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COMPARISON OF INSTABILITY THEORY WITH SIMULATION RESULTS*

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Introduction

For the past year and a half, a parallel effort of analytic work and simulation computations has been in progress for the purpose of understanding the nature of instabilities which arise in the transport of intense ion beams and finding safe regions of operation.¹ The two approaches have agreed qualitatively in that simulation runs using initial conditions predicted by the linearized theory to be highly unstable indeed distinctly show instability, and subsequent saturation may occur only after an increase in emittance by a factor of two or three. Since the theory predicted pronounced instability for a large number of modes concurrently, no attempt was made at a quantitative comparison. Only recently we found a combination of parameters for which the theory predicts only one unstable but rapidly growing mode, and the corresponding simulation run shows an orderly growth in the early stages (presumably the linear regime) with the expected qualitative appearance. Since the linear theory provides definite predictions of the evolution of various moments of the distribution and the distortion of the boundaries of the various two-dimensional projections of the distribution, we undertook a quantitative comparison as a means of simultaneously establishing the validity of the theory and the reliability of the simulation procedure.

Method²

The KV distribution is the only one we have found to be amenable to theoretical analysis for a quadrupole-focussed beam³ and consequently has been used most often also in the simulation work. The case to be discussed is a FODO lattice with a phase advance of 90° per cell at zero intensity, but depressed to $\sigma \cong 45^\circ$ by the beam loading. According to the theory, a fluctuation of charge density from the unperturbed uniform density giving rise to a deviation from the quadratic electrostatic potential of the form $V = Ax^3 + Bxy^2$, or $Ay^3 + Bxy^2$, should alternate in sign and grow in magnitude each period of the transport structure. The specific comparison between theoretical and simulation results that we report here for this "third-order" mode considered odd moments (such as y^3 , y^2p_y , ...) and the y, p_y projected

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distribution obtained for a 2048-particle sample available from the 16000-particle simulation run mentioned above. According to the theory, the boundary of the y, p_y projection should develop as described by the equation

$$\begin{aligned} \left(\frac{y}{b}\right)^2 + \left(\frac{p_y}{p_{y,\max}}\right)^2 + C \left[r_1 \left(\frac{y}{b}\right)^3 + r_2 \left(\frac{y}{b}\right)^2 \left(\frac{p_y}{p_{y,\max}}\right) + r_3 \left(\frac{y}{b}\right) \left(\frac{p_y}{p_{y,\max}}\right)^2 \right. \\ \left. + r_4 \left(\frac{p_y}{p_{y,\max}}\right)^3 + t_1 \left(\frac{y}{b}\right) + t_2 \left(\frac{p_y}{p_{y,\max}}\right) \right] = 1, \end{aligned} \quad (1)$$

where C should grow, as the instability develops from the noise, in direct proportion to λ^P -- λ being the theoretical eigenvalue of the unstable mode ($|\lambda| > 1$) and P the period number. The coefficients $r_1, r_2, \dots, t_2$, however, are uniquely given by the theory, once the tunes σ_0 and σ are specified, as values that are rather insensitive to the fractional amount by which the lengths of the quadrupole lenses occupy the transport lattice. The growing y -moments, moreover, similarly are expected to bear a definite ratio to one another and to the factor C in Eq. (1), so that in principle only one adjustable parameter is available for fitting simulation results of the type mentioned.

Results²

For purposes of the comparison, theoretical coefficients were evaluated for a 90-degree lattice, one-sixth occupied by quadrupole lenses, in which (as estimated from trajectories of the simulation particles) the space-charge forces depressed the tune to $\sigma = 45.85$ degrees (near the expected center of the predicted strong third-order instability). The theoretical value of λ under these circumstances is $\lambda = -1.2676$ (real, but negative)⁴, and the expected ratios (normalized so that $\langle y^3 \rangle = 1$) are as shown by the first line of entries on Table I.

The growth of moments obtained from the simulation data was examined only beginning with period number 8 or 9 (where the results began to emerge somewhat consistently from the noise) through period 19 or 20 (at which point some indication of incipient saturation became apparent). Taking into account the possible presence of some constant background noise in these moments, we estimated their growth as characterized by λ values in the range between -1.22 and -1.28 (average value for the six moments examined = -1.26, in good agreement with the theoretical value) and obtained the moment ratios shown by the second line of entries in Table I.

The coefficient C in Eq. (1) for the y, p_y projection was adjusted for a best fit of the simulation data only for periods 8, 13, and 18. The values so obtained suggested a growth rate characterized by $\lambda \cong -1.2$ -- i.e., somewhat less than estimated from the growth of the moments. To relate the magnitude and sign of C to values found for the moments at period 18, the observed moment $\langle p_y^3 \rangle$ reduced to "scaled variables"⁵ is approximately $+4.07 \times 10^{-3}$ and the corresponding expected value of the dimensionless parameter C is close to this value, while the best fit to the boundary of the y, p_y projection

TABLE I

Normalized Moments

	$\langle y^3 \rangle$	$\langle y^2 p_y \rangle$	$\langle y p_y^2 \rangle$	$\langle p_y^3 \rangle$	$\langle y^5 \rangle$	$\langle p_y^5 \rangle$
Theoretical	1.	1.35	-0.665	-0.94	111.	-66.
Simulation	1.	1.23	-0.63	-1.08	111.	-67.

at period 18 gave the similar value 3.84×10^{-3} . Although it is our expectation to examine the simulation data in further detail, the evidence cited above indicates that the theory for the third-order perturbation modes of a Kapchinskij-Vladimirskij distribution provides a good account of the initial instability exhibited by simulation computations for a similar initial distribution.

Conclusion

Numerical calculations based on the theory just mentioned suggest that, with a zero-intensity tune $\sigma_0 = 90$ deg., stability cannot be assured for beam intensities that depress the tune to below $\sigma \cong 57.5$ degrees. It is found, however, that this limitation is relieved by designing the transfer lattice so that $\sigma_0 < 60$ deg., because third-order instabilities then are not to be expected. Accordingly, with $\sigma_0 < 60$ deg., intensity limitations would be imposed only through the action of other, higher-order instabilities. Specifically, with $\sigma_0 = 60$ deg., intensities that result in a tune depression to σ in the neighborhood of 24 or 28.2 deg. (where substantial instabilities of 6th or higher order may appear) may be permissible and permit the transport of significantly greater currents than would be possible with a lattice designed for $\sigma_0 = 90$ degrees. The use of a transport lattice designed for $\sigma_0 = 60$ deg., and a comparison of its characteristics with those of a lattice with $\sigma_0 = 90$ deg., will be discussed in detail (with tables) in a LBL report now in preparation ["Concerning an A-G Transport System with $\sigma_0 = 60$ Degrees", LBL Report HI-FAN-56 (1978)]. Simulation runs will be made to determine whether or not the 60° - 24° range is indeed free of unstable regions.

Acknowledgements

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References

1. See Laslett, p. 112 and Haber and Maschke, p. 122 and references in those articles, BNL workshop.
2. This work is described in greater detail in HI-FAN-43.
3. See HI-FAN-13, 14, and 15.

4. The distinctive signature of this instability (associated with a negative real eigenvalue) is that the odd moments (of order 3, or greater) and the boundary in phase space (or of any projected distribution) oscillates with a "frequency" of one half period of the transport lattice and with growing amplitude, but the particles distribute themselves within the boundary so that their centroid remains stationary.
5. G. Lambertson, L.J. Laslett, and L. Smith, "Transport of Intense Ion Beams", IEEE Trans. NS-24, 993 (1977).

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