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BEAM EXCITATION AND DAMPING WITH THE TRANSVERSE FEEDBACK SYSTEM

J.-L. Pellegrin and J. R. Rees

1. Introduction

The questions often come up, "What is the strength of the beam excitation system? How much damping can the transverse feedback provide?" The design is now advanced enough to answer these questions; also, laboratory tests of some components have been conducted and we know what can be expected of the hardware. The system functional diagram is:

Figure 1.

8-79

3670A

The amplifier symbol represents, in effect, several components such as a multiplexer, AM modulator, RF preamplifier and, finally, a tuned power amplifier. However, for all practical purposes we will look at it as a box which produces a maximum of ± 20 amperes in the deflection electrode when ± 1 volt is applied to its input. The power amplifier specifications have been written by Genova et al., in PTM-191.

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The magnetic field produced on the beam centerline by the deflection electrodes, i.e., the kicker, is

$$B_0 = \frac{\mu_0 I}{2\pi b} \left(1 - \frac{b^2}{a^2} \right) \quad (1)$$

where I is the instantaneous current at the moment of the bunch passage, a is the radius of the cylindrical vacuum chamber and b is the distance of an electrode from the center line. (See Figure 2.) In the PEP kicker $a = 74.3$ mm and $b = 57$ mm, so a current of 20A gives a field of 30×10^{-6} Tesla.

Figure 2. Geometry of the beam deflection electrode.

2. Beam Excitation for Diagnostics

In the Appendix the response of the beam to sinusoidal excitation at one of the resonant frequencies [Eq. (A16)] is calculated to be

$$Y = \frac{e c \hat{B} l R \omega_r}{14 \pi E_0 \alpha_{eff} v} \quad (2)$$

The symbols are defined in the Appendix. In terms of the average beta

function $\bar{\beta}$, the orbital period T_0 and the effective damping time T_{eff} , this expression takes the form,

$$Y = \frac{e c \hat{B} \ell}{4 \pi E_0} \frac{T_{eff}}{T_0} \bar{\beta} \quad . \quad (3)$$

All of the factors of this equation are well defined for a given storage ring "configuration" except T_{eff} which depends on the stored beam current for reasons mentioned in the Appendix. As the beam current is reduced toward zero, T_{eff} approaches the radiation damping time; conversely, at high beam currents T_{eff} is likely to be much shorter if the chromaticity is overcorrected by the sextupole system because of the strong damping of the head-tail effect. Consider maximum excitation of the kicker,

$$\hat{B} \ell = (30 \times 10^{-6} \text{ Tesla}) (2.8 \text{ m}) = 84 \times 10^{-6} \text{ Tesla-m}$$

and, as an example,

$$E_0 = 4.0 \text{ GeV}$$

$$\bar{\beta} = 7 \text{ m}$$

$$T_{eff} = 1 \text{ msec} \quad .$$

We get

$$Y = 3 \text{ mm} \quad .$$

In this example, the radiation damping time is 1,000 milliseconds, so the damping assumed must come from effects other than radiation. With a sufficiently tiny beam current, this same excitation would produce

$$Y = 3 \text{ meters !}$$

3. Damping by the Closed-Loop System

Equation (A24) of the Appendix gives the damping rate of the closed-loop feedback system for coherent bunch oscillations. That equation may be rewritten in the same terms we used in Eq. (3):

$$\alpha_f = - \frac{ec l J}{2 E_0 T_0} (\beta_M \beta_k)^{1/2} \sin \Delta\psi , \quad (4)$$

where J is the transfer function of the feedback system from beam position at the monitor at "pick-up time" to magnetic field in the kicker at the later "action time". The quantity J is just the product of S , the detector sensitivity in volts per meter and A , the transfer function of the electronics in Tesla per volt, so

$$\alpha_f = - \frac{ec l S A}{2 E_0 T_0} (\beta_M \beta_k)^{1/2} \sin \Delta\psi . \quad (5)$$

Using the following values:

$$\beta_M = \beta_k = 7m$$

$$S = 500 \text{ V/m (independent of beam current)}$$

$$A = 30 \times 10^{-6} \text{ Tesla/V (minimum)}$$

$$l = 2.8 \text{ m}$$

$$E_0 = 4.0 \text{ GeV}$$

$$\sin \Delta\psi = 1$$

We obtain

$$\alpha_f = 1500 \text{ sec}^{-1}$$

for a damping time of 0.66 msec.

