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Nuclear-Transmission Coefficients for Calculation of the Absorption Cross Section in the Adiabatic Coupled-Channel Approximation Method

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FOREWORD

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NUCLEAR TRANSMISSION
COEFFICIENTS FOR CALCULATION OF THE
ABSORPTION CROSS SECTION IN THE
ADIABATIC COUPLED-CHANNEL
APPROXIMATION METHOD

ABSTRACT

This report derives formulas which are needed to calculate transmission coefficients for the adiabatic coupled-channel approximation method. In terms of these coefficients, nuclear absorption cross sections may be obtained. First, derivations are given of various cross sections for a system of coupled inelastic channels in terms of the S matrix. The adiabatic approximation method is discussed for a rotational band, and the dynamical nuclear S matrix is obtained from the S matrix for scattering from a static rotor. The formulas are valid for a spheroidal rotor, with or without an extra-core particle, which does not interact with the projectile but does provide angular momentum to the target.

INTRODUCTION

The adiabatic coupled-channel (ACC) approximation method is a very convenient technique for rapid calculation of cross sections for excitation of states of a rotational band of a heavy deformed nucleus. The ACC method consists of making the strong-coupling approximation for the scattered projectile and then neglecting the excitation energies of states of the rotational band. The problem reduces mathematically to that of scattering of the projectile from a static rotor. In order to calculate cross sections for nuclear transitions, one must obtain the dynamic nuclear S matrix from the static S matrix (for scattering from a static deformed potential). Various total cross sections are calculated in terms of the nuclear S matrix for scattering of a projectile from a nucleus, the adiabatic approximation is derived, and the dynamic nuclear S matrix in the adiabatic approximation is calculated from the S matrix for scattering from a static deformed potential.

This S matrix can then be used to calculate absorption elastic and inelastic cross sections. The parameters in this report are related to those used by Tamura.¹

ABSORPTION CROSS SECTIONS FROM COUPLED-CHANNEL WAVE FUNCTIONS

The asymptotic wave function for a nuclear target including several inelastically coupled-channels, n , can be written

$$\psi + \sum_{JM\gamma} (2ik_0 r)^{-1} \left[\Phi_{nJ}(\xi) y_{j\ell s}(\hat{r}) \right]_{JM} \left[B_{\gamma}^{JM} H_{\ell}(k_n r) - A_{\gamma}^{JM} H_{\ell}^*(k_n r) \right], \quad (1)$$

where

$\Phi_{nJ}(\epsilon)$ = target state of energy E_n and angular momentum J ,

$y_{j\ell s}(\tau) = \left[i^{\ell} Y_{\ell}(\hat{r}) \phi_s \right]_j$ = projectile spin-orbit wave function with projectile spin, s , angular momentum, ℓ , and total angular momentum, j ,

$H_{\ell,n}$ = outgoing Coulomb function,

$$\lim_{r \rightarrow \infty} \exp \left\{ i \left[k_n r - \eta_n \ln(2k_n r) - \frac{\ell\pi}{2} + \sigma_{\ell n} \right] \right\} \equiv \exp \left\{ i \Sigma_{\ell n} \right\},$$

$$\gamma = \{n, J, j, \ell\},$$

$$k_n = \left(\frac{2\mu}{\hbar^2} \right)^{1/2} (E - E_n)^{1/2}, \quad k_0 = \text{incident wave number},$$

and A_{γ} and B_{γ} are coefficients of the incoming and outgoing waves, respectively. They are related by

$$B_{\gamma}^{JM} = \sum_{\gamma'} S_{\gamma\gamma'}^J A_{\gamma'}^{JM}, \quad (2)$$

which also defines the S (scattering) matrix.

