TITLE: NONLINEAR PULSATION MASSES

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NONLINEAR PULSATION MASSES

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I. INTRODUCTION

The advent of nonlinear pulsation theory really coincides with the development of the large computers after the Second World War. Christy and Stobbie were the first to make use of finite difference techniques on computers to model the "bumps" observed in the classical Cepheid light and velocity curves, the so-called "Hertzsprung" sequence. Following this work a more sophisticated analysis of the light and velocity curves from the models was made by Simon and Davis (1983) using Fourier techniques. Recently a simpler (semianalytic) amplitude equation formalism has been developed that helps explain this resonance mechanism (Buchler and Goupil, 1984). The determination of Population I Cepheid masses by nonlinear methods will be discussed in Section II. For the lower mass objects, such as RR Lyrae and BL Her stars, we find general agreement using evolutionary masses and nonlinear pulsation theory (Section III). An apparent difficulty of nonlinear pulsation theory occurs in the understanding of "double" mode pulsation, which will also be discussed in Section III. Recent studies in nonlinear pulsation theory have dealt with the question of mode selection, period doubling and the trends towards chaotic behavior such as is observed in the transition from W Virginis to RV Tauri-like stars (IV). Some conclusions are given in Section V.

II. CEPHEIDS AND LONG-PERIOD VARIABLES

We have successfully modeled the Hertzsprung sequence of observed "bumps" on the velocity and luminosity curves for Cepheids with periods from 3 to 10 days. In a comparison by Simon and Davis (1983), using Fourier analysis and the function, $\phi_{21} = \phi_2 - 2\phi_1$, they find general agreement with this same quantity as obtained from observed Cepheids. The actual resonance peak observed, at $P_1/P_0 = 0.5$, is higher than calculated. A more recent study by Buchler, Moskalik and
Kovacs (1990) shows better agreement with the resonance peak. A simple explanation of this resonance is found using the semianalytic amplitude formalism of Klapp, Goupil, and Buchler (1985) and comparing these results with those of Simon and Davis. The problem is in the masses used in these comparisons to observation. These masses are some 60% of the evolutionary masses as first observed by Christy and Stobie. In developing models for longer period Cepheids, where the "bumps" are less distinguishable and actual "dips" occur in their light curves before light maximum, evolutionary masses had to be used (Davis, Moffet and Barnes, 1980). Possible evidence in support of these results comes from the work of Fernly, Skillen and Jameson (1989). The following table is from their paper and is shown as Table I.

<table>
<thead>
<tr>
<th>star</th>
<th>M_E</th>
<th>Mw/Me</th>
</tr>
</thead>
<tbody>
<tr>
<td>T VUL</td>
<td>5.60</td>
<td>0.61 ± 0.20</td>
</tr>
<tr>
<td>δ CEPHEI</td>
<td>5.70</td>
<td>0.44 ± 0.15</td>
</tr>
<tr>
<td>η AGL</td>
<td>6.44</td>
<td>0.69 ± 0.23</td>
</tr>
<tr>
<td>S SGE</td>
<td>7.20</td>
<td>0.52 ± 0.27</td>
</tr>
<tr>
<td>ψ CYG</td>
<td>8.89</td>
<td>0.89 ± 0.30</td>
</tr>
</tbody>
</table>

Their results are based on a Baade-Wesselink method utilizing the uv portion of the light curve and relevant velocity measurements.

