulation because it reduces the quantum mechanical QCD radiation to a classical branching. Unfortunately, QCD is not so simple. Probably the most important defect of the branching approximation is that it neglects coherence effects. For example, in e^+e^- annihilation the complete final state is a color singlet, and a low-momentum gluon sees this zero net color charge rather than the color charges of the individual partons. Therefore, the radiation of soft partons, and hence the multiplicity, is overestimated.

To construct a Monte Carlo algorithm it is necessary to formulate the coherence effect in a way which does not involve negative probabilities. Marchesini and Webber⁸ found an approximate solution some time ago for final state radiation. In QCD gluons can be described as pairs of quark lines, so any final state can be described by a set of connected quark lines ij , where the ends i and j may be either quarks or parts of gluons. The radiation of a gluon with momentum $k \rightarrow 0$ from such a line including coherence effects can be found by, for example, taking the soft limit of the $e^+e^- \rightarrow q\bar{q}g$ cross section. It is given by a simple eikonal formula:

$$\begin{aligned}\sigma &= \sigma_0 \times g^2 \left(\frac{p_i \cdot p_j}{p_i \cdot k p_j \cdot k} \right) \\ &= \sigma_0 \times \frac{g^2}{2k^2} \left(\frac{\zeta_{ij}}{\zeta_i \zeta_j} + \frac{1}{\zeta_i} - \frac{1}{\zeta_j} \right)\end{aligned}\quad (4.1)$$

where

$$\zeta_i = \hat{p}_i \cdot \hat{k}, \quad \zeta_{ij} = \hat{p}_i \cdot \hat{p}_j \quad (4.2)$$

An elementary integration shows that Eq. (4.1) averaged over the azimuthal angle ϕ of k relative to p_i gives

$$\sigma = \sigma_0 \times \left[\frac{g^2}{k^2 \zeta_i} \theta(\zeta_{ij} - \zeta_i) + (i \rightarrow j) \right] \quad (4.3)$$

That is, the coherent radiation cross section, Eq. (4.1), reduces after angular averaging to Eq. (4.3), which involves only radiation off of each line separately but with θ functions that order the emission angles as well as the masses. Such an angular restriction yields the correct asymptotic form for the multiplicity in QCD perturbation theory. The full leading-log mass singularity can be obtained by replacing the $1/\zeta_i$ factor with the appropriate Altarelli-Parisi function. This angle-ordering approximation was first used in a Monte Carlo program by Marchesini and Webber, who showed that it can reproduce many features of the experimental data, including in particular the asymmetries in the particle distribution seen in 3-jet e^+e^- events. A similar coherent radiation algorithm is included in PYTHIA.

Ellis, Marchesini, and Webber³ have recently extended the analysis to soft gluon radiation from both the initial and the final state by studying the $k \rightarrow 0$ limit of the exact $2 \rightarrow 3$ jet cross sections. They find after angular averaging that there is a similar angular-ordered region around each initial and final parton within which the radiation has the simple, angular-ordered form. In addition there is a wide-angle region in which the results are more complicated and process dependent. Consider for example

$$q(p_1) + q'(p_2) \rightarrow q(p_3) + q'(p_4) + g(k) \quad (4.4)$$

In the soft limit after angular averaging the cross section for this process reduces to

$$\sigma = \sigma_0 \left\{ \frac{8}{3} [(14) + (23) + (32) + (41)] + \frac{1}{3} (12; 34) \right\} \quad (4.5)$$

where σ_0 is the lowest order cross section for $q+q' \rightarrow q+q'$, the symbol (ij) is a shorthand notation for the first term in Eq. (4.3), and $(ij; kl)$ represents the combination

$$(ij; kl) = 2(ij) - (ik) - (il) + 2(ji) - (jk) - (jl) + 2(kl) - (ki) - (kj) + 2(lk) - (li) - (lj) \quad (4.6)$$

and has no mass singularities. Thus Eq. (4.5) is just a sum of angular ordered radiation from each parton line plus a finite term; some of the other $2 \rightarrow 3$ processes give more complicated answers for the angular averaged soft radiation.

It is clearly important to implement all of the QCD coherence effects in the Monte Carlo programs, especially for SSC energies, so as to obtain the correct asymptotic behavior of the multiplicity. This is now being done for QCD jets by Ellis, Marchesini, and Webber.

