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ENGINEERING MEMORANDUM NO. 0203

STANDPIPE-BUBBLER PUMP LEVEL-CONTROL STUDY,
SODIUM TEMPERATURE DISTRIBUTION (TASK B)

W.P. 7373

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ABSTRACT

A thermal analysis was performed for the CRBRP Primary Pump 1E3862 to determine the temperature distribution within the pump cavities which would result from a severe thermal transient at the suction port. The inlet transient, defined by the equipment specification, consists primarily of a sudden decrease in temperature of the sodium entering the pump from 1015°F to 580°F within a period of 150 seconds and with the pump operating at pony motor speed.

Temperatures are calculated and plotted for five different fluid cavities for periods up to 600 seconds. All cavities experienced considerable temperature lag due to the thermal capacitances within the pump. The temperature within the pump shaft would decrease most rapidly, 392 degrees in 600 seconds, while the upper pump tank had a predicted drop of only 77 degrees for the same period.

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I. INTRODUCTION

As a part of the general study of the operation of the standpipe-bubbler, this particular analysis was performed to examine the local temperature changes within the pump due to thermal transients in the main fluid flow. The equipment specification (Reference 7, Appendix B) defines a number of transient cases, and the one selected for this analysis appears to result in the largest thermal gradients.

The selected transient is designated PP-2E, "Loss of a Primary Pump Pony Motor with Failure of the Check Valve to Shut," and the temperature of the fluid at the pump suction port is shown in Reference 7, Appendix B, Figure B-70. The figure shows a temperature ramp starting at a steady state temperature of 1015°F and decreasing to 580°F in a period of 150 seconds, where it subsequently remains nearly constant for the next 700 seconds.

The analysis considers the conditions within a pump which continues to operate at pony motor speed, thereby maintaining the internal fluid circulation. Due to various lag processes, several sequential steps can be defined and analyzed. The flows between the various pump cavities are defined by the Internal Fluid Management Analysis (Reference 8), and upper level external heat transfer conditions are described in the Preliminary Thermal Analysis (Reference 9). The pump configuration which was analyzed was that shown in the Design Layout Drawing 1E3862 (Reference 11).

The baffle chamber at the bottom of the upper pump tank is examined first, followed by analysis of conditions in the lower pump tank. The thermal lag in the IHX return flow is evaluated in order to determine the thermal transient within the hollow pump shaft. Finally, the temperature within the upper pump tank is calculated.

II. SUMMARY

The analytical approach was to consider several pump cavities individually, in each case writing equations for an energy balance which included fluid flows, heat

transfer, and other physical processes as appropriate. The sequential procedure which was followed in the analysis was found to be justified by the results. Thus, for example, the initial assumption of constant temperature in the upper pump tank was found to be sufficiently accurate so as not to materially affect the calculations for the baffle chamber or the lower pump tank.

The temperature transient for the baffle chamber is shown in Figure 3 and consists of an initial decrease from 1015°F to 833°F in the first 200 seconds, with a further decline to 774°F at the end of 600 seconds.

The return flow from the intermediate heat exchanger (IHX) is directed to the pump through an insulated 2-inch line over 71 feet long. Under the transient conditions the flow is only 15 gpm, and the thermal capacitance of the line results in a lag of about 62 seconds before the cooling transient is evidenced at the discharge in the pump tank. The result is shown in Figure 9.

The lower pump tank receives fluid from several sources at different temperatures and discharges fluid to the suction line and up the pump shaft. There is also heat transfer from the pump to the surrounding guard vessel under steady state conditions, but as the temperature of the pump tank falls during a transient case, the guard vessel becomes a heat source. The transient effects in the lower pump are thus moderated so that there is only a gradual decline in temperature, as shown in Figure 8, to a level of 730°F at an elapsed time of 600 seconds.

The return flow from the IHX is ducted directly into the hollow pump shaft. This IHX return flow is mixed with some fluid from the lower pump tank, as the combined flow is pumped out of the shaft bleed holes into the upper pump tank. The fluid temperature within the shaft is predominantly affected by the temperature of the IHX return flow. Therefore, after an initial delay, there is a drop to 673°F over a period of 270 seconds, then a gradual decline to 623°F at 600 seconds. This transient is shown in Figure 12.

Under steady state conditions the fluid in the upper pump tank would be a few degrees cooler than the temperature of the main fluid flow due to the large amount of heat transfer to the surroundings. An initial fluid temperature of 1000°F was assumed, and the analysis indicated a very small decline to 923°F after 600 seconds. This transient is shown in Figure 15.

III. CONCLUSIONS AND RECOMMENDATIONS

The thermal lags in the various pump cavities will result in certain portions of the pump structure being exposed on one side to maximum temperature transients, while on the other side there is a substantial thermal lag. Temperature differences across some structural elements will be as high as 300°F for periods as long as 100 seconds.

Only one system transient case was analyzed, but the trends are sufficiently clear to indicate the general effect of other types of transients. Since the equations are first developed in general form before numerical substitutions are made, the method can be applied to other transients which need detailed evaluation.

The results of this analysis should be evaluated for the effects on structural design problems. Structural changes or variation in the hydraulic parameters affecting the analytical results may then be evaluated for their effect on the temperature distribution, providing criteria for design optimization.

IV. DISCUSSION

In the case of a system transient where the fluid entering the pump becomes progressively cooler, this fluid quickly fills the impeller and volute, and the cavities of the hydrostatic bearings. This relatively cooler fluid then passes through the various leakage paths described in the Internal Fluid Management Analysis (Reference 8) and enters into the baffle chamber and the lower pump tank. However, the lower pump tank also receives flow from the baffle chamber, requiring that attention first be directed to the baffle chamber, and then to the lower pump tank.

Some fluid from the lower pump tank flows up the hollow pump shaft, mixed with the IHX return flow. Therefore, an analysis of the IHX return flow will precede the examination of conditions in the pump shaft. Finally, the temperature in the upper pump tank will be analyzed, considering the effects of the cooling flow issuing from the pump shaft through the bleed holes.

The thermal transient, PP-2E (Reference 7, Appendix B), will be stated as starting from a steady state temperature of 1015°F and then decreasing to 580°F within 150 seconds, after which the temperature of the incoming fluid remains constant at 580°F. Fluid flows will be those obtained at pony motor speed.

A. Temperature Transient in Baffle Chamber

The baffle chamber receives flow not only from the upper bearing, but also from the upper pump tank. These flows are mixed in passage through the chamber and are discharged through peripheral holes leading to the lower pump tank (See Figure 1). Since the fluid in the upper pump tank is relatively isolated from the initial effects of the temperature transient, it will remain warmer than the baffle chamber and will transmit heat to the baffle chamber through the baffle wall.

1. Flow Balance

Since the baffle chamber is filled with fluid the volume flow balance is

$$Q_{\text{BEAR}} + Q_S = Q_C$$

Where

Q_{BEAR} = Volumetric flow rate through
upper bearing leakage paths

Q_S = Volumetric flow rate from upper
tank to baffle chamber

Q_C = Volumetric flow rate from baffle
chamber to lower pump tank

FIGURE 1 - BAFFLE CHAMBER FLOWS

2. Energy Balance

Some simplifying assumptions can be justified by an examination of Figure 1:

- a) Differences in velocity and elevation of fluid entering and leaving the chamber are not significant
- b) Energy input due to shaft work is not significant
- c) There is perfect mixing of the fluid within the chamber, and the fluid is at a uniform temperature, T .

An energy balance for the baffle chamber is

$$\frac{d}{dt} (\rho V_B c_p T) = \rho_1 c_{p1} T_1 Q_{BEAR} + \rho_2 c_{p2} T_2 Q_S - \rho c_p T Q_C + q$$

Where

t = Time

ρ = Density of fluid

c_p = Specific heat of fluid

T = Temperature of flows as noted

q = Heat flow through the walls

V_B = Volume of baffle chamber

Since the fluid properties do not change drastically in the temperature range being considered, the computation will be simplified by assuming constant fluid properties

$$\rho_1 = \rho_2 = \rho$$

$$c_{p1} = c_{p2} = c_p$$

Thus, the equation may be written

$$V_B \frac{dT}{dt} = Q_{BEAR} T_1 + Q_S T_2 + \frac{q}{\rho c_p} - Q_C T$$

3. Heat Transfer

If the heat transfer is primarily from the upper wall at a uniform temperature level, we may write

$$q = U_B A_B (T_2 - T)$$

Where

U_B = Overall coefficient of heat transfer

A_B = Area of heat transfer surface

Substituting

$$V_B \frac{dT}{dt} = Q_{BEAR} T_1 + Q_S T_2 + \frac{U_B A_B}{\rho c_p} (T_2 - T) - Q_C T$$

Rearranging terms

$$\begin{aligned} \frac{dT}{dt} + \left(\frac{U_B A_B}{\rho c_p V_B} + \frac{Q_C}{V_B} \right) T &= \left(\frac{Q_{BEAR}}{V_B} \right) T_1 \\ &+ \left(\frac{Q_S}{V_B} + \frac{U_B A_B}{\rho c_p V_B} \right) T_2 \end{aligned}$$

4. Assumed Conditions

As written there are several implicit assumptions in the equation:

- a) Flow from the upper pump tank into the baffle chamber is constant
- b) The temperature of the fluid in the upper tank remains constant, at least for the time during the transient in the baffle chamber which is being considered
- c) The overall coefficient of heat transfer for the baffle surface is assumed to be constant, which follows from an assumption that the controlling mechanisms of heat transfer on the two sides of the baffle remain constant
- d) The lower surface of the baffle chamber will be cooled from below by the same fluid which subsequently flows into the baffle chamber. Therefore, heat transfer through the lower surface need not be considered, and this surface is assumed to be in thermal equilibrium with the fluid in the baffle chamber
- e) Flow from the bearing is assumed to be constant. Although the temperature is also indicated as constant, this term will be modified for the transient condition.

5. Temperature Equation

The temperature of the bearing flow will vary as

$$T_1 = T_{10} - \beta t$$

Where

$$\beta = \frac{\Delta T_1}{\Delta t} = \text{constant}$$

Substituting

$$\begin{aligned} \frac{dT}{dt} + \left(\frac{U_B A_B}{\rho c_p V_B} + \frac{Q_c}{V_B} \right) T &= \frac{Q_{BEAR}}{V_B} (T_{10} - \beta t) \\ &+ \left(\frac{Q_S}{V_B} + \frac{U_B A_B}{\rho c_p V_B} \right) T_2 \end{aligned}$$

Expanding

$$\begin{aligned} \frac{dT}{dt} + \left(\frac{U_B A_B}{\rho c_p V_B} + \frac{Q_c}{V_B} \right) T &= \left[\left(\frac{Q_{BEAR}}{V_B} \right) T_{10} \right. \\ &+ \left. \left(\frac{Q_S}{V_B} + \frac{U_B A_B}{\rho c_p V_B} \right) T_2 \right] \\ &- \left(\frac{Q_{BEAR} \beta}{V_B} \right) t \end{aligned}$$

Which may be written in the form

$$\frac{dT}{dt} + XT = Y - Zt$$

Using an initial condition $T = T_0$ for $t = 0$
the solution may be written as

$$T = \left(\frac{Y}{X} + \frac{Z}{X^2} \right) - \left(\frac{Z}{X} \right) t + \left(T_0 - \frac{Y}{X} - \frac{Z}{X^2} \right) e^{-Xt}$$

Where

$$X = \frac{U_B A_B}{\rho c_p V_B} + \frac{Q_c}{V_B}$$

$$Y = \left(\frac{Q_{BEAR}}{V_B} \right) T_{10} + \left(\frac{Q_S}{V_B} + \frac{U_B A_B}{\rho c_p V_B} \right) T_2$$

$$Z = \frac{Q_{BEAR} \beta}{V_B}$$

6. Baffle Chamber Geometry

The dimensions of the baffle chamber are shown in Figure 2.

FIGURE 2 - Baffle Chamber Geometry

The volume of the chamber will be approximated as a spherical segment

$$\begin{aligned}
 V_B &= \frac{\pi n_1^2}{3} (3r_s - n_1) - \pi r_1^2 n_1 \\
 &= (\pi/3)(8.00)^2 [3(72.00) - 8.00] \\
 &\quad - \pi(8.00)^2(8.00) = 12332 \text{ in}^3 = 7.14 \text{ ft}^3
 \end{aligned}$$

Similarly, the upper area is approximated

$$\begin{aligned}A_B &= 2\pi r_s(n_1 - n_2) - \pi r_2^2 \\&= 2\pi(72.00)(8.00 - 3.25) - \pi(5.375)^2 \\&= 2058 \text{ in}^2 = 14.29 \text{ ft}^2\end{aligned}$$

7. Transient Temperature Ramp

For the PP-2E transient temperature

$$T_0 = 1015^\circ\text{F}$$

$$T_{150} = 580^\circ\text{F at 150 seconds}$$

$$\beta = \frac{1015 - 580}{150} = 2.9^\circ\text{F/sec}$$

The mid-time temperature is

$$T_{\text{mid}} = \frac{1015 + 580}{2} = 798^\circ\text{F}$$

The fluid properties at 798°F are found from Reference 5

$$c_p = 0.304 \text{ BTU/lb-}^\circ\text{F}$$

$$\rho = 53.07 \text{ lb/ft}^3$$

8. Evaluation of Flow Factors

Using data from the Fluid Management Analysis (Reference 8)

$$Q_{\text{BEAR}} = 21.75 \text{ GPM} = 0.048 \text{ ft}^3/\text{sec}$$

$$Q_S = 14.5 \text{ GPM} = 0.032 \text{ ft}^3/\text{sec}$$

$$Q_C = 0.048 + 0.032 = 0.080 \text{ ft}^3/\text{sec}$$

$$\frac{Q_{\text{BEAR}}}{V_B} = \frac{0.048}{7.14} = 0.0067 \text{ 1/sec}$$

$$\frac{Q_S}{V_B} = \frac{0.032}{7.14} = 0.0045 \text{ 1/sec}$$

$$\frac{Q_C}{V_B} = \frac{0.080}{7.14} = 0.0112 \text{ 1/sec}$$

9. Evaluation of Heat Transfer Coefficients

The temperature of the fluid in the upper tank is assumed constant in the normal operating temperature range

$$T_2 = 1000^\circ\text{F}$$

The heat transfer coefficient for the baffle must be evaluated for fluid conditions on each side of the surface as well as the effect of the wall itself

$$\frac{1}{U_B} = \frac{1}{h_{\text{upper}}} + \frac{x_{\text{wall}}}{k_{\text{wall}}} + \frac{1}{h_{\text{lower}}}$$