The coefficient A_Y is determined by matching Eq. (1) to incident Coulomb waves corresponding to the boundary conditions for a particular scattering problem. Typically, one would have an incident wave on the ground state of the target, but it is not advantageous to impose this restriction. The asymptotic wave function for a wave incident on the nucleus in state $n_0 J_0$ is

$$\psi_{M_0 m_s} + \psi_c(r) \phi_{n_0 J_0 M_0} \phi_{sm_s} + \text{scattered wave} \quad (3)$$

where

$$\psi_c(r) = \sum_{\ell, m} 4\pi i^\ell e^{i\sigma_\ell} n_0 Y_{\ell m}^*(\hat{k}_0) Y_{\ell m}(\hat{r}) \sin \Sigma_{n_0 \ell} / (k_0 r) \quad (4)$$

Using Eq. (4) in Eq. (3) and using a product-to-coupled-angular-momentum-state transformation gives

$$\begin{aligned} \psi_{M_0 m_s} + \sum_{\ell m} 4\pi i^\ell e^{i\sigma_\ell} Y_{\ell m}^*(\hat{k}_0) \sin \Sigma_{n_0 \ell} / (k_0 r) \sum_{j m_j} \sum_{JM} \langle \ell s m_s | j m_j \rangle \langle J_0 j m_0 m_j | JM \rangle \\ \times \left[\phi_{n_0 J_0} \left[Y_\ell(\hat{r}) \phi_s \right]_j \right]_{JM} + \text{scattered wave} \quad (5) \end{aligned}$$

Comparing the incoming part of Eq. (5) with that of Eq. (1) gives the expansion coefficient

$$A_Y^{JM} = \sum_m 4\pi e^{i\sigma_\ell} n_0 Y_{\ell m}^*(\hat{k}_0) \sum_{m_j} \langle \ell s m_s | j m_j \rangle \langle J_0 j m_0 m_j | JM \rangle \delta_{nn_0} \delta_{JJ_0} \quad (6)$$

If $\hat{k}_0$ is in the forward direction so that only $m = 0$ contributes, the m sums disappear giving

$$A_Y^{JM} = \sqrt{4\pi(2\ell+1)} e^{i\sigma_\ell} n_0 \langle \ell s 0 m_s | j m_s \rangle \langle J_0 j m_0 m_s | JM \rangle \delta_{nn_0} \delta_{JJ_0} \quad (7)$$

and

$$\begin{aligned} \psi_{M_0 m_s} + \sum_{JM\gamma} (2ik_0 r)^{-1} \left[\phi_J(\xi) y_{jls}(\hat{r}) \right]_{JM} \sum_{\gamma'} \left[s_{\gamma'}^J, e^{i\Sigma_{nl}} - \delta_{\gamma'}, e^{-i\Sigma_{nl}} \right] \\ \times \sqrt{4\pi(2l'+1)} e^{i\sigma_{l',n'}} \langle l's_0 m_s | j'm_s \rangle \langle J'j'm_0 m_s | JM \rangle \delta_{n'n_0} \delta_{J'J_0} . \quad (8) \end{aligned}$$

To obtain the elastic cross section, we rewrite Eq. (1) in terms of an incident plane wave plus a scattered wave.

$$\begin{aligned} \psi + \sum_{JM\gamma} (k_0 r)^{-1} \left[\phi_{nJ}(\xi) y_{jls}(\hat{r}) \right]_{JM} \\ \times \sum_{\gamma'} \left[\left(s_{\gamma'}^J - \delta_{\gamma'} \right) \frac{e^{i\Sigma_{nl}}}{2i} + \delta_{\gamma'} \sin k_0 r \right] A_{\gamma'}^{JM} . \quad (9) \end{aligned}$$

The total elastic and total inelastic cross sections are obtained by dividing, respectively, the total scattered current in the elastic channel and all explicit inelastic channels by the incident current density. The outgoing current in channel n_1 is