4. Detector Noise and Kicker Amplifier Saturation

The damping produced by the feedback system at 4 GeV was calculated in the preceeding section for a given value of the loop gain factor $S \times A$, and with the assumption that we had a noiseless detector. As the energy of the beam is increased, the damping provided by the feedback system will be reduced and provision has been made for raising the value of $S \times A$ such as to maintain the same damping factor. However, it is necessary to estimate the contribution of the detector noise at the power amplifier output to make sure that saturation does not occur as the gain is increased. A measurement for the evaluation of this noise has been performed.

A model for the detector was connected to one of the SPEAR horizontal strip-lines. These electrodes are five times less sensitive to the beam position than the ones planned for PEP, and for this test the detector sensitivity was estimated to be 100 V/m. The machine was running for physics with two 10-mA colliding beams but the detector gate was set to observe the electron beam only. Figure 3 is a photograph of the unfiltered detector output spectrum. The center line is at the revolution frequency; the first pair of side-bands (the dipole mode) are approximately at a level of -36 dBm, or 0.25×10^{-6} watt,

Figure 3. Detector Output Spectrum in response to a SPEAR Colliding Electron Beam. Reference level is -20 dBm. Vertical scale is 10 dB/div. Horizontal scale is 50 KHz/div. Resolution is 3 KHz.

and correspond to a peak voltage of 5 mV, or a bunch center-of-mass oscillation of 0.05 mm peak. One notes that the noise floor of the detector is about 14 dB below the level of these side-bands and is equivalent to an input noise of approximately ± 0.01 mm. Dividing this figure into the maximum kicker field, 30×10^{-6} Tesla, one obtains an upper limit for the gain factor $S \times A$ of 3 Tesla/m, a factor of 200 larger than the value used for the calculation of the damping. It is concluded that even at the maximum PEP energy, the damping time of the feedback system can be made as small as a few machine turns, without producing saturation of the power amplifier.

APPENDIX

1. Equation of Motion

In the smooth approximation the equation of betatron motion for either the horizontal or the vertical degree of freedom has the form

$$\left(\frac{d^2 y}{ds^2} \right)_f = - \left(\frac{v}{R} \right)^2 y \quad (A1)$$

The term on the right represents the focusing forces provided by the guide field. Any damping which is present has been omitted in the preceeding equation. Radiation damping introduces another term,

$$\left(\frac{d^2 y}{ds^2} \right)_r = -2 \frac{\alpha_r}{c} \frac{dy}{ds} \quad (A2)$$

where α_r is the radiation damping rate (coefficient) in inverse time for the degree of freedom in question. The usual approximate equation of transverse motion for a single particle under the influence of radiation damping is obtained by adding Equations (A1) and (A2) to get the whole second derivative.

Now if we introduce a new transverse bending field which is externally controlled, $B_k(s,t)$, we add another term.

$$\left(\frac{d^2 y}{ds^2} \right)_k = \frac{e c B_k(s,t)}{E_0} \quad (A3)$$

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We use the subscript "k" because this field will kick the beam particles, and the sign convention for $B_k(s,t)$ is defined by Eq. (A3). The total equation of motion including the influence of this field is

$$\frac{d^2 y}{ds^2} = -2 \frac{\alpha_r}{c} \frac{dy}{ds} - \left(\frac{v}{R}\right)^2 y + \frac{ec B_k(s,t)}{E_0} \quad (A4)$$

When we measure the betatron tune we kick all of the particles in a bunch and detect the motion of the center of charge $\bar{y}$.