For the wall material, stainless steel type 316, the properties at 1000°F are found from Reference 5. Thus,

$$x_{\text{wall}} = 1.25 \text{ in} = 0.104 \text{ ft}$$

$$k_{\text{wall}} = 12.3 \text{ BTU/hr-ft-}^\circ\text{F}$$

$$\frac{x_{\text{wall}}}{k_{\text{wall}}} = \frac{0.104}{12.3} = 0.0085 \frac{\text{hr-ft}^2}{\text{BTU-}^\circ\text{F}}$$

The upper surface will be assumed to have mixed flow in the proximity due to the jets from the shaft bleed holes. Using the Lyon-Martinelli equation for uniform heat flux (Reference 2, page 391)

$$\frac{hD}{k} = 7 + 0.025(Re)^{0.8}(Pr)^{0.8}$$

The value for Pr at 1000°F for liquid sodium

$$Pr = 0.0044$$

Since the evaluation of the Reynolds number is somewhat arbitrary, based on a representative dimension, we will instead assume:

$$0.025(Re)^{0.8}(Pr)^{0.8} \ll 1$$

$$\text{Then } \frac{hD}{k} = 7.0$$

$$\text{Using } D_s = 2(25.60) = 51.20 \text{ in} = 4.27 \text{ ft}$$

For liquid sodium at 1000°F (Reference 5)

$$k = 37.62 \text{ BTU/hr-ft-}^\circ\text{F}$$

Then

$$\frac{1}{h_{\text{upper}}} = \frac{4.27}{(7.0)(37.62)} = 0.0162 \frac{\text{hr-ft}^2\text{-}^\circ\text{F}}{\text{BTU}}$$

The lower surface of the baffle chamber is exposed to the full flow through the chamber, which will be evaluated at a mean Reynolds number

$$Re = \frac{D_{eq} u \rho}{\mu}$$

Using

$$d_{\text{mean}} = 32.0 \text{ in and } n_{\text{mean}} = 6.5 \text{ in}$$

$$\begin{aligned} A_{\text{mean}} &= \pi d_{\text{mean}} n_{\text{mean}} = \pi (32.0) (6.5) \\ &= 653.5 \text{ in}^2 = 4.54 \text{ ft}^2 \end{aligned}$$

$$u = \frac{Q_C}{A_{\text{mean}}} = \frac{0.080}{4.54} = 0.0176 \text{ ft/sec}$$

$$\begin{aligned} D_{\text{eq}} &= \frac{4A_{\text{mean}}}{2\pi d_{\text{mean}} + 2n} = \frac{4(4.54)}{2\pi(32.0/12) + 2(6.5/12)} \\ &= 1.02 \text{ ft} \end{aligned}$$

for liquid sodium at 798°F (Reference 5)

$$\mu = 0.644 \text{ lb/hr-ft} = 0.000179 \text{ lb/sec-ft}$$

$$\rho = 53.07 \text{ lb/ft}^3$$

$$Re = \frac{(1.02)(0.0176)(53.07)}{0.000179} = 5322$$

Using the Lyon-Martinelli equation for uniform flux, as was used on the upper surface,

$$\frac{hD}{k} = 7 + 0.025(Re)^{0.8}(Pr)^{0.8}$$

For liquid sodium at 798°F (Reference 5)

$$k = 40.62 \text{ BTU/hr-ft-}^\circ\text{F}$$

$$Pr = 0.0048$$

Then

$$\begin{aligned} h_{\text{lower}} &= \frac{k}{D} [7.0 + 0.025(Re)^{0.8}(Pr)^{0.8}] \\ &= \frac{40.62}{1.02} [7.0 + 0.025(5322)^{0.8}(0.0048)^{0.8}] \\ &= 292.1 \text{ BTU/hr-ft}^2\text{-}^\circ\text{F} \end{aligned}$$

$$\frac{1}{h_{\text{lower}}} = 0.0034 \frac{\text{hr-ft}^2\text{-}^\circ\text{F}}{\text{BTU}}$$

Evaluating

$$\frac{1}{U_B} = 0.0034 + 0.0085 + 0.0162 = 0.0281$$

$$\begin{aligned} U_B &= 35.59 \text{ BTU/hr-ft}^2\text{-}^\circ\text{F} \\ &= 0.010 \text{ BTU/sec-ft}^2\text{-}^\circ\text{F} \end{aligned}$$

10. Evaluation of Temperature Equation Parameters

Substituting the terms previously evaluated:

$$\begin{aligned} \frac{U_B A_B}{\rho c_p V_B} &= \frac{(0.010) \text{ BTU/sec-ft}^2\text{-}^\circ\text{F} (14.29) \text{ ft}^2}{(53.07) \text{ lb/ft}^3 (0.304) \text{ BTU/lb-}^\circ\text{F} (7.14) \text{ ft}^3} \\ &= 0.0012 \text{ 1/sec} \end{aligned}$$

$$\begin{aligned} X &= \frac{U_B A_B}{\rho c_p V_B} + \frac{Q_C}{V_B} = 0.0012 + 0.0112 \\ &= 0.0124 \text{ 1/sec} \end{aligned}$$

$$\begin{aligned} Y &= \frac{Q_{\text{BEAR}}}{V_B} T_{10} + \left(\frac{Q_S}{V_B} + \frac{U_B A_B}{\rho c_p V_B} \right) T_2 \\ &= (0.0067) (1015) + (0.0045 + 0.0012) (1000) \\ &= 12.54^\circ\text{F/sec} \end{aligned}$$

$$Z = \frac{Q_{\text{BEAR}}^\beta}{V_B} = (0.0067) (2.9) = 0.0194^\circ\text{F/sec}^2$$

Substituting in the temperature equation

$$\begin{aligned}
 T &= \left(\frac{Y}{X} + \frac{Z}{X^2} \right) - \left(\frac{Z}{X} \right)t + \left(T_0 - \frac{Y}{X} - \frac{Z}{X^2} \right)e^{-Xt} \\
 &= \left[\frac{12.54}{0.0124} + \frac{0.0194}{(0.0124)^2} \right] - \left[\frac{0.0194}{0.0124} \right]t \\
 &\quad + \left[1015 - \frac{12.54}{0.0124} - \frac{0.0194}{(0.0124)^2} \right]e^{-0.0124t} \\
 T &= (1137.5) - (1.56)t - (122.5)e^{-0.0124t}
 \end{aligned}$$

11. Initial Transient in Baffle Chamber

The fluid flow from the bearing decreases in temperature for 150 seconds then holds steady at 580°F.

$$\begin{aligned}
 \text{For } t &= 150 \text{ sec} \\
 T &= 884^\circ\text{F}
 \end{aligned}$$

The temperature plot is shown in Figure 3.

12. Bearing Flow at Constant Temperature

The bearing flow temperature is constant after the initial 150 second transient, at a value of 580°F. The temperature equation for the baffle chamber may be rewritten by noting that for

$$\beta = 0 \quad \text{and} \quad Z = 0$$

then

$$T = \frac{Y}{X} + \left(T_0 - \frac{Y}{X} \right)e^{-Xt}$$

where

$$X = 0.0124 \text{ 1/sec (unchanged)}$$

$$Y = \left(\frac{Q_{\text{BEAR}}}{V_B} \right) T_1 + \left(\frac{Q_S}{V_B} + \frac{U_B A_B}{\rho c_p V_B} \right) T_2$$

Evaluating for $T_1 = 580^\circ\text{F}$

$$\begin{aligned} Y &= (0.0067)(580) + (0.0045 + 0.0012)1000 \\ &= 9.59^\circ\text{F/sec} \end{aligned}$$

Since $T_o = 884$ at $t = 0$

$$T = \left(\frac{9.59}{0.0124} \right) + \left(884 - \frac{9.59}{0.0124} \right) e^{-0.0124t}$$

Reducing:

$$T = 773.4 + 110.6e^{-0.0124t}$$

In this case, $T = 884$ at the new time scale $t = 0$. For a total elapsed time of 300 seconds, using

$$t = 150 \text{ seconds}$$

Then

$$T = 791^\circ\text{F}$$

The temperature plot for an elapsed time up to 600 seconds is presented in Figure 3.

B. Temperature Transient in Lower Pump Tank

The lower pump tank receives flow from a number of sources and discharges flow to the suction line, as shown in the Fluid Management Analysis (Reference 8). These sources will be at different temperatures depending on the various thermal lags in the system.

FIGURE 3 - TEMPERATURE TRANSIENT IN BAFFLE CHAMBER

1. Flow Balance

The flow model for the pump tank is shown in Figure 4, the applicable rates being those at pony motor speed. The lower tank is filled with fluid, and the volume flow balance may be written by inspection.

$$Q_N + Q_{SH} = Q_A + Q_C + Q_{BEAR} + Q_{LEAK}$$

2. Energy Balance

To develop the energy equation, the same simplifying assumptions as used previously for the baffle chamber will be applied:

- a) Differences in velocity and elevation of fluid entering and leaving the tank are not significant.
- b) There is perfect mixing of the fluid within the tank and the fluid is at a uniform temperature.
- c) Fluid properties will be considered constant over the temperature range considered.

With regard to the temperature of the inlet flows, the following apply:

- a) Flow from the upper tank, Q_A , is assumed to be at a constant temperature due to the large thermal capacitance of the fluid in the upper tank.
- b) The two flows, Q_{BEAR} and Q_{LEAK} , are both assumed to have the same temperature as the fluid at the pump inlet.
- c) The flow from the baffle chamber will decrease in temperature as previously calculated.

FIGURE 4 - PUMP TANK FLOW MODEL

An energy balance for the lower pump tank is

$$\begin{aligned} \frac{d}{dt} (\rho V_{LT} c_p T) = & \rho c_p T_1 (Q_{BEAR} + Q_{LEAK}) \\ & + \rho c_p T_2 Q_A + \rho c_p T_3 Q_C \\ & - \rho c_p T (Q_N + Q_{SH}) + q \end{aligned}$$

Where

t = Time

ρ = Density of fluid

c_p = Specific heat of fluid

T = Temperature of flows as noted

V_{LT} = Volume of fluid in lower pump tank

q = Heat transfer to fluid from surrounding walls

To simplify the equation, let

$$Q_{BL} = Q_{BEAR} + Q_{LEAK}$$

and

$$Q_{OUT} = Q_N + Q_{SH}$$

Thus the equation may be written

$$V_{LT} \frac{dT}{dt} = Q_{BL} T_1 + Q_A T_2 + Q_C T_3 - Q_{OUT} T + \frac{q}{\rho c_p}$$

3. Heat Transfer Components

The heat transfer may be regarded as having two principal components:

- a) Heat flow through the outer wall to the environment in the guard vessel.
- b) Heat flow through the internal structure to the fluid in the primary flow passages.

4. Internal Heat Transfer

The internal heat transfer is a function of the temperature driving potential $(T - T_1)$ such that

$$q_{in} = K(T - T_1)$$

However, the proportionality may be a complicated factor, affected by the thermal capacitance of the structure. Also, this internal heat flow has the same general effect as a factor of the fluid flow, Q_{BL} . Since this leakage flow is subject to considerable variation due to variations in clearances, it will be convenient to represent the effects of internal heat flow as an increase in the leakage flow. Some simple approximations can be used to estimate the magnitude of proportionality.

5. External Heat Transfer

The heat flow through the outer wall may be represented by an expression of the form

$$q_{out} = U_T A_{LT} (T - T_A)$$

Where

U_T = overall coefficient of heat transfer

A_{LT} = External area of lower pump tank

T_A = Temperature of environment in guard vessel

The expression implies heat transfer by convection, although it may be that an appreciable amount is due to radiative action. However, there are means of evaluating the coefficient, U , to represent a combined mechanism of convection and radiation, and such a procedure gives a more convenient mathematical expression than the fourth power law for radiation (Reference 1, page 165).