$$\begin{aligned} i_{sc}^{n_1} &= \sum_{JM\gamma\gamma'} (4k_0^2)^{-1} \delta_{nn_1} \int d\Omega \hbar k_n / \mu \sum_{JM\bar{M}} B_Y^{JM} B_{\bar{Y}}^{\bar{J}\bar{M}*} \left\langle \left[\phi_{nJ} y_{jls} \right]_{JM} \mid \left[\phi_{\bar{n}\bar{J}} y_{\bar{j}\bar{l}\bar{s}} \right]_{\bar{J}\bar{M}} \right\rangle \\ &= \sum_{JM\gamma} \delta_{nn_1} (4k_0^2)^{-1} (\hbar k_n / \mu) \left| \sum_{\gamma'} \left(s_{\gamma'}^J - \delta_{\gamma'} \right) A_{\gamma'}^{JM} \right|^2 \\ &= \sum_{JM\gamma} \pi \hbar k_n / (k_0^2 \mu) \sum_{\gamma', \gamma''} \left(s_{\gamma'}^J - \delta_{\gamma'} \right) \left(s_{\gamma''}^J - \delta_{\gamma''} \right)^* \\ &\quad \times e^{i(\sigma_{l',n'} - \sigma_{l'',n''})} \hat{l}' \cdot \hat{l}'' \langle l's_0 m_s | j'm_s \rangle \langle J_0 j'm_0 m_s | JM \rangle \langle l'' s_0 m_s | j'' m_s \rangle \\ &\quad \times \langle J_0 j'' m_0 m_s | JM \rangle \delta_{nn_1} \delta_{JJ_1} \delta_{n'n_0} \delta_{J'J_0} \delta_{n''n_0} \delta_{J''J_0} . \quad (10) \end{aligned}$$

Averaging over initial projection quantum numbers M_0 , m_s and summing over M gives

$$\begin{aligned} \left\langle i_{sc}^{n_1} \right\rangle &= \sum_{J\gamma} \left[\pi \hbar k_n / (k_0^2 \mu) \right] (2J+1) (2J_0+1)^{-1} (2s+1)^{-1} \\ &\times \sum_{\gamma'} |s_{\gamma\gamma'} - \delta_{\gamma\gamma'}|^2 \delta_{nn_1} \delta_{JJ_1} \delta_{n'n_0} \delta_{J'J_0} . \quad (11) \end{aligned}$$

The total elastic cross section, σ_{el} , is then

$$\begin{aligned} \sigma_{el} &= \left\langle i_{sc}^{n_0} \right\rangle / (\hbar k_0 / \mu) = \sum_{J\gamma\gamma'} \left(\pi / k_0^2 \right) (2J+1) \\ &\times (2J_0+1)^{-1} (2s+1)^{-1} |s_{\gamma\gamma'} - \delta_{\gamma\gamma'}|^2 \delta_{nn_0} \delta_{JJ_0} \delta_{n'n_0} \delta_{J'J_0} . \quad (12) \end{aligned}$$

The total inelastic cross section, σ_{inel} , is obtained by summing Eq. (11) over inelastic channels and dividing by the incident current density,

$$\begin{aligned} \sigma_{inel} &= \sum_{n_i} \left\langle i_{sc}^{n_i} \right\rangle / (\hbar k_0 / \mu) \\ &= \sum_{J\gamma\gamma'} \left(\pi k_n / k_0^3 \right) (2J+1) (2J_0+1)^{-1} (2s+1)^{-1} \\ &\times |s_{\gamma\gamma'}^J|^2 (1 - \delta_{nn_0} \delta_{JJ_0}) \delta_{n'n_0} \delta_{J'J_0} . \quad (13) \end{aligned}$$

We define the absorption cross section, σ_A , as the current lost from all channels, n , explicitly included in the γ' sum of Eq. (1), divided by the incident current density. The outgoing and incoming currents are, respectively,

$$\begin{aligned} i_{out} &= \sum_{J\gamma} \left(4k_0^2 \right)^{-1} \int d\Omega \hbar k_n / \mu \sum_{J\bar{J}\bar{M}} B_Y^{JM} B_Y^{J\bar{M}} \delta_{n\bar{n}} \left\langle [\Phi_{nJ^y j l s}]_{JM} [\Phi_{\bar{n}J^y \bar{j} \bar{l} \bar{s}}]_{J\bar{M}} \right\rangle \\ &= \sum_{JMY} \left(4k_0^2 \right)^{-1} (\hbar k_n / \mu) |B_Y^{JM}|^2 , \quad (14) \end{aligned}$$