$$\bar{y} = \frac{1}{N_b} \sum_{\text{bunch}} y_i \quad (A5)$$

Similarly when we use the kicker for damping coherent bunch oscillations, we sense the center of charge motion and use it as a feedback signal to control the time dependence of $B_k(s,t)$. Just substituting Eq. (A5) into Eq. (A4) we might be tempted to conclude that $\bar{y}$ obeys Eq. (A4); however, that is not exactly true. Equation (A4) omits the effects of various macroscopic fields induced in the vacuum chamber by the bunches and also experienced by them. In general, these fields alter both the frequency of oscillation and the damping rate, but their effects on the damping rate are more important, because the damping rate is much smaller than the frequency. If we call the altered damping rate α_{eff} and ignore the alteration of frequency, we can write for the center of charge

$$\bar{y}'' + \frac{2\alpha_{eff}}{c} \bar{y}' + \left(\frac{v}{R}\right)^2 \bar{y} = \frac{ec B_k(s,t)}{E_0} \quad (A6)$$

where primes denote differentiation with respect to s .

Next we shall specialize the equation to the particular form of $B_k(s,t)$ which is appropriate to the beam excitation electrodes, i.e., kickers, which are confined to an interval of s which is short compared to the betatron wavelength.

$$B_k(s,t) = B_0(t) P(s) \quad (A7)$$

where $P(s)$ is a dimensionless rectangular pulse of amplitude one and length l (see Figure 1). $P(s)$ is a periodic function of s which can therefore be expanded as a Fourier series and, if we choose the center of the kicker as the origin of s -coordinates, the series takes the form,

$$P(s) = \sum_{n=-\infty}^{\infty} a_n e^{\frac{ins}{R}},$$

with

$$a_n = \frac{1}{2\pi R} \int_{-R}^{R} P(s) e^{-\frac{ins}{R}} ds$$

Using the properties of $P(s)$, we get

$$a_n = \frac{1}{n\pi} \sin \frac{n l}{2R}. \quad (A8)$$

For values of n small enough that

$$n l \ll 2\pi R, \quad (A9)$$

the values of a_n are all the same.

$$a_n = \frac{l}{2\pi R}. \quad (A10)$$

Considering only these values of n , we have for Eq. (A7)

$$B_k(s, t) = \frac{B_0(t) l}{2\pi R} \sum_n e^{\frac{ins}{R}}.$$

and we can write our equation of motion

$$\ddot{y} + \frac{2\alpha_{eff}}{c} \dot{y} + \left(\frac{v}{R}\right)^2 y = \frac{ec B_0(t) l}{2\pi R E_0} \sum_n e^{\frac{ins}{R}}. \quad (A11)$$

Now, for the bunch, $s = ct$, so for time as the independent variable,

$$\ddot{y} + 2\alpha_{eff} \dot{y} + (v\omega_r)^2 y = \frac{ec l R \omega_r^2 B_0(t)}{2\pi E_0} \sum_n e^{in\omega_r t}, \quad (A12)$$

where ω_r is the angular frequency of revolution. This is the equation of motion for the center of charge of the bunch which we shall use to calculate the response of the beam to the transverse feedback system.

2. Response to External Sinusoidal Input

For the excitation of the beam for tune measurement we apply an external sinusoidal signal to produce

$$B_0(t) = \hat{B} e^{i\omega t},$$

with both $\hat{B}$ and ω under the control of the operator. The equation of motion becomes

$$\ddot{y} + 2\alpha_{eff} \dot{y} + (v\omega_r)^2 y = \frac{ec \hat{B} l R \omega_r^2}{2\pi E_0} \sum_n e^{i(\omega + n\omega_r)t}. \quad (A13)$$

With the application of a single input frequency ω , the beam sees a whole ensemble of drivers at frequencies $(\omega + n\omega_r)$. (Remember that n can be both negative and positive, so this spectrum contains both upper and lower sidebands separated by ω from multiples of the revolution frequency.) The steady state solution,

$$\bar{y} = Y_n e^{i(\omega + n\omega_r)t},$$

to any one of the drivers is

$$Y_n = \frac{(ec \hat{B} L R \omega_r^2) / (2\pi E_0)}{(v\omega_r)^2 - (\omega + n\omega_r)^2 + i2\alpha_{eff}(\omega + n\omega_r)} \quad (A14)$$

Resonance occurs whenever

$$\omega = (v - n) \omega_r, \quad (A15)$$

and the amplitude of motion at resonance is

$$Y(res) = \frac{ec \hat{B} L R \omega_r}{14\pi E_0 \alpha_{eff} v}, \quad (A16)$$

independent of n if it satisfies Eq. (A9).