Since the energy equation is written in terms of positive inflow of heat, we must write

$$q = U_L A_{LT} (T_A - T)$$

6. Input Temperatures

With regard to the temperatures, the main flow transient is

$$T_1 = T_{10} - \beta t$$

Where

$$\beta = \frac{\Delta T_1}{\Delta t} = \text{constant}$$

Also, as was stated in the assumptions

$$T_2 = \text{constant}$$

The baffle chamber temperature was found as

$$T_3 = \left(\frac{Y}{X} + \frac{Z}{X^2} \right) - \left(\frac{Z}{X} \right) t + \left(T_0 - \frac{Y}{X} - \frac{Z}{X^2} \right) e^{-Xt}$$

Defining new constants, let

$$T_3 = L - Mt + Ne^{-Xt}$$

7. Temperature Equation

Substituting the expressions for temperature and heat flow in the energy equation and rearranging:

$$\begin{aligned} \frac{dT}{dt} + \left(\frac{Q_{OUT}}{V_{LT}} \right) T &= \frac{Q_{BL}}{V_{LT}} (T_{10} - \beta t) + \frac{Q_A}{V_{LT}} T_2 \\ &+ \frac{Q_C}{V_{LT}} (L - Mt + Ne^{-Xt}) \\ &+ \frac{U_{TAL} T}{\rho_c V_{LT}} (T_A - T) \end{aligned}$$

$$\begin{aligned} \frac{dT}{dt} + \left(\frac{Q_{OUT}}{V_{LT}} + \frac{U_{TAL} T}{\rho_c V_{LT}} \right) T &= \left[\frac{Q_{BL}}{V_{LT}} T_{10} + \frac{Q_A}{V_{LT}} T_2 \right. \\ &+ \frac{Q_C L}{V_{LT}} + \frac{U_{TAL} T}{\rho_c V_{LT}} T_A \left. \right] \\ &- \left(\frac{Q_{BL} \beta}{V_{LT}} + \frac{Q_C M}{V_{LT}} \right) t \\ &+ \frac{Q_C N}{V_{LT}} e^{-Xt} \end{aligned}$$

Writing the equation in the form

$$\frac{dT}{dt} + AT = B - Dt + Fe^{-Xt}$$

From which the solution is found for $T = T_0$
when $t = 0$

$$T = \left[\frac{B}{A} + \frac{D}{A^2} \right] - \left(\frac{D}{A} \right)t + \frac{F}{A-X} e^{-Xt} \\ + \left[T_0 - \frac{B}{A} - \frac{D}{A^2} - \frac{F}{A-X} \right] e^{-At}$$

Where

$$A = \frac{Q_{OUT}}{V_{LT}} + \frac{U_{T_{LT}} A_{LT}}{\rho c_p V_{LT}}$$

$$B = \frac{Q_{BL}}{V_{LT}} T_{10} + \frac{Q_A}{V_{LT}} T_2 + \frac{Q_C}{V_{LT}} L + \frac{U_{T_{LT}} A_{LT}}{\rho c_p V_{LT}} T_A$$

$$D = \frac{Q_{BL}}{V_{LT}} \beta + \frac{Q_C}{V_{LT}} M$$

$$F = \frac{Q_C}{V_{LT}} N$$

8. Principal Constants

From the analysis of the baffle chamber, the following values are defined

$$X = 0.0124 \text{ 1/sec}$$

$$L = 1137.5^\circ\text{F}$$

$$M = 1.56^\circ\text{F/sec}$$

$$N = 122.5^\circ\text{F}$$

$$\beta = 2.9^\circ\text{F/sec}$$

$$T_{10} = 1015^\circ\text{F}$$

$$T_2 = 1000^\circ\text{F}$$

9. Fluid Flows

Evaluating the flows, based on data from the Fluid Management Analysis (Reference 8) adjusted for pony motor speed

$$Q_{\text{BEAR}} = 21.75 \text{ GPM}$$

$$Q_{\text{LEAK}} = 34.0 \text{ GPM}$$

Then

$$\begin{aligned} Q_{\text{BL}} &= Q_{\text{BEAR}} + Q_{\text{LEAK}} = 21.75 + 34.0 = 55.75 \text{ GPM} \\ &= 0.124 \text{ ft}^3/\text{sec} \end{aligned}$$

$$Q_{\text{H}} = 22.7 \text{ GPM}$$

$$Q_{\text{IHX}} = 15.0 \text{ GPM}$$

$$Q_{\text{SH}} = Q_{\text{IHX}} - Q_{\text{H}} = 22.7 - 15.0 = 7.7 \text{ GPM}$$

$$Q_{\text{N}} = 100.0 \text{ GPM}$$

$$\begin{aligned} Q_{\text{OUT}} &= Q_{\text{N}} + Q_{\text{SH}} = 100.0 + 7.7 = 107.7 \text{ GPM} \\ &= 0.240 \text{ ft}^3/\text{sec} \end{aligned}$$

From the baffle chamber analysis

$$Q_{\text{C}} = 36.25 \text{ GPM} = 0.081 \text{ ft}^3/\text{sec}$$

Also

$$Q_{\text{A}} = 15.7 \text{ GPM} = 0.035 \text{ ft}^3/\text{sec}$$

This flow includes an estimate of the flow due to the gradual descent of the upper tank fluid surface as calculated in the Standpipe Bubbler Report (Reference 10).

10. Tank Geometry Calculations

The fluid volume in the lower tank will be approximated by the following simplified geometrical calculations: based on the dimensions shown in Figure 5.

Volume of Bottom Segment

$$\begin{aligned} V_1 &= \pi/3 n^2(3r-n) \\ &= \pi/3(15.0)^2[(3)(45.0) - 15.0] \\ &= 28274 \text{ in}^3 = 16.36 \text{ ft}^3 \end{aligned}$$

Volume of Total Side Ring

$$\begin{aligned} V_2 &= \pi/6 c^3 = \pi/6 (60.0)^3 = 113,097 \text{ in}^3 \\ &= 65.45 \text{ ft}^3 \end{aligned}$$

Volume of Suction Nozzle Cone

$$\begin{aligned} V_3 &= \pi/12 a^2 s [1 + (b/a) + (b/a)^2] \\ &= \pi/12 (44.0)^2 (15.0) [1 + (35.0/44.0) \\ &\quad + (35.0/44.0)^2] \\ &= 18461 \text{ in}^3 = 10.68 \text{ ft}^3 \end{aligned}$$

Volume of Discharge Nozzle Cone

$$\begin{aligned} V_3 &= \pi/12 a^2 s [1 + (b/a) + (b/a)^2] \\ &= \pi/12 (29.0)^2 (15.0) [1 + (23.0/29.0) \\ &\quad + (23.0/29.0)^2] \\ &= 7999 \text{ in}^3 = 4.63 \text{ ft}^3 \end{aligned}$$

Lower Tank Volume

$$\begin{aligned} V_{LT} &= V_1 + V_2 - V_3 - V_4 \\ &= 16.36 + 65.45 - 10.68 - 4.63 = 66.50 \text{ ft}^3 \end{aligned}$$

FIGURE 5 - LOWER PUMP TANK GEOMETRY

In a similar fashion, the area of the lower tank exposed to external heat transfer can be found

Total Sphere Area

$$A_1 = 4\pi r_s^2 = 4\pi(45.0)^2 = 25447 \text{ in}^2 = 176.72 \text{ ft}^2$$

Suction Cone Area

$$\text{Using } d_{\text{mean}} = \frac{35.0 + 44.0}{2} = 39.5 \text{ in}$$

$$\begin{aligned} A_2 &= \pi/4 d_{\text{mean}}^2 = \pi/4(39.5)^2 = 1225 \text{ in}^2 \\ &= 8.51 \text{ ft}^2 \end{aligned}$$

Discharge Cone Area

$$\text{Using } d_{\text{mean}} = \frac{23.0 + 29.0}{2} = 26.0 \text{ in}$$

$$\begin{aligned} A_3 &= \pi/4 d_{\text{mean}}^2 = \pi/4(26.0)^2 = 531 \text{ in}^2 \\ &= 3.69 \text{ ft}^2 \end{aligned}$$

Upper Tank Intersection

$$\begin{aligned} A_4 &= 2\pi r_s^2 - \pi \sqrt{4r_s^4 - (r_s d)^2} \\ &= 2\pi(45.0)^2 - \pi \sqrt{4(45.0)^4 - (45.0)^2(72.4)^2} \\ &= 5165 \text{ in}^2 = 35.87 \text{ ft}^2 \end{aligned}$$

Lower Tank Exposed Area

$$\begin{aligned} A_{\text{LT}} &= A_1 - A_2 - A_3 - A_4 \\ &= 176.72 - 8.51 - 3.69 - 35.87 = 128.65 \text{ ft}^2 \end{aligned}$$

11. Consideration of Internal Heat Transfer

To relate internal heat transfer to the effects of leakage flow, we may compare the two energy transport equations

$$q = Q\rho c_p \Delta T$$

$$q = UA\Delta T$$

A leakage flow equivalent to the internal heat transfer may be defined

$$\Delta Q_{\text{LEAK}} = \frac{UA}{\rho c_p}$$

Using values previously established for the baffle chamber analysis

$$\rho = 53.07 \text{ lb/ft}^3 @ 798^\circ\text{F}$$

$$c_p = 0.304 \text{ BTU/lb-}^\circ\text{F} @ 795^\circ\text{F}$$

$$U = 35.59 \text{ BTU/hr-ft}^2\text{-}^\circ\text{F}$$

And since the internal area of the structure is less than the tank area, use

$$A = 0.9 A_{\text{LT}} = 0.9(128.65) = 116 \text{ ft}^2$$

Then

$$\Delta Q_{\text{LEAK}} = \frac{(35.59)(116)}{3600(53.07)(0.304)} = 0.071 \text{ ft}^3/\text{sec}$$

$$= 32 \text{ GPM}$$

Therefore, the internal heat transfer may be approximated by increasing the leakage flow by 32 GPM. Then

$$Q_{\text{BL}} = 0.124 + 0.071 = 0.195 \text{ ft}^3/\text{sec}$$

$$Q_{\text{OUT}} = 0.240 + 0.071 = 0.311 \text{ ft}^3/\text{sec}$$

12. Evaluating the Fluid Flow Factors

$$\frac{Q_{OUT}}{V_{LT}} = \frac{0.311}{65.50} = 0.00468 \text{ 1/sec}$$

$$\frac{Q_{BL}}{V_{LT}} T_{10} = \frac{0.195}{66.50} 1015 = 2.976^\circ\text{F/sec}$$

$$\frac{Q_A}{V_{LT}} T_2 = \frac{0.035}{66.50} 1000 = 0.526^\circ\text{F/sec}$$

$$\frac{Q_{CL}}{V_{LT}} = \frac{0.081}{66.50} 1137.5 = 1.386^\circ\text{F/sec}$$

$$\frac{Q_{BL}^\beta}{V_{LT}} = \frac{0.195}{66.50} 2.9 = 0.00850^\circ\text{F/sec}^2$$

$$\frac{Q_{CM}}{V_{LT}} = \frac{0.081}{66.50} 1.56 = 0.00190^\circ\text{F/sec}^2$$

$$\frac{Q_{CN}}{V_{LT}} = \frac{0.081}{66.50} 122.5 = 0.149^\circ\text{F/sec}$$

To complete the evaluation of the factors, the external heat transfer parameters must be established. The principal difficulty is that the environment in the guard vessel is not known. However, the guard vessel will be insulated, and the ultimate intention will be to provide sufficient insulation so that the heat loss at the normal operating level will not be large. We may thus make a few assumptions as to the environmental conditions with some assurance that the ultimate error cannot be large in relation to the other factors involved.

13. External Heat Transfer Potential

The pump is installed in a guard vessel, as shown in Figure 7. The guard vessel is insulated and is located in a large pit with concrete walls. The Equipment Specification (Reference 7) places a limit of 150°F on the concrete, and the analysis of the upper pump structure (Reference 9) indicates an equilibrium temperature of 118°F in the surrounding concrete. The lower concrete will undoubtedly be somewhat cooler since it is not as near the structure and has a better ultimate heat sink in the earth. As a convenient approximation we will use the temperatures of

Fluid in lower pump tank: $T_0 = 1015^\circ\text{F}$

Concrete wall of pit: $T_C = 115^\circ\text{F}$

The total temperature potential is thus

$$\Delta T = T_0 - T_C = 900^\circ\text{F}$$

14. Heat Transfer Resistance

We may derive a relation expressing the heat transfer from the pump to the concrete as

$$q = \frac{\Delta T}{R_{eq}}$$

The factor, R_{eq} , which is a composite thermal resistance, will represent the combined effect of a number of heat transfer processes.

- a) Convective heat transfer from the fluid to the tank wall
- b) Heat conduction through the pump tank wall
- c) Convective heat transfer to the gas surrounding the pump tank and from the gas to the guard vessel. The gas, called "depleted air," has the properties of nitrogen (Reference 4)

FIGURE 6 - SCHEMATIC OF HEAT TRANSFER RESISTANCES

- d) Radiative heat transfer from the pump tank to the guard vessel
- e) Heat conduction through the guard vessel wall
- f) Heat transfer by both conduction and radiation across the air gap between the guard vessel and the insulation. The insulation has a polished steel sheath to impede radiative heat transfer
- g) Heat conduction through the insulation
- h) Heat transfer by convection and radiation through the "depleted air" in the pit from the sheathing on the insulation to the surrounding concrete, which is considered to be a sink.

All these processes will be evaluated then expressed as heat flow through a thermal resistance, and the various resistances may be combined into an overall resistance to calculate the temperatures.

15. Heat Transfer Geometry

To simplify the various area relationships, we will treat the lower pump as a full sphere of external diameter

$$D_0 = 93.0 \text{ in}$$

The corresponding area is

$$A_{P0} = \pi(D_0)^2 = \pi(93.0)^2 = 27172 \text{ in}^2 = 189 \text{ ft}^2$$

In subsequent calculations for the lower pump tank alone, we will use the area exposed to heat transfer which was calculated for just the lower pump tank. The current calculation is more general in scope.

Also, the guard vessel will be treated as an equivalent sphere with a diameter based on the dimensions shown in Figure 7

$$D_{GI} = 190.0 \text{ in}$$

FIGURE 7 - GUARD VESSEL GEOMETRY

The corresponding area is then

$$\begin{aligned} A_G &= \pi(D_{GI})^2 = \pi(190.0)^2 = 113,411 \text{ in}^2 \\ &= 788 \text{ ft}^2 \end{aligned}$$

The area of the insulation will also be calculated at an equivalent spherical diameter.

16. Heat Transfer Assumptions

A number of simplifying assumptions will be made which are deemed appropriate considering the uncertainty of many of the factors involved.

- a) Radiation will be treated as an interchange between a sphere and a much larger sphere which totally encloses it. It will also be assumed that both surfaces are gray bodies and that the gas between the surfaces is non-absorbing. Both surfaces are also assumed to be at a uniform temperature.
- b) The effect of the suction line and discharge line on heat transfer from the pump tank will not be considered significant.
- c) Gas convection heat transfer coefficients will be calculated on the basis of natural convection from vertical walls or large spheres.
- d) Fluid flow conditions within the lower pump tank are not known; therefore, as a reasonable approximation, the surface heat transfer coefficient calculated for the interior of the baffle chamber will be used for the interior of the pump tank.