$$i_{in} = \sum_{JM_Y} (4k_0^2)^{-1} (\hbar k_0/\mu) |A_Y^{JM}|^2 . \quad (15)$$

Thus, the absorption cross section is

$$\begin{aligned} \sigma_A(n_0, J_0) &= (|i_{in}| - |i_{out}|) / (\hbar k_0/\mu) \\ &= \sum_{JM_Y} k_n / (4k_0^3) \left[|A_Y^{JM}|^2 - |B_Y^{JM}|^2 \right] \\ &= \sum_{JM_Y} k_n / (4k_0^3) \left[\sum_{Y'Y''} \delta_{YY'} \delta_{YY''} - S_{YY'}^J S_{YY''}^{J*} \right] A_Y^{JM} A_{Y''}^{JM*} . \end{aligned} \quad (16)$$

Using Eq. (7), averaging over incident projection quantum numbers M_0 , m_s , and summing over M gives finally

$$\sigma_A(n_0, J_0) = \sum_{JY} \pi k_n / k_0^3 \sum_{Y'} \left[\delta_{YY'} - |S_{YY'}^J|^2 \right] \frac{2J+1}{(2s+1)(2J_0+1)} \delta_{n'n_0} \delta_{J'J_0} . \quad (17)$$

Analogously to Eq. (14), one can also define an outgoing current in the incident (elastic) channel only

$$i_{out}^{(el)} = \sum_{JM_Y} (4k_0^2)^{-1} (\hbar k_0/\mu) \delta_{nn_0} \delta_{JJ_0} |B_Y^{JM}|^2 . \quad (18)$$

The total reaction cross section, σ_R , is then

$$\begin{aligned} \sigma_R &= (|i_{in}| - |i_{out}^{(el)}|) / (\hbar k_0/\mu) \\ &= (2s+1)^{-1} (2J_0+1)^{-1} \pi k_0^{-2} \sum_{JY'} (2J+1) \\ &\quad \times \left[\delta_{YY'} - |S_{YY'}^J|^2 \right] \delta_{nn_0} \delta_{n'n_0} \delta_{JJ_0} \delta_{J'J_0} . \end{aligned} \quad (19)$$

It is easy to verify from Eqs. (13), (17), and (19) that

$$\sigma_R = \sigma_A + \sigma_{inel} . \quad (20)$$

Equation (17) may also be written

$$\begin{aligned} \sigma_A(n_0, J_0) &= (2s+1)^{-1} (2J_0+1)^{-1} \pi k_0^{-2} \sum_{J'} (2J+1) \\ &\times (1 - \sum_Y (k_n/k_0) |s_{Y'}|^2) \delta_{n, n_0} \delta_{J, J_0} \\ &= \pi k_0^{-2} (2s+1)^{-1} (2J_0+1)^{-1} \sum_{J'j'l} (2J+1) T_{n_0 J_0 j' l}^J , \quad (21) \end{aligned}$$

where

$$T_{n_0 J_0 j' l}^J = 1 - \sum_{nJj l} (k_n/k_0) |s_{nJj l, n_0 J_0 j' l}^J|^2 . \quad (22)$$

is the transmission coefficient in the target channel $n_0 J_0$.

THE ADIABATIC APPROXIMATION METHOD

For a deformed rotor we may write a Hamiltonian in the body-fixed system as a sum of rotor, valence, coriolis, and projectile parts,

$$\underline{H} = \underline{H}_r + \underline{H}_v + \underline{H}_c + \underline{H}_p .$$

For a highly deformed nucleus, $\underline{H}_{\text{coriolis}}$ is neglected (strong-coupling approximation). Additionally, in the adiabatic approximation, $\underline{H}_r$ is neglected on the grounds that the rotational energies are very small compared

to valence or projectile energies. The problem is then reduced to solving the Schrödinger equation for the scattering of the projectile from a statically deformed core with, perhaps, a valence particle, for which the strong-coupling wave function has the form²

$$\psi_{J M \Omega} = \sum_{j \ell} j(4\pi)^{-1} u_{j \ell \Omega_1 \Omega}^{J M}(r) \left[D_{M K}^{J*} \chi_{\Omega_1} y_{j \ell \Omega} + D_{M -K}^{J*} \chi_{\Omega_1} y_{j \ell -\Omega} (-1)^{\Omega_1 - j} \right]. \quad (23)$$