3. Damping by Feedback

To use the system for damping coherent oscillations, we derive a signal proportional to $\dot{\bar{y}}$ from the beam and apply it to drive the system in order to enhance the damping already provided by the α_{eff} term of Eq. (A12). Leaving until later a discussion of how we obtain the $\dot{\bar{y}}$

signal we write, for this case,

$$B_0(t) = h \dot{\bar{y}}(t) , \quad (A17)$$

where h is some constant proportional to the gain of the feedback path.

Using this expression in Eq. (A12) we obtain

$$\ddot{\bar{y}} + 2\alpha_{\text{eff}} \dot{\bar{y}} + (\nu\omega_r)^2 \bar{y} = \frac{ec l R \omega_r^2 h}{2\pi E_0} \dot{\bar{y}} \sum_n e^{in\omega_r t} .$$

The oscillatory terms on the right hand side will not produce long-term damping, so we retain only the $n=0$ term, whence

$$\ddot{\bar{y}} + 2\left(\alpha_{\text{eff}} - \frac{ec l R \omega_r^2 h}{4\pi E_0}\right) \dot{\bar{y}} + (\nu\omega_r)^2 \bar{y} = 0 . \quad (A18)$$

We see immediately that the damping coefficient due to the feedback system alone is

$$\alpha_f = - \frac{ec l R \omega_r^2 h}{4\pi E_0} . \quad (A19)$$

Turning to the constant h , it is just the ratio of the field in the kicker to the $\dot{\bar{y}}$ of the bunch at the kicker, $\dot{\bar{y}}_k$. This transverse velocity is related to the slope of the bunch's trajectory at the kicker, simply

$$\bar{y}'_k = \dot{\bar{y}}_k / c .$$

However, we do not measure this slope directly, but rather we measure the transverse displacement of the bunch at an earlier time and at a betatron

phase $\Delta\psi$ earlier. Call this displacement $\bar{y}_M$; M stands for monitor. It can be readily shown that $\bar{y}_M$ is related to the subsequent position and slope of the trajectory at the kicker as follows

$$\bar{y}_M = \left(\frac{\beta_M}{\beta_k} \right)^{\frac{1}{2}} (\cos \Delta\psi - \alpha_k \sin \Delta\psi) \bar{y}_k - (\beta_M \beta_k)^{\frac{1}{2}} \sin \Delta\psi \bar{y}_k' , \quad (\text{A20})$$

where α_k is a parameter describing betatron motion, and not a damping coefficient. If we characterize the transfer function of the feedback path in terms of the instantaneous field produced in the kicker (at the time of the bunch passage) per unit displacement at the monitor,

$$B_0 = J \bar{y}_M , \quad (\text{A21})$$

then we see that Eq. (A20) gives us two terms contributing to the kicker field: one proportional to $\bar{y}_k$ and one proportional to $\bar{y}_k'$. The former was not contemplated when we stated Eq. (A17) and plugged it into Eq. (A12), but, if it had been considered, it would have produced a frequency-shifting term in Eq. (A18) and no damping; so we shall discard it now. This leaves us with

$$B_0 = -J (\beta_M \beta_k)^{\frac{1}{2}} \sin \Delta\psi \frac{\dot{\bar{y}}_k}{c} , \quad (\text{A22})$$

from which we see

$$h = -\frac{J}{c} (\beta_M \beta_k)^{\frac{1}{2}} \sin \Delta\psi . \quad (\text{A23})$$

Then finally

$$\alpha_f = \frac{e J \ell R \omega_r^2 (\beta_M \beta_K)^{1/2} \sin \Delta\psi}{4 \pi E_0} \quad (A24)$$

We close the Appendix with a table of selected symbols and units:

$\alpha_r, \alpha_{eff}, \alpha_f$	damping coefficients	sec^{-1}
β_K, β_M	betatron functions	m
E_0 / e	beam energy / e	volts
J	system transfer function	Tesla / m
R	gross radius of storage ring	
ν	betatron wave number	