17. Thermal Gradient Approximation

Since the various heat transfer parameters are temperature dependent, an approximate thermal gradient must be established to be used in the individual calculations. The largest temperature difference should be across the insulation, and the heat flow will be

$$q = \frac{k}{x} A \Delta T$$

where, based on the conditions defined in Reference 9

k = Thermal conductivity of insulation,
approx. 0.035 BTU-ft/hr-ft²-°F

A = Area of insulation, approx. 900 ft²

x = Thickness of insulation, 14 in. =
1.167 ft

then

$$q = \frac{(0.035)}{1.167} (900) \Delta T = 27.0 \Delta T \text{ BTU/hr}$$

At high temperatures radiant heat transfer should predominate in the air spaces. Using the approximate relation (Reference 1)

$$q = \sigma A \epsilon_1 T_1^3 \Delta T$$

where

σ = Stefan-Boltzmann constant
0.1714 X 10⁻⁸ BTU/ft²hr°R⁴

A = Area of pump tank = 189 ft²

ϵ = Emissivity of steel, approx. 0.85

T₁ = Absolute temperature of pump tank,
1475°R

then

$$\begin{aligned} q &= (0.1714 \times 10^{-8}) (189) (0.85) (1475)^3 \Delta T \\ &= 884 \Delta T \end{aligned}$$

There are three air spaces between the tank and pit wall; therefore, we may divide the total temperature difference as follows:

$$\Delta T_{\text{TOTAL}} = 3 \Delta T_{\text{RADIATION}} + \Delta T_{\text{INSULATION}}$$

Thus

$$\Delta T_{\text{INSULATION}} = 900 - 3 \Delta T_{\text{RADIATION}}$$

Since the heat flow is the same through each element

$$884 \Delta T_{\text{RADIATION}} = 27.0 \Delta T_{\text{INSULATION}}$$

Substituting

$$884 \Delta T_{\text{RADIATION}} = 27.0 [900 - 3 \Delta T_{\text{RADIATION}}]$$

Thus

$$\Delta T_{\text{RADIATION}} = \frac{(27.0)(900)}{884 + (27.0)(3)} = 25^{\circ}\text{F}$$

$$\Delta T_{\text{INSULATION}} = 900 - 3(25) = 825^{\circ}\text{F}$$

Based on this rough approximation, the following temperature gradient is assumed.

Fluid in Tank	T_0	1015°F
Tank (outside)	T_1	1010°F
Guard Vessel (inside)	T_A	990°F
Guard Vessel (outside)	T_B	988°F
Insulation (inside)	T_{SI}	965°F
Insulation (outside)	T_{SO}	140°F
Concrete Wall	T_C	115°F

18. Heat Transfer Through Tank Wall

For the fluid within the tank, the heat transfer coefficient calculated for the baffle chamber will be used

$$h = 292 \text{ BTU/hr-ft}^2\text{-}^\circ\text{F}$$

For the resistance to heat flow, since

$$q = hA\Delta T$$

using

$$A = \text{Area of surface} = 189 \text{ ft}^2$$

Then

$$R_F = \frac{\Delta T}{q} = \frac{1}{hA} = \frac{1}{(292)(189)} = 0.18 \times 10^{-4} \frac{\text{Hr-}^\circ\text{F}}{\text{BTU}}$$

The pump tank wall will transmit heat by conduction. Since the wall thickness is small in comparison to the diameter of the tank, the wall will be treated as a uniform slab for which the heat transfer equation is

$$q = \frac{k}{x} A\Delta T$$

And the resistance is then

$$R_W = \frac{x}{kA}$$

Where

$$\begin{aligned} x &= \text{Wall thickness} = 1.5 \text{ in} = 0.125 \text{ ft} \\ k &= \text{Thermal conductivity of stainless steel type 316 at } 1000^\circ\text{F} \\ &= 12.44 \text{ BTU/hr-ft-}^\circ\text{F} \end{aligned}$$

Evaluating

$$R_W = \frac{0.125}{(12.44)(189)} = 0.53 \times 10^{-4} \frac{\text{Hr-}^\circ\text{F}}{\text{BTU}}$$

19. Convection at Tank Wall

From the external surface of the pump, heat will be transferred to the guard vessel through convection to the atmosphere and from the atmosphere to the guard vessel. Assuming the atmosphere is nonabsorbing for radiation, there will also be direct radiative heat transfer to the guard vessel. These two processes will act in parallel.

For natural convection, the heat transfer is found by use of the relation (Reference 3, page 6-14)

$$Nu = 0.12(GrPr)^{1/3}$$

where

$$Gr = \frac{g\beta\rho^2 D^3 \Delta T}{\mu^2}$$

For the gas in the guard vessel which has the properties of nitrogen, the properties are found for 1000°F (Reference 4)

$$\rho = \text{density} = 0.0272 \text{ lb/ft}^3$$

$$\mu = \text{viscosity} = 0.0858 \text{ lb/hr-ft}$$

$$\begin{aligned} \beta &= \text{bulk expansion coeff.} = \frac{1}{^\circ R} = \frac{1}{1460} \\ &= 6.8 \times 10^{-4} \text{ 1/}^\circ R \end{aligned}$$

$$g = 32.17 \text{ ft/sec}^2 = 4.17 \times 10^8 \text{ ft/hr}^2$$

$$k = \text{thermal conductivity} = 0.0324 \text{ BTU/ft-hr-}^\circ F$$

$$c_p = \text{specific heat} = 0.268 \text{ BTU/lb-}^\circ F$$

$$Pr = \text{Prandtl number} = 0.708$$

Then for

$$\Delta T = 10^\circ$$

$$\begin{aligned} Gr &= \frac{(4.17 \times 10^8)(6.8 \times 10^{-4})(0.0272)^2(7.75)^3(10)}{(0.0858)^2} \\ &= 1.33 \times 10^8 \end{aligned}$$

Since

$$Nu = \frac{hD}{k}$$

Then

$$\begin{aligned} h &= \frac{k}{D} (0.12) (GrPr)^{1/3} \\ &= \frac{0.0324}{7.75} (0.12) [(1.33 \times 10^8)(0.708)]^{1/3} \\ &= 0.228 \text{ BTU/hr-ft}^2\text{-}^\circ\text{F} \end{aligned}$$

Similarly, for the guard vessel

$$D = 190.0 \text{ in} = 15.83 \text{ ft}$$

$$\begin{aligned} Gr &= \frac{(4.17 \times 10^8)(6.8 \times 10^{-4})(0.0272)^2(15.83)^3(10)}{(0.0858)^2} \\ &= 1.13 \times 10^9 \end{aligned}$$

$$\begin{aligned} h &= \frac{0.0324}{15.83} (0.12) [(1.13 \times 10^9)(0.708)]^{1/3} \\ &= 0.228 \text{ BTU/hr-ft}^2\text{-}^\circ\text{F} \end{aligned}$$

The total convective heat transfer resistance from pump tank to guard vessel will be

$$\begin{aligned} R_1 + R_2 &= \frac{1}{h_1 A_1} + \frac{1}{h_2 A_2} \\ &= \frac{1}{(0.228)(189)} + \frac{1}{(0.228)(788)} \\ &= 0.0232 + 0.0056 = 0.0288 \text{ Hr-}^\circ\text{F/BTU} \end{aligned}$$

20. Radiation from Tank Wall

Examining the radiation from pump tank to guard vessel, both surfaces will be considered as oxidized so that the emissivity is

$$\epsilon_1 = \epsilon_2 = 0.85$$

The radiative interchange is expressed as (Reference 1)

$$q = \sigma A \frac{1}{1/\epsilon_1 + 1/\epsilon_2 - 1} (T_1^4 - T_A^4)$$

Using $\sigma = 0.1714 \times 10^{-8}$ BTU/ft²hr°R⁴

$$A = 189 \text{ ft}^2$$

$$T_1 = 1010 + 460 = 1470 \text{ }^\circ\text{R}$$

$$T_A = 990 + 460 = 1450^\circ\text{R}$$

$$\epsilon_1 = \epsilon_2 = 0.85$$

$$\text{then } q = (0.1714 \times 10^{-8})(189) \frac{1}{1/0.85 + 1/0.85 - 1} \cdot (1470^4 - 1450^4) = 5.96 \times 10^4 \text{ BTU/hr}$$

The resistance is then

$$R_3 = \frac{\Delta T}{q} = \frac{1470 - 1450}{5.96 \times 10^4} = 0.000335 \frac{\text{Hr-}^\circ\text{F}}{\text{BTU}}$$

Combining the convective and radiative resistances in parallel

$$\frac{1}{R_A} = \frac{1}{R_1 + R_2} + \frac{1}{R_3} = \frac{1}{0.0288} + \frac{1}{0.000335}$$

$$= 3016 \text{ BTU/Hr-}^\circ\text{F}$$

$$R_A = 3.32 \times 10^{-4} \text{ Hr-}^\circ\text{F/BTU}$$

21. Heat Transfer Through Guard Vessel Wall

The guard vessel wall will transmit heat by conduction. Since the wall thickness is small in comparison to the diameter of the vessel, the wall will be treated as a uniform slab for which the heat transfer equation is

$$q = \frac{k}{x} A \Delta T$$

The resistance is found as

$$R_4 = \frac{\Delta T}{q} = \frac{x}{kA}$$

where

$$x = \text{Wall thickness} = 2.0 \text{ in} = 0.167 \text{ ft}$$

$$A = \text{Guard vessel wall area} = 788 \text{ ft}^2$$

$$k = \text{Thermal conductivity of stainless steel type 316 @ } 990^\circ\text{F} = 12.40 \text{ BTU/hr-ft-}^\circ\text{F}$$

Evaluating

$$R_4 = \frac{0.167}{(12.40)(788)} = 0.17 \times 10^{-4} \frac{\text{Hr-}^\circ\text{F}}{\text{BTU}}$$

22. Heat Transfer from Guard Vessel

Since it was found that heat transfer by convection was very small as compared to that due to radiation in the guard vessel, convection will be ignored in the air gap between guard vessel and insulation. The area on the outside of the guard vessel will be considered a sphere of diameter:

$$D = 194.0 \text{ in}$$

Thus

$$A = \pi D^2 = \pi (194.0)^2 = 118237 \text{ in}^2 = 821 \text{ ft}^2$$

For the guard vessel, as previously,

$$\epsilon_1 = 0.85$$

For the insulation, which is covered by a polished shield of stainless steel, use (Reference 5)

$$\epsilon_2 = 0.28$$

Then the radiation equation may be evaluated

$$q = \sigma A \frac{1}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} (T_B^4 - T_{SI}^4)$$

Where, from the approximate thermal gradient,

$$T_B = 988 + 460 = 1448^\circ R$$

$$T_{SI} = 965 + 460 = 1425^\circ R$$

$$q = (0.174 \times 10^{-8})(821) \frac{1}{\frac{1}{0.85} + \frac{1}{0.28} - 1} (1448^4 - 1425^4) = 10.24 \times 10^5 \text{ BTU/hr}$$

For which the resistance is

$$R_7 = \frac{\Delta T}{q} = \frac{1448 - 1425}{10.24 \times 10^5} = 2.25 \times 10^{-4} \frac{\text{Hr} \cdot ^\circ F}{\text{BTU}}$$

23. Heat Transfer Through Insulation

Heat conduction through the insulation will be treated in the same manner as was the guard vessel wall, but with a diameter of

$$D = 190.0 + 2(1.0) + 2(14.0/2) = 206.0 \text{ in}$$

for which the area is

$$A = \pi(D^2) = \pi(206.0)^2 = 133317 \text{ in}^2 = 926 \text{ ft}^2$$

The resistance is

$$R_8 = \frac{x}{kA}$$

where

$$x = \text{Wall thickness} = 14.0 \text{ in} = 1.167 \text{ ft}$$

$$\begin{aligned} k &= \text{Thermal conductivity of WRP-X} \\ &\quad \text{insulation at } 553^\circ\text{F (Reference 9)} \\ &= 0.032 \text{ BTU-ft/Hr-ft}^2\text{-}^\circ\text{F} \end{aligned}$$

$$R_8 = \frac{1.167}{(0.032)(926)} = 393.83 \times 10^{-4} \frac{\text{Hr-}^\circ\text{F}}{\text{BTU}}$$

24. Heat Transfer in Pit

For heat transfer in the pit environment, the coefficients of convective heat transfer will not be much different than for the pump tank. Therefore, we will calculate the radiative resistance; and if it is small, we may ignore the effects of convection. The outer area of the insulation will be considered a sphere of diameter

$$D = 190.0 + 2(1.0) + 2(14.0) = 220.0 \text{ in}$$

For which the area is

$$A = \pi D^2 = \pi (220.0)^2 = 152053 \text{ in}^2 = 1055 \text{ ft}^2$$

For the sheathing on the insulation, at 140°F , use, as previously,

$$\epsilon_1 = 0.20$$

For the concrete in the surrounding pit use (Reference 1)

$$\epsilon_2 = 0.93$$

Then the radiation equation may be evaluated

$$q = \sigma A \frac{1}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} (T_{SO}^4 - T_C^4)$$

Where

$$T_{SO} = 140 + 460 = 600^\circ R$$

$$T_C = 115 + 460 = 575^\circ R$$

$$q = (0.1714 \times 10^{-8})(1055) \frac{1}{\frac{1}{0.20} + \frac{1}{0.93} - 1} (600^4 - 575^4) = 7.23 \times 10^3 \text{ BTU/hr}$$

For which the resistance is

$$R_{11} = \frac{\Delta T}{q} = \frac{600 - 575}{7.23 \times 10^3} = 34.59 \times 10^{-4} \frac{\text{Hr} \cdot ^\circ F}{\text{BTU}}$$

25. Overall Equivalent Resistance

Adding the significant resistances, which are in series,

$$R_F = 0.18 \times 10^{-4} \frac{\text{Hr} \cdot ^\circ F}{\text{BTU}}$$

$$R_W = 0.53 \times 10^{-4}$$

$$R_A = 3.32 \times 10^{-4}$$

$$R_4 = 0.17 \times 10^{-4}$$

$$R_7 = 2.25 \times 10^{-4}$$

$$R_8 = 393.83 \times 10^{-4}$$

$$R_{11} = 34.59 \times 10^{-4}$$

$$R_{eq} = 434.87 \times 10^{-4} \frac{\text{Hr} \cdot ^\circ F}{\text{BTU}}$$

For the total temperature difference of

$$T_O - T_C = 900^\circ\text{F}$$

The heat transfer is

$$\begin{aligned} q &= \frac{T_O - T_C}{R_{eq}} = \frac{900}{434.87 \times 10^{-4}} = 2.07 \times 10^4 \text{ BTU/hr} \\ &= 5.75 \text{ BTU/sec} \end{aligned}$$