In Eq. (23)

$$\chi_{\Omega_1} = \sum_{j_1} c_{j_1 \Omega_1} \phi_{j_1 \Omega_1}, \quad (24a)$$

and

$$\chi_{-\Omega_1} = \sum_{j_1} (-1)^{j_1 - \Omega_1} c_{j_1 \Omega_1} \phi_{j_1 -\Omega_1} \quad (24b)$$

are intrinsic wave functions for the valence particle in the deformed well of the rotor. The Schrödinger equation in the adiabatic approximation is then

$$(\underline{E} - \underline{E}_v - \underline{E}_p) \psi_{J M \Omega} = 0. \quad (25)$$

Multiplying by $\chi_{\Omega_1} y_{j', \ell', \Omega}$ and integrating the valence coordinate and projectile spin-angle coordinate gives

$$(\underline{E} - \underline{E}_{\Omega_1} - \underline{E}_p) u_{j \ell \Omega_1 \Omega}^{J M}(r) = \sum_{j', \ell'} \langle j \ell \Omega | v_p | j' \ell' \Omega \rangle u_{j', \ell', \Omega_1 \Omega}^{J M}(r) \quad (26)$$

Since the Hamiltonian contains no operator on the Euler angles α, β, γ , the D function is common on both sides of Eq. (26) and has been divided out.

Equation (26) is the adiabatic coupled-channel equation. Only $j\ell$ projectile quantum numbers are mixed by the non-spherical projectile-core interaction. Also, the valence particle is not involved in any way in Eq. (26), since the projectile-valence interaction has been neglected on the grounds that the projectile-core interaction is much larger. The valence particles play the role of spectators, contributing to the angular momentum but not taking part in the collision.

THE S MATRIX AND TRANSMISSION COEFFICIENT IN THE ACC APPROXIMATION METHOD

Since Eq. (26) involves no nuclear dynamics, the scattering wave functions involve only the projectile quantum numbers. The asymptotic radial wave function is

$$U_{j\ell\Omega_1\Omega}^{JM} \rightarrow \frac{b_{j\ell\Omega_1\Omega}^{JM} e^{i\Sigma_{n_0}\ell} - a_{j\ell\Omega_1\Omega}^{JM} e^{-i\Sigma_{n_0}\ell}}{2ik_0 r} \quad (27)$$

where

$$b_{j\ell\Omega_1\Omega}^{JM} = \sum_{j'\ell'} S_{j\ell, j'\ell'}^{\Omega} a_{j'\ell'\Omega_1\Omega}^{JM} \quad (28)$$

This S matrix describes the scattering from a static core with a valence nucleon in a good $j_3 = \Omega_1$ state for the conserved projectile quantum number $j_3 = \Omega$. We now introduce what is left of nuclear dynamics by transforming to the space-fixed axes. The constant $a_{j'\ell'\Omega_1\Omega}^{JM}$ is to be chosen to match an incoming plane wave in the space-fixed axes when $\chi_{\Omega_1\Omega}^{JM}$ is summed over $JM\Omega$.