4.2 Multijet cross sections

The existing algorithms reproduce the multijet QCD cross sections within about a factor of two over all of phase space.¹⁷ This is probably adequate for estimating multijet backgrounds to new signatures, but for other purposes one would like to obtain the correct 3-jet QCD cross sections (and perhaps the correct multi-jet cross sections as well). This could be done by rewriting the exact cross section as a sum of terms each containing one of the mass singularities. The difficulties are that there are several overlapping mass singularities and that the separate terms become negative in some regions of phase space. To date no one has found a general solution to this problem.

Alternatively, one might try to reweight the events after generation. The problem here is to identify the Monte Carlo final state, which may contain tens of partons, with a corresponding 3-jet state. It is not sufficient just to take the primary $2 \rightarrow 2$ process plus the hardest branching because this is not in general kinematically consistent with a $2 \rightarrow 3$ process. One could of course run a cluster algorithm on the final state to determine an equivalent $2 \rightarrow 3$ process, but the results may have more to do with the way the clustering is done than with QCD.

5. Minijets

The QCD-improved parton model, Eq. (2.1), predicts a growing cross section for $s \rightarrow \infty$ with $\hat{s}$ fixed because

$$x_1 \sim x_2 \sim \sqrt{\frac{\hat{s}}{s}} \quad (5.1)$$

and the structure functions $x f(x, Q^2)$ grow quite rapidly as $x \rightarrow 0$. The slowest possible growth is obtained by assuming that at $Q^2 = Q_0^2$ the gluon structure function $x g(x, Q_0^2)$ is regular as $x \rightarrow 0$. This corresponds in Regge language to a Pomeron which is a simple pole at $j = 1$. Then for any $Q^2 > Q_0^2$ the Altarelli-Parisi equations imply that¹⁸

$$x g(x, Q^2) \sim \exp \left[\frac{48}{11 - \frac{2}{3} n_f} \log \left(\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \right) \log \left(\frac{1}{x} \right) \right]^{1/2}, \quad (5.2)$$

so that $xg(x, Q^2)$ grows faster than any power of $\log(1/x)$ although slower than any power of $1/x$.

It has recently been suggested that $xg(x, Q_0^2)$ cannot approach a constant as $x \rightarrow 0$ but must be more singular. The simplest argument¹⁹ is that this behavior is inconsistent with the Altarelli-Parisi equations, which when used backwards with such a boundary condition produce negative structure functions for $Q^2 < Q_0^2$. Both the stability of the Altarelli-Parisi equations and a resummation of QCD perturbation theory give similar behavior:

$$xg(x, Q_0^2) \sim x^{-p}, \quad p \approx .3 - .5 \quad (5.3)$$

If such a power law behavior is correct at Q_0^2 , then it must hold asymptotically for $x \rightarrow 0$ at any $Q^2 > Q_0^2$, leading to much larger cross sections for small $\hat{s}$ at high energy.

Even the slower growth of Eq. (5.2) predicts at the SSC a minijet cross section equal to the total cross section for $p_T \gtrsim 3\text{GeV}$. (There is of course no reason why the inclusive jet cross section cannot exceed the total cross section.) Perhaps surprisingly, perturbation theory is believed to remain valid even for such large cross sections, although it must fail eventually. The first failure occurs when the parton densities become so large that the partons become densely packed in the proton and start to reinteract, generating absorption.²⁰ Since the number of partons per unit rapidity at a scale Q is $xf(x, Q^2)$ and each of these has a size $1/Q$, the Altarelli-Parisi evolution fails when

$$\frac{xf(x, Q^2)}{Q^2} \gtrsim \pi R_p^2 \quad (5.4)$$

where R_p is the proton radius. This relationship can be made more quantitative²¹ by computing the first correction term to the Altarelli-Parisi equations, which comes from two parton interactions. This has a small numerical coefficient, so the simple evolution remains valid over a larger domain than the estimate in Eq. (5.4).

The study of minijets is of course not limited to the SSC. The UA1 Collaboration²² has reported a minijet cross section of about 10mb for $p_T > 5\text{GeV}$ at $\sqrt{s} = 900\text{GeV}$. It is important to realize that the quoted value of p_T has not been corrected for the contribution of the spectator jets. For minimum bias events these would give 1.2GeV in $\Delta R < 1$ on average; fluctuations will make the actual difference between the jet and parton p_T larger than this. The small- x behavior of the parton distributions is also important. The best opportunity for measuring this in the near future is probably to measure $\mu^+\mu^-$ pairs at the Tevatron Collider in the forward direction; muons from $b\bar{b}$ pairs in the same region may also be useful to determine the behavior of the gluon distribution. A great deal more analysis is needed before minijet cross sections can be directly compared with QCD predictions.

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