The temperature of the guard vessel may be found

$$\begin{aligned} T_O - T_A &= q(R_F + R_W + R_A) \\ &= (2.07 \times 10^4)(0.18 + 0.53 + 3.32) \times 10^{-4} \\ &= 8.3^\circ\text{F} \end{aligned}$$

Since

$$T_O = 1015^\circ\text{F}$$

Then

$$T_A = 1015 - 8.3 = 1006.7^\circ\text{F}$$

Thus we will use $T_A = 1007^\circ\text{F}$ as the sink temperature for thermal transients, and also

$$\begin{aligned} UA &= \frac{1}{R_F + R_W + R_A} = \frac{10^4}{0.18 + 0.53 + 3.32} \\ &= 2481 \text{ BTU/Hr}^\circ\text{F} = 0.689 \text{ BTU/sec}^\circ\text{F} \end{aligned}$$

26. Evaluating Heat Transfer Factors

For external heat transfer the estimated coefficient is

$$UA = 0.689 \text{ BTU/sec}^\circ\text{F}$$

However, this is based on a total spherical area of

$$A_{PO} = 189 \text{ ft}^2$$

Restricting the area to just the amount interfacing with the fluid in the lower pump tank, the area previously found was

$$A_{LT} = 128.65 \text{ ft}^2$$

Therefore

$$U_{T_{LT}} A_{LT} = UA \frac{A_{LT}}{A_{PO}} = 0.689 \frac{128.65}{189} = 0.470 \text{ BTU/sec}^\circ\text{F}$$

Then

$$\frac{U_{T_{LT}} A_{LT}}{\rho c_p V_{LT}} = \frac{0.470}{(53.07)(0.304)(66.50)} = 0.000438 \text{ 1/sec}$$

$$\frac{U_{T_{LT}} A_{LT}}{\rho c_p V_{LT}} T_A = (0.000438)(1007) = 0.441 \text{ }^\circ\text{F/sec}$$

27. Lower Pump Tank Temperatures

The temperature equation is

$$T = \left[\frac{B}{A} + \frac{D}{A^2} \right] - \left(\frac{D}{A} \right) t + \frac{F}{A-X} e^{-Xt} \\ + \left[T_0 - \frac{B}{A} - \frac{D}{A^2} - \frac{F}{A-X} \right] e^{-At}$$

Then

$$A = \frac{Q_{OUT}}{V_{LT}} + \frac{U_{T^A_{LT}}}{\rho c_p V_{LT}} = 0.00468 + 0.000438$$

$$= 0.00512 \text{ 1/sec}$$

$$B = \frac{Q_{BL}}{V_{LT}} T_{10} + \frac{Q_A}{V_{LT}} T_2 + \frac{Q_{CL}}{V_{LT}} + \frac{U_{T^A_{LT}}}{\rho c_p V_{LT}} T_A$$

$$= 2.976 + 0.526 + 1.386 + 0.441$$

$$= 5.329 \text{ }^\circ\text{F/sec}$$

$$D = \frac{Q_{BL}^\beta}{V_{LT}} + \frac{Q_C^M}{V_{LT}} = 0.00850 + 0.00190$$

$$= 0.0104 \text{ }^\circ\text{F/sec}^2$$

$$F = \frac{Q_C^N}{V_{LT}} = 0.149 \text{ }^\circ\text{F/sec}$$

$$\frac{B}{A} = \frac{5.329}{0.00512} = 1040.8 \text{ }^\circ\text{F}$$

$$\frac{D}{A} = \frac{0.0104}{0.00512} = 2.03 \text{ }^\circ\text{F/sec}$$

$$\frac{D}{A^2} = \frac{0.0104}{(0.00512)^2} = 396.7^\circ\text{F}$$

$$X = 0.0124 \text{ 1/sec}$$

$$\frac{F}{A-X} = \frac{0.0149}{(0.00512) - (0.0124)} = -20.5^\circ\text{F}$$

$$T = (1040.8 + 381.7) - (2.03)t - (20.5)e^{-0.0124t}$$

$$+ [1015 - 1040.8 - 396.7 + 20.5]e^{-0.00512t}$$

Reducing:

$$T = 1437.5 - 2.03t - 20.5e^{-0.0124t} - 402.0e^{-0.00512t}$$

Evaluating for

$$\begin{array}{ll} t = 50 \text{ sec} & T = 1014^{\circ}\text{F} \\ t = 100 \text{ sec} & T = 988^{\circ}\text{F} \\ t = 150 \text{ sec} & T = 943^{\circ}\text{F} \end{array}$$

At an elapsed time of 150 seconds, the temperature ramp ends and the pump inlet temperature is constant at 580°F. We may modify the equation by noting that at 150 seconds

$$T_0 = 943^{\circ}\text{F}$$

$$T_1 = 580^{\circ}\text{F}$$

$$\beta = 0$$

$$L = 773.4^{\circ}\text{F}$$

$$N = 110.6^{\circ}\text{F}$$

$$M = 0$$

Therefore:

$$\frac{Q_{BL}}{V_{LT}} T_1 = \frac{0.195}{66.50} 580 = 1.701^{\circ}\text{F/sec}$$

$$\frac{Q_{BL}}{V_{LT}} \beta = 0$$

$$\frac{Q_{BL}}{V_{LT}} M = 0 \quad \frac{Q_{CM}}{V_{LT}} = 0$$

$$\frac{Q_{CN}}{V_{LT}} = \frac{0.081}{66.50} 110.6 = 0.135^{\circ}\text{F/sec}$$

$$\frac{Q_{CL}}{V_{LT}} = \frac{0.081}{66.50} 773.4 = 0.942 \text{ } ^\circ\text{F}/\text{sec}^2$$

$$A = 0.00512 \text{ 1/sec (unchanged)}$$

$$\begin{aligned} B &= 1.701 + 0.526 + 0.942 + 0.438 \\ &= 3.607 \text{ } ^\circ\text{F}/\text{sec} \end{aligned}$$

$$D = 0$$

$$F = 0.135 \text{ } ^\circ\text{F}/\text{sec}$$

$$\frac{F}{A-X} = \frac{0.135}{(0.00512) - (0.0124)} = -18.5 \text{ } ^\circ\text{F}$$

$$\frac{B}{A} = \frac{3.607}{0.00512} = 704.5 \text{ } ^\circ\text{F}$$

Substituting

$$\begin{aligned} T &= 704.5 - (18.5)e^{-0.0124t} + (943 - 704.5 \\ &\quad + 18.5)e^{-0.00512t} \end{aligned}$$

Reducing

$$T = 704.5 - 18.5e^{-0.0124t} + 257.0e^{-0.00512t}$$

This equation starts a new time scale so that when

$$T = 943^\circ\text{F} \quad t = 0$$

For a total elapsed time of 300 seconds, the equation is solved for

$$t = 150 \text{ sec}$$

yielding:

$$T = 821 \text{ } ^\circ\text{F}$$

The values for the temperature transient are plotted in Figure 8 for a period of 600 seconds.

FIGURE 8 - TEMPERATURE TRANSIENT IN LOWER PUMP TANK

C. Temperature Transient in IHX Return Line

Due to the cooling proceeding in the IHX, the fluid in the IHX return line will be cooled as well. This return fluid is from the top of the heat exchanger and has some entrained gas. The temperature of this fluid within the heat exchanger is not known, but since it is so substantially affected by the flow directly from the pump, it will be assumed that the fluid entering the IHX return line has the same temperature as that indicated for the pump inlet.

1. Return Flow

With the pump operating at pony motor speed, the IHX return flow can be found from Reference 8 as:

$$Q_{\text{IHX}} = 15.0 \text{ GPM} = 0.0334 \text{ ft}^3/\text{sec}$$

2. Return Line

The return line is 2-inch Schedule 40 pipe, 71.5 feet long, with cross section dimensions of (Reference 2):

$$D_o = 2.375 \text{ in}$$

$$D_i = 2.067 \text{ in}$$

The flow area is

$$A_{\text{inside}} = \pi/4(2.067)^2 = 3.356 \text{ in}^2 = 0.0233 \text{ ft}^2$$

The wall area is

$$\begin{aligned} A_{\text{wall}} &= \pi/4(2.375^2 - 2.067^2) = 1.075 \text{ in}^2 \\ &= 0.00746 \text{ ft}^2 \end{aligned}$$

3. Initial Temperature Lag

At the start of the transient, the pipe is filled with fluid at 1015°F. The time for this fluid to traverse the pipe will be:

$$\Delta t = \frac{A_{\text{inside}} L}{Q_{\text{IHX}}} = \frac{(0.0233)(71.5)}{0.0334} = 49.9 \text{ sec}$$

Since the return line is insulated, the pipe is a simple heat capacitor which will further delay the decline in temperature of the returning fluid. The pipe is fabricated of stainless steel type 316 for which the properties are evaluated at a mean temperature of

$$T = \frac{1015 + 580}{2} = 798^{\circ}\text{F}$$

Thus for the steel (Reference 5)

$$c_{pS} = 0.1319 \text{ BTU/lb-}^{\circ}\text{F}$$

$$\rho_S = 486.3 \text{ lb/ft}^3$$

4. Heat Transfer Coefficient

To evaluate the heat transfer conditions within the pipe, the fluid properties will also be used at the mean temperature, as in previous calculations (Reference 5)

$$\mu = 0.644 \text{ lb/hr-ft}$$

$$\rho = 53.07 \text{ lb/ft}^3$$

$$c_p = 0.304 \text{ BTU/lb-}^{\circ}\text{F}$$

$$k = 40.62 \text{ BTU/hr-ft-}^{\circ}\text{F}$$

$$\text{Pr} = 0.0048$$

The Reynolds number is

$$\text{Re} = \frac{D_i u \rho}{\mu}$$

Where

$$D_i = 2.067 \text{ in} = 0.172 \text{ ft}$$

$$u = Q_{\text{IHX}} / A_{\text{inside}} = \frac{0.0334}{0.0233} = 1.433 \text{ ft/sec}$$

$$Re = \frac{(0.172)(1.433)(53.07)}{(0.644/3600)} = 20$$

The value of the Reynolds number indicates laminar flow. From Reference 4, page 9-62, the limiting Nusselt number tends to a minimum

$$Nu = 3.66$$

then

$$\frac{hD}{k} = 3.66$$

$$h = (3.66) \frac{40.62}{0.172} = 864.4 \text{ BTU/hr-ft}^2\text{-}^\circ\text{F}$$

The discharge temperature from the pipe may be calculated by a method described in Reference 6.

5. Numerical Procedure

Using the notation as in Reference 6, we may calculate the coefficients

$$\begin{aligned} V &= Q_{IHX} \rho c_p = (0.0334)(3600 \text{ sec/hr})(53.07)(0.304) \\ &= 1939.9 \text{ BTU/hr } ^\circ\text{F} \end{aligned}$$

$$\begin{aligned} W &= A_{\text{wall}} \rho_S c_{p_S} L = (0.00746)(486.3)(0.1319)L \\ &= (0.4785)L \text{ BTU/}^\circ\text{F} \end{aligned}$$

$$H = h\pi D_i L = (864.4)\pi(0.172)L = (467.1)L \text{ BTU/hr}^\circ\text{F}$$

Since the required limits are

$$H \leq 2V$$

$$(467.1)L \leq 2(1939.9)$$

$$\text{thus: } L \leq 8.31$$

Total pipe length is 71.5 ft; therefore, dividing the total length into 9 segments

$$L = \frac{71.5}{9} = 7.94 \text{ ft}$$

Then

$$W = (0.4785)(7.94) = 3.800 \text{ BTU/}^\circ\text{F}$$

$$H = (467.1)(7.94) = 3708.8 \text{ BTU/hr}^\circ\text{F}$$

$$\frac{W}{H} = \frac{3.80}{3708.8} = 0.00102 \text{ hr} = 3.69 \text{ sec}$$

Since the time increment must meet the requirement

$$\Delta\tau < \frac{W}{H}$$

let

$$\Delta\tau = 3.0 \text{ sec} = 0.000833 \text{ hr}$$

$$\begin{aligned} t_2 &= \frac{2H}{2V + H} t_a + \frac{2V - H}{2V + H} t_1 \\ &= \frac{2(3708.8)}{2(1939.9) + 3708.8} t_a + \frac{2(1939.9) - 3708.8}{2(1939.9) + 3708.8} t_1 \end{aligned}$$

$$t_2 = (0.9775)t_a + (0.0225)t_1$$

$$\begin{aligned} t'_a &= \left(1 - \frac{H\Delta\tau}{W}\right)t_a + \frac{H\Delta\tau}{2W} t_1 + \frac{H\Delta\tau}{2W} t_2 \\ &= \left[1 - \frac{3708.8}{3.80} (0.000833)\right] t_a \\ &\quad + \frac{(3708.8)(0.000833)}{2(3.80)} t_1 \\ &\quad + \frac{(3708.8)(0.000833)}{2(3.80)} t_2 \end{aligned}$$

FIGURE 9 - TEMPERATURE TRANSIENT IN IHX RETURN LINE

$$t'_a = (0.1870)t_a + (0.4065)t_1 + (0.4065)t_2$$

The two equations permit progressive calculations of the temperatures in the nine elements selected. The initial temperature is

$$T_o = 1015^\circ\text{F}$$

and the transient ramp is

$$\beta = 2.9^\circ\text{F/sec}$$

For ease of calculation we will use the temperature transforms:

$$t_a = 1015 - T_o$$

$$t_1 = 1015 - T_o - \beta\Delta T$$

6. Discharge Temperatures

Successive application of the numerical procedure yields the discharge temperature at intervals of 3 seconds, and the results are shown in Figure 9. The numerical method tends to overstate the temperature change in the first few steps, as explained in Reference 6, but comes progressively closer as time advances; therefore, the curve was faired into the line of constant temperature for the initial lag of 49.9 seconds to give a smooth plot. As indicated on the plot, the effect of the IHX line is a displacement of the transient ramp by 62 seconds.