The quantity in the brackets in Eq. (23) is transformed to

$$\begin{aligned}
& \sum_{j_1 m_1 m} c_{j_1}^{\Omega_1} \left[D_{mK}^{J^*} D_{m_1 \Omega_1}^{j_1} D_{m\Omega}^j + D_{m-K}^{J^*} D_{m_1 - \Omega_1}^{j_1} D_{m-\Omega}^j (-1)^{J-j-j_1} \right] \phi_{j_1 m_1}(\vec{r}_1) y_{jm}(\hat{r}) \\
&= \sum_{j_1 m_1 m} c_{j_1}^{\Omega_1} \left[(-1)^{M-K} \sum_{JMIN} \langle Jj -Mm | J -M \rangle \langle Jj -K\Omega | J -\Omega_1 \rangle \langle Jj_1 -Mm_1 | I -N \rangle \right. \\
&\quad \times \langle Jj_1 -\Omega_1 \Omega_1 | IO \rangle + (-1)^{M+K+J-j-j_1} \sum_{JMIN} \langle Jj -Mm | J -M \rangle \langle JjK -\Omega | J\Omega_1 \rangle \\
&\quad \times \langle Jj_1 -Mm_1 | I -N \rangle \langle Jj_1 \Omega_1 -\Omega_1 | IO \rangle \left. \right] D_{-NO}^I \phi_{j_1 m_1}(\vec{r}_1) y_{jm}(\hat{r}) \\
&= 2 \sum_{I=J-1}^{J+1} (-1)^{2M-K+j+j_1} \langle Jj -K\Omega | J -\Omega_1 \rangle \\
&\quad \times \langle Jj_1 -\Omega_1 \Omega_1 | IO \rangle \left[\left[\sum_{j_1 I \text{ even}} c_{j_1}^{\Omega_1} D_{j_1 O}^{I*} \phi_{j_1}(\vec{r}_1) \right] y_j(\hat{r}) \right]_{JM} , \quad (29)
\end{aligned}$$

where D_{O}^I is $D_{M,O}^I$ with the M suppressed. Substituting Eq. (29) in Eq. (23), using the asymptotic form Eq. (27) and the connection Eq. (28), and summing over JMK to form a scattering wave gives

$$\begin{aligned}
& \psi_{r \rightarrow \infty} (2ik_0 r)^{-1} \sum_{JMK j_l j' l'} (-1)^{2M+j-\Omega} \langle Jj -K\Omega | J -\Omega_1 \rangle \left[\phi_J(\alpha, \beta, \gamma, \vec{r}_1) y_j(\hat{r}) \right]_{JM} \\
& \quad \times \left[s_{j_l j', l}^{\Omega} e^{i \Sigma_{n_0 l}} - \delta_{jj}, \delta_{ll}, e^{-i \Sigma_{n_0 l}} \right] a_{j, l, \Omega_1}^{JM} \quad (30)
\end{aligned}$$

where

$$\phi_J(\alpha, \beta, \gamma, \vec{r}_1) \equiv \sum_{j_1 I \text{ even}} \sqrt{2} (-1)^{j_1 - \Omega_1} \langle Jj_1 -\Omega_1 \Omega_1 | IO \rangle c_{j_1}^{\Omega_1} \frac{1}{\sqrt{8\pi^2}} \left[D_{j_1 O}^{I*} \phi_{j_1} \right]_J \quad (31)$$

is a properly normalized, configuration-mixed nuclear wave function. Equation (30) must be made to agree with the general form Eq. (5), in which the

incoming wave has the restriction $\delta_{\gamma\gamma'} = \delta_{nn'} \delta_{JJ'} \delta_{jj'} \delta_{\ell\ell'}$.

Thus, we must have

$$\sum_{\Omega'K'} \langle Jj - K' \Omega' | J - \Omega_1 \rangle a_{j', \ell', \Omega_1 \Omega'}^{JM} \delta_{jj'} \delta_{\ell\ell'} (-1)^{2M+j-\Omega} \\ = A_{n'J'j'\ell'}^{JM} \delta_{JJ'} \delta_{jj'} \delta_{\ell\ell'}. \quad (32)$$

Multiplying by $\langle Jj - K \Omega | J - \Omega_1 \rangle$ and summing over J gives

$$a_{j', \ell', \Omega_1 \Omega}^{JM} = \langle Jj' - K \Omega | J' - \Omega_1 \rangle (-1)^{2M+j-\Omega} A_{n'J'j'\ell'}^{JM}. \quad (33)$$

Equation (19) then becomes

$$\psi_{r \rightarrow \infty}^{(2ik_0 r)^{-1}} \sum_{JMj\ell j'\ell'} \left[\phi_J(\alpha, \beta, \gamma, r_1) y_j(\hat{r}) \right]_{JM} \\ \times \left[\sum_{\Omega} \langle Jj - K \Omega | J - \Omega_1 \rangle \langle Jj' - K \Omega | J' - \Omega_1 \rangle \right. \\ \left. \times s_{j\ell, j'\ell'}^{\Omega} e^{i \Sigma_{n_0 \ell}} - \delta_{JJ'} \delta_{jj'} \delta_{\ell\ell'} e^{-i \Sigma_{n_0 \ell}} \right] A_{n'J'j'\ell'}^{JM}. \quad (34)$$