III. RR LYRAE, BL HER AND "DOUBLE" MODE PULSATORS

Nonlinear modelling of RR Lyrae stars has been quite successful using appropriate masses, effective temperatures, and luminosities. From our earlier work (Davis, 1975) we believe that an improvement in the use of radiative transfer techniques can be achieved mainly in support of the uv excess observed. Bump models for BL Her stars have also been successful using evolutionary masses. Recently many double mode RR Lyrae and "beat" Cepheids have been observed. Because these pulsations are motions in the gravitational field of the star, the mean density and, therefore, the mass of the star can be determined independent of pulsation theory. The restraints on models of double mode stars from the observations are \( P_1/P_0 = 0.746 \) and \( P_0 = 0.55 \) days for the double mode RR Lyraes and \( 0.696 > P_1/P_0 < 0.711 \) and periods from
2-6 days for the "beat" Cepheids. The masses obtained for the double mode RR Lyrae is like 0.65 \( M_0 \), in agreement with evolution theory, but the masses obtained for the "beat" Cepheids is only like one-third of the evolutionary masses. There are no successful nonlinear models with the above parameters but it is encouraging that models for RR Lyrae and Cepheids have been obtained that pulsate nonlinearly in two modes. Until our nonlinear models show pulsation in multiple modes with the correct periods and period ratios as observed, they will not be useful in fixing the masses for these peculiar stars.

IV. W VIRGINIS AND SEMIREGULAR STARS

W Virginis are low mass population II stars pulsating in their fundamental modes. Their masses are therefore from 0.6 to 0.8 \( M_0 \) but with high luminosity to mass ratios. Nonlinear models for these stars have been developed by Christy, Davis, Briger, Fadeyev and more recently by Kovacs and Buchler. A general consensus is that a condition of semiregularity occurs in their alternation of minimum and maximum luminosity. There is an indication of a trend towards bifurcation and ultimate chaos, as in RV Tauri-like stars. In models for these stars we believe that a correct treatment of radiative transfer is needed in order to obtain reasonable light curves. This view is supported by a recent paper by Fokin (1989). As a start of a study of high luminosity to mass stars we have applied our variable Eddington radiative transfer hydrodynamic code(SPEC) to models from Kovacs and Buchler (1988). The details of the models selected are shown in Table II.

<table>
<thead>
<tr>
<th>( T_{\text{eff}}(^\circ K) )</th>
<th>( P_0 )</th>
<th>( P_1/P_0 )</th>
<th>( \eta_0 )</th>
<th>( \eta_1 )</th>
<th>Type</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>5500</td>
<td>9.158</td>
<td>0.541</td>
<td>0.98</td>
<td>-0.23</td>
<td>1P</td>
<td>BK</td>
</tr>
<tr>
<td>5500</td>
<td>9.256</td>
<td>0.543</td>
<td>0.93</td>
<td>-0.52</td>
<td>1P</td>
<td>CG</td>
</tr>
<tr>
<td>5000</td>
<td>13.494</td>
<td>0.487</td>
<td>1.34</td>
<td>-0.10</td>
<td>CP</td>
<td>BK</td>
</tr>
<tr>
<td>5000</td>
<td>13.872</td>
<td>0.477</td>
<td>1.33</td>
<td>-0.13</td>
<td>?</td>
<td>CG</td>
</tr>
</tbody>
</table>

\( X = 0.70, Y = 0.299 \) (King IA)

Because we are using tables (King IA) instead of fits to population II mixtures, we expect the differences as noted.
So far we have only reached two conclusions. One is that the lower temperature model shows some irregularity as compared with the higher temperature model (obs Figure 1). This is in general agreement with Kovacs et. al.. The other point is that the radiative transfer luminosities show little effects to changes in mass from 0.6 to 0.8 $M_\odot$. These new luminosities differ from some of our earlier work (Davis, 1972), and we are still looking into these differences. It is apparent that an adaptive mesh scheme (ala DYN) will have to be applied to this problem. The new luminosities show the effect of the strong shock front exiting the stars extended atmospheres as a decided "dip" before maximum light.

V. CONCLUSIONS

The field of nonlinear stellar pulsation is being revitalized by the efforts of Buchler, and his associates, as well as Fadeev and his associates in the Soviet Union, and Takeuti.

Fig. 1. Phase plane for 5000 K (upper) and 5500 K (lower) models of W Vir stars.
and his associates in Tokyo. Developments in time dependent convection, shock resolving methods, radiative transfer, new opacities and further insights into multiple mode pulsation, and the trend towards bifurcation and chaos as observed in some stars is reopening interest in this field. To support the understanding of the masses of these various pulsating stars, we will need to improve our nonlinear models and because of this it should be an exciting time to work in stellar pulsation theory.

REFERENCES


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