D. Temperature Transient Within the Pump Shaft

The lower portion of the pump shaft is integral with the impeller and is directly exposed to the cooled fluid entering the pump. Within the hollow shaft the IHX return flow is injected and mixes with fluid from the lower pump tank (Figure 10).

FIGURE 10 - PUMP SHAFT GEOMETRY

1. Flow Balance

The fluid within the pump shaft rises until it is pumped out to the upper tank through the bleed holes. Since the flow through the bleed holes is greater than the IHX return flow, the difference is supplied by flow from the lower pump tank (Figure 4)

$$Q_H = Q_{IHX} + Q_{SH}$$

where

Q_H = Volumetric flow rate through bleed holes

Q_{IHX} = Volumetric return flow rate from IHX

Q_{SH} = Volumetric flow rate supplied from lower tank

2. Energy Balance

The temperatures of the flows Q_{IHX} and Q_{SH} have been found for the transient case, and both decrease with time. The flow through the shaft is actually induced by the pumping action of the bleed holes; therefore, the difference in elevation, velocity, and pressure between the fluid entering and leaving the shaft has some significance. However, the fluid is pumped into the upper tank where its energy is dissipated and appears as heat. Therefore, the results of this pumping action will be treated simply as an equivalent heat energy input to the fluid in the shaft.

In writing the energy equation the following simplifying assumptions will be adopted:

- (1) There is perfect mixing of the fluid within the shaft so that the fluid is at a uniform temperature
- (2) Fluid properties will be considered constant over the temperature range considered.

An energy balance for the fluid in the shaft may be written:

$$\frac{d}{dt} (\rho V_{FS} c_p T) = \rho c_p Q_{IHX} T_{IHX} + \rho c_p Q_{SH} T_{SH} - \rho c_p Q_H T + q + W$$

where

t = Time

ρ = Density of fluid

c_p = Specific heat of fluid

T = Temperatures of flows as noted

V_{FS} = Volume of fluid in shaft

q = Heat transfer to fluid from surrounding walls

W = Work done on fluid by rotating shaft (pumping)

Since fluid properties are constant

$$V_{FS} \frac{dT}{dt} = Q_{IHX} T_{IHX} + Q_{SH} T_{SH} - Q_H T + \frac{1}{\rho c_p} (q + W)$$

3. Input Temperatures

The temperatures for both the IHX flow and the flow from the lower tank may be approximated by ramps, to provide some mathematical simplification, using the form

$$T_{IHX} = T_{XO} - \psi t$$

and

$$T_{SH} = T_{SHO} - \lambda t$$

4. Heat Transfer

The heat transfer through the walls may be expressed as

$$q = U_S A_{FS} (T_1 - T)$$

where

U_S = Overall coefficient of heat transfer

A_{FS} = Area of shaft wall exposed to heat transfer

T = Temperature of fluid in shaft

T_1 = Temperature of fluid outside shaft

The equation is written for transfer of heat to the fluid in the shaft, but since the external fluid is colder, the evaluation will reveal that the temperature difference is negative, and heat is transferred from the shaft fluid to the external fluid. The external fluid is actually the main pump flow which has a temperature transient expressed as

$$T_1 = T_{10} - \beta t$$

5. Temperature Equation

Substituting the terms for temperature ramps and heat transfer in the energy equation

$$\begin{aligned} \frac{dT}{dt} + \left(\frac{Q_H}{V_{FS}} + \frac{U_S A_{FS}}{\rho c_p V_{FS}} \right) T \\ = \left[\frac{Q_{IHX}}{V_{FS}} T_{XO} + \frac{Q_{SH}}{V_{FS}} T_{SHO} + \frac{U_S A_{FS}}{\rho c_p V_{FS}} T_{10} + \frac{W}{\rho c_p V_{FS}} \right] \\ - \left[\frac{Q_{IHX}}{V_{FS}} \psi + \frac{Q_{SH}}{V_{FS}} \lambda + \frac{U_S A_{FS}}{\rho c_p V_{FS}} \beta \right] t \end{aligned}$$

This may be written in the form

$$\frac{dT}{dt} + XT = Y - Zt$$

Using an initial condition $T = T_o$ for $t = 0$
the solution may be written as

$$T = \left(\frac{Y}{X} + \frac{Z}{X^2} \right) - \left(\frac{Z}{X} \right)t + \left(T_o - \frac{Y}{X} - \frac{Z}{X^2} \right) e^{-Xt}$$

where

$$X = \frac{Q_H}{V_{FS}} + \frac{U_{S^A_{FS}}}{\rho c_p V_{FS}}$$

$$Y = \frac{Q_{IHX}}{V_{FS}} T_{XO} + \frac{Q_{SH}}{V_{FS}} T_{SHO} + \frac{U_{S^A_{FS}}}{\rho c_p V_{FS}} T_{10} + \frac{W}{\rho c_p V_{FS}}$$

$$Z = \frac{Q_{IHX}}{V_{FS}} \psi + \frac{Q_{SH}}{V_{FS}} \lambda + \frac{U_{S^A_{FS}}}{\rho c_p V_{FS}} \beta$$

6. Evaluating the Constants

The flow values as previously established are

$$Q_{IHX} = 15.0 \text{ GPM} = 0.0334 \text{ ft}^3/\text{sec}$$

$$Q_{SH} = 7.7 \text{ GPM} = 0.0172 \text{ ft}^3/\text{sec}$$

$$Q_H = 22.7 \text{ GPM} = 0.0506 \text{ ft}^3/\text{sec}$$

Examining the temperature plots as shown on
Figure 11, in the initial 62 seconds

$$\psi = 0 \quad \text{and} \quad T_{XO} = 1015^\circ\text{F}$$

$$\lambda = 0 \quad \text{and} \quad T_{SHO} = 1015^\circ\text{F}$$

while, as previously defined

$$\beta = 2.9^{\circ}\text{F/sec and } T_0 = 1015^{\circ}\text{F}$$

After the initial 62 second period,

$$\psi = \beta = 2.9^{\circ}\text{F/sec}$$

$$\lambda = \frac{\Delta T}{\Delta t} = \frac{1015 - 815}{300 - 62} = 0.84^{\circ}\text{F/sec}$$

As established in the previous calculations:

$$\text{at 150 seconds } \beta = 0 \text{ and } T_1 = 580^{\circ}\text{F}$$

$$\text{at 212 seconds } \psi = 0 \text{ and } T_{\text{IHX}} = 580^{\circ}\text{F}$$

From Figure 11, as previously calculated for 300 seconds:

$$T_{\text{SH}} = 815^{\circ}\text{F}$$

$$\lambda = \frac{815 - 730}{600 - 300} = 0.28^{\circ}\text{F/sec}$$

7. Shaft Geometry

The volume is calculated as the sum of several sections

- a) Considering the shaft a pipe of 7.00 in diameter, 76.00 in long, the volume is

$$\begin{aligned} V_1 &= \pi/4 D_i^2 L \\ &= \pi/4 (7.00)^2 (76.00) = 2925 \text{ in}^3 = 1.69 \text{ ft}^3 \end{aligned}$$

- b) For the triangular section in the center of the impeller

$$\begin{aligned} A &= 1/2 bn \\ &= 1/2 (16.00)(6.00) = 48.00 \text{ in}^2 \end{aligned}$$

The center of gravity is at $n/3 = 6.00/3 = 2.00$

FIGURE 11 - TEMPERATURE RAMPS OF INPUT FLOWS TO SHAFT

Diameter of rotation of the center of gravity is

$$D_{c.g.} = D_i + 2n/3 = 7.00 + 2(2.00) = 11.00 \text{ in}$$

Volume of segment is

$$\begin{aligned} V_2 &= A\pi D_{c.g.} \\ &= (48.00)\pi(11.00) = 1659 \text{ in}^3 = 0.96 \text{ ft}^3 \end{aligned}$$

c) The total volume is

$$V_{FS} = V_1 + V_2 = 1.69 + 0.96 = 2.65 \text{ ft}^3$$

The area is calculated in a similar manner.

a) For cylindrical section

$$\begin{aligned} A_1 &= \pi D(L - b) \\ &= \pi(7.00)(76.00 - 16.00) = 1319 \text{ in}^2 = 9.16 \text{ ft}^2 \end{aligned}$$

b) For triangular section

$$\begin{aligned} A_2 &= \pi D_{c.g.} [2\sqrt{(b/2)^2 + n^2} + b] \\ &= \pi(11.00) [2\sqrt{(16.00/2)^2 + (6.00)^2} + 16.00] \\ &= 1244 \text{ in}^2 = 8.63 \text{ ft}^2 \end{aligned}$$

c) For base of triangle

$$\begin{aligned} A_3 &= \pi Db \\ &= \pi(7.00)(16.00) = 352 \text{ in}^2 = 2.44 \text{ ft}^2 \end{aligned}$$

d) The total lateral area is

$$A_{FS} = A_1 + A_2 - A_3 = 9.16 + 8.63 - 2.44 = 15.35 \text{ ft}^2$$

e) The cross-section area for flow is

$$A_{XS} = \pi/4 D^2 = \pi/4(7.00)^2 = 38.48 \text{ in}^2 = 0.267 \text{ ft}^2$$

8. Heat Transfer Coefficient

The overall heat transfer coefficient for the shaft must be evaluated for fluid conditions on both sides of the shaft as well as the effect of the shaft wall itself.

$$\frac{1}{U_S} = \frac{1}{h_{\text{inside}}} + \frac{x_{\text{wall}}}{k_{\text{wall}}} + \frac{1}{h_{\text{outside}}}$$

9. Heat Transfer in Wall

For the wall, stainless steel type 316, evaluating the properties at the convenient temperature of 1000°F (Reference 5)

$$k_{\text{wall}} = 12.3 \text{ BTU/hr-ft-}^\circ\text{F}$$

The wall thickness varies somewhat, but using a typical value

$$x_{\text{wall}} = 2.00 \text{ in} = 0.167 \text{ ft}$$

then

$$\frac{x_{\text{wall}}}{k_{\text{wall}}} = \frac{0.167}{12.3} = 0.0136 \frac{\text{Hr-ft}^2\text{-}^\circ\text{F}}{\text{BTU}}$$

10. Heat Transfer Within the Shaft

Inside the shaft the fluid velocity is found as

$$u = \frac{Q_H}{A_{XS}} \quad \text{where } Q_H = 0.0506 \text{ ft}^3/\text{sec}$$

$$A_{XS} = 0.267 \text{ ft}^2$$

$$u = \frac{0.0506}{0.207} = 0.189 \text{ ft/sec}$$

The Reynolds number is then

$$Re = \frac{Du\rho}{\mu}$$

Using

$$D = \text{Inside shaft diameter} = 7.00 \text{ in} = 0.583 \text{ ft}$$

Evaluating the properties of liquid sodium at an average 798°F

$$\rho = \text{Density} = 53.07 \text{ lb/ft}^3$$

$$\begin{aligned}\mu &= \text{Viscosity} = 0.644 \text{ lb/Hr-ft} \\ &= 0.000179 \text{ lb/sec-ft}\end{aligned}$$

$$\text{Re} = \frac{(0.583)(0.189)(53.07)}{0.000179} = 32668$$

Then using the Lyon-Martinelli equation for uniform heat flux (Reference 2, page 391)

$$\frac{hD}{k} = 7 + 0.025(\text{Re})^{0.8}(\text{Pr})^{0.8}$$

For liquid sodium at 798°F

$$k = \text{Thermal Conductivity} = 40.62 \text{ BTU/Hr-ft-}^{\circ}\text{F}$$

$$\text{Pr} = \text{Prandtl No.} = 0.0048$$

Then

$$\begin{aligned}h &= \frac{k}{D} [7 + 0.025(\text{Re})^{0.8}(\text{Pr})^{0.8}] \\ &= \frac{37.62}{0.583} [7 + 0.025(32668 \times 0.0048)^{0.8}] \\ &= 580 \text{ BTU/Hr-ft}^2\text{-}^{\circ}\text{F}\end{aligned}$$

11. Heat Transfer Outside the Shaft

Outside the shaft there are considerable variations in local flow conditions ranging from low flow in the bearing pockets to high velocity in the impeller area. It would appear that the high velocity areas would predominate and that the heat transfer coefficient would be higher than that on the inside of the shaft. In such a case, the inside coefficient would be the controlling factor, and a large coefficient on the outside would not substantially affect the result. Therefore, it will be assumed that

$$h_{\text{inside}} = h_{\text{outside}}$$

then

$$\begin{aligned} \frac{1}{U_S} &= \frac{2}{h_{\text{inside}}} + \frac{x_{\text{wall}}}{k_{\text{wall}}} \\ &= \frac{2}{580} + 0.0136 = 0.0171 \frac{\text{Hr-ft-}^\circ\text{F}}{\text{BTU}} \end{aligned}$$

And

$$U_S = 58.5 \frac{\text{BTU}}{\text{Hr-ft}^2\text{-}^\circ\text{F}}$$

Hence

$$\begin{aligned} U_S A_{FS} &= (58.5)(15.35) = 897.6 \text{ BTU/Hr-}^\circ\text{F} \\ &= 0.249 \text{ BTU/sec-}^\circ\text{F} \end{aligned}$$

12. Pumping Losses

Based on data from the Internal Fluid Management Analysis (Reference 8) the total theoretical head due to pumping is

$$H = \frac{\omega^2}{2g}$$

where

$$\omega = \text{Shaft angular velocity which @}$$

$$110 \text{ RPM} = 11.5 \text{ rad/sec}$$

then

$$\Delta H = \frac{(11.5)^2}{2(32.17)} = 2.06 \text{ ft}$$

The work is then

$$W = \rho Q_H H J$$

where

$$Q_H = \text{Volumetric flow rate} = 0.0506 \text{ ft}^3/\text{sec}$$

$$\rho = \text{Fluid density @ } 1000^\circ\text{F} = 53.07 \text{ lb/ft}^3$$

$$J = \text{Heat equivalent of work}$$

$$1.286 \times 10^{-3} \text{ BTU/ft-lb}$$

Evaluating

$$W = (53.07)(0.0506)(2.06)(1.286 \times 10^{-3})$$

$$= 0.00711 \text{ BTU/sec}$$

This term is so small it will have little effect on the result.

13. Shaft Temperatures

Evaluating the factors for the first 62 seconds

$$X = \frac{Q_H}{V_{FS}} + \frac{U_S A_{FS}}{\rho c_p V_{FS}}$$

$$= \frac{0.0506}{2.65} + \frac{0.249}{(53.07)(0.304)(2.65)} = 0.0249 \text{ 1/sec}$$

$$\begin{aligned}
Y &= \frac{Q_{IHX}}{V_{FS}} T_{XO} + \frac{Q_{SH}}{V_{FS}} T_{SHO} + \frac{U_{S^A_{FS}}}{\rho c_p V_{FS}} T_{10} + \frac{W}{\rho c_p V_{FS}} \\
&= \frac{0.0334}{2.65} (1015) + \frac{0.0172}{2.65} (1015) \\
&\quad + \frac{0.249(1015)}{(53.07)(0.304)(2.65)} + \frac{0.00711}{(53.07)(0.304)(2.65)} \\
&= 25.3 \text{ } ^\circ\text{F/sec} \\
Z &= \frac{Q_{IHX}}{V_{FS}} \psi + \frac{Q_{SH}}{V_{FS}} \lambda + \frac{U_{S^A_{FS}}}{\rho c_p V_{FS}} \beta \\
&= 0 + 0 + \frac{0.249}{(53.07)(0.304)(2.65)} 2.9 \\
&= 0.0169 \text{ } ^\circ\text{F/sec}^2
\end{aligned}$$

For $t = 62$

$$\begin{aligned}
T &= \left(\frac{Y}{X} + \frac{Z}{X^2} \right) - \left(\frac{Z}{X} \right) t + \left(T_o - \frac{Y}{X} - \frac{Z}{X^2} \right) e^{-Xt} \\
&= \left[\frac{25.3}{0.0249} + \frac{0.0169}{(0.0249)^2} \right] - \left(\frac{0.0169}{0.0249} \right) (t) \\
&\quad + \left[1015 - \frac{25.5}{0.0251} - \frac{0.0175}{(0.0251)^2} \right] e^{-0.0251t} \\
&= (1043.3) - (0.68)t - (28.3)e^{-0.0249t} \\
&= 995^\circ\text{F}
\end{aligned}$$

Evaluating the factors for the range 62 seconds to 150 seconds

$$t = 0 \text{ to } t = 88 \text{ seconds}$$

$$T_0 = 995^\circ\text{F}$$

$$T_{10} = 1015 - (2.9)(62) = 835^\circ\text{F}$$

$$X = 0.0249 \text{ 1/sec}$$

$$Y = \frac{0.0334}{2.65} (1015) + \frac{0.0172}{2.65} (1015) + \frac{0.249(835)}{(53.07)(0.304)(2.65)} = 24.2 \text{ }^\circ\text{F/sec}$$

$$Z = \frac{0.0334}{2.65} (2.9) + \frac{0.0172}{2.65} (0.84) + \frac{0.249}{(53.07)(0.304)(2.65)} (2.9) = 0.0589 \text{ }^\circ\text{F/sec}^2$$

$$T = \left[\frac{24.2}{0.0249} + \frac{0.0589}{(0.0249)^2} \right] - \left(\frac{0.0589}{0.0249} \right) t + \left[995 - \frac{24.2}{0.0249} - \frac{0.0589}{(0.0249)^2} \right] e^{-0.0249t} = (1066.9) - (2.37)t - (71.9)e^{-0.0251t}$$

for $t = 88$ seconds

$$T = 850^\circ\text{F}$$

Values are plotted in Figure 12.

Evaluating the factors for the range 150 seconds to 212 seconds

$$t = 0 \text{ to } t = 62$$

$$T_0 = 850^\circ\text{F}$$

$$T_{10} = 580^\circ\text{F}$$

$$T_{\text{SHO}} = 1015 - (0.84)(88) = 941^\circ\text{F}$$

$$T_{\text{XO}} = 1015 - (2.9)(88) = 760^\circ\text{F}$$

$$\beta = 0$$

$$X = 0.0249 \text{ 1/sec}$$

$$Y = \frac{0.0334}{2.65} (760) + \frac{0.0172}{2.65} (941) + \frac{0.249(580)}{(53.07)(0.304)(2.65)} = 19.06^\circ\text{F/sec}$$

$$Z = \frac{0.0334}{2.65} (2.9) + \frac{0.0172}{2.65} (0.84) + 0 = 0.042^\circ\text{F/sec}^2$$

$$T = \left[\frac{19.06}{0.0249} + \frac{0.042}{(0.0249)^2} \right] - \left(\frac{0.042}{0.0249} \right) t + \left[850 - \frac{19.06}{0.0249} - \frac{0.042}{(0.0249)^2} \right] e^{-0.0249t} = (833.2) - (1.69)t + (16.8)e^{-0.0249t}$$

For $t = 62$

$$T = 732$$

Values are plotted in Figure 12.

Evaluating the factors for the range 212 seconds to 300 seconds

$$t = 0 \text{ to } t = 88 \text{ seconds}$$

$$T_0 = 732^\circ\text{F}$$

$$T_{10} = 580^\circ\text{F}$$

$$T_{\text{SHO}} = 1015 - (0.84)150 = 889^\circ\text{F}$$

$$T_{\text{XO}} = 580^\circ\text{F}$$

$$\beta = 0$$

$$\psi = 0$$

$$X = 0.0249 \text{ 1/sec}$$

$$Y = \frac{0.0334}{2.65} (580) + \frac{0.0172}{2.65} (889)$$

$$+ \frac{0.249(580)}{(53.07)(0.304)(2.65)} = 16.46^\circ\text{F/sec}$$

$$Z = 0 + \frac{0.0172}{2.65} (0.84) + 0 = 0.00545^\circ\text{F/sec}^2$$

$$T = \left[\frac{16.49}{0.0249} + \frac{0.00545}{(0.0249)^2} \right] - \left(\frac{0.00545}{0.0249} \right) t$$

$$+ \left[732 - \frac{16.49}{0.0249} - \frac{0.00545}{(0.0249)^2} \right] e^{-0.0249t}$$

$$= 671.0 - (0.22)t + (61.0)e^{-0.0249t}$$

For $t = 88$ seconds

$$T = 658^\circ\text{F}$$

Values are plotted in Figure 12.

Evaluating the factors for the range 300 seconds to 600 seconds

$$t = 0 \text{ to } t = 300$$

$$T_0 = 658^\circ\text{F}$$

$$T_{10} = 580^\circ\text{F}$$

$$T_{\text{SHO}} = 815^\circ\text{F}$$

$$T_{\text{XO}} = 580^\circ\text{F}$$

$$\beta = 0$$

$$\psi = 0$$

$$\lambda = \lambda_2 = 0.28^\circ\text{F/sec}$$

$$X = 0.0249 \text{ 1/sec}$$

$$Y = \frac{0.0334}{2.65} (580) + \frac{0.0172}{2.65} (815)$$

$$+ \frac{0.249(580)}{(53.07)(0.304)(2.65)} = 15.98^\circ\text{F/sec}$$

$$Z = 0 + \frac{0.0172}{2.65} (0.28) + 0 = 0.00182^\circ\text{F/sec}^2$$

$$T = \left[\frac{15.98}{0.0249} + \frac{0.00182}{(0.0249)^2} \right] t - \left(\frac{0.00182}{0.0249} \right) t$$

$$+ \left[658 - \frac{15.98}{0.0249} - \frac{0.00182}{(0.0249)^2} \right] e^{-0.0249t}$$

$$= (644.7) - (0.073)t + (13.3)e^{-0.0249t}$$

for $t = 300$

$$T = 623$$

The temperature transient for a total elapsed time of 600 seconds is presented in Figure 12.

FIGURE 12 - TEMPERATURE TRANSIENT IN PUMP SHAFT

E. Temperature Transient in Upper Pump Tank

The factors affecting the temperature of the fluid in the upper pump tank are less well defined than for the lower portion of the pump. The fluid level in the tank is subject to some variation, as described in the Standpipe Bubbler Study (Reference 10). Heat transfer from the upper pump tank will be both lateral and vertical, the latter process being described in the Preliminary Thermal Analysis (Reference 9). The preceding calculations have been based on the assumption that the temperature in the upper tank changes slowly, and this assumption will be retained as we develop the analytical substantiation for this approach.

1. Flow Balance

The flow balance for the tank will be

$$Q_H - Q_A - Q_S = \frac{dV}{dt}$$

Where the flows were previously defined. The rate of change of tank volume is small and will be assumed constant. Thus

$$\frac{dV}{dt} = \phi$$

And then the tank volume decreases as

$$V_{UT} = V_{TO} - \phi t$$

2. Assumed Conditions

To develop the energy equation, the same simplifying assumptions will be used as in previous cases:

- a) Differences in velocity and elevation of fluid entering and leaving the tank are not significant.
- b) Energy input to the fluid from the rotating shaft will not be considered significant.

FIGURE 13 - GEOMETRY OF UPPER PUMP TANK

- c) There is perfect mixing of the fluid within the tank, and the fluid is at a uniform temperature.
- d) Fluid properties will be considered constant over the temperature range considered and will be evaluated at 1000°F.

3. Energy Balance

An energy balance for the upper tank is

$$\frac{d}{dt} (\rho c_p V_{UT} T) = \rho c_p Q_H T_H - \rho c_p (Q_A + Q_S) T + q$$

where

t = Time

ρ = Density of fluid

c_p = Specific heat of fluid

T = Temperatures as noted

V_{UT} = Volume of fluid in upper pump tank

q = Heat transfer to fluid from surrounding walls

Since the fluid properties are constant for all flows

$$\frac{d}{dt} (V_{UT} T) = Q_H T_H - (Q_A + Q_S) T + \frac{q}{\rho c_p}$$

Since the volume is not constant

$$V_{UT} \frac{dT}{dt} + T \frac{dV_{UT}}{dt} = Q_H T_H - (Q_A + Q_S) T + \frac{q}{\rho c_p}$$

And

$$\begin{aligned} (V_{T0} - \phi t) \frac{dT}{dt} + T \frac{d}{dt} (V_{T0} - \phi t) \\ = Q_H T_H - (Q_A + Q_S) T + \frac{q}{\rho c_p} \end{aligned}$$

which reduces to

$$\frac{dT}{dt} + \left[\frac{Q_A + Q_S - \phi}{V_{T0} - \phi t} \right] T = \frac{Q_H T_H}{V_{T0} - \phi t} + \frac{q}{\rho c_p (V_{T0} - \phi t)}$$

4. Heat Transfer

Although the heat transfer, q , is a function of the temperature of the fluid in the tank, it is also a function of the temperature of the heat sinks. There are several sinks at various temperatures. However, preliminary calculations indicate that none of the temperatures will change rapidly, and the heat transfer will not be a significant variable. Therefore, it will be assumed that heat transfer is a constant and

$$\frac{q}{\rho c_p} = \text{constant}$$

5. Input Temperature

The temperature of the fluid from the bleed holes will have an initial period of nearly constant temperature and then will decrease as a function of time. The decrease in temperature may be approximated to reduce complexity and be expressed as a ramp

$$T_H = T_{HI} - \delta t$$

6. Temperature Equation

Substituting the temperature expression in the energy equation:

$$\frac{dT}{dt} + \left[\frac{Q_A + Q_S - \phi}{V_{T0} - \phi t} \right] T = \frac{Q_H (T_{HI} - \delta t)}{(V_{T0} - \phi t)} + \frac{q}{\rho c_p (V_{T0} - \phi t)}$$

Writing the equation in the form

$$\frac{dT}{dt} + \left[\frac{Q_A + Q_S - \phi}{V_{T0} - \phi t} \right] T = \frac{(Q_H T_{HI} + q/\rho c_p) - Q_H \delta t}{V_{T0} - \phi t}$$

The solution is found for $T = T_0$ when $t = 0$

$$T = A - B(V_{T0} - \phi t) + C(V_{T0} - \phi t)^X$$

where

$$A = \frac{Q_H \delta V_{T0} - (Q_H T_{HI} \phi + q\phi/\rho c_p)}{\phi^2 - \phi(Q_A + Q_S)}$$

$$B = \frac{Q_H \delta}{2\phi^2 - \phi(Q_A + Q_S)}$$

$$C = [T_0 + BV_{T0} - A](V_{T0})^{-X}$$

$$X = \frac{Q_A + Q_S - \phi}{\phi}$$

7. Tank Geometry

Evaluating the volume as the frustrum of a cone less the volume displaced by the shaft, and using the dimensions in Figure 13

$$\begin{aligned} V_{T0} &= \pi/3 r^2 n [1 + r_2/r_1 + (r_2/r_1)^2] - \pi/4 D_0^2 n \\ &= \pi/3 (42.0)^2 (52.0) [1 + (\frac{35.0}{42.0}) + (\frac{35.0}{42.0})^2] \\ &\quad - \pi/4 (10.75)^2 (52.0) \\ &= 238092 \text{ in}^3 = 137.8 \text{ ft}^3 \end{aligned}$$

The lateral area below the liquid level is

$$\begin{aligned}
 A_{UT} &= \pi(r_1 + r_2) \sqrt{(r_1 - r_2)^2 + n^2} \\
 &= \pi(42.0 + 35.0) \sqrt{(42.0 - 35.0)^2 + (52.0)^2} \\
 &= 12,692 \text{ in}^2 = 88.1 \text{ ft}^2
 \end{aligned}$$

The lateral area of the tank between the liquid surface and the bottom of the shield plug is

$$\begin{aligned}
 A_{GT} &= \pi(r_1 + r_2) \sqrt{(r_1 - r_2)^2 + n^2} \\
 &= \pi(42.0 + 45.75) \sqrt{(45.75 - 42.0)^2 + (31.3)^2} \\
 &= 8690 \text{ in}^2 = 60.3 \text{ ft}^2
 \end{aligned}$$

8. Heat Transfer to Upper Environment

The fluid in the upper pump tank is exposed to several heat sinks which should cause its initial steady-state temperature to be somewhat lower than that of the fluid in the lower portion of the pump. The assumed temperature for this analysis was taken as 1000°F. The standard operating temperature of 995°F was used in the previous heat transfer analysis of the upper pump as covered in the Preliminary Thermal Analysis report (Reference 9). In that report the following heat rejection rates were calculated.

Inner Pump Cover = 1571 BTU/hr @ $\Delta T = 103^\circ\text{F}$

Outer Pump Cover = 2546 BTU/hr @ $\Delta T = 80^\circ\text{F}$

Pump Shaft = 237 BTU/hr @ $\Delta T = 55^\circ\text{F}$

Support Flange = 5199 BTU/hr @ $\Delta T = 38^\circ\text{F}$

These values will all be assumed to increase with an increase in fluid temperature to 1000°F, the amount being assumed as due to an increase in the ΔT of 5°F. Thus:

$$\text{Inner Pump Cover} = 1571 \frac{108}{103} = 1647 \text{ BTU/Hr}$$

$$\text{Outer Pump Cover} = 2546 \frac{85}{80} = 2705 \text{ BTU/Hr}$$

$$\text{Pump Shaft} = 237 \frac{60}{55} = 259 \text{ BTU/Hr}$$

$$\text{Support Flange} = 5199 \frac{43}{38} = \underline{5883 \text{ BTU/Hr}}$$

$$\text{TOTAL} \qquad \qquad \qquad 10,494 \text{ BTU/Hr}$$

9. Heat Transfer Above the Fluid Level

Using the same procedure as above, the tank wall above the fluid level will also be assumed to be 5° warmer. Using

$$T_{UW} = 923^{\circ}\text{F}$$

Using the same assumption for the environment

$$T_{GE} = 840^{\circ}\text{F}$$

The convective heat transfer coefficient used previously was

$$h_f = 0.3125 \text{ BTU/Hr-ft}^2\text{-}^{\circ}\text{F}$$

which was calculated for the external wall of the pump tank. If we also consider the guard vessel and assume it has the same convective coefficient, then the combined coefficient is

$$\frac{1}{U_{\text{conv}}} = \frac{2}{h_f}$$

The convective heat transfer is then

$$\begin{aligned} q &= \frac{h_f}{2} A_{GT} (T_{UW} - T_{GE}) \\ &= \frac{0.3125}{2} (60.3) (923 - 840) \\ &= 781 \text{ BTU/Hr} \end{aligned}$$

The preliminary analysis did not consider radiative transfer to the guard vessel. Using the same approach as employed for the lower pump tank the radiative transfer is

$$q = \sigma A_{GT} \frac{1}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} (T_{UW}^4 - T_{GE}^4)$$

where

$$\sigma = 0.1714 \times 10^{-8} \text{ BTU/ft}^2\text{-Hr-}^\circ\text{R}^4$$

$$A_{GT} = 60.3 \text{ ft}^2$$

$$T_{UW} = 923 + 460 = 1383 \text{ }^\circ\text{R}$$

$$T_{GE} = 840 + 460 = 1300 \text{ }^\circ\text{R}$$

$$\epsilon_1 = \epsilon_2 = 0.85$$

then

$$\begin{aligned} q &= (0.1714 \times 10^{-8}) (60.3) \frac{1}{\frac{1}{0.85} + \frac{1}{0.85} - 1} (1383^4 \\ &\quad - 1300^4) = 61288 \text{ BTU/Hr} \end{aligned}$$

The radiative heat transfer is in parallel with the convective heat transfer and is directly additive.

10. Heat Transfer Below the Fluid Level

For the liquid filled portion of the upper tank, we will use the same overall coefficient as used in the lower tank but adjusted for tank lateral area

$$UA = 2481 \text{ BTU/Hr-}^\circ\text{F for } A = 189 \text{ ft}^2$$

then

$$U_{UT}A_{UT} = 2481 \frac{88.1}{189} = 1156 \text{ BTU/Hr-}^\circ\text{F}$$

If we assume a guard vessel temperature in this region as an average of the upper and lower values

$$T_{GV} = \frac{840 + 1007}{2} = 923^\circ\text{F}$$

Then the heat transfer is

$$\begin{aligned} q &= U_{UT}A_{UT}(1000 - T_{GV}) = 1156(1000-923) \\ &= 89012 \text{ BTU/Hr} \end{aligned}$$

11. Heat Transfer to the Baffle Chamber

For the heat transfer to the baffle chamber, the average baffle temperature will be used for the first 300 seconds

$$T_{BC} = \frac{1015 + 821}{2} = 918^\circ\text{F}$$

The overall heat transfer coefficient was calculated as

$$U_B = 35.59 \text{ BTU/Hr-ft}^2\text{-}^\circ\text{F}$$

and the area is

$$A_B = 14.29 \text{ ft}^2$$

The heat transfer is thus

$$\begin{aligned} q_B &= U_B A_B (1000 - T_{BC}) \\ &= (35.59)(14.29)(1000 - 918) = 41,704 \text{ BTU/Hr} \end{aligned}$$

12. Combined Heat Transfer

The total heat transfer is then

Heat transfer to upper structure	=	10,494 BTU/Hr
Convective heat transfer above fluid level	=	781 BTU/Hr
Radiation above fluid level	=	61,288 BTU/Hr
Heat transfer from fluid to guard vessel	=	89,012 BTU/Hr
Heat transfer to baffle chamber	=	41,704 BTU/Hr
TOTAL		<hr/> 203,279 BTU/Hr

Hence,

$$q = -203,279 \text{ BTU/Hr} = -56.5 \text{ BTU/sec}$$

The minus sign must be used since it is a heat loss.

13. Evaluating Constants

Evaluating the factors, the flows have been established as

$$\begin{aligned} Q_H &= 22.7 \text{ GPM} = 0.0506 \text{ ft}^3/\text{sec} \\ Q_A &= 15.7 \text{ GPM} = 0.0350 \text{ ft}^3/\text{sec} \\ Q_S &= 14.5 \text{ GPM} = 0.0323 \text{ ft}^3/\text{sec} \\ \phi &= 7.5 \text{ GPM} = 0.0167 \text{ ft}^3/\text{sec} \end{aligned}$$

The temperatures previously established are

$$T_0 = 1000^\circ\text{F}$$

$$T_{\text{HI}} = 1015^\circ\text{F}$$

The ramp for the fluid from the bleed holes will be divided for three periods as shown on Figure 14:

- a) For the period up to 45 seconds, $\delta = 0$
- b) For the period 45 to 285 seconds, $T_{\text{HI}} = 1015^\circ\text{F}$ and

$$\delta = \frac{1015 - 650}{285 - 45} = 1.52^\circ\text{F/sec}$$

- c) For the period 285 to 600 seconds, $T_{\text{HI}} = 650^\circ\text{F}$ and

$$\delta = \frac{650 - 623}{600 - 285} = 0.086^\circ\text{F/sec}$$

Also, as calculated

$$V_{\text{TO}} = 137.8 \text{ ft}^2$$

$$q = -56.5 \text{ BTU/sec}$$

The sodium properties (Reference 5) at 1000°F are

$$\rho = 51.39 \text{ lb/ft}^3$$

$$c_p = 0.3006 \text{ BTU/lb-}^\circ\text{F}$$

14. Tank Temperature

For the period of the first 45 seconds

$$\begin{aligned} A &= \frac{Q_H \delta V_{\text{TO}} - (Q_H T_{\text{HI}} \phi + q \phi / \rho c_p)}{\phi^2 - \phi(Q_A + Q_S)} \\ &= \frac{0 - [(0.0506)(1015)(0.0167) + \frac{(-56.5)(0.0167)}{(51.39)(0.3006)}]}{(0.0167)^2 - (0.0167)(0.0350 + 0.0323)} \\ &= \frac{0 - [0.8577 - 0.0611]}{-0.000845} = 942.7^\circ\text{F/sec} \end{aligned}$$

FIGURE 14 - TEMPERATURE RAMPS FOR FLOW FROM PUMP SHAFT

$$B = \frac{Q_H \delta}{2\phi^2 - \phi(Q_A + Q_S)} = 0$$

$$X = \frac{Q_A + Q_S - \phi}{\phi} = \frac{0.0305 + 0.0323 - 0.0167}{0.0167} = 3.03$$

$$\begin{aligned} C &= [T_0 + BV_{T0} - A](V_{T0})^{-X} \\ &= [1000 + 0 - 942.7](137.8)^{-3.03} \\ &= (57.3)(137.8)^{-3.03} = 1.89 \times 10^{-5} \text{ } ^\circ\text{F} \end{aligned}$$

for $t = 45 \text{ sec}$

$$\begin{aligned} T &= A - B(V_{T0} - \phi t) + C(V_{T0} - \phi t)^X \\ &= (942.7) - 0 + (1.89 \times 10^{-5})[(137.8) \\ &\quad - (0.0167)(45)]^{3.03} \\ &= 942.7 + 56.4 = 999.1 \text{ } ^\circ\text{F} \end{aligned}$$

Values are plotted in Figure 15.

Evaluating the factors for the period of 45 seconds to 285 seconds

$$T_0 = 999 \text{ } ^\circ\text{F}$$

$$\begin{aligned} A &= \frac{(0.0506)(1.52)(137.8) - [0.8577 - 0.0611]}{-0.000845} \\ &= \frac{10.5985 - 0.8577 + 0.0611}{-0.000845} = -11599.9 \text{ } ^\circ\text{F} \end{aligned}$$

$$\begin{aligned} B &= \frac{(0.0506)(1.52)}{2(0.0167)^2 - (0.0167)(0.0350 + 0.0323)} \\ &= \frac{0.0769}{-0.000566} = -135.9 \text{ } ^\circ\text{F/sec} \end{aligned}$$

$$C = [999.1 - (135.9)(137.8) + 11599.9](137.8)^{-3.03}$$

$$= (6128)(3.297 \times 10^{-7}) = -0.00202 \text{ } ^\circ\text{F}$$

for $t = 285 - 45 = 240$ seconds

$$(V_{T0} - \phi t) = [137.8 - (0.0167)(240)] = 133.8 \text{ ft}^3$$

$$T = (-11599.9) - (-135.9)(133.8)$$

$$+ (-0.00202)(133.8)^{3.03}$$

$$= -11599.9 + 18183.4 - 5604.2$$

$$= 979.3 \text{ } ^\circ\text{F}$$

Values are plotted in Figure 15.

Evaluating the factors for the period 285 seconds to 600 seconds

$$T_0 = 979.3 \text{ } ^\circ\text{F}$$

$$A = \frac{(0.0506)(0.086)(137.8) - [0.8577 - 0.0611]}{-0.000845}$$

$$= 233.1 \text{ } ^\circ\text{F}$$

$$B = \frac{(0.0506)(0.086)}{-0.000566} = -7.68 \text{ } ^\circ\text{F/sec}$$

$$C = [979.3 - (7.68)(137.8) - 233.1](137.8)^{-3.03}$$

$$= (-312.1)(137.8)^{-3.03} = -0.0001029 \text{ } ^\circ\text{F}$$

for $t = 600 - 285 = 315$ sec

$$(V - \phi t) = 137.8 - (0.086)(315) = 110.7$$

$$T = 233.1 + (7.68)(110.7) - (0.0001029)(110.7)^{3.03}$$

$$= 922.5 \text{ } ^\circ\text{F}$$

The temperature transient for 600 seconds is plotted in Figure 15.

FIGURE 15 - TEMPERATURE TRANSIENT IN UPPER PUMP TANK

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VI. NOMENCLATURE

A, B, C, D, F, L, M, N	Constants in equations
$A, A_1, A_2 \dots A_B, A_G \dots$	Area, as indicated by subscript
$D_o, D_i \dots$	Diameter, as indicated by subscript
Gr	Grashof number (dimensionless)
H	Pump head
J	Physical constant (heat equivalent of work)
Nu	Nusselt number (dimensionless)
Pr	Prandtl number (dimensionless)
$Q_A, Q_C, Q_{LEAK} \dots$	Volumetric flow rate, as indicated by subscript
$R_1, R_2 \dots R_A, R_W \dots$	Heat transfer resistance, as indicated by subscript
Re	Reynolds number (dimensionless)
$T, T_A \dots T_1, T_2 \dots$	Temperature, as indicated by subscript
$U, U_A, U_T \dots$	Overall coefficient of heat transfer, as indicated by subscript
$V_A, V_B \dots V_1, V_2 \dots$	Volume, as indicated by subscript
W	Rate of pumping work
X, Y, Z	Constants in equations

a, b, c, s	Dimensions of a cavity
c_p	Specific heat at constant pressure
d	Derivative of a variable
$d_1, d_2 \dots d_{\text{mean}} \dots$	Diameter as indicated by subscript
e	Base of natural logarithms, 2.71828
g	Acceleration of gravity
$h, h_{\text{upper}}, h_{\text{lower}} \dots$	Surface heat transfer coefficient
k	Thermal conductivity
n	Height dimension in a cavity
$q, q_1, q_2 \dots$	Heat transfer rate, as indicated by subscript
$r_1, r_2 \dots r_S \dots$	Radius, as indicated by subscript
t	Elapsed time
u	Local fluid velocity
x, x_{wall}	Thickness dimension

β	Constant rate for temperature ramp
δ	Constant rate for temperature ramp
Δ (Δt , ΔT ...)	Finite increment of change
ϵ	Emissivity for radiation
λ	Constant rate for temperature ramp
μ	Fluid viscosity
π	Mathematical constant, 3.14159
ρ	Fluid density
σ	Stefan-Boltzman constant for radiation
ϕ	Constant rate for volume change
ψ	Constant rate for temperature ramp
ω	Angular velocity