We have assumed that neither the projectile nor the target undergoes any change in the intrinsic wave function in the scattering. Then Eq. (34) is equivalent to Eq. (8). From comparison, we see that

$$s_{\gamma\gamma'}^J = \sum_{\Omega K} s_{j\ell, j'\ell'}^{\Omega} \langle Jj - K \Omega | J - \Omega_1 \rangle \langle Jj' - K \Omega | J' - \Omega_1 \rangle. \quad (35)$$

Since $k_n \approx k_0$, the transmission coefficient Eq. (22) for this case is then

$$T_{J, j, \ell}^J = 1 - \sum_{j\ell} \left| \sum_{\Omega K} s_{j\ell, j'\ell'}^{\Omega} \langle Jj - K \Omega | J - \Omega_1 \rangle \langle Jj' - K \Omega | J' - \Omega_1 \rangle \right|^2 \\ = 1 - \sum_{j\ell \Omega K} \left| s_{j\ell, j'\ell'}^{\Omega} \right|^2 \langle Jj' - K \Omega | J' - \Omega_1 \rangle^2. \quad (36)$$

This Clebsch-Gordan coefficient appearing in Eq. (36) is all that remains of the nuclear dynamics in the transmission coefficient. In its absence, one has the transmission coefficient for absorption on a static deformed target. Since the nucleus always must start in some target state J' , there is no way of getting the static limit from Eq. (36).

In the limit of a spherical nucleus not only is $S_{j\ell,j'\ell'}^\Omega$ proportional to $\delta_{jj'}$, $\delta_{\ell\ell'}$, but also $J = J'$ is always zero. Then $J = j'$ and

$$T_{j'\ell'} = 1 - |S_{j\ell}|^2 \sum_{\Omega} \langle j'j'\Omega - \Omega | 00 \rangle^2 = 1 - |S_{j\ell}|^2. \quad (37)$$

Equation (37) is the correct result for scattering from an absorbing spherical potential.

CONNECTION TO TAMURA'S C COEFFICIENTS

Comparing our Eq. (5) with Eq. (46) of Ref. 1, we see that Tamura¹ has included an extra factor of $(\hat{\ell}/\hat{\ell}') (k_0/k_n)^{1/2}$ in his outgoing wave compared to Eq. (5). Thus, Tamura's matrix and our S matrix are related as

$$\begin{aligned} S_{\gamma\gamma'}^J &= \frac{\hat{\ell}}{\hat{\ell}'} \left(\frac{k_0}{k_n} \right)^{1/2} S_{\gamma'\gamma}^J \text{ (Tamura)} \\ &= \frac{\hat{\ell}}{\hat{\ell}'} \left(\frac{k_0}{k_n} \right)^{1/2} \left[2iC_{\ell'j',n\ell j} + \delta_{nn'} \delta_{jj'} \delta_{\ell\ell'} \right]. \end{aligned} \quad (38)$$

Therefore rewritten in Tamura's notation, Eq. (22) becomes

$$T_{n'j'\ell's'}^J = 1 - \sum_{n j \ell s} \frac{\hat{\ell}^2}{\hat{\ell}'^2} |S_{\gamma'\gamma}^J \text{ (Tamura)}|^2, \quad (39)$$

which is Eq. (56) of Ref. 1.

SUMMARY

The transmission coefficient for coupled-channel wave functions, the adiabatic coupled-channel approximation (ACC) method, and the transmission coefficient in the adiabatic approximation method have been derived in this report.

The transmission coefficient, Eq. (36) in the ACC approximation method, is the essential result of this report. The S matrix for scattering from a static deformed nucleus can be obtained from an adiabatic coupled-channel code, and Eq. (36) can then be used to calculate absorption cross sections into states J' . The main physics is in Eq. (35) relating the S matrix for scattering from a static deformed potential to the dynamic nuclear S matrix in the adiabatic approximation method. Such a relation for a spinless projectile has been given previously by Chase